Granular Comparative Advantage

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Exports are Granular

  
  Across 32 developing countries, the largest exporting firm accounts on average for 17% of total manufacturing exports

- Our focus: French manufacturing

<table>
<thead>
<tr>
<th>Average export share of the largest firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing 1 industry 7%</td>
</tr>
<tr>
<td>— 2-digit 23 sectors 18%</td>
</tr>
<tr>
<td>— 3-digit 117 sectors 26%</td>
</tr>
<tr>
<td>— 4-digit 316 sectors 37%</td>
</tr>
</tbody>
</table>
Granularity

- Firm-size distribution is:
  1. fat-tailed (Zipf’s law)
  2. discrete

\[ \implies \text{Granularity} \]

- Canonical example: power law (Pareto) with shape \( \theta < 2 \)

- Intuitions from Gaussian world fail, even for very large \( N \):
  - a single draw can shape \( \sum_{i=1}^{N} X_i \)
  - average can differ from expectation (failure of LLN)
Granularity

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  2. discrete  \implies \text{Granularity}

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- Intuitions from Gaussian world fail, even for very large $N$
  - a single draw can shape $\sum_{i=1}^{N} X_i$
  - average can differ from expectation (failure of LLN)

- Most common application: aggregate fluctuations

- The role of granularity for \textit{comparative advantage} of countries is a natural question, yet has not been explored
  - Can a few firms shape country-sector specialization?
Trade Models

- Trade models acknowledge fat-tailed-ness but not discreteness
  - emphasis on firms, but each firm is infinitesimal (LLN applies)
  - hence, no role of individual firms in shaping sectoral aggregates

- Exceptions with discrete number of firms
  1. One-sector model of Eaton, Kortum and Sotelo (EKS, 2012)
  2. Literature on competition/markups
     (e.g., AB 2008, EMX 2014, AIK 2014, 2019, Neary 2015)

- Our focus: can granularity explain sectoral trade patterns?
  1. Sector-level comparative advantage (like DFS)
  2. Firm heterogeneity within sectors (like Melitz)
  3. Granularity within sectors (like EKS)

\[ \rightarrow \] relax the LLN assumption in a multi-sector Melitz model

In a typical French sector, there are 350 firms, with the largest firm commanding a 20% market share.
Trade Models

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  1. sector-level comparative advantage (like DFS)
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  3. granularity within sectors (like EKS)

→ relax the LLN assumption in a multi-sector Melitz model
  take seriously that a typical French sector has 350 firms
  with the largest firm commanding a 20% market share
Granularity

Our approach

Productivity draws, $\varphi$

- Fundamental vs Granular: Why do we care?
Granularity
Our approach

- Fundamental vs Granular

Productivity draws, $\varphi$

T(z)

percentiles
draws
Granularity

Our approach

- **Fundamental** vs **Granular**: Why do we care?
This paper

- Roadmap:
  1. Basic framework with granular comparative advantage
  2. GE Estimation Procedure
     - SMM using French firm-level data
  3. Explore implications of the estimated granular model
     - many continuous-world intuitions fail
     - dynamic and policy counterfactuals

- Highlights of the results from the estimated model:
  1. A parsimonious granular model fits many empirical patterns.
  2. Moments of firm-size distribution explain trade patterns
  3. Granularity accounts for 20% of variation in export shares
     - most export-intensive sectors tend to be granular
  4. Granularity can explain much of the mean reversion in CA
     - more granular sectors are more volatile
     - death of a single firm can alter considerably the CA
  5. Policy in a granular economy: mergers and tariffs
     - the role of markups
This paper

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     — the role of markups
Modeling Framework
Model Structure

1. Two countries: Home and Foreign
   — inelastically-supplied labor $L$ and $L^*$

2. Continuum of sectors $z \in [0, 1]$:
   $$Q = \exp \left\{ \int_0^1 \alpha_z \log Q_z \, dz \right\}$$

3. Sectors vary in comparative advantage: $\log \frac{T_z}{T^*_z} \sim \mathcal{N}(\mu_T, \sigma_T)$
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   $\log \frac{T_z}{T^*_z} \sim \mathcal{N}(\mu_T, \sigma_T)$

4. Within a sector, a finite number of firms (varieties) $K_z$: 
   \[
   Q_z = \left[ \sum_{i=1}^{K_z} \frac{q_{z,i}}{\sigma} \right]^{\frac{\sigma}{\sigma-1}}
   \]

5. Each sector has an EKS market structure
• Productivity draws in a given sector $z$:
  — Number of (shadow) entrants: $\text{Poisson}(M_z)$
  — Entrants' productivity draws: $\text{Pareto} (\theta; \varphi_z)$

• Denote $N_\varphi$ number of firms with productivity $\geq \varphi$

$$N_\varphi \sim \text{Poisson}(T_z \cdot \varphi^{-\theta}), \quad T_z \equiv M_z \varphi_z^\theta$$

with $T_z/T_z^*$ shaping sector-level CA
EKS Sectors

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- Marginal cost: $c = w/\varphi$ at home and $\tau w/\varphi$ abroad

- Fixed cost of production and exports: $F$ in local labor
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• Marginal cost: $c = w/\varphi$ at home and $\tau w/\varphi$ abroad

• Fixed cost of production and exports: $F$ in local labor

• Oligopolistic (Bertrand) competition and variable markups
  — Atkeson-Burstein (2008): $\{c_i\} \mapsto \{s_i, \mu_i, p_i\}_{i=1}^{K_z}$
Market Entry and GE

- Assumption: sequential entry in increasing order of unit cost

\[ c_1 < c_2 < \ldots < c_K < \ldots, \quad \text{where} \quad c_i = \begin{cases} \frac{w}{\varphi_i}, & \text{if Home,} \\ \frac{\tau w^*}{\varphi_i^*}, & \text{if Foreign} \end{cases} \]

\[ \rightarrow \] unique equilibrium

- Profits: \( \Pi_i = \frac{s_i}{\varepsilon(s_i)} \alpha_z Y - wF \)
Market Entry and GE

• Assumption: sequential entry in increasing order of unit cost

\[ c_1 < c_2 < \ldots < c_K < \ldots, \text{ where } c_i = \begin{cases} \frac{w}{\varphi_i}, & \text{if Home,} \\ \frac{\tau w^*/\varphi^*_i}{i}, & \text{if Foreign} \end{cases} \]

→ unique equilibrium

• Profits: \(\Pi_i = \frac{s_i}{\varepsilon(s_i)} \alpha z Y - wF\)

• Entry: \(\Pi^K_K \geq 0 \text{ and } \Pi^K_{K+1} < 0 \iff \text{determines } K_z\)
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\[ c_1 < c_2 < \ldots < c_K < \ldots, \quad \text{where} \quad c_i = \begin{cases} \frac{w}{\varphi_i}, & \text{if Home,} \\ \tau \frac{w^*}{\varphi_i^*}, & \text{if Foreign} \end{cases} \]

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- Entry: \( \Pi^K_K \geq 0 \) and \( \Pi^K_{K+1} < 0 \) determines \( K_z \)

- General equilibrium:
  - GE vector \( X = (Y, Y^*, w, w^*) \)
  - Within-sector allocations \( Z = \{ K_z, \{ s_z, i \}_{i=1}^{K_z} \}_{z \in [0,1]} \)
  - Labor market clearing and trade balance (linear in \( X \))
  - Fast iterative algorithm
Estimation and Model Fit
Estimation procedure

- Data: French firm-level data (BRN) and Trade data
  - Firm-level domestic sales and export sales
  - Aggregate import data (Comtrade)
  - 119 4-digit manufacturing sectors

- Parametrize sector-level comparative advantage:
  - $T(z)/T^*(z) \sim \log \mathcal{N}(\mu_T, \sigma_T)$ (and robustness with Laplace)
  - Based on empirical distribution shown in Hanson et al. (2015)

- Stage 1: calibrate Cobb-Douglas shares \{$\alpha_z$\} and \(w/w^*$
  - CD shares read from domestic sales + imports, by sector
  - \(w/w^* = 1.13\), trade-weighted wage of France’s trade partners
  - Normalizations: \(w = 1\) and \(L = 100\)

- Stage 2: SMM procedure to estimate \{\(\sigma, \theta, \tau, F, \mu_T, \sigma_T\}\},
  while \((Y, Y^*, L^*/L)\) are pinned down by GE
## Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
<th>Auxiliary variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>5</td>
<td>—</td>
<td>$\kappa = \frac{\theta}{\sigma-1}$ 1.077</td>
</tr>
<tr>
<td>$\theta$</td>
<td>4.307</td>
<td>0.246</td>
<td>$w/w^*$ 1.130</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.341</td>
<td>0.061</td>
<td>$L^*/L$ 1.724</td>
</tr>
<tr>
<td>$F \times 10^5$</td>
<td>0.946</td>
<td>0.252</td>
<td>$Y^*/Y$ 1.526</td>
</tr>
<tr>
<td>$\mu_T$</td>
<td>0.137</td>
<td>0.193</td>
<td>$\Pi/Y$ 0.211</td>
</tr>
<tr>
<td>$\sigma_T$</td>
<td>1.422</td>
<td>0.232</td>
<td></td>
</tr>
</tbody>
</table>
## Moment Fit

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data, $\hat{m}$</th>
<th>Model, $\hat{M}(\hat{\Theta})$</th>
<th>Loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Log number of firms, mean log $\tilde{M}_z$</td>
<td>5.631</td>
<td>5.624</td>
<td>0.1</td>
</tr>
<tr>
<td>2. — st. dev. log $\tilde{M}_z$</td>
<td>1.451</td>
<td>1.222</td>
<td>7.9</td>
</tr>
<tr>
<td>3. Top-firm market share, mean $\tilde{s}_{z,1}$</td>
<td>0.197</td>
<td>0.206</td>
<td>3.5</td>
</tr>
<tr>
<td>4. — st. dev. $\tilde{s}_{z,1}$</td>
<td>0.178</td>
<td>0.149</td>
<td>3.8</td>
</tr>
<tr>
<td>5. Top-3 market share, mean $\sum_{j=1}^{3} \tilde{s}_{z,j}$</td>
<td>0.356</td>
<td>0.343</td>
<td>2.0</td>
</tr>
<tr>
<td>6. — st. dev. $\sum_{j=1}^{3} \tilde{s}_{z,j}$</td>
<td>0.241</td>
<td>0.175</td>
<td>11.5</td>
</tr>
<tr>
<td>7. Imports/dom. sales, mean $\tilde{\Lambda}_z$</td>
<td>0.365</td>
<td>0.351</td>
<td>2.2</td>
</tr>
<tr>
<td>8. — st. dev. $\tilde{\Lambda}_z$</td>
<td>0.204</td>
<td>0.268</td>
<td>14.8</td>
</tr>
<tr>
<td>9. Exports/dom. sales, mean $\tilde{\Lambda}_z^{*f}$</td>
<td>0.328</td>
<td>0.350</td>
<td>6.0</td>
</tr>
<tr>
<td>10. — st. dev. $\tilde{\Lambda}_z^{*f}$</td>
<td>0.286</td>
<td>0.346</td>
<td>6.5</td>
</tr>
<tr>
<td>11. Fraction of sectors with exports&gt;dom. sales $\mathbb{P}{\tilde{X}_z &gt; \tilde{Y}_z - \tilde{X}_z}$</td>
<td>0.185</td>
<td>0.092</td>
<td>37.9</td>
</tr>
</tbody>
</table>

### Regression coefficients†

<table>
<thead>
<tr>
<th></th>
<th>$\hat{b}_1$</th>
<th>$\hat{b}_3$</th>
<th>$\hat{b}_4^*$</th>
<th>$\hat{b}_5^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. export share on top-firm share</td>
<td>0.215</td>
<td>0.254</td>
<td>-0.016</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
<td>(0.108)</td>
<td>(0.097)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>13. export share on top-3 share</td>
<td>0.243</td>
<td>0.232</td>
<td>-0.020</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.104)</td>
<td>(0.090)</td>
<td>(0.079)</td>
<td>(0.069)</td>
</tr>
<tr>
<td>14. import share on top-firm share</td>
<td>-0.016</td>
<td>-0.020</td>
<td>0.002</td>
<td>-0.005</td>
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<td></td>
<td>(0.097)</td>
<td>(0.079)</td>
<td>(0.074)</td>
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<tr>
<td>15. export share on top-3 share</td>
<td>0.002</td>
<td>-0.005</td>
<td>0.002</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.069)</td>
<td>(0.074)</td>
<td>(0.069)</td>
</tr>
</tbody>
</table>
Non-targeted Moments

- Correlation between top market share and number of firms:

\[ \tilde{s}_{z,1} = const + \gamma_M \cdot \log \tilde{M}_z + \gamma_Y \cdot \log \tilde{Y}_z + \epsilon_s \]

<table>
<thead>
<tr>
<th>Data:</th>
<th>-0.094</th>
<th>0.018</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Model:</td>
<td>-0.064</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.006)</td>
</tr>
</tbody>
</table>

- Extensive margin of sales:

\[ \log \tilde{M}_z = c_d + \chi_d \cdot \log(\tilde{Y}_z - \tilde{X}_z^*) + \epsilon_d \]

<table>
<thead>
<tr>
<th>Data:</th>
<th>0.563</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.082)</td>
</tr>
<tr>
<td>Model:</td>
<td>0.861</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
</tr>
</tbody>
</table>
Equilibrium markups

- Oligopolistic (Bertrand) markups: averages (blue bars) and 10–90% range (red intervals) across industry
- Monopolistic competition markup: $\frac{\sigma}{\sigma - 1} = 1.25$ is lower bound for all oligopolistic markups
Quantifying Granular Trade
Properties of the Granular Model

- **Foreign share:**
  \[ \Lambda_z \equiv \frac{X^*_z}{\alpha_z Y} = \sum_{i=1}^{K_z} (1 - \iota_{z,i}) s_{z,i} \]

- **Expected foreign share:**
  \[ \Phi_z = \mathbb{E}\{\Lambda_z \mid \frac{T_z}{T^*_z}\} = \frac{1}{1 + (\tau \omega)^\theta \cdot \frac{T_z}{T^*_z}} \]

- **Granular residual:**
  \[ \Gamma_z \equiv \Lambda_z - \Phi_z : \quad \mathbb{E}_T\{\Gamma_z\} = \mathbb{E}_T\{\Lambda_z - \Phi_z\} = 0 \]

- **Aggregate exports:**
  \[ X^* = Y \int_0^1 \alpha_z \Lambda_z dz = \Phi Y, \quad \Phi \equiv \int_0^1 \alpha_z \Phi_z dz \]
Decomposition of Trade Flows

- Variance decomposition of \( X_z = \Lambda^*_z \alpha_z Y^* \) with \( \Lambda^*_z = \Phi^*_z + \Gamma^*_z \):
  \[
  \text{var}(\Lambda^*_z) = \text{var}(\Phi^*_z) + \text{var}(\Gamma^*_z), \\
  \text{var}(\log X_z) \approx \text{var}(\log \alpha_z) + \text{var}(\log \Lambda^*_z)
  \]

Table: Variance decomposition of trade flows

<table>
<thead>
<tr>
<th></th>
<th>Common ( \theta )</th>
<th>Sector-specific ( \theta_z )</th>
<th></th>
<th></th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>Granular contribution</td>
<td>[\frac{\text{var}(\Gamma^<em>_z)}{\text{var}(\Lambda^</em>_z)}]</td>
<td>17.0%</td>
<td>22.3%</td>
<td>26.0%</td>
<td>28.4%</td>
</tr>
<tr>
<td>Export share contribution</td>
<td>[\frac{\text{var}(\log \Lambda^*_z)}{\text{var}(\log X_z)}]</td>
<td>57.2%</td>
<td>59.2%</td>
<td>62.5%</td>
<td>63.9%</td>
</tr>
<tr>
<td>Pareto shape parameter</td>
<td>( \kappa_z = \frac{\theta_z}{\sigma - 1} )</td>
<td>1.08</td>
<td>1.00</td>
<td>1.02</td>
<td>0.96</td>
</tr>
<tr>
<td>Estimated Pareto shape</td>
<td>( \hat{\kappa}_z )</td>
<td>1.10</td>
<td><strong>1.02</strong></td>
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<tr>
<td>Top-firm market share</td>
<td>( s_{z,1} )</td>
<td><strong>0.21</strong></td>
<td>0.25</td>
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<td>0.29</td>
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Decomposition of Trade Flows

- Variance decomposition of $X_z = \Lambda^*_z \alpha_z Y^*$ with $\Lambda^*_z = \Phi^*_z + \Gamma^*_z$:
  
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  $\text{var}(\log X_z) \approx \text{var}(\log \alpha_z) + \text{var}(\log \Lambda^*_z)$

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Extensions: (i) $T_z / T_z^* \sim \text{Laplace}$ (two-sided Pareto)
(ii) $\log \varphi_{z,i} \sim \mathcal{N}(\mu, \theta)$
Export Intensity and Granularity

- Granularity does not create additional trade on average
- Yet, granularity creates skewness across sectors in exports
  - most export-intensive sectors are likely of granular origin

(a) Fraction of granular sectors

(b) Granular contribution to trade
Export Intensity and Granularity

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  - most export-intensive sectors are likely of granular origin

(a) Distribution of $\Lambda_z^* \mid \Phi_z^*$

(b) Distribution of $\Phi_z^* \mid \Lambda_z^*$
Properties of Granular Exports

- $\Gamma^*_z = \Lambda^*_z - \Phi^*_z$ are orthogonal with $\Phi^*_z$, $\log(\alpha^*_z Y^*)$ and $\log \tilde{M}_z$
- Best predictor of $\Gamma^*_z$ is $\tilde{s}_{z,1}$, the relative size of the largest firm

<table>
<thead>
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<th>Table: Projections of granular exports $\Gamma^*_z$</th>
</tr>
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<tbody>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>$\tilde{s}_{z,1}$</td>
</tr>
<tr>
<td>$\tilde{s}^*_z$</td>
</tr>
<tr>
<td>$\log \tilde{M}_z$</td>
</tr>
<tr>
<td>$\log(\alpha^*_z Y)$</td>
</tr>
<tr>
<td>$\Phi^*_z$</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
</tbody>
</table>
Identifying Granular Sectors

- Which sectors are granular? Neither $\Phi^*_z$, nor $\Gamma^*_z$ are observable

$$\mathbb{P}\{\Gamma^*_z \geq \vartheta \Lambda^*_z \mid \Lambda^*_z, r_z\} = \frac{\int_{\Lambda^*_z - \Phi^*_z \geq \vartheta \Lambda^*_z} g(\Phi^*_z, \Lambda^*_z, r_z) d\Phi^*_z}{\int_0^1 g(\Phi^*_z, \Lambda^*_z, r_z) d\Phi^*_z},$$
Dynamics of Comparative Advantage
Dynamic Model

- Use the granular model with firm dynamics to study the implied time-series properties of aggregate trade
  - Shadow pull of firms in each sector with productivities \( \{ \varphi_{it} \} \)
  - Productivity of the firms follows a random growth process:
    \[
    \log \varphi_{it} = \mu + \log \varphi_{i,t-1} + \nu \varepsilon_{it}, \quad \varepsilon_{it} \sim iid \mathcal{N}(0, 1)
    \]
    with reflection from the lower bound \( \varphi \) and \( \mu = -\theta \nu^2 / 2 \)
  - Each period: static entry game and price setting equilibrium

- Calibrate idiosyncratic firm dynamics (volatility of shocks \( \nu \)) using the dynamic properties of market shares
Dynamic Model

- Use the granular model with firm dynamics to study the implied time-series properties of aggregate trade
  - Shadow pull of firms in each sector with productivities \( \{\varphi_{it}\} \)
  - Productivity of the firms follows a random growth process:
    \[
    \log \varphi_{it} = \mu + \log \varphi_{i,t-1} + \nu \varepsilon_{it}, \quad \varepsilon_{it} \sim iid \mathcal{N}(0, 1)
    \]
    with reflection from the lower bound \( \varphi \) and \( \mu = -\theta \nu^2 / 2 \)
  - Each period: static entry game and price setting equilibrium

- Calibrate idiosyncratic firm dynamics (volatility of shocks \( \nu \)) using the dynamic properties of market shares

- Extension with aggregate shocks: \( \varepsilon_{it} = \sqrt{\rho} \cdot \nu_t + \sqrt{1 - \rho} \cdot u_{it} \)
Firm Dynamics and CA

- Empirical evidence in Hanson, Lind and Muendler (2015):
  1. Hyperspecialization of exports
  2. High Turnover of export-intensive sectors

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HLM</td>
<td>France</td>
</tr>
<tr>
<td>SR persistence std(Δ̇(\tilde{s}_{z,i,t+1}))</td>
<td>—</td>
<td>0.0018</td>
</tr>
<tr>
<td>LR persistence corr((\tilde{s}<em>{z,i,t+10}, \tilde{s}</em>{z,i,t}))</td>
<td>—</td>
<td>0.86</td>
</tr>
<tr>
<td>Top-1% sectors export share</td>
<td>21%</td>
<td>17%</td>
</tr>
<tr>
<td>Top-3% sectors export share</td>
<td>43%</td>
<td>30%</td>
</tr>
<tr>
<td>Turnover I: remain in top-5% after 20 years</td>
<td>52%</td>
<td>—</td>
</tr>
<tr>
<td>Turnover II: remain in top-5% after 10 years</td>
<td>—</td>
<td>80%</td>
</tr>
</tbody>
</table>

- Idiosyncratic firm productivity dynamics explains the majority of turnover of top exporting sectors over time
Mean Reversion in CA

- Idiosyncratic firm dynamics in a granular model predicts mean reversion in comparative advantage
- In addition, granular sectors are more volatile

(a) Mean reversion in $\Lambda^*_z$

(b) Volatility of $\Delta \Lambda^*_z$
Death of a Large Firm

- Death (sequence of negative productivity shocks) of a single firm can substantially affect sectoral comparative advantage.
- In the most granular sectors, death of a single firm can push the sector from top-5% of CA into comparative disadvantage.
Granularity and reallocation

- Sectoral labor allocation:
  \[
  \frac{L_z}{L} \approx \alpha_z + \frac{\theta}{\sigma \kappa - 1} \frac{NX_z}{Y}
  \]

- Interaction between trade openness and granularity results in sectoral reallocation and aggregate volatility

Figure: Total and Sectoral Labor Reallocation (fraction of total \(L\))
Empirical Analysis
## Granularity and Exports

Cross section and Dynamic panel

### Table: Regression Results

<table>
<thead>
<tr>
<th></th>
<th>Cross-section, 2005</th>
<th>Panel, 1997–2008</th>
<th>Dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum_{i=1}^{3} \tilde{s}_z,i )</td>
<td>(1) 0.802*** (0.290)</td>
<td>(2) 0.833** (0.293)</td>
<td>(3) 0.846*** (0.302)</td>
</tr>
<tr>
<td>( \log D_z )</td>
<td>(1) 0.895*** (0.050)</td>
<td>(2) 0.933*** (0.051)</td>
<td>(3) 0.909*** (0.051)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.512</td>
<td>0.652</td>
<td>0.520</td>
</tr>
<tr>
<td>( R^2_{adj} )</td>
<td>0.509</td>
<td>0.623</td>
<td>0.518</td>
</tr>
<tr>
<td>( N )</td>
<td>316</td>
<td>316</td>
<td>3,409</td>
</tr>
<tr>
<td>( N ) clusters</td>
<td>316</td>
<td>316</td>
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<tr>
<td>Fixed effects:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-digit</td>
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<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sector</td>
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<tr>
<td>Year</td>
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<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Notes
- Significance levels: *** p < 0.001, ** p < 0.01, * p < 0.05
- Standard errors in parentheses.
Predictive Regressions

Mean reversion in exports

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{10} \log X_z$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log X_z$</td>
<td>$-0.116^{***}$</td>
<td>$-0.092^{**}$</td>
</tr>
<tr>
<td></td>
<td>$(0.040)$</td>
<td>$(0.040)$</td>
</tr>
<tr>
<td>$\sum_{i=1}^{3} \tilde{s}_{z,i}$</td>
<td>$-0.660^{***}$</td>
<td>$-0.559^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.199)$</td>
<td>$(0.203)$</td>
</tr>
<tr>
<td>$\log D_z$</td>
<td>$0.101^{**}$</td>
<td>$-0.057$</td>
</tr>
<tr>
<td></td>
<td>$(0.049)$</td>
<td>$(0.036)$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.146</td>
<td>0.153</td>
</tr>
<tr>
<td>$R^2_{adj}$</td>
<td>0.075</td>
<td>0.083</td>
</tr>
<tr>
<td>$N$</td>
<td>316</td>
<td>316</td>
</tr>
<tr>
<td>2-digit FE</td>
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<td>✓</td>
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</table>
Policy Counterfactuals
Policy counterfactuals

1. Misallocation and trade policy
   — policies that hinder growth of granular firms
   — why trade barriers often target individual foreign firms?

2. Merger analysis
Policy counterfactuals

1. Misallocation and trade policy
   — policies that hinder growth of granular firms
   — why trade barriers often target individual foreign firms?

2. Merger analysis

   • Welfare analysis of a policy:

     \[ \hat{W} \equiv d \log \frac{Y}{P} \]
     \[ = \frac{wL}{Y} d \log w + \frac{dTR}{Y} + \int_{0}^{1} \alpha_z \frac{d\Pi_z}{\alpha_z Y} dz - \int_{0}^{1} \alpha_z d \log P_z dz \]

     and across sectors \[ \hat{W} = \int_{0}^{1} \alpha_z \hat{W}_z dz \]

     — In partial equilibrium: \[ \hat{W}_z = \frac{dTR_z + d\Pi_z}{\alpha_z Y} - d \log P_z \]
     — In general equilibrium: spillovers to other sectors via \((w, Y)\)
Merger

- Merger is more beneficial:
  1. The larger is the productivity spillover $\varrho \uparrow$
     \[ \varphi'_{z,2} = \varrho \varphi_{z,1} + (1 - \varrho) \varphi_{z,2}. \] Baseline $\varrho = 0.5$. For low $\varrho = 0.1$
  2. The more open is the economy $\tau \downarrow$
  3. The more granular is the sector $\Gamma^*_z \uparrow$

(a) Welfare effect of a merger, $\hat{W}_Z$

(b) Decomposition of $\hat{W}_Z$, $\tau = 1.34$

Quintiles of sectors by granular $\Gamma^*_z$
Import Tariff

- Tariff on the top importer $\varsigma_{z,1}$ vs a uniform import tariff $\bar{\varsigma}_z$
  - yielding the same tariff revenue
  - $\varsigma_{z,1} \succ \bar{\varsigma}_z$, particularly in the foreign granular industries ($\Gamma_z \uparrow$)

(a) Uniform tariff

(b) Granular tariff
Conclusion
Conclusion

• The world is granular!  \textit{(at least, at the sectoral level)}  
  We better develop tools and intuitions to deal with it

• Applications:
  1. Innovation, growth and development
  2. Misallocation
  3. Industrial policy
  4. Cities and agglomeration
APPENDIX
Granularity

Illustration

- The role of top draw, as the number of draws $N$ increases

\[
\text{corr} \left( \max_i X_i, \sum_i X_i \right)
\]

\[
\frac{\max_i X_i}{\sum_i X_i}
\]
Sectoral equilibrium

- Sectoral equilibrium system:

\[ p_i = \mu_i c_i, \]

\[ \mu_i = \frac{\varepsilon_i}{\varepsilon_i - 1}, \]

where \( \varepsilon_i = \sigma (1 - s_i) + s_i, \)

\[ s_i = \left( \frac{p_i}{P} \right)^{1-\sigma}, \]

where \( P = \left( \sum_{i=1}^{K} p_i^{1-\sigma} \right)^{1/(1-\sigma)}. \)
(a) Pareto shape, $\kappa_z = \frac{\theta_z}{\sigma - 1}$

(b) Estimated Pareto, $\hat{\kappa}_z$

(c) Top-firm sales share, $\tilde{s}_{z,1}$

(d) Number of firms, $\tilde{M}_z$
Probability a sector remains among top-5% of export-intensive sectors
Trade effects of individual firm exit

(a) All sectors, deciles of $\Gamma_z^*$

(b) All sectors, deciles of $\Lambda_z^*$
Merger

Low spillover $\varrho = 0.1$

Welfare effect of a merger, $\hat{W}_Z$

Decomposition of $\hat{W}_Z$, $\tau = 1.34$