Does quantum theory kill time?

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October 24, 2010

If you want to understand the nature of time, you might want to look at our most fundamental physical theories. There are two such: general relativity, and quantum field theory. Any accurate description of the universe — and in particular, of the nature of time — must somehow combine the insights of these two theories.

Some experts (e.g. Earman 2002, Rovelli 1991, Rovelli 2008) argue that a universe that is both relativistic and quantum-theoretic must, of necessity, be an unchanging, essentially timeless universe. But the arguments for this claim often depend on intricacies of the the Hamiltonian formulation of Einstein's field equations.

In this note, I provide a more elementary argument that any quantum theory of spacetime will dispense with the traditional notion of the passage of time. In particular, I prove that quantum theory rules out the possibility of any quantity that one might call "the time interval between two events."

The mathematical fact on which my philosophical argument is based has long been known (see e.g. Pauli 1933), although we give a more concise and transparent proof. Nonetheless, metaphysicians seem to have been ignorant of this fact, or to have judged it unimportant. I hypothesize that metaphysicians ignored these results because they were usually presented as showing that, "time is not an *observable*" — more a problem for an empirical theory of observability than for metaphysics. But that description of the results is an understatement. The results shows that, insofar as quantities are represented by operators, time is not a quantity at all — not even an unobservable quantity.

Assume then that we have a quantum theory whose state space is a vector space H with inner-product, and whose time evolution is represented by a group $\{u_t = e^{ith} : t \in \mathbb{R}\}$ of symmetries of H, where h is an operator that has spectrum bounded from below. In other words, we assume that there is a lower bound on energy.

Suppose now for reductio ad absurdum that for any interval (a, b) of real numbers, there is a subspace $s(a, b)$ of states that come about during that interval. Let $e(a, b)$ to represent the projection onto the subspace $s(a, b)$.

For any state v, applying the time-evolution operator u_t to v evolves the state forward by t (in whatever units of time we are using). Thus, if a state v is in the subspace $s(a, b)$, the evolved state $u_t v$ should be in $s(a + t, b + t)$. But a unitary operator u maps a subspace s onto a subspace s', i.e. $u(s) = s'$, if and only if $u^*(s') = s$. Hence, we should have $u_t e(a, b) = e(a + t, b + t)u_t$ for all a, b, t in $\mathbb{R}$. We are now ready to derive a contradiction from the assumption that there is a quantity called "time," whose value changes.

Lemma (Hegerfeldt 1994)**.** *Suppose that $u_t = e^{ith}$, where h is a half-bounded operator. Let e be a projection onto a subspace, and let $f(t) = \langle u_t v, e u_t v\rangle$. Then either $f(t) \neq 0$ on a dense open set, or $f(t) = 0$ for all t.*

Theorem. *Suppose that there is an assignment $(a, b) \mapsto s(a, b)$ of temporal intervals to subspaces of state space H such that:*

1. *$u_t s(a, b) = s(a + t, b + t)u_t$ for all $t \in \mathbb{R}$, and*

2. *$s(a, b)$ is orthogonal to $s(c, d)$ when (a, b) is disjoint from (c, d).*

Then $s(a, b) = 0$ for all (a, b).

[In the physics literature, this theorem might be phrased as follows: "There is no operator that is canonically conjugate to the energy operator" or more precisely as "there is no system of imprimitivity whose unitary operators generate time evolution."]

Proof. Let v be a vector in $s(a, b)$. We will show that $v = 0$, in particular, $s(a, b)$ contains no unit vectors. Consider the function defined by

$$f(t) = \langle u_t v, e(a, b)u_t v\rangle = \langle v, e(a + t, b + t)v\rangle, \qquad (t \in \mathbb{R}).$$

Clearly f satisfies the hypotheses of Hegerfeldt's lemma. Furthermore, for all $t > |b - a|$,

$$f(t) = \langle v, e(a + t, b + t)v\rangle = \langle e(a, b)v, e(a + t, b + t)v\rangle = 0,$$

since the subspaces $s(a, b)$ and $s(a + t, b + t)$ are orthogonal. In particular, $f(t) = 0$ on an open set, and Hegerfeldt's lemma entails that $f(t) = 0$ for all $t \in \mathbb{R}$. Thus,

$$0 = f(0) = \langle v, e(a, b)v\rangle = \langle v, v\rangle,$$

from which it follows that $v = 0$. □

Some have already grappled with the implications of this result (see Hilgevoord 2001). A common response is to claim that time *is* a quantity in quantum theory, but that it is represented by a parameter (c-number) rather than by an operator. But that distinction is merely verbal, and does nothing to help us understand the special role of time in quantum theory.[1] What is the difference between quantities that can be represented by operators, and those — such as 'amount of time' — that cannot? And why is time the only such parametric quantity? What is special about time?

References

[1] J. Earman, "Thoroughly modern McTaggart" Philosopher's Imprint 2, 1–28 (2002)

[2] G. Hegerfeldt, "Causality problems for Fermi's two-atom system" Physical Review Letters 72, 596–99 (1994)

[3] J. Hilgevoord, "Time in quantum mechanics" `http://philsci-archive.pitt.edu/368` (2001)

[4] W. Pauli "Die allgemeine Prinzipien der Wellenmechanik", in *Handbuch der Physik*, 83–272 (1933)

[5] C. Rovelli, "Time in quantum gravity: an hypothesis," Physical Review D 43, 442–456 (1991)

[6] C. Rovelli, "Forget time" Essay for FXQI contest (2008)

[1]Perhaps we said too much: there are good arguments that we should expect time to emerge as a parameter (a classical, superselected quantity) because of decoherence processess (see Zeh 2008). But these arguments indicate that time belongs more to the realm of appearances than to the realm of being.

[7] H.-D. Zeh, "Time in quantum theory" in *Compendium of Quantum Physics - Concepts, Experiments, History and Philosophy*, ed by F. Weinert et al Springer (2008)