Interest rate version of the model of Section 2 of "Equilibrium Contracts for the Central Bank of a Monetary Union"

Avinash Dixit
Princeton University

Henrik Jensen
University of Copenhagen

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Abstract

This note provides a variant of the simple two-country model of monetary union of Section 2 of our paper "Equilibrium Contracts for the Central Bank of a Monetary Union." Instead of treating inflation as the policy instrument (as in that paper), we consider the nominal interest rate as the instrument. It turns out that one can express all relevant economic quantities in terms of the actual and expected nominal interest rate.

1 A two-country model of monetary union with the nominal interest rate as policy instrument

The notation follows largely Dixit and Jensen (2000), and will only be explained if necessary. Production functions:

$$y^s = \alpha\, n, \qquad y^{*s} = \alpha^*\, n^*,$$

(superscript s denotes supply, $*$ denotes foreign country).

Factor demand equations:

$$w - p = -(1-\alpha)\, n, \qquad w^* - p^* = -(1-\alpha^*)\, n^*.$$

Wages are set in advance to aim for employment targets $\overline{n}, \overline{n}^*$, respectively. So,

$$w - p^e = -(1-\alpha)\,\overline{n}, \qquad w^* - p^{*e} = -(1-\alpha^*)\,\overline{n}^*,$$

where expectations are rational:

$$p^e = \mathbf{E}[p], \qquad p^{*e} = \mathbf{E}[p^*].$$

Combining the factor demand and wage-setting equations,

$$n = \overline{n} + \frac{1}{1-\alpha}(p - p^e), \qquad n^* = \overline{n}^* + \frac{1}{1-\alpha^*}(p^* - p^{*e})$$

Union-wide consumer price index is $\pi = \theta\,p + (1-\theta)\,p^*$, and the real exchange rate is $\rho = p^* - p$. So,

$$p = \pi - (1-\theta)\,\rho, \qquad p^* = \pi + \theta\,\rho$$

Demand equations:

$$\begin{aligned} y^d &= \overline{y}^d + \delta\,\rho - \eta\,(i - p^e) + d \\ y^{*d} &= \overline{y}^{*d} - \delta^*\,\rho - \eta^*\,(i - p^{*e}) - d \end{aligned}$$

where i is the nominal interest rate (the common central bank's policy instrument), and d is a stochastic shock with $\mathbf{E}[d] = 0$. The constants $\overline{y}^d$, $\overline{y}^d$ can also be stochastic shocks. In this one-period model, the previous period's (log) prices can be normalized to equal 0 in both countries, so this period's price levels and inflation rates coincide.

Let $\gamma = \alpha/(1-\alpha)$. Then in equilibrium, $y^d = y^s = y$ and $y^d = y^s = y$ become

$$\begin{aligned} y &= \alpha\,n = \alpha\,\overline{n} + \gamma\,(p - p^e) = \overline{y}^d + \delta\,\rho - \eta\,(i - p^e) + d, \\ y^* &= \alpha^*\,n^* = \alpha^*\,\overline{n}^* + \gamma^*\,(p^* - p^{*e}) = \overline{y}^{*d} - \delta^*\,\rho - \eta^*\,(i - p^{*e}) - d. \end{aligned}$$

Taking expectations,

$$\begin{aligned} \alpha\,\overline{n} &= \mathbf{E}[\overline{y}^d] + \delta\,\mathbf{E}[\rho] - \eta\,(\mathbf{E}[i] - p^e), \\ \alpha^*\,\overline{n}^* &= \mathbf{E}[\overline{y}^{*d}] - \delta^*\,\mathbf{E}[\rho] - \eta^*\,(\mathbf{E}[i] - p^{*e}). \end{aligned}$$

But $\mathbf{E}[\rho] = p^{*e} - p^e$. Therefore

$$\frac{\alpha\,\overline{n} - \mathbf{E}[\overline{y}^d] - \delta\,\mathbf{E}[\rho]}{\eta} - \frac{\alpha^*\,\overline{n}^* - \mathbf{E}[\overline{y}^{*d}] + \delta^*\,\mathbf{E}[\rho]}{\eta^*} + \mathbf{E}[\rho] = 0$$

Except when $\delta/\eta + \delta^*/\eta^* = 1$, which is an exceptional case, this equation gives a unique solution for $\mathbf{E}[\rho]$, and the solution is independent of anything to do with monetary policy. Then we can go back to the expectations equations and get the solution

$$\begin{aligned} p^e &= \mathbf{E}[i] + k \\ p^{*e} &= \mathbf{E}[i] + k^* \end{aligned}$$

where k, k^* are again independent of anything to do with monetary policy. Therefore

$$\pi^e = \mathbf{E}[i] + \theta\,k + (1-\theta)\,k^*, \qquad \rho^e = k^* - k$$

Now the output equilibrium equations can be regarded as determining p and p^*. Write them as:

$$\begin{aligned} (\gamma + \delta)\,p - \delta\,p^* &= \gamma\,\pi^e - \eta\,(i - \pi^e) + \text{constants} \\ -\delta^*\,\bar{p} + (\gamma^* + \delta^*)\,p^* &= \gamma^*\,\pi^e - \eta^*\,(i - \pi^e) + \text{constants} \end{aligned}$$

where "constants" refers to things that are independent of monetary policy — some expectations like those included in k, k^* as well as some realizations of stochastic shocks. The terms involving the demand shift shock d can be shown explicitly if needed.

The solution to this is

$$\begin{bmatrix} p \\ p^* \end{bmatrix} = \frac{1}{\gamma\,\gamma^* + \gamma\,\delta^* + \gamma^*\,\delta} \begin{bmatrix} \gamma^* + \delta^* & \delta \\ \delta^* & \gamma + \delta \end{bmatrix} \begin{bmatrix} \gamma\,\pi^e - \eta\,(i - \pi^e) + \text{constants} \\ \gamma^*\,\pi^e - \eta^*\,(i - \pi^e) + \text{constants} \end{bmatrix}$$

Therefore

$$p = \pi^e - \frac{\eta\,(\gamma^* + \delta^*) + \eta^*\,\delta}{\gamma\,\gamma^* + \gamma\,\delta^* + \gamma^*\,\delta}\,(i - \pi^e) + \text{constants}$$

and

$$p^* = \pi^e - \frac{\eta^*\,(\gamma + \delta) + \eta\,\delta^*}{\gamma\,\gamma^* + \gamma\,\delta^* + \gamma^*\,\delta}\,(i - \pi^e) + \text{constants}$$

Combining the two, we also have

$$\pi = \pi^e - \phi\,(i - \pi^e) + \text{constants}$$

where ϕ is a θ-weighted average of the coefficients in the p and p^* equations just above.

We see that a 1 percentage point decrease in the interest rate policy rule — the whole function $i(z)$ expressing the nominal interest rate as a function of the vector of stochastic shocks z — will lower π^e by 1 percentage point. This will leave the real interest rate and the real exchange rate unchanged. But a "surprise" 1 percentage point decrease in the interest rate for any one realization, keeping $\mathbf{E}[i]$ unchanged, will raise p and p^* according to the equations above. It will also change the real exchange rate:

$$\rho = \frac{\eta\,\gamma^* - \eta^*\,\gamma}{\gamma\,\gamma^* + \gamma\,\delta^* + \gamma^*\,\delta}\,(i - \pi^e)$$

Therefore, the monetary surprise will affect employment and output in the two countries differently.

Finally,

$$\begin{aligned} n &= \bar{n} + \frac{1}{1-\alpha}\,[\pi - (1-\theta)\rho - p^e] \\ &= \bar{n} + \frac{1}{1-\alpha}\left[\mathbf{E}[i] - \left(\phi + (1-\theta)\frac{\eta\gamma^* - \eta^*\gamma}{\gamma\,\gamma^* + \gamma\,\delta^* + \gamma^*\,\delta}\right)(i - \mathbf{E}[i]) + \text{constants}\right] \end{aligned}$$

All relevant economic quantities can now be expressed as functions of i and $\mathbf{E}[i]$, or with some recombination of terms, as functions of i and $i - \mathbf{E}[i]$.

References

[1] Dixit, Avinash and Henrik Jensen, 2000, Equilibrium Contracts for the Central Bank of a Monetary Union, mimeo, Princeton University and University of Copenhagen.