Symbiosis of Monetary and Fiscal Policies in a Monetary Union*

by
Avinash Dixit, Princeton University
and
Luisa Lambertini, UCLA

First draft — August 13, 1999
This draft — February 20, 2002

A Appendix: Microfounded model

We consider a two-country general equilibrium monetary model of the Blanchard and Kiyotaki (1987) type, popularized by Obstfeld and Rogoff (1996, chapter 10). There are N goods in the global economy, which are imperfect substitutes, and money. Each good is produced by a producer who acts as a monopolistic competitor facing a downward sloping demand curve and chooses the nominal price and the level of production of her good. Production makes only use of labor and, since labor supply is elastic, production is endogenously determined. Each producer is also a consumer, who derives utility from the consumption of all goods and real money balances but derives disutility from the effort put in production. For simplicity of notation only, we assume that the two countries have equal population; hence, $N/2$ goods are produced in country 1 and the other $N/2$ in country 2. The two countries are in a monetary union; hence, there is a single currency circulating and a common central bank that decides monetary policy.

Producer-consumer (producer for short) j in country i has the following utility function

$$U_{j,i} = \left(\frac{C_{j,i}}{\gamma}\right)^{\gamma} \left(\frac{M_{j,i}/P}{1-\gamma}\right)^{1-\gamma} - \left(\frac{d_i}{\beta}\right) (Y_{j,i})^{\beta}, \quad \gamma \in (0,1), \ \ d_i > 0, \ \ \beta \geq 1, \tag{A.1}$$

*We thank Michael Woodford, Roel Beetsma the editors and two anonymous referees for useful suggestions. Dixit thanks the National Science Foundation and Lambertini thanks the UCLA Senate for financial support.

where $j = 1, \ldots, N/2, i = 1, 2$ and the variable $C_{j,i}$ is a real consumption index

$$C_{j,i} = N^{\frac{1}{1-\theta}} \left[\sum_{z=1}^{N} \left(C_{z,j,i} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \tag{A.2}$$

where $C_{z,j,i}$ is the $j-$th individual consumption of good z in country i. The price deflator for nominal money is the consumption-based money price index corresponding to the consumption index (A.2)

$$P = \left[\frac{1}{N} \left(\sum_{z=1}^{N} P_z^{1-\theta} \right) \right]^{\frac{1}{1-\theta}}, \tag{A.3}$$

where P_z is the price of good z. The interpretation of equations (A.1) to (A.3) is completely standard – see Blanchard and Kiyotaki (1987). Producer j in country i has the following budget constraint:

$$\sum_{z=1}^{N} P_z C_{z,j,i} + M_{j,i} = P_{j,i} Y_{j,i} (1 - \tau_i) - PT_i + \overline{M}_{j,i} \equiv I_{j,i}, \tag{A.4}$$

which says that nominal consumption expenditure plus the demand for money must equal nominal income. It is assumed that taxes τ_i are proportional to sales; individuals also pay lump-sum taxes PT_i and have an initial holding of money, $\overline{M}_{j,i}$. Hence, nominal income is equal to nominal after-tax revenues from selling the produced good, minus lump-sum taxes, plus the initial money holding. Both τ_i and T_i can be either positive or negative.

There is a government in each country that runs fiscal policy and a central bank that runs monetary policy for both countries. The government of country i has the budget constraint:

$$I_{g,i} \equiv \sum_{j=1}^{N/2} P_{j,i} Y_{j,i} \tau_i + \frac{N}{2} PT_i = \chi_i NPG_i + (1 - \chi_i) X_i. \tag{A.5}$$

χ_i is an indicator function that is equal to 1 if government revenues are used to purchase the goods produced in the economy, and equal to 0 otherwise. Tax revenues, either from sale and/or lump-sum taxation, can be used either to purchase the per-capita amount G_i of the same composite good consumed by private individuals[1] ($\chi_i = 1$), or they can be rebated

[1] This implies that the government in country i chooses consumption $G_{z,i}$ so as to

$$\max G_i = N^{\frac{1}{1-\theta}} \left[\sum_{z=1}^{N} G_{z,i}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1,$$

subject to its budget constraint. Hence, the government's demand for good z will have the same form of the individual demand for good z.

back the consumers ($\chi = 0, X_i = 0$), or simply wasted ($\chi_i = 0$ and $X_i > 0$). Notice that money supply does not enter the government budget constraints: the monetary and the fiscal authorities does not share (A.5) and the monetary authority is truly policy independent.

The solution of this model is briefly sketched here. The first order condition with respect to $C_{z,j,i}$ and $M_{j,i}$, respectively, imply

$$C_{z,j,i} = \left(\frac{P_z}{P}\right)^{-\theta} \frac{\gamma I_{j,i}}{NP}, \tag{A.6}$$

$$M_{j,i} = (1-\gamma) I_{j,i}. \tag{A.7}$$

As usual, the demand for each good is linear in wealth and depends on its relative price with elasticity $-\theta$. The demand for money is also linear in wealth. Let $W \equiv \gamma I/(NP) + \sum_{i=1}^{2} \chi_i G_i$, where $I \equiv I_1 + I_2$, with $I_i = \sum_{j=1}^{N/2} I_{j,i}$. Hence, $\chi_i G_i$ is country i's government demand for goods. The demand facing producer z can be obtained by aggregating individual demand over consumers and governments

$$Y_z^d = \sum_{j=1}^{N/2} \sum_{i=1}^{2} C_{z,j,i} + G_{z,i} = \left(\frac{P_z}{P}\right)^{-\theta} W. \tag{A.8}$$

The price, and therefore output, chosen by producer j in country i is found by maximizing her indirect utility function

$$U_{j,i} = (1-\tau_i) W^{\frac{1}{\theta}} Y_{j,i}^{\frac{\theta-1}{\theta}} - T_i + \frac{\overline{M}_{j,i}}{P} - \left(\frac{d_i}{\beta}\right) Y_{j,i}^{\beta}$$

with respect to the relative price, which gives

$$\frac{P_{j,i}}{P} = \left[\frac{\theta d_i}{(\theta-1)(1-\tau_i)} W^{\beta-1}\right]^{\frac{1}{1+\theta(\beta-1)}}. \tag{A.9}$$

The higher the wealth W and the disutility of effort d_i, the higher the relative price set by producer j in country i.

Suppose the parameters d_i, θ, β are stochastic with variances $\sigma_d, \sigma_\theta, \sigma_\beta$, respectively; for simplicity, we normalize $\sigma_\beta = 1$ and assume that these stochastic variables are independent. We also assume that the d_i have equal mean. In both countries, a fraction $\phi \in (0,1)$ of the goods prices remain unchanged each period, while new prices are chosen for the other $1-\phi$ goods; the probability that any given price will be adjusted in any given period is assumed to be independent of the length of time since the price was changed and independent of what the good's current price may be. This implies that, in any period and in each country, a fraction ϕ of the prices is given from the past and constant; we denote the preset price of the $z-$th good in country i as $\bar{P}_{z,i}$ and the average of such prices as $E\bar{P}_i$. A fraction $1-\phi$ of the prices is set freely after uncertainty and policy are resolved and we denote the price of the $z-$th good in country i $\tilde{P}_{z,i}$; due to the symmetry of the model, all new freely set prices are equal. For simplicity, we set $\phi = 1/2$; then, the price level is

$$P^{1-\theta} = \frac{1}{4}\left[\sum_{i=1}^{2}\left(E\bar{P}_i^{1-\theta} + \tilde{P}_{z,i}^{1-\theta}\right)\right]. \tag{A.10}$$

It is convenient to define

$$\lambda \equiv \frac{1}{2}\left[\left(\frac{E\bar{P}_1}{P}\right)^{1-\theta} + \left(\frac{\tilde{P}_{z,1}}{P}\right)^{1-\theta}\right] \tag{A.11}$$

as country 1's relative price. Notice that country 2's relative price is equal to $2-\lambda$. We define aggregate output in country i as

$$Y_i \equiv \sum_{j=1}^{N/2}\frac{P_{j,i}Y_{j,i}}{P} \tag{A.12}$$

so that $Y_1 = N\lambda W/2$ and $Y_2 = N(2-\lambda)W/2$. Aggregate output in the economy is

$$Y \equiv \sum_{i=1}^{2} Y_i = WN. \tag{A.13}$$

Let a 0 subscript indicate the value at the steady state; we assume $\tau_{i,0} = T_{i,0} = G_{i,0} = 0$. We consider the following fiscal policies in country i:

1. **Supply Side: Reduction in distortionary taxation.** The government uses distortionary taxes $\tau_i > 0$ to finance its budget; the revenues are wasted. An expansionary fiscal policy is a reduction in τ_i: $x_i \equiv -d\tau_i > 0$.

2. **Mercantilist: Production subsidy.** The government uses lump-sum taxes $T_i > 0$ to finance its budget; the revenues are redistributed to the producers via a production transfer $\tau_i < 0$. An expansionary fiscal policy is a reduction in τ_i: $x_i \equiv -d\tau_i > 0$.

3. **Keynesian: Balanced-budget expenditure.** The government raises distortionary taxes $\tau_i > 0$ and spends the revenues to purchase the composite good G_i. An expansionary fiscal policy is an increase in $\tau_i : x_i \equiv d\tau_i > 0$.

Consider fiscal policy 1. In this case, $\chi_i = 0, X_i > 0$. If both countries use fiscal policy 1, it is easy to show that

$$W = \frac{\gamma \bar{M}}{NP} \frac{1}{\left\{1-\gamma+\frac{\gamma}{2}\left[\lambda \tau_1+(2-\lambda)\tau_2\right]\right\}} \tag{A.14}$$

where $\overline{M} \equiv \sum_{j=1}^{N/2} \sum_{i=1}^{2} \overline{M}_{j,i}$.

Consider now fiscal policy 2. In this case, $\chi_i = X_i = G_i = 0, \tau_i < 0, T_i > 0$. Suppose both country 1 and 2 follow fiscal policy 2. We obtain that

$$W = \frac{\gamma \bar{M}}{(1-\gamma)\, NP} \tag{A.15}$$

Consider now fiscal policy 3. In this case, $\chi_i = 1$ and $\tau_i > 0$ as long as $G_i > 0$. Notice that $I_i/P = Y_i(1-\tau_i) + N\bar{M}_{j,i}/(2P)$ and $I_{g,i}/P = Y_i \tau_i$, so that, if both countries follow fiscal policy 3,

$$W = \frac{\gamma \bar{M}}{(1-\gamma)\, NP \left(1-\frac{2-\lambda}{2}\tau_2-\frac{\lambda}{2}\tau_1\right)}. \tag{A.16}$$

We now proceed to find the optimal price for those producers who get a chance to update their prices. Let $m = d\bar{M}/\bar{M}_0, \pi = dP/P_0, \bar{\pi}_{j,i} = d\bar{P}_{j,i}/P_{j,i,0}, y_i = dY_i/Y_{i,0}$ and x_i as described earlier; the log of the optimal price satisfies the following log-linear approximation

$$\tilde{\pi}_{z,i} = (1-\phi\eta)\left[\pi_{z,i} + \frac{\phi\eta}{1-\phi\eta}\bar{\pi}_{z,i}\right], \tag{A.17}$$

where η is the personal discount factor and $\pi_{z,i}$ is the optimal price for the current period only in country i. Intuitively, the newly set price is an average of the price that is optimal in the current period and of the price that is expected to be optimal in future periods. The former depends on the realization of the current shocks; the latter depends on the expected realization of shocks and policies and, thanks to the law of large numbers, is equal to the average of the preset prices already existing in country i. We first find $\bar{\pi}_{z,i}$ that maximizes future expected indirect utility. Consider fiscal policy 1. Under the assumption that $(1-\tau_i)W(\theta-1)\bar{P}_{j,i}^{-\theta}P^{\theta-1}$ and $d_i W^{\beta}\theta\bar{P}_{j,i}^{-\theta\beta-1}P^{\theta\beta}$ are lognormally distributed and after several manipulations, the first order condition with respect to $\tilde{P}_{z,i}$ gives

$$\bar{\pi}_{z,i} = \chi_{0,i} + \bar{e}E\pi + (1-\bar{e})Em + \bar{f}_i Ex_i + \bar{f}_{-i}Ex_{-i} \tag{A.18}$$

where $-i$ stands for $\neq i$ and

$$\chi_{0,i} = \frac{1}{E[1+\theta(\beta-1)]}\left\{E\left[\log d_i + \log\frac{\theta}{\theta-1} + (\beta-1)\left(\log\frac{2\gamma}{N(1-\gamma)}\right)\right] +\right.$$

$$+\frac{1}{2}(Var_{0,i} - Var_{1,i}) + Cov(m,\beta) + Cov(\pi,(\theta-1)(\beta-1))\Big\},$$

$$Var_{0,i} = Var\left[\log\left(d_i W^{\beta}\theta P^{\theta\beta}\right)\right]$$

$$Var_{1,i} = Var\left[\log\left((1-\tau_i)W(\theta-1)P^{\theta-1}\right)\right]$$

$$\bar{e} = \frac{E[1+(\theta-1)(\beta-1)]}{E[1+\theta(\beta-1)]}, \quad \bar{f}_i = \frac{E\left[-1+\frac{\gamma(\beta-1)}{2(1-\gamma)}\right]}{E[1+\theta(\beta-1)]}, \quad \bar{f}_{-i} = \frac{E\left[\frac{\gamma(\beta-1)}{2(1-\gamma)}\right]}{E[1+\theta(\beta-1)]},$$

where $Var_{0,i}, Var_{1,i}$ are constants. Now we find that $\pi_{z,i}$, which is the price that maximize the current period indirect utility. This is given by

$$\pi_{z,i} = \chi_{1,i} e\pi + (1-e)m + f_i x_i + f_{-i}x_{-i} \tag{A.19}$$

with

$$\chi_{1,i} = \frac{1}{1+\theta(\beta-1)}\left\{\log\frac{\theta d_i}{\theta-1} + (\beta-1)\left[\log\frac{\gamma}{N(1-\gamma)}\right]\right\},$$

$$e = \frac{1+(\theta-1)(\beta-1)}{1+\theta(\beta-1)}, \quad f_i = \frac{-1+\frac{\gamma(\beta-1)}{2(1-\gamma)}}{[1+\theta(\beta-1)]}, \quad f_{-i} = \frac{\frac{\gamma(\beta-1)}{2(1-\gamma)}}{[1+\theta(\beta-1)]}.$$

The price level in the economy is an average of pre-set and newly changed prices; log-linearization of (A.10) gives

$$\pi = \frac{1}{4}\left[\sum_{i=1}^{2}(\bar{\pi}_{z,i} + \tilde{\pi}_{z,i})\right].$$

Using (A.17), we can write the price level as

$$\pi = \frac{1}{4}\left[\left(1+\frac{\eta}{2}\right)(\bar{\pi}_1 + \bar{\pi}_2) + \left(1-\frac{\eta}{2}\right)(\pi_1+\pi_2)\right]. \tag{A.20}$$

It is useful to write the price level as a function of monetary and fiscal policies:

$$\pi = \pi_0 + c_i x_i + c_{-i}x_{-i} \tag{A.21}$$

with

$$\pi_0 = \frac{\beta-1}{1+(\theta+1)(\beta-1)}m + \frac{1+\theta(\beta-1)}{2[1+(\theta+1)(\beta-1)]}(\bar{\pi}_{j,i} + \bar{\pi}_{j,-i})$$

$$c_i = c_{-i} = -\frac{1-\gamma\beta}{2(1-\gamma)[1+(\theta+1)(\beta-1)]}.$$

A fiscal expansion in either country, consisting in a reduction of τ and therefore an increase of x, lowers prices as long as c_i is negative, which requires $1 > \gamma\beta$. Output in country i is derived from (A.12) and is given by

$$y_i = \bar{y}_i + b_i(\pi - \pi^e) + a_i x_i + a_{-i}x_{-i} \tag{A.22}$$

with

$$b_i = \frac{1+\theta(\beta-1)}{\beta-1} > 0, \qquad \bar{y}_i = \frac{1}{\beta-1}\log\frac{N(\theta-1)}{2\theta d_i}$$

$$a_i = \frac{1}{2(\beta-1)} + \frac{\theta-1}{4[1+\theta(\beta-1)]} > 0, \qquad a_{-i} = \frac{1}{2(\beta-1)} - \frac{\theta-1}{4[1+\theta(\beta-1)]} > 0.$$

The first term in (A.22) is the natural rate of output; the second term is output effect of surprise inflation. The last two terms capture the effect of fiscal policy in country i and in country $-i$ on the output of country i. The first term in a_i is the effect of fiscal policy on the relative price of the goods produced at home; the second term in a_i is the effect of fiscal policy on the demand of the goods produced at home. Notice that $a_i > a_{-i} > 0$ and the difference between the two coefficients is the effect of fiscal policy in country $-i$ on its relative price. The direct effect on own output of a reduction in distortionary taxation, a_i, is unambiguously positive; the overall effect on own output of a reduction in distortionary taxation, namely $a_i + b_i c_i$, is also unambiguously positive.

Consider now fiscal policy 2. The prices set in advance are given by (A.18) with

$$\bar{f}_i = -\frac{1}{E[1+\theta(\beta-1)]}, \qquad \bar{f}_{-i} = 0.$$

Similarly, the flexible prices in country i are as in (A.19) with

$$f_i = -\frac{1}{1+\theta(\beta-1)}, \qquad f_{-i} = 0.$$

The price level in the economy is still given by (A.21), where the monetary policy variable π_0 is as in fiscal policy 1 and

$$c_i = c_{-i} = -\frac{1-\gamma}{2(1-\gamma)[1+(\theta+1)(\beta-1)]} < 0.$$

A fiscal expansion in either country, namely a larger production subsidy, unambiguously lowers the price level. Notice that the effect on the price level of a fiscal expansion is larger under fiscal policy 1 than under fiscal policy 2, namely $c_i^{FP1} \geq c_i^{FP2}$ as long as $\alpha \geq 0$. Intuitively, the deadweight losses inherent in the redistribution process reduce the impact of fiscal policy on the price level. Output in country i is given by (A.22) with $b_i, \bar{y}_i, a_i, a_{-i}$ as in fiscal policy 1. The overall effect on own output of a fiscal expansion is unambiguously positive, namely $a_i + b_i c_i > 0$.

Consider fiscal policy 3. Pre-set prices in country i are as in (A.18) with

$$\bar{f}_i = \frac{E(\beta+1)}{2E[1+\theta(\beta-1)]}, \qquad \bar{f}_{-i} = \frac{E(\beta-1)}{2E[1+\theta(\beta-1)]}.$$

Flexible prices are as in (A.19) with e as in fiscal policy 1 and

$$f_i = \frac{\beta + 1}{2[1 + \theta(\beta - 1)]} > 0, \quad f_{-i} = \frac{\beta - 1}{2[1 + \theta(\beta - 1)]} > 0.$$

The general price level is (A.21) with π_0 as in fiscal policy 1 and

$$c_i = c_{-i} = \frac{\beta}{2[1 + (\theta + 1)(\beta - 1)]} > 0.$$

An increase in government spending financed with higher distortionary taxes raises the price level. Output in country i is (A.22) with $b_i, \bar{y}_i$ as in fiscal policy 1 and with

$$a_i = -\frac{\theta - 1}{4[1 + \theta(\beta - 1)]} - \frac{1}{2(\beta - 1)} < 0, \quad a_{-i} = \frac{\theta - 1}{4[1 + \theta(\beta - 1)]} - \frac{1}{2(\beta - 1)} < 0.$$

Notice that $0 > a_{-i} > a_i$: higher country i's government spending is more recessionary on country i's output than higher government $-i$'s spending. This is because higher government spending financed by higher distortionary taxation lowers labor supply and lowers aggregate demand. The overall impact on output of higher government spending, $a_i + b_i c_i$, is positive as long as $\theta^2[2\beta^2 - 5\beta + 3] + \theta(2\beta - 3) + \beta > 0$, which is satisfied for θ large and $\beta < 1.5$.

The model studied above is symmetric. In a non-symmetric case or in the case where all the stochastic shocks are country specific, namely θ_i, β_i and d_i, the price and output equations follow similarly with different coefficients for different countries. Hence, in a more general model, $c_i \neq c_{-i}$ and b_i depends on parameters specific to country i.

References

Blanchard, Olivier and Nobu Kiyotaki. 1987. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review,* 77(4), September, 647-666.

Obstfeld, Maurice, and Kenneth Rogoff. 1996. *Foundations of International Macroeconomics.* Cambridge, MA: MIT Press.