On Optimal Transition to Green Power Generation

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Green technologies and their equipment for electric power generation (windmills, solar panels etc.) used to be very costly and inefficient. But experience (learning by doing; see Arrow 1962) is bringing down the cost of manufacturing and installing the equipment and increasing its efficiency to the point where it is starting to come into widespread use even on a purely commercial basis without any government subsidies. This raises the question of whether a transition from investing in traditional polluting power plants (coal or gas powered etc.) to the new methods should be gradual or sudden, and at what point of cost differences the transition should begin or end. This note attempts to model the issue in an optimal control framework.

My main result is a "clean break" theorem, with two parts. First, at the point in time when the transition becomes optimal, it should be a sudden and complete change from polluting to green investment; no further traditional polluting power plants at all should be installed thereafter. Second, the transition becomes optimal at a point where the cost of the new technologies per unit of the power they generate is still higher than that for traditional technologies, as a kind of "investment" in the learning by doing that will bring it down in the future. The learning by doing is a benefit available to the society as a whole. Therefore while the installation cost of the new clean plants exceeds that of the traditional polluting kinds, a subsidy equal to (or perhaps just slightly greater than) the cost difference should be offered to private manufacturers or owners of the clean plants.

I should clarify and emphasize up front that the sudden "clean break" transition applies to construction of and investment in *new* plants; it does not mean abandonment of the

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existing capital stock of polluting power generation plants and equipment. In fact the older capital will continue to play a useful role in providing back-ups to cover interruptions in the operation of solar and wind power. Gradually, as research into methods of storage and long-distance transmission of cleanly generated power comes to fruition, this role will be needed less and less. Also, as time progresses, the stock of polluting power generation capital will gradually depreciate. It is an important part of planning for the ultimate transition to clean energy that these two dynamics operate in harmony – ensuring that enough older stock remains to provide the back-up capacity. I have not included optimization of this balance in the model of this paper to keep the initial exploration simple, but hope to do so in subsequent research.

The costs of clean power generation vary widely across countries or regions or situations depending on the amount and strength of sunshine or the speed and persistence of wind, whether the windmill farm is on land or offshore, whether the solar panels are in a large "farm" or on residential rooftops, and so on. Costs of polluting technologies depend on the type of fuel (relatively clean versus highly polluting coal and gas), on whether this is locally available or transported over large distances at considerable costs including the added CO_2 emissions in the process, and so on. Therefore the optimal time for the "clean break" can differ greatly from one specific instance to another. But the data presented here suggest that in an average-clean to average-polluting comparison, the switch should already have occurred, perhaps some time between 2011 and 2015!

The model is deliberately simple, and generally leans toward the cautious side. The statement and discussion of the assumptions in the next section suggests that the case for making the clean break is even stronger than what the results of the model yield. However, reality differs from both parts of the clean break theorem. Investments in dirty and clean power generation coexist and continue. Some investments in solar farms and windmills were made even when they was far more costly than coal and gas; otherwise the learning by doing would never have got started. On the other hand, in many places the costs of clean power generation are already lower than those of polluting kinds, and yet the latter persist. Obviously, the real world is not optimally designed as a mathematician or industrial engineer would have it; instead it is a complex political interaction involving voters, lobbyists, entrepreneurs, and many other influences. Employment in coal-mining areas, contributions from oil and gas companies and their suppliers who have huge sunk stakes in the activities of exploration, mining, drilling and fracking, and such other considerations of private profit help explain the continuance of polluting power generation. Contractual obligations may also keep polluting power plants in operation longer than firms or governments would freely choose. Countering these influences, we have some pressure from the scientific community concerned with climate change, and investments by some visionary entrepreneurs who looked sufficiently far into the future. These forces launched research and development projects, often with some public funding or subsidies, that gained momentum for the solar panel industry and led to major technological improvements in electricity generation from windmills. The political game continues, and I hope that the stark findings of this simple optimization exercise will be an input that tilts the balance toward an overall social good.

The Model

Notation:

$$\begin{aligned} P &= \text{stock of traditional polluting power generation capital} \\ I &= \text{investment in traditional power generation} \\ K &= \text{stock of non-polluting ``clean'' power generation capital} \\ G &= \text{investment in clean power generation} \end{aligned}$$

These entities are measured in the units of power they generate, say megawatts. We begin at time $t = 0$ with $P(0) > 0$ and $K(0) = 0$.

The dynamics of the system is given by

$$\left.\begin{aligned} \dot{P} &= I - \delta\, P \\ \dot{K} &= G - \delta\, K \end{aligned}\right\} \qquad (1)$$

where δ is the rate of depreciation which, in the absence of any clear reason to believe otherwise, I assume to be the same for the two types of capital.[1]

I assume that the technology of power generation by traditional polluting methods (coal, gas etc.) has stabilized, so the cost of installing each unit (measured in gigawatts of capacity) of such investment is constant, denoted by ϕ. In reality there is some technical progress (through R&D or other exogenous developments) as well as learning by doing in the traditional technology, but these forces are evidently faster in newer less-polluting technologies, so the qualitative results of the model will remain valid. We can also expect the costs of

[1] If equipment embodying clean capital, being newer and having the benefit of more modern materials and techniques, has a lower rate of depreciation, that will only further strengthen the results.

these methods to rise over time, as their inputs are resources in finite supply, whose scarcity will tighten over time, and which are in the long run subject to Hotelling's rule that their price should rise at the rate of interest. The data presented later support the assumption of a constant ϕ as compared to a declining cost for clean technologies.

Clean power generation technologies (wind, solar etc.) become less costly over time, through technical progress as well as learning by doing. I focus on the latter in the theoretical model; the empirical work will recognize both. I specify the cost of installing each unit of investment in clean technologies as $\psi(K)$, where

$$\psi'(K) < 0, \quad \text{and} \quad \psi(0) > \phi > \psi(\infty) > 0 \tag{2}$$

This cost decreases with K, i.e. the accumulated experience of previous investments (learning by doing). Initially the cost of clean investment exceeds that of polluting investment (measured per unit of power they will generate), but if enough clean capital is accumulated, this cost will fall below the level of the polluting investment.[2]

Then the consumption flow is given by

$$C = \beta\,(P + K) - \gamma\,P - \phi\,I - \psi(K)\,G \tag{3}$$

where β is the consumption generated by each megawatt of installed power, and γ is the current cost of the polluting technology – expenses needed to counter the warming, health problems caused by the smoke etc. I will give traditional technology some benefit of doubt by assuming $0 \leq \gamma < \beta$; if $\gamma > \beta$ that technology should never be used even if nothing else is available.

The optimal control problem is to choose the time-path of the investments to maximize the discounted present value of the utility of consumption:

$$\int_0^\infty U(C)\; e^{-\rho\,t}\,dt \tag{4}$$

where the utility function satisfies $U'(C) > 0$, $U''(C) < 0$, and $\rho > 0$ is the rate of time-discounting.

[2] I am assuming that as the clean capital depreciates, the learning it contributed is lost also. Instead, perhaps gross accumulated past investment ($\int_0^t G\,dt$) should be used as the argument of ψ. But that requires introduction of a third state variable, which has proved intractable so far. In any case it would only further strengthen the case for the transition, which this paper shows to be already quite strong.

Clean Break Theorem

To solve the control problem I use Pontrjagin's method (Pontrjagin et al. 1962), familiar to economists from Cass and Shell (1976) and others. Define the current value Hamiltonian

$$H = U(C) + p\,(I - \delta\, P) + q\,(G - \delta\, K) \tag{5}$$

where p and q are the respective current value shadow prices of polluting and clean capital. Then at each instant I and G are chosen to maximize H, and the shadow prices evolve according to

$$\left.\begin{array}{rcl} \dot{p} &=& (\rho+\delta)\,p - \partial H/\partial P = (\rho+\delta)\,p - U'(C)\;(\beta-\gamma) \\ \dot{q} &=& (\rho+\delta)\,q - \partial H/\partial K = (\rho+\delta)\,q - U'(C)\;[\beta - \psi'(K)\,G] \end{array}\right\} \tag{6}$$

I begin by asking whether any transition can be gradual, i.e. whether a phase with both I and G positive can continue for any interval of time. Temporarily assume this to be so. Over this interval, the interior optimum should satisfy the first order (necessary) conditions

$$\left.\begin{array}{rcl} \partial H/\partial I &=& -U'(C)\;\phi - p = 0 \\ \partial H/\partial G &=& -U'(C)\;\psi(K) - q = 0 \end{array}\right\} \tag{7}$$

Define the ratio of the two shadow prices $s = q/p$. Then the equations in (7) imply

$$s = \psi(K)\,/\,\phi \tag{8}$$

This defines a downward-sloping curve in (K, s) space, starting above 1 and eventually falling below that level because of (2). This is shown in the attached figure.

Along any interval of positive investment in both technologies, the point in (K, s) space would have to move along this curve, i.e.

$$\dot{s}\,/\,\dot{K} = ds/dK = \psi'(K)\,/\,\phi \tag{9}$$

From the dual dynamics equations (6), we have

$$\frac{\dot{s}}{s} = \frac{\dot{q}}{q} - \frac{\dot{p}}{p} = U'(C)\;\left[\frac{\beta-\gamma}{p} - \frac{\beta - \psi'(K)\,G}{q}\right]$$

Using the first-order conditions (7) in this,

$$\frac{\dot{s}}{s} = \frac{\beta-\gamma}{\phi} - \frac{\beta - \psi'(K)\,G}{\psi(K)}$$

And log-differentiating the definition of s (8)

$$\frac{\dot{s}}{s} = \frac{\psi'(K)\,(G - \delta\,K)}{\psi(K)}$$

Subtracting the second of these equations from the first, we have

$$0 = \frac{\beta - \gamma}{\phi} - \frac{\beta}{\psi(K)} + \frac{\delta\,K}{\psi(K)}$$

Multiplying through by $\psi(K)$ and using the definition of s again,

$$s = \frac{\beta - \delta\,K}{\beta - \gamma} \tag{10}$$

This downward-sloping line can intersect the curve (8) only in isolated points (there are two in the figure).

Thus my tentative assumption – coexistence of positive investment in both technologies over an interval of time – implies two things: the point in (K, s) space has to move along the curve (8), and it has to satisfy (10) at all times. But the two are compatible only at isolated instants. Therefore the tentative assumption is invalidated. Any transition between the technologies must be sudden and complete.[3]

An intuition for this can be presented as follows. Suppose at an instant of time t we have exact indifference between investing in the two types of power generation, polluting and green, so positive investment levels in both types are justified and both are undertaken. Consider another instant t' that is $> t$, however slightly so. The cost of the polluting equipment is unchanged, but that of the green equipment has gone down because the positive investment at t has generated some learning. That breaks the indifference at t' in favor of the green investment. Therefore positive investment in both types cannot coexist over any interval of time of positive duration, no matter how small.

Let us examine what kind of transitions can occur. If we are in a phase where only the clean technology is being used, its cost advantage can only increase over time because of the learning by doing. Therefore it will remain optimal in the future - a transition from clean to polluting generation should not happen.

[3]Since learning by doing brings with it some increasing returns to scale (non-convexities), first-order conditions may not yield an optimum, and it would seem necessary to check second-order (Legendre) conditions. However, here I am proving that the first-order conditions do *not* yield an interior optimum for other reasons, and the true optimum is to be found by direct binary comparison of two extreme paths as argued below. Therefore I do not need to be concerned with second-order conditions.

Next consider a transition from polluting to clean investment at time T. Before this, there was no investment in clean generation, so $P(T) > 0$ and $K(T) = 0$. There was no learning by doing, so cost conditions were stationary at ϕ for polluting and $\psi(0)$ for clean. With unchanged cost conditions, if a transition is desirable at time T, it is also desirable at any time $t < T$. Therefore if there is to be a transition from polluting to clean investment, it should occur right at the outset $t = 0$, or else not at all. Which is better depends on the $\psi(K)$ function and its relation with ϕ.

Thus the optimization problem boils down to a simple binary comparison: (i) solve the problem of optimal investment in polluting capital as if the clean technology didn't exist, (ii) separately solve the problem of investment in clean technology alone, keeping the polluting capital starting at the given $P(0)$ and then gradually depreciating, and (iii) compare the discounted present values of the utilities from the two separate calculations. That is best left to numerical work based on actual data for the two technologies. The answer will depend, among other things, on the cost ϕ of the polluting generation, the path of the learning curve $\psi(K)$ for the clean technology, how dirty the polluting technology is (which determines the negative externality parameter γ) and the initial level $P(0)$ of the polluting capital stock.

However, one general principle can be stated. If $\psi(0) \leq \phi$, then clean technology dominates polluting technology in all respects: less costly, and without the bad effects on consumption via γ. Therefore by continuity, even if $\psi(0)$ is raised slightly above ϕ, it will remain optimal to transition to clean generation at once. This is the second part of the Clean Break theorem stated above: It is optimal to make an "investment in the learning to come" that clean generation provides, even if it is currently somewhat more costly.

To make these general propositions precise needs numerical solutions, and the answers will vary across applications. In this paper I develop a simple and broad analysis to illustrate and apply the theoretical analysis above. The data presented next are for averages of the cost parameters. They suggest that in this average sense the optimal switch to green generation is already overdue. But many variations around these averages should be investigated in future research directed to specific contexts of contemplated switches.

Data Analysis

There exists a voluminous literature estimating learning by doing in various industries, dating back to the classic paper of Wright (1936) on airframes. A recent review in the context of solar and wind generation is Castrejon-Campos, Aye and Hui (2022). My scope and method in this paper are far simpler, but different in some crucial ways from much of the literature:

Following Wright, most of the literature specifies a power law. In my notation, starting with an initial installed capacity K_0, we are to have, for $K > K_0$,

$$\psi(K) \,/\, \psi(K_0) = [\, K \,/\, K_0 \,]^{-\omega}$$

This is unsatisfactory in at least two ways. (1) It does not allow the process to start with no solar or wind capacity installed ($K_0 = 0$): the cost is infinite at that point, so no investments will be made, and no learning will occur. More importantly, it forces $\psi(K) \to 0$ as K gets ever larger, which is patently untrue: the LCOE tables in the Lazard publication show that their projections for out-years level out at costs significantly greater than 0. (2) The decline of LCOEs seen in the data from one year to the next is likely to be some combination of exogenous technical progress through R&D or other influences, and learning by doing, and as Nordhaus (2009) and others have argued, a specification attributing the entire effect to learning by doing can yield very misleading results.

My specification avoids both these problems. I assume that $\psi(K)$ declines from a high value $\overline{\psi}$ at $K = 0$ and $t = 0$[4] toward its low asymptotic level $\underline{\psi}$ as a product negative exponentials in t and K:

$$\psi(K) - \underline{\psi} = \; e^{-\sigma\, t} \; (\overline{\psi} - \underline{\psi}) \; e^{-\theta\, K}$$

where the first factor on the right hand side reflects other exogenous technical progress and the second is the learning effect. I transform this into an equation to fit to the data:

$$\ln[\, \psi(K) - \underline{\psi} \,] = \text{Constant} - \sigma\, t - \theta\, K \tag{11}$$

Finally, although the existing literature uses more disaggregated data for longer time periods, I focus on the last dozen years, when the cost reduction in clean power generation have been most dramatic, and therefore most instructive for future planning, and I keep the analysis at an aggregate level to bring out the main ideas in the simplest possible way.

The concept used in measuring and comparing the cost of electricity generation from various sources is the Levelized Cost of Electricity (LCOE), measured in dollars per megawatt-hour. To get this, the capital cost of the equipment is spread out over its projected lifetime

[4] This is just the choice of origin on the time axis, and affects only the magnitude of the constant term in the regression.

using standard discounted present value methods. To this are added the forecast costs of materials, maintenance etc. in each year of the lifetime. Dividing the annual cost by the output (taking into account the capacity of the plant and its forecast utilization) gives the LCOE.

Tables 1 and 2 show the LCOEs for solar and wind for the years 2009-2021.[5] A range of high, average, and low estimates is shown. For the purpose of my illustrative calculations I have used the average, but the other cases should also be experimented with. The costs vary widely across countries and regions. The cost for wind power differs substantially between land and offshore windmills. Therefore the optimal transition dates from polluting to clean wind power will also depend on where the windmills are to be located. The theoretical model above can readily be adapted and applied to calculations for these specific situations.

On the same aggregate and average basis, I guessed $\underline{\psi}$ to be 30 for solar and 35 for wind, by looking at where the numbers in the out-year projections in the Lazard publication seemed to be leveling off; variations around these guesses do not make a great deal of difference.

The results of the regressions are shown in Tables 3 and 4 for solar and wind respectively. The overall fit is very good in both cases. For solar the coefficients θ and σ are very tightly estimated. For wind they are similar in magnitude to solar but their standard errors are much larger; in fact θ is not significantly different from 0. Of course many variations around these numbers should be explored, both because of the uncertainty and guesswork involved, and because of the variations across countries, specific locations and different sizes of solar farms, windmills etc.

For settled LCOEs of polluting technologies (ϕ in the notation of my model), the data (see the chart on p.9 of the Lazard publication cited in the Footnote just above) give a range of \$64 to \$171 for coal (average \$117), and \$33 to \$108 for gas (average \$70). Most importantly, the curves for these costs are much flatter over time, justifying my assumption in the theoretical model.

[5] The cost data are from a Lazard publication *LCOE Lazard, Version 16.0*, April 2023, available at `www.lazard.com/research-insights/levelized-cost-of-energyplus/` as the file `lazards-lcoeplus-april-2023.pdf`; see the charts on p.13 of the PDF file (p.10 in the document's page numbering at the bottom right of each page) for solar and wind, and the bar chart on p.7 of the PDF (page numbered 4 in the document) for coal and gas. The solar and wind capacity data are from an Ember publication, from the site `ember-climate.org/data-catalogue/yearly-electricity-data/`. This is a very large file in `.csv` format, for numerous countries and years. I have used the capacity numbers for the whole world, since learning about technologies spreads fast among the relevant engineering and equipment manufacturing communities. Both data sets are publicly available and downloadable.

As I have already emphasized, energy investment policies in the real world are not guided solely, or even substantially, by aims of social optimization. But suppose we were starting an optimization exercise at the highest costs $\overline{\psi}$ for clean technologies, in the year 2009 or thereabout. The costs of solar and wind in that year were substantially higher than those of coal and gas. It is not clear whether a switch would have been optimal at that time. But as costs declined over time through R&D and other exogenous forces, a transition would become optimal soon thereafter. Comparing the average path of the costs for solar and wind with the average costs of polluting technologies, it seems that "on average" in 2023 it is already well past time for the "clean break" from investment in coal and gas based electricity generation to solar or wind. Since the clean break should occur while $\psi(K)$ is still greater than ϕ, the numbers in Tables 1 and 2 suggest that perhaps the optimal switch should have been made some time around 2011-15! Of course calculations for specific instances of contemplated switches should use the actual cost figures in each context; the theoretical model gives a method for policy-makers to perform these calculations and choose policies and investments.

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Table 1: Solar cost and capacity data

Year	LCOE - $/MW Max	Avg	Min	Capacity GW
2009	394	359	323	23.6
2010	270	248	226	41.59
2011	166	157	148	73.91
2012	149	125	101	104.05
2013	104	104	91	140.89
2014	86	79	72	179.92
2015	70	64	58	227.72
2016	61	55	49	299.86
2017	53	50	46	394.93
2018	46	43	40	488.47
2019	44	40	36	590.69
2020	42	37	31	716.39
2021	41	36	30	849.06

Table 2: Wind cost and capacity data

Year	LCOE - $/MW Max	Avg	Min	Capacity GW
2009	169	135	101	150.12
2010	148	124	99	180.83
2011	92	71	50	220.10
2012	95	72	48	266.84
2013	95	70	45	299.76
2014	81	59	37	349.23
2015	77	55	32	416.09
2016	62	47	32	466.78
2017	60	45	30	514.11
2018	56	42	29	563.36
2019	54	41	28	621.14
2020	54	40	26	731.63
2021	50	38	26	824.60

Table 3: Regression analysis - Solar

ln(Avg-30)	Time	ln(Capacity)
5.7961	0	3.1612
5.3845	1	3.7279
4.8442	2	4.3028
4.5539	3	4.6449
4.3041	4	4.9480
3.8918	5	5.1925
3.5264	6	5.4281
3.2189	7	5.7033
2.9957	8	5.9787
2.5649	9	6.1913
2.3026	10	6.3813
1.9459	11	6.5742
1.7918	12	6.7441

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9991
R Square	0.9983
Adjusted R S	0.9980
Standard Err	0.0589
Observations	13

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	20.2794	10.1397	2924.7613	1.4477E-14
Residual	10	0.0347	0.0035		
Total	12	20.3141			

	Coefficients	*Standard Error*	*t Stat*	*Lower 95%*	*Upper 95%*
Intercept	7.0396	0.3080	22.8544	6.3533	7.7259
Time	-0.2229	0.0243	-9.1890	-0.2770	-0.1689
ln(Capacity)	-0.3915	0.0847	-4.6196	-0.5803	-0.2027

Table 4: Regression analysis - Wind

ln(Avg-35)	Time	ln(Capacity)
4.6052	0	5.0114
4.4886	1	5.1976
3.5835	2	5.3941
3.6109	3	5.5866
3.5553	4	5.7030
3.1781	5	5.8557
2.9957	6	6.0309
2.4849	7	6.1459
2.3026	8	6.2424
1.9459	9	6.3339
1.7918	10	6.4316
1.6094	11	6.5953
1.0986	12	6.7149

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9896
R Square	0.9793
Adjusted R S	0.9752
Standard Err	0.1731
Observations	13

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	14.1829	7.0914	236.5643	3.7992E-09
Residual	10	0.2998	0.0300		
Total	12	14.4826			

	Coefficients	*Standard Error*	*t Stat*	*Lower 95%*	*Upper 95%*
Intercept	6.4183	4.3010	1.4923	-3.1649	16.0016
Time	-0.2288	0.1160	-1.9725	-0.4872	0.0296
ln(Capacity)	-0.3670	0.8401	-0.4368	-2.2388	1.5049

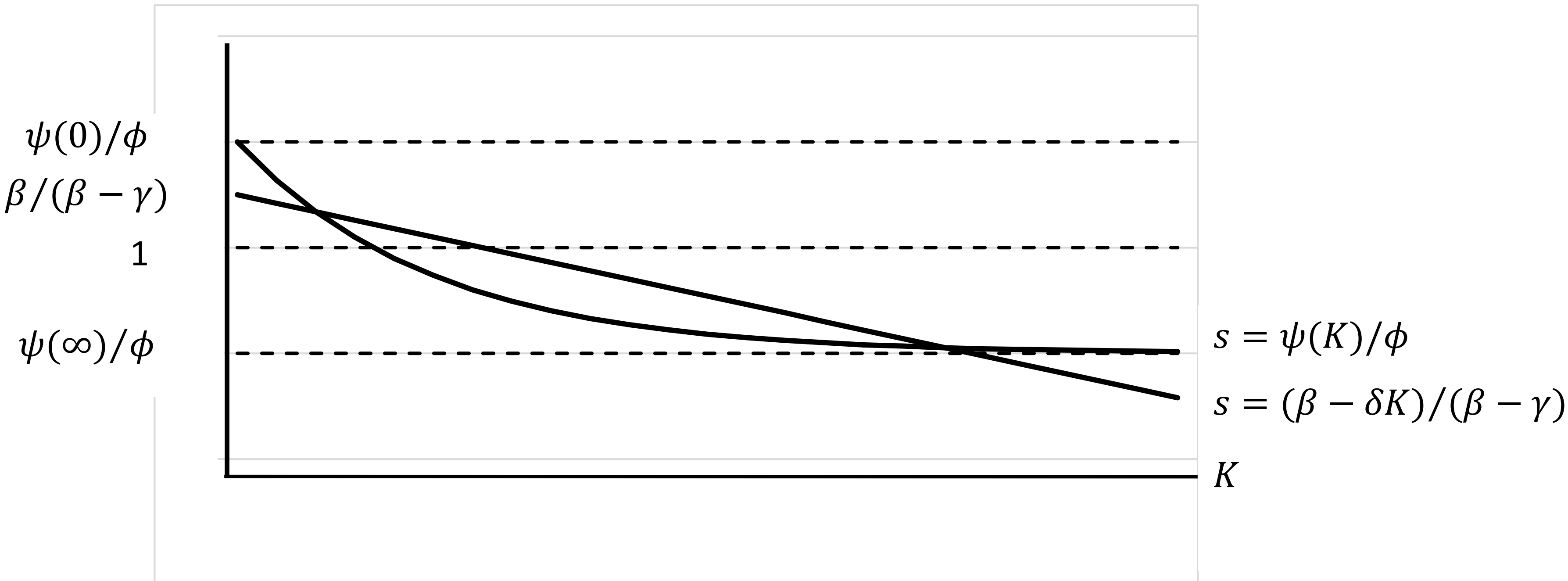

Figure 1 – Impossibility of interior optimum

Source: Author’s construction from calculations in text