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How early should you plan to arrive for a flight?

Avinash Dixit, Princeton University[1]

"If you have never missed a flight, you are not optimizing". This statement is usually attributed to George Stigler, one of the founders of the very rational Chicago school of economics. Ellenberg (2014, ch.12) expands on this. He points out that it would be irrational to plan your arrival at the airport so early as to reduce the risk of missing a flight literally to zero. Allowing a very tiny probability of missing, and therefore a very tiny loss, would save you enough time gained for other activities to justify that trade-off. Ellenberg uses this as a way to introduce the idea of expected utility and an interior optimum (2014, p.235): "[Planning to show up] fifteen minutes before the plane leaves is going to slam you with a very high probability of missing the plane, with all the negative utility that implies. On the other hand, arriving many hours before also costs you many utils. The optimal course of action falls somewhere in between." But he does not pursue this to give any useable formula for the optimum. In this brief note I do that.

There are two important considerations to incorporate in a quantitative model. (1) You plan on a target arrival time, but the actual time is subject to several random shocks: the traffic on the roads to the airport may be heavy, or there may be an accident holding up traffic, or your car may break down; you may miss a bus or train connection; the baggage check-in line or security line may be unexpectedly long. (2) There is an important asymmetry in the loss function or disutility from arriving too early versus too late. If you arrive a minute too early, you have wasted a minute; if you arrive a minute too late, you have lost much more: an hour even if flights are very frequent as on a New York to Washington shuttle, several hours if your flight is transatlantic, or a whole day if it is transpacific. To this time loss you should add the monetary costs (airline fees, extra meals or hotel accommodation etc.) of changing the flight. There is also the professional cost of missing a lecture or a conference, or the personal cost of missing a day at home or a day of vacation.[2] Your optimization should take into account this asymmetry in formulating the expected loss function to be minimized.

Suppose you plan to arrive x minutes early at the gate. Let y denote the random delay in your trip, so your actual arrival time is $(x - y)$ minutes early. The "delay" may be negative: the traffic on roads may be exceptionally light, you may catch a train or bus scheduled for an earlier time, or the security line may move very fast. So for simplicity of notation let the support of the distribution of y be $(-\infty, \infty)$. Let the cumulative distribution function of y be $F(y)$, the probability density function $f(y)$, and mean 0 (absorbing any non-zero mean into x).

1 I thank Lars Svensson for useful comments on a first draft, especially for pointing me to the Ellenberg reference.

2 Sometimes the time waiting at the airport after an early arrival can be put to productive use; I wrote one of my better-cited papers (on investment and entry-deterrence, Dixit 1980) when I had gone to Geneva airport too early. If you miss a flight and reschedule it for the next day, you can go back to your office and do some work. I have left out such possibilities for this simple note. Readers can make their personal adjustments to the parameters.

If $x - y > 0$, you arrive too early and waste $(x - y)$ minutes; suppose the cost of this is $a(x - y)$. If $x - y < 0$, you miss the flight. Even the smallest miss carries a lump-sum cost; suppose this cost is k. So the expected loss from your choice is

$$L(x) = \int_{-\infty}^{x} a(x - y)f(y)dy + \int_{x}^{\infty} kf(y)dy.$$

Then

$$L'(x) = \int_{-\infty}^{x} af(y)dy + a(x - x)f(x) - kf(x)$$

or

$$L'(x) = aF(x) - kf(x).$$

Further,

$$L''(x) = af(x) - kf'(x).$$

In most cases of practical interest, we should expect $x > 0$ and in that region $f'(x) < 0$. [3] Then $L''(x) > 0$, so the second-order condition to minimize $L(x)$ is fulfilled and the first-order condition yields the optimum. This is simply

$$f(x)/F(x) = a/k.$$

Many random shocks add up to form the total delay, and the different sources of these are at most imperfectly correlated. Therefore a normal distribution may be a good approximation for the total delay. Using the symmetry of this distribution, we can express the above equation in terms of its hazard rate $H(x)$:

$$H(-x) = f(-x)/[1 - F(-x)] = f(x)/F(x) = a/k.$$

Since the normal distribution has a monotone hazard rate, this equation has a unique solution. So long as that solution has $x > 0$, we have $f'(x) < 0$ so the second-order condition is met.

Some numerical examples will illustrate the results of this algebra. Suppose the distribution of y is normal with mean 0 and standard deviation $\sigma = 15$ minutes. Then the probability of your arrival within 30 minutes of your intended time is over 95%, which seems reasonable, but may be wrong in some specific situations (as for example if there is an accident on the Belt Parkway on your way to JFK Airport in New York).

[3] Exceptional cases can arise: if the distribution of y is skewed in such a way that $x < 0$ (it is optimal to plan to arrive too late because negative shocks are sufficiently more likely). If there are jumps in the probability density function of y, say because of the scheduling of trains or buses at discrete intervals, then we may have $f'(x) > 0$ for some values of x so some first-order conditions yield a local minimum. I leave out such possibilities here.

By tabulating $f(x)/F(x)$ for a range of x, and comparing it to this value of a/k, we can find the optimal x. A table is attached, and the Excel spreadsheet is available from the author.

When you know your value of a/k, locate the row in the table where the last column shows this value (or a number close to it). The x in the first column of this row is your optimal choice, and the number in the column second from the right is the resulting probability of missing your flight. Consider three types of flights, shown bolded in the table for convenience:

Hourly Shuttle: Missing it delays you by only an hour, so a/k=1/60= $1.667 * 10^{-2}$. The optimal x is between 16 and 17 minutes, and the probability of missing the flight is about 14%.

Transatlantic flight: Suppose missing this means delay of six hours, so $k = 360\ a$, even leaving out all the non-time costs of missing the flight. Then a/k= $2.78 * \ 10^{-3}$. The optimal x is 32 minutes, and the probability of missing the flight is 1.64%.

Transpacific flight: Missing this usually entails delay of a whole day, so a/k=1/(24*60)= $6.94 * \ 10^{-4}$. The optimal x is between 40 and 41 minutes, and the probability of missing the flight is only about 0.35%.

The numbers suggest that rational people will occasionally miss their shuttle flights, but most may never miss an international flight in their lifetime while being perfect optimizers! As Ellenberg says (2014, p.236): "a probability that's close to zero can be hard to distinguish from a probability that actually is zero. If you are a global jet-setting economist, accepting a 1% risk of missing a plane might really mean missing a flight every year. For most people, such a low risk might well mean going your whole life without missing a plane." This note adds quantitative content to that. It also shows that for any one person, the optimal "1% (or whatever) risk" is not the same regardless of the type of flight: even jet-setting economists should set very different target arrival times, and very different probabilities of missing, for different types of flights.

Equipped with this model, everyone can channel their calculations of their personal optimum for any specific flight through the three parameters: σ, which depends on the local ground traffic conditions, the time of day of the flight (the likely variation in traffic conditions will depend on this, as will the length of the security line), and so on; and a and k, which depend on your personal opportunity costs of time, the policies and fees of your airline about rescheduling passengers who miss flights, and your psychological attitudes to wasting idle time and to the trauma of missing the flight.

REFERENCES

Dixit, Avinash. 1980. The role of investment in entry deterrence. *The Economic Journal*, 90(1), March, 95-106.

Ellenberg, Jordan. 2014. *How Not To Be Wrong: The Power of Mathematical Thinking*. New York: Penguin Books.

x	F(x)	f(x)	1-F(x)	f(x)/F(x)
0.0	5.000E-01	2.660E-02	5.000E-01	5.319E-02
1.0	5.530E-01	2.636E-02	4.470E-01	4.767E-02
2.0	5.530E-01	2.636E-02	4.470E-01	4.767E-02
3.0	5.793E-01	2.607E-02	4.207E-01	4.500E-02
4.0	6.051E-01	2.567E-02	3.949E-01	4.242E-02
5.0	6.306E-01	2.516E-02	3.694E-01	3.990E-02
6.0	6.554E-01	2.455E-02	3.446E-01	3.746E-02
7.0	6.796E-01	2.385E-02	3.204E-01	3.510E-02
8.0	7.031E-01	2.307E-02	2.969E-01	3.281E-02
9.0	7.257E-01	2.221E-02	2.743E-01	3.061E-02
10.0	7.475E-01	2.130E-02	2.525E-01	2.849E-02
11.0	7.683E-01	2.033E-02	2.317E-01	2.645E-02
12.0	7.881E-01	1.931E-02	2.119E-01	2.450E-02
13.0	8.069E-01	1.827E-02	1.931E-01	2.264E-02
14.0	8.247E-01	1.721E-02	1.753E-01	2.086E-02
15.0	8.413E-01	1.613E-02	1.587E-01	1.917E-02
16.0	**8.569E-01**	**1.506E-02**	**1.431E-01**	**1.757E-02**
17.0	**8.715E-01**	**1.399E-02**	**1.285E-01**	**1.606E-02**
18.0	8.849E-01	1.295E-02	1.151E-01	1.463E-02
19.0	8.974E-01	1.192E-02	1.026E-01	1.329E-02
20.0	9.088E-01	1.093E-02	9.121E-02	1.203E-02
21.0	9.192E-01	9.982E-03	8.076E-02	1.086E-02
22.0	9.288E-01	9.072E-03	7.123E-02	9.768E-03
23.0	9.374E-01	8.209E-03	6.260E-02	8.757E-03
24.0	9.452E-01	7.395E-03	5.480E-02	7.823E-03
25.0	9.522E-01	6.632E-03	4.779E-02	6.965E-03
26.0	9.585E-01	5.921E-03	4.152E-02	6.178E-03
27.0	9.641E-01	5.263E-03	3.593E-02	5.460E-03
28.0	9.690E-01	4.658E-03	3.097E-02	4.807E-03
29.0	9.734E-01	4.104E-03	2.660E-02	4.216E-03
30.0	9.772E-01	3.599E-03	2.275E-02	3.683E-03
31.0	9.806E-01	3.143E-03	1.938E-02	3.205E-03
32.0	**9.836E-01**	**2.732E-03**	**1.645E-02**	**2.778E-03**
33.0	9.861E-01	2.365E-03	1.390E-02	2.398E-03
34.0	9.883E-01	2.038E-03	1.171E-02	2.062E-03
35.0	9.902E-01	1.748E-03	9.815E-03	1.765E-03
36.0	9.918E-01	1.493E-03	8.198E-03	1.505E-03
37.0	9.932E-01	1.269E-03	6.819E-03	1.278E-03
38.0	9.944E-01	1.075E-03	5.649E-03	1.081E-03
39.0	9.953E-01	9.055E-04	4.661E-03	9.098E-04
40.0	**9.962E-01**	**7.597E-04**	**3.830E-03**	**7.627E-04**
41.0	**9.969E-01**	**6.346E-04**	**3.135E-03**	**6.366E-04**
42.0	9.974E-01	5.277E-04	2.555E-03	5.290E-04
43.0	9.979E-01	4.369E-04	2.074E-03	4.378E-04
44.0	9.983E-01	3.601E-04	1.677E-03	3.607E-04
45.0	9.987E-01	2.955E-04	1.350E-03	2.959E-04