The integrated world equilibrium diagram from international trade theory

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Factor price equalization means that under certain conditions unrestricted international trade in goods equalizes prices of factors across countries, even though the factors cannot move across borders so there are only separate national markets for them. When Victor Norman and I started our work on various generalizations of the 2-by-2 Heckescher-Ohlin-Samuelson model of trade, the dominant approach to the factor price equalization issue was "univalence", namely whether the mapping from the prices of factors to the unit costs of goods was globally invertible. The mathematical conditions for this were stringent and abstruse, and relevant only in the case where the number of goods exactly equaled the number of factors. No one knows the exact number of either, but surely it is extremely unlikely that the two are equal. As Victor Norman remarked, what is really special about the 2-by-2 case is not that 2 is a small number, but that 2 = 2.

The univalence approach had an even more basic flaw. It enabled us to find unique factor prices *given* the prices of goods. But where do these prices of goods come from? Under free trade, they come from the equilibrium conditions of the world markets for goods. Trade theory has always prided itself on being a showcase for general equilibrium theory. The study of factor price equalization should not disregard the fact that a trading equilibrium is a *general* equilibrium.

This implies some restrictions on the prices of goods. We should look for solutions to the set of simultaneous equations "Unit cost = Price" for all the goods, not for arbitrary prices on the right hand side, but for equilibrium prices. How do we find

them? The answer is immediate when we look back at the basic question in this light. If factor prices are equal across countries, no further trade in factors will occur even if country borders are opened up for factor movements. In other words, goods trade will achieve the same equilibrium as when factors are also internationally mobile, so that there is a world market for each factor. Therefore we should find that equilibrium, and the prices of goods it generates.

Once we set up that equilibrium, we realize that there is no need to examine univalence. Instead, we can directly ask whether the equilibrium quantities of goods can be split up between the separate countries in such a way as to employ fully the endowments of factors in each country, making it unnecessary for factors to move.

The integrated world equilibrium diagram immediately emerges as a simple way to illustrate this process.[1] Figure ??.1 below shows the case of three goods and two factors; this is a reproduction of Fig. 4.6 from Dixit and Norman (1980). The lengths OB_1 and OB_2 are the world's total endowments of the two factors v_1 and v_2; then OO' becomes the vector of these endowments. In a hypothetical equilibrium where factors can be traded freely across countries, suppose the directions OA_1, OA_2 and OA_3 represent the factor proportions used in the production of the three goods. (If input substitution in production is possible, these proportions are endogenous, being cost-minimizing at the equilibrium prices of factors. The figure merely illustrates the outcome of the equilibrium that has been calculated in the background.) Similarly, the vectors OC_1, OC_2 and OC_3 represent the factor quantities used in the production of the three goods in the equilibrium. For the world factor markets to clear, these vectors must add up to OO'.

[1] The diagram gives an appealing visualization and intuition. Of course the rigorous proof that the conditions that emerge from the procedure are both necessary and sufficient for factor price equalization requires algebraic reasoning; see Dixit and Norman (1980, pp. 108-110).

With GO' equal and parallel to OC_1, C_1H and C_3G each equal and parallel to OC_2, and HO' equal and parallel to OC_3, this completes a hexagon $OC_1HO'GC_3O$.

Now suppose factors are not internationally tradable. Consider a point Q inside the hexagon, and suppose OQ is the vector of factor endowments in one country, so that QO' is the vector of factor endowments in the other country. Now it is possible to split up the production of goods in the integrated equilibrium between the two countries in such a way as to clear the markets for factors in each country. In fact it is possible to do so in a continuous infinity of ways. The figure shows two examples. In the first way, one country produces enough of good 3 to use up factors OD, enough of good 2 to use up factors DQ, and nothing of good 1; the other country produces the rest. In the second way, one country produce enough of good 1 to use up factor endowments OE, good 3 to use up EF, and good 2 to use up FQ; again the other country produces the rest. Any of these assignments replicate the integrated equilibrium with separate factor markets in each country. Incidentally the construction also shows how quantities of goods produced and traded cannot be determined uniquely when there are more goods than factors.

Next consider a point such as X outside the hexagon. If one country has the factor endowment OX, then any attempt to assign the quantities of goods in the integrated equilibrium will require this country to produce enough of the third good to use up factors somewhere between OY and OZ. But that will leave the other country producing negative amounts of the third good. This is not possible, therefore the integrated equilibrium cannot be replicated with separate factor markets in the two countries, and factor price equalization cannot occur.

It is even more evident that nonnegative production quantities cannot be assigned to the two countries to replicate the integrated equilibrium if the factor endowment point lies outside the cone spanned by the directions OA_1 and OA_3. In other words, if the relative factor endowment composition of the two countries differs by more than the differences in the relative factor requirements of the goods' production, we cannot get factor price equalization. Point X lies inside this cone, yet it cannot be replicated; so factor price equalization in fact requires a little bit more.

Our construction is very similar to that in another showcase of general equilibrium, namely finance. In Arrow-Debreu theory, people trade in contracts for commodities contingent on each of the conceivable states of the world. In reality, markets exist only for various financial assets, which are particular linear combinations of the state-by-state contingent claims. Then the question is whether or when markets are complete, that is, trading in assets can replicate the equilibrium of an Arrow-Debreu world. The answer is that the Arrow-Debreu equilibrium can be replicated with trade in assets if the vectors of state-contingent payoffs to the available assets *span* the full space of state-contingent commodities.

In the standard model of international trade, countries differ in their factor endowments, and would like to trade factor services. They cannot do so directly, but can trade linear combinations of factors that are embodied in goods. The question is whether trade in these packages of factors can replicate direct trade in all individual factors. The analogy would suggest that answer is yes if the vectors of factor proportions embodied in goods span the space of factors. But there is a further complication. In finance, the packages of Arrow-Debreu state-contingent claims that constitute the assets can be traded

in positive or negative amounts by taking long or short positions in the assets. But in international trade a country cannot produce negative quantities of a good. Therefore we need a theory of *non-negative* spanning. And that is exactly what the construction of the integrated world equilibrium diagram achieves.

This is how Victor and I thought about factor price equalization and the integrated world equilibrium diagram. Let me say a little bit about precedents and subsequent developments. So much of modern international trade theory comes from Samuelson that it is hardly surprising to find a nice verbal statement of the idea in one of his early papers on the subject (Samuelson, 1949):

> "Let us suppose that in the beginning all factors were perfectly mobile and nationalism had not yet reared its ugly head. … [T]here would be one world price of food and clothing, one real wage, one real rent, and the world's land and labor would be divided between food and clothing production in a determinate way. … Now suppose that an angel came down from heaven and notified some fraction of all the labour and land units producing clothing that they were to be called Americans, the rest to be called Europeans; and some different fraction of the food industry that henceforth they were to carry American passports. Obviously, just giving people and areas national labels does not alter anything: it does not change commodity or factor prices or production patterns.
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> "But now turn a recording geographer loose, and what will he report? Two countries with quite different factor proportions, but with identical wages and rents and identical modes of commodity production."

Here is the integrated world equilibrium and its split into two national equilibria with factor price equalization, for the 2-by-2 case but in very simple words. Our diagram just shows how to do it for more goods and factors. Indeed, we later discovered that Travis (1964) had anticipated some simple cases of our general theory of the integrated world equilibrium and its diagrammatic representation.

What about subsequent developments? No new idea or technique can meet the fondest hopes of its authors without the work of others who use, extend, modify, and improve upon it. We have been lucky. It would take too long to list all the advances, so we will mention just two. Helpman and Krugman (1985) used the integrated equilibrium to great effect in their analyses of trade under various kinds of imperfect competition. Deardorff (1994) produced an elegant generalization with many goods and factors, clarifying the general sense in which factor price equalization requires endowments to differ less across countries than factor intensities vary across industries.

The only correct answer to the question whether economics is a science or an art is “Both”. Therefore Victor Norman and I are especially pleased to have a contribution on the art side that is deemed worthy of inclusion in this volume, and to have the opportunity to tell the story behind it.

REFERENCES

Deardorff, Alan V. (1994), 'The possibility of factor price equalization, revisited', *Journal of International Economics,* 36(1-2), 161-175.

Dixit, Avinash and Victor Norman (1980), *Theory of International Trade,* Cambridge, UK: Cambridge University Press.

Helpman, Elhanan and Paul Krugman (1985), *Market Structure and Foreign Trade,* Cambridge, MA: MIT Press.

Samuelson, Paul A. (1948), 'International factor-price equalization once again', *Economic Journal,* 58( ), 163-184.

Travis, William P. (1964), *The Theory of Trade and Protection,* Cambridge, MA: Harvard University Press.

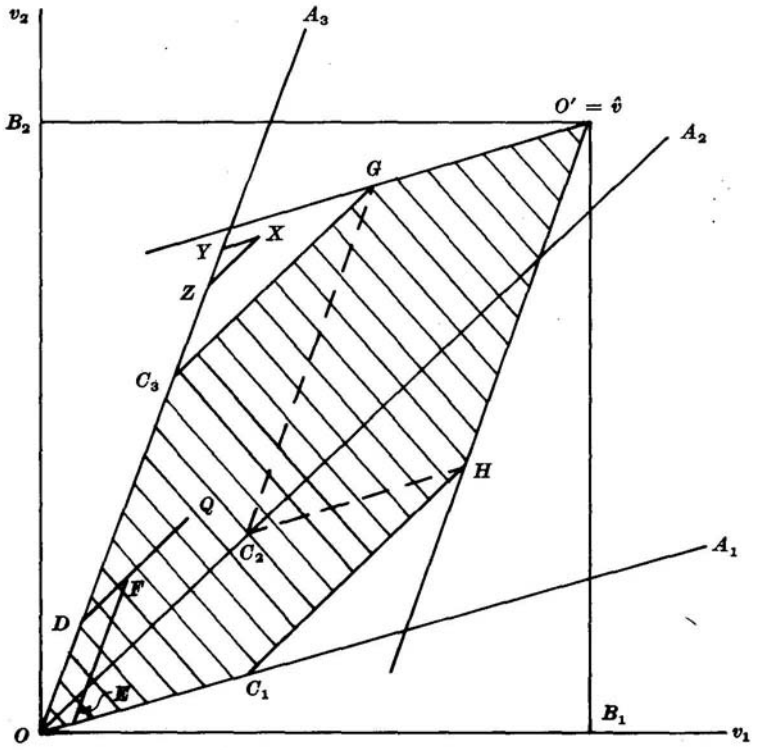

Figure 4.6