Risk and Liquidity in a System Context*

Hyun Song Shin
Princeton University
Bendheim Center for Finance
hsshin@princeton.edu

Final Version, February 2008

Abstract

This paper explores the pricing of debt in a financial system where the assets that borrowers hold to meet their obligations include claims against other borrowers. Assessing financial claims in a system context captures features that are missing in a partial equilibrium setting, such as liquidity spillovers across financial institutions resulting from expansions and contractions of balance sheets. Aggregate liquidity can be seen as the rate of growth of financial sector balance sheets.

*I am grateful to Gara Minguez Afonso, Tobias Adrian, Claudio Borio, Mauro Grande, Dick Herring, Stephen Morris and Kostas Tsatsaronis for encouragement and advice. I thank S. Viswanathan and a referee for many constructive suggestions. Earlier versions of this paper were presented at the BIS conference on "Accounting, Risk Management and Prudential Regulation", and the NAS-FRBNY conference on systemic risk. This research forms part of the ESRC research programme on *World Economy and Finance.*

1 Introduction

Many assets in the financial system are claims against other borrowers. As such, the value of such assets fluctuate with the strength of the borrowers' balance sheets. When the web of claims and obligations link financial entities together into a tightly knit system, the relative values of liabilities and assets (and hence net worth), the availability of credit, and asset prices are interrelated and fluctuate together. We may also expect feedback elements that serve to magnify the responses to shocks. Balance sheet changes will affect asset prices and asset price changes will affect balance sheets. The loop thus created may generate amplified responses to any shocks to the financial system.

In a financial system with interlinked claims and obligations, one party's obligation is another party's asset. When calculating the equity value of the financial system as a whole, such claims and obligations cancel out. What remains as the equity value of the financial system as a whole is the marked-to-market value of the "fundamental" assets - assets that are not the obligation of some other party. The larger is the value of fundamental assets, the larger is the equity value of the financial system as a whole, and the stronger is the average balance sheet in the system. In this sense, an increase in the value of fundamental assets is a rising tide that lifts all boats.

As a case in point, consider the following passage from a commentary in the *Wall Street Journal*.[1]

> While many believe that irresponsible borrowing is creating a bubble in housing, this is not necessarily true. At the end of 2004,

[1] "Mr. Greenspan's Cappuccino" Commentary by Brian S. Wesbury, *Wall Street Journal,* May 31, 2005. The title makes reference to Alan Greenspan's comments on the "froth" in the U.S. housing market.

> U.S. households owned \$17.2 trillion in housing assets, an increase of 18.1% (or \$2.6 trillion) from the third quarter of 2003. Over the same five quarters, mortgage debt (including home equity lines) rose \$1.1 trillion to \$7.5 trillion. The result: a \$1.5 trillion increase in net housing equity over the past 15 months.

The author minimizes the dangers from the \$1.1 trillion increase in debt by appealing to the marked-to-market value of housing equity. By valuing the entire housing stock at the current marginal transaction price, the marked-to-market value of housing may be a poor indicator of the potential liquidation value when a substantial chunk of the housing stock is put up for sale. However, the marked-to-market value is the relevant measure when assessing the market price of claims (such as mortgages) backed by the housing stock. The effects are then transmitted further up the chain to mortgage-backed securities, and then to collateralized debt obligations (CDOs), and then to claims against financial institutions that hold such CDOs. At each step in the chain, obligations are backed by claims further down the chain.

In a tightly-knit financial system, externalities transmitted through balance sheets are unavoidable and have far-reaching consequences. A transaction in the market affects more than the parties directly involved in the transaction itself, since the price determined in the transaction is used to price other assets and obligations. As such, the transaction has a spillover effect on the balance sheets of other entities in the financial system.[2]

The externalities cut both ways, however. Just as house buyers paying exorbitant prices exert positive externalities that buoy up others' balance sheets, the distressed selling by defaulting home owners exert negative ex-

[2]Plantin, Sapra and Shin (2005) examine the spillover effects of this kind associated with wider adoption of fair value (mark-to-market) accounting rules.

ternalities that undermine others' balance sheets. In this sense, asset price booms fuelled by lax credit conditions and slumps fuelled by financial distress can be seen as mirror images of each other. The common thread is the feedback from asset price changes to changes in the strength of balance sheets, and the spillover effects across market participants. One of the main tasks of the paper is provide a unifying framework that can accommodate both types of phenomena, and to chart the propagation mechanisms both "on the way up" and "on the way down".

I begin by outlining a framework that can be used to assess the value of claims and obligations in a system of interlocking balance sheets. The basic problem can be posed in the following way. The marked-to-market value of my claim against A depends on A's creditworthiness, and so depends on the value of A's claims against B, C, etc. However, B or C may have a claim against me, and so we are back full circle. The task of valuing claims in a financial system thus entails solving for a consistent set of prices - that is, solving a fixed point problem.

I show that such a problem has a well-defined solution, and the value of all claims can be priced uniquely as a function of the underlying parameters of the financial system - the level and seniority profile of debt, the structure of balance sheet interconnections, and (crucially) the current marginal transaction price of fundamental assets. When the level of debt is treated as a choice variable, it is possible to undertake comparative statics exercises on the externalities that are transmitted through the financial system. I do this both "on the way up" and "on the way down" by showing how the framework can be used to study lending booms and market crashes.

I draw on several strands in the literature. The formal apparatus owes much to the literature on lattice-theoretic ideas made popular in economics by Topkis (1978) and Milgrom and Roberts (1990, 1994). Eisenberg and Noe (2001) showed how tools from lattice theory can be applied to solve an allocation problem in bankruptcy, and the framework reported here builds on their results.[3]

To the extent that balance sheets serve as the conduit for the transmission of shocks, my framework is related to the large literature on the collateral role of assets in amplifying shocks to the financial system. The research to date has distinguished two distinct channels. The first is the increased credit that operates through the *borrower's* balance sheet, where increased lending comes from the greater creditworthiness of the borrower (Bernanke and Gertler (1989), Kiyotaki and Moore (1998, 2001)). The second is the channel that operates through the *banks'* balance sheets, either through the liquidity structure of the banks' balance sheets (Bernanke and Blinder (1988), Kashyap and Stein (2000)), or the cushioning effect of the banks' capital (Van den Heuvel (2002)). All these features make an appearance in my framework.

The results reported here are also related to the developing theoretical literature on the role of liquidity in asset pricing (Acharya and Pedersen (2005), Brunnermeier and Pedersen (2005a, 2005b), Morris and Shin (2004)). Although my framework has little to contribute to the study of market microstructure issues, the common thread is the relationship between funding conditions and the resulting market prices of assets.

The theme of financial distress examined here is also closely related to the literature on liquidity drains that deal with events such as the stock market crash of 1987 and the LTCM crisis in the summer of 1998. Gennotte and Leland (1990) and Geanakoplos (2003) provide analyses that are based on competitive equilibrium. The framework to be presented below could be seen

[3] Boss, Summer and Thurner (2005) have used the Eisenberg and Noe approach to assess the potential for contagious defaults in the Austrian banking sector.

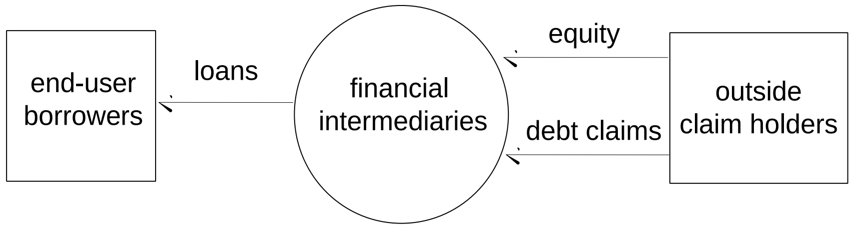

Figure 1: Stylized Financial System

as a reduced-form representation of a fully-fledged competitive analysis. It provides a simplified representation that draws attention to the key balance sheet spillovers without having to flesh out the full competitive economy. Provided that the shortcomings of such a reduced-form representations are borne in mind, the simplicity of the framework can aid our understanding of the balance sheet propagation effects. I begin by outlining the formal apparatus. Applications to liquidity drains and lending booms then follow.

2 Model

There are n leveraged financial intermediaries that we call "banks" for convenience but in principle, they encompass intermediaries such as broker dealers and other entities involved in the securitization process. The banks are indexed by $i \in \{1, \cdots, n\}$. In addition, there is a non-leveraged sector which we label by $n + 1$. The non-leveraged sector has equity and debt claims against the banks. In turn, the banks have claims against end-users of credit such as firms and households. The stylized financial system is depicted in figure 1.

There are two dates, 0 and 1. Loans are granted at date 0 and repaid at date 1. Bank $i \in \{1, \cdots, n\}$ has the following balance sheet in notional values.

Assets	Liabilities
$\bar{y}_i$	$\bar{e}_i$
$\sum_{j=1}^{n} \bar{x}_j \pi_{ji}$	$\bar{x}_i$

Here, $\bar{y}_i$ is the face value of loans to end-users. In general, we use the upper bar notation to denote face values. $\bar{x}_i$ is the face value of bank i's debt and π_{ji} is the proportion of bank j's debt held by i. Hence, $\sum_{j=1}^{n} \bar{x}_j \pi_{ji}$ is the face value of the claim that bank i has against other leveraged institutions in the financial system. Finally, $\bar{e}_i$ is the book value of bank i's equity, defined as the residual item that sets the two sides of the balance sheet equal. The balance sheet identity in terms of face values is thus

$$\bar{y}_i + \sum_{j=1}^{n} \bar{x}_j \pi_{ji} = \bar{x}_i + \bar{e}_i \tag{1}$$

We impose one regularity condition at this stage. We assume that the non-leveraged sector $n+1$ holds a piece of every bank's debt. In other words, $\pi_{i,n+1} > 0$ for all i. This regularity condition is stronger than necessary, but simplifies the argument. I return to this issue shortly.

2.1 Credit Risk

The banks face credit risk in the sense that they are repaid less than face value of the loan in some states of the world. There is a random variable Y that serves as the common factor in the credit risk, and determines the realized values of all loans to the end-users. Y has support in the unit interval $[0, 1]$. We will use the hat notation "^" to denote realized values at date 1. So, we denote by $\hat{y}_i$ the realized repayment on bank i's loans at date 1. Assume that

$$\hat{y}_i = \bar{y}_i \hat{Y}$$

Denote by $\hat{x}_i$ the realized repayment by bank i on its debt. All debt is of equal seniority, so that if bank i defaults on a part of its debt, then its creditors receive payments in proportion to face values. In other words, if $\hat{x}_i < \bar{x}_i$, then bank j receives payment $x_i \pi_{ij}$ from bank i.

Given the priority of debt over equity, the realized values of debt satisfy:

$$\begin{aligned}
\hat{x}_1 &= \min\left(\hat{a}_1\left(\hat{x}\right), \bar{x}_1\right) \\
\hat{x}_2 &= \min\left(\hat{a}_2\left(\hat{x}\right), \bar{x}_2\right) \\
&\vdots \\
\hat{x}_n &= \min\left(\hat{a}_n\left(\hat{x}\right), \bar{x}_n\right)
\end{aligned}$$

where $\hat{x} = (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n)$, and $\hat{a}_i$ is the realized value of bank i's assets:

$$\hat{a}_i = \hat{y}_i + \sum_j \hat{x}_j \pi_{ji}$$

So, there is non-decreasing function $F(.)$ that maps a repayment vector $\hat{x}$ to the repayment vector that results when others repay according to $\hat{x}$. The ex post allocation is a fixed point of $F(.)$. Eisenberg and Noe (2001) have shown that there is a unique solution for the realized values $(\hat{x}_1, \cdots, \hat{x}_n)$ that respects priority of debt over equity, and proportional settlement. We summarize the argument for uniqueness here, and refer the reader to Eisenberg and Noe (2001) for the full proof. We can define a complete lattice as the ordered set $(X, \leq)$ where

$$X \equiv [0, \bar{x}_1] \times [0, \bar{x}_2] \times \cdots \times [0, \bar{x}_n]$$

and the ordering $\leq$ is given by the usual component-wise order. Since $F(.)$ is non-decreasing, it has a largest and smallest fixed point by Tarski's fixed point theorem. Hence, if there is more than one fixed point, there are fixed points $\hat{x}$ and $\hat{x}'$ such that $\hat{x} \leq \hat{x}'$ and $\hat{x}_i < \hat{x}'_i$ for some i. From our regularity condition that the non-leveraged sector holds a piece of every bank's debt, the asset value of the non-leveraged sector is strictly higher at $\hat{x}'$ than at $\hat{x}$. The equity value of all banks are (weakly) higher at $\hat{x}'$ than at $\hat{x}$. Hence, the equity value of the whole financial system is strictly higher at $\hat{x}'$ than at $\hat{x}$. But the equity value of the system is the total value of realized repayments, $\sum_i \hat{y}_i$, which must be invariant across any fixed point of $F(.)$. Hence, we have a contradiction if we suppose that there is more than one fixed point of $F(.)$. This proves uniqueness of the fixed point of F.

The comparative statics result on lattices due to Milgrom and Roberts (1994, Theorem 3) implies that the fixed point $\hat{x}$ is weakly increasing in $\hat{y}_j$ for any j. In our case, the realized values $\{\hat{y}_i\}$ are increasing functions of the realized value of the common factor Y. Hence, the realized value of assets of bank i is a well defined increasing function of the common factor Y. We summarize this feature of our model as follows.

Lemma 1 *For each bank i, the realized value of its assets $\hat{a}_i$ is an increasing function of Y.*

3 Balance Sheet Management

We now turn to the core of our paper - the balance sheet management of the banks at the ex ante stage. We reserve the notation y_i (without any hats or bars) to denote the expected value at date 0 of the repayment $\hat{y}_i$. Similarly, denote by x_i the expected value at date 0 of the realized claim $\hat{x}_i$, and so on. We refer to these ex ante values as "market values", or "marked-to-market values". The total marked-to-market assets of bank i is

$$a_i = y_i + \sum_j x_j \pi_{ji}$$

Total marked-to-market value of liabilities is

$$e_i + x_i$$

Denote the leverage of bank i as λ_i, where leverage is defined as the ratio of total assets to equity. That is

$$\frac{a_i}{a_i - x_i} = \lambda_i \tag{2}$$

Then, for $\delta_i = 1 - \frac{1}{\lambda_i}$, we have

$$\begin{aligned} x_i &= \delta_i \left(y_i + \sum_j x_j \pi_{ji} \right) \\ &= \delta_i y_i + \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix} \begin{bmatrix} \delta_i \pi_{1i} \\ \vdots \\ \delta_i \pi_{ni} \end{bmatrix} \end{aligned} \tag{3}$$

Let $x = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}$, $y = \begin{bmatrix} y_1 & \cdots & y_n \end{bmatrix}$, and

$$\Delta = \begin{bmatrix} \delta_1 & & \\ & \ddots & \\ & & \delta_n \end{bmatrix} \tag{4}$$

Then we can write (3) in vector form as:

$$x = y\Delta + x\Pi\Delta$$

Solving for x,

$$\begin{aligned} x &= y\Delta \left(I - \Pi\Delta\right)^{-1} \\ &= y\Delta \left(I + \Pi\Delta + (\Pi\Delta)^2 + (\Pi\Delta)^3 + \cdots\right) \end{aligned} \tag{5}$$

The matrix $\Pi\Delta$ is given by

$$\Pi\Delta = \begin{bmatrix} 0 & \delta_2\pi_{12} & \cdots & \delta_n\pi_{1n} \\ \delta_1\pi_{21} & 0 & & \delta_n\pi_{2n} \\ \vdots & & \ddots & \vdots \\ \delta_1\pi_{n1} & \delta_2\pi_{n2} & \cdots & 0 \end{bmatrix} \tag{6}$$

The infinite series in (5) converges since the rows of $\Pi\Delta$ sum to a number strictly less than 1. Hence, the inverse $(I - \Pi\Delta)^{-1}$ is well-defined.

3.1 Value at Risk

For bank i its *value at risk* at confidence level c relative to the face value of its assets $\bar{a}_i$, is defined as the smallest non-negative number V such that

$$\Pr\left(\hat{a}_i < \bar{a}_i - V\right) \leq 1 - c$$

where $\hat{a}_i$ is the realized value of assets of bank i at date 1. In other words, the value at risk V can be seen as the "approximately worst case" loss that the bank may suffer, where "approximately worst case" is defined so that anything worse than this approximately worst case happens with probability less than some benchmark $1 - c$.

The concept of value at risk has been adopted widely among financial institutions in their risk management practices. The annual reports and regulatory filings of major banks devote a substantial part to a discussion of their value at risk estimates. Moreover, value at risk has been enshrined in the regulatory framework for capital since the 1996 Market Risk Amendment of the Basel Accord, and in the Basel II regulations.

The underlying microeconomics of the contracting problem is of independent theoretical interest. Indeed, whether risk management rules based on the value at risk concept can be obtained as the solution to an optimal contracting problem is an important theoretical question. Important as this question is, I leave it to a separate analysis. Here, I will simply recognize the widespread use of the value at risk notion in risk management practices and investigate the consequences of such actions.

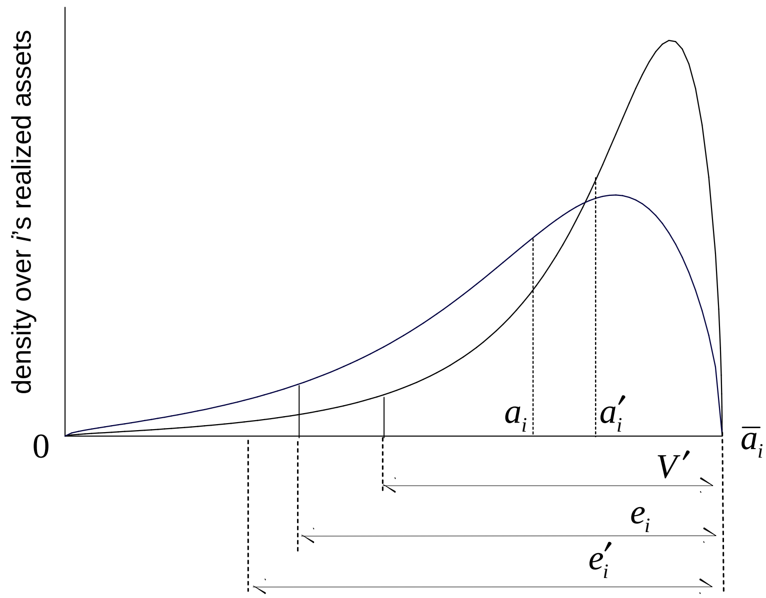

Figure 2: Economic Capital and Market Equity

3.2 Determination of Leverage

Risk management is intimately tied to the leverage of the bank. I assume that a bank aims to adjust its balance sheet so that its market equity e_i is set equal to its value at risk. The term "economic capital" is sometimes used interchangeably with the bank's value at risk. We will follow this convention and define the *economic capital* of bank i as its value at risk. The following feature of our model is then immediate.

Lemma 2 *If the density of Y shifts to the right in the sense of first-degree stochastic dominance, market equity exceeds economic capital at all banks.*

Figure 2 illustrates the argument for lemma 2. Initially, the probability density over realized assets $\hat{a}_i$ is such that the market value of assets is a_i, and the value at risk is given by initial market equity e_i. When there is a first degree stochastic dominance shift in the density of Y, there is a first degree stochastic dominance shift in the density over possible asset value realizations $\hat{a}_i$. Figure 2 illustrates the shift. The new market value of assets is given by a_i', and economic capital falls to the bank's value at risk V', while the market value of equity rises to e'. We have $V' < e_i'$, since the area under the density to the left of $\bar{a}_i - e_i$ under the old density must be equal to the area to the left of $\bar{a}_i - V'$ under the new density. Thus, market equity e' after the shift exceeds economic capital, given by value at risk V'.

3.3 Recovering Face Values

From equation (5), the vector of market values of debt x satisfies

$$x = y\Delta \left(I + \Pi\Delta + (\Pi\Delta)^2 + (\Pi\Delta)^3 + \cdots \right) \tag{7}$$

From the full set of market values (y, x), we can recover the face value $\bar{x}_i$ of each bank's debt. Using the option pricing insight of Merton (1974), note that x_i is the price of portfolio consisting of a cash holding of $\bar{x}_i$ and a short put written on the assets of bank i with strike price $\bar{x}_i$. From Breeden and Litzenberger (1978), we know that the second derivative of the option price with respect to its strike price gives the state price density over realizations. Hence, for given a_i, x_i is an increasing, concave function of $\bar{x}_i$. From the full set of market prices (y, x), we have a well-defined profile of marked-to-marked asset values $a = (a_1, \cdots, a_n)$. Each pair (x_i, a_i) then defines uniquely the face value of debt $\bar{x}_i$ from the fact that x_i is monotonic in $\bar{x}_i$. Figure 3 illustrates the argument. The vertical axis plots the market value x_i and the horizontal axis plots the face value $\bar{x}_i$. Letting $x_i = h(\bar{x}_i; a_i)$, we can recover $\bar{x}_i$ from $\bar{x}_i = h^{-1}(x_i; a_i)$.

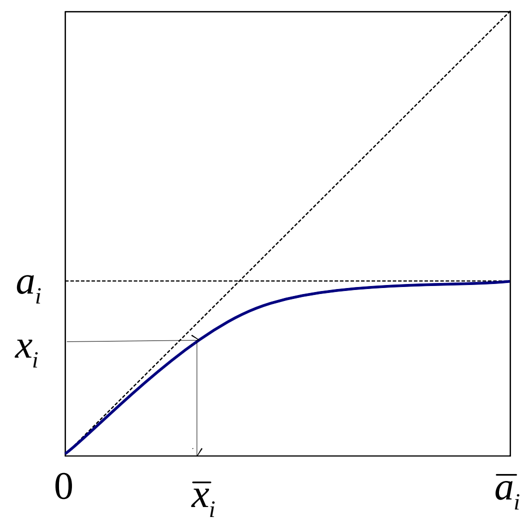

Figure 3: Determination of $\bar{x}_i$ given x_i and a_i

3.4 Comparative Statics of Debt and Leverage

Let us now see the consequences of banks aiming to set its *actual capital* (its market equity) to be equal to its *economic capital* (its value at risk). Banks adjust their leverage by adjusting the notional values $\{\bar{y}_i, \bar{x}_i\}$. If current leverage is too low relative to desired leverage, then the bank will increase the face value of debt in order to raise leverage up to the desired level. Conversely, if current leverage is too high, the bank will reduce leverage by reducing the face value of debt.[4]

Let us examine the comparative statics of debt and leverage when there is a first-degree stochastic dominance shift in the realizations of the fundamentals Y. We use the prime ($'$) notation to denote market values after the shift in the density but before any adjustment of face values. Hence, a_i' is the

[4] I do not consider other ways of adjusting leverage - such as equity buy-backs using debt. The reason for this restriction arises from the interpretation of debt as very short term repurchase agreements (repos) that forms the bulk of broker dealer balance sheets. Equity buybacks are not usually funded in this way.

market value of assets of bank i after the shift in the density of Y, but at the initial profile of face values, and similarly for e_i', x_i' and so on. Following the first degree stochastic dominance shift in the density of Y the value at risk (VaR) of each bank falls. Meanwhile, the equity value of all banks increase due to the shift. If we denote by e_i the initial equity of bank i and denote by e_i' the equity after the shift, we have

$$e_i' > e_i > \mathrm{VaR}'$$

Hence,

$$\frac{a_i'}{e_i'} < \frac{a_i'}{\mathrm{VaR}'} \equiv \lambda_i^*$$

so that banks wish to adjust their leverage upward to λ_i^*. Denote by Δ^* the diagonal matrix whose ith diagonal entry is $1 - 1/\lambda_i^*$. Denote by x^* the profile of debt values defined as

$$x^* \equiv v\bar{y}\Delta^* \left(I + \Pi\Delta^* + (\Pi\Delta^*)^2 + (\Pi\Delta^*)^3 + \cdots \right)$$

where $v = E(Y)$, the market price of a loan with one dollar face value and $\bar{y} = (\bar{y}_1, \cdots, \bar{y}_n)$. In other words, x^* is the profile of debt values when the fundamental asset values are given by the initial profile $v\bar{y}$, while the leverage ratios are given by $\{\lambda_i^*\}$. Then, from (7) and the fact that $\Delta^* > \Delta'$, we have

$$\begin{aligned} x^* &= v\bar{y}\Delta^* \left(I + \Pi\Delta^* + (\Pi\Delta^*)^2 + (\Pi\Delta^*)^3 + \cdots \right) \\ &> v\bar{y}\Delta' \Big(I + \Pi\Delta' + (\Pi\Delta')^2 + (\Pi\Delta')^3 + \cdots \Big) \\ &= x' \end{aligned} \tag{8}$$

It should be stressed that x^* is not necessarily an equilibrium profile of debt values, since changes in the debt profile will have consequences for the liabilities matrix Π as well as the total supply of credit to end-users. Instead, x^*

should be seen as a comparative statics shift in the debt profile to shifts in desired leverage.

Let us examine the comparative statics of credit to the end-users such as firms and households as implied by x^*. Our measure of the credit to end-users is $\sum_i \bar{y}_i$, the total face value of loans granted to the end-user sector. Our result is that when economic conditions become more benign in the sense that credit risk declines, there is a lending boom. More formally, we have the following result.

Proposition 3 *The total face value of credit to end-users implied by x^* is larger than that implied by x'.*

Proof. Consider the aggregate balance sheet of the banking sector as a whole (for banks 1 to n) after the shift in the density of Y, but before any adjustment of face values. The interbank claims and obligations cancel out in the aggregation. The balance sheet of the banking sector as a whole is given by

Assets	Liabilities
$v\sum_{i=1}^n \bar{y}_i$	$\sum_{i=1}^n e_i'$ $\sum_{i=1}^n \pi_{i,n+1} x_i'$

(9)

In contrast, the desired balance sheet implied by the desired leverage ratios $\{\lambda_i^*\}$ is given by

Assets	Liabilities
$v\sum_{i=1}^n \bar{y}_i^*$	$\sum_{i=1}^n e_i'$ $\sum_{i=1}^n \pi_{i,n+1} x_i^*$

(10)

The equity value in (10) remains unchanged from (9), since any adjustment of the balance sheet will affect the value of assets and debt equally. However, from (8), we have

$$\sum_{i=1}^n \pi_{i,n+1} x_i^* > \sum_{i=1}^n \pi_{i,n+1} x_i'$$

Therefore, comparing (9) and (10), we have

$$\sum_{i=1}^{n} \bar{y}_i^* > \sum_{i=1}^{n} \bar{y}_i$$

so that the desired supply of total credit by banks to end-users is higher. ■

3.5 Liquidity

The difference between the debt profile x^* and the debt values x' resulting from the shift in the density of Y could be seen as the additional willingness to lend. Thus, a natural definition of liquidity in the current context is in terms of the difference:

$$x^* - x'$$

When this difference is positive and large, banks wish to expand their balance sheet by taking on more debt, and supplying credit both to end-users and to other banks. Conversely, when this difference is small, liquidity is low.

The case of particularly topical interest is when the difference $x^* - x'$ is *negative*, so that banks wish to contract their balance sheets. This case would be consistent with a credit squeeze such as the one experienced in the interbank markets during the credit crisis of 2007. In our model, banks find that their equity is too low to support the current size of their balance sheets, and wish to contract their balance sheets to a size that is small enough to be supported by current equity. Equity is large enough to support the balance sheet once the face value of assets can be reduced sufficiently that the bank's value at risk is once again small enough to be met by current market equity.

4 Example of Credit Boom and Bust

So far, we have treated the probability of default as being exogenous. However, if we introduce an additional element into the model so that the probability of default depends on the supply of credit to end-users, then there is the potential for a feedback effect where greater supply of credit raises asset prices, which then induces a greater supply of credit, and so on.

In what follows, I present an informal example, motivated by the subprime mortgage market. The example rests on the following key assumption.

Assumption 4. An increase in aggregate credit $\sum_{i=1}^{n} \bar{y}_i$ to end-users shifts the density of Y to the right in the sense of first degree stochastic dominance.

This assumption is motivated by the role of house prices in household mortgage finance. It rests on the consideration that when credit to end-users grows, borrowers who were previously unable to buy houses gain access to credit and are able to purchase houses. If they face a fixed upward-sloping supply curve for housing, then an increase in total credit leads to an increase in the transations price for houses.

An increase in house prices will reduce credit risk in two ways. First, the housing equity of existing borrowers increases, reducing the probability of default. Second, the higher value of collateral reduces the loss given default. For both reasons, the assumption that credit risk declines in the aggregate credit $\sum_{i=1}^{n} \bar{y}_i$ is a natural one.

Denote by p the probability of default implied by market prices of loans. That is,

$$p \equiv 1 - v$$

Figure 4 charts two relationships between the probability of default and total credit to end-users given by $\sum_{i=1}^{n} \bar{y}_i$. The first (the dark line) is the relationship implied by Proposition 3, which states that total credit is increasing as the probability of default declines. The second relationship is the one

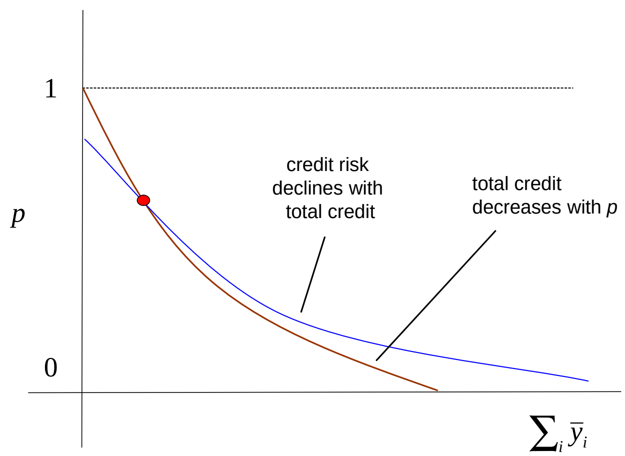

Figure 4: Probability of default and credit to end-users

assumed above in Assumption 4 that the probability of default declines with total credit, via its effect on house prices. The initial point is where the two curves intersect.

Figure 5 illustrates the feedback effect during a credit boom following an exogenous shock to house prices that initially lowers the probability of default. The initial change is for p to decline. However, at the lower value of p, the aggregate credit increases, in line with Proposition 3. Then, the probability of default falls further, which induces a further increase in credit, and so on. The new settling point is to the right, associated with a much larger stock of credit as well as a lower probability of default.

However, the feedback mechanism works in reverse, too. Figure 6 illustrates a housing bust scenario. Starting from a benign environment of low probability of default and large stock of outstanding loans, suppose that a negative shock hits the housing market, pushing up the probability of default. This move corresponds to the first arrow facing up from the initial

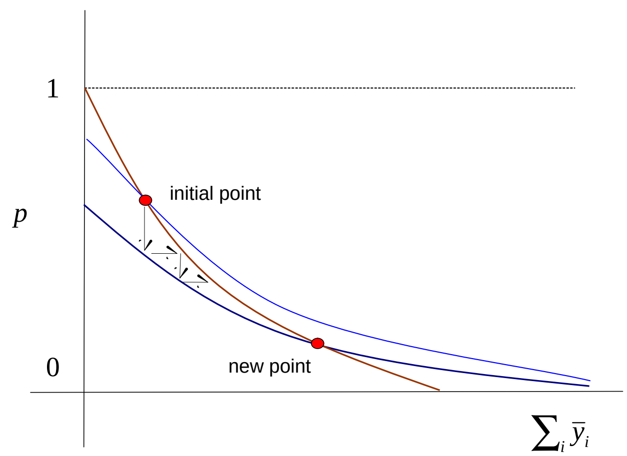

Figure 5: Housing Boom Fuelled by Credit

point. However, at the higher probability of default, the desired supply of credit to the end-users is lower. This leads to a contraction of credit (associated with the first left-pointing arrow). But then, a lower supply of credit raises the probability of default, and so on. In each round of the iteration, the probability of default increases and credit is lower. The new settling point is to the upper lieft, associated with a large probability of default and lower credit.

Our example reinforces the importance placed on asset prices in the recent theoretical literature on banking and financial crises (see Allen and Gale (2004), Cifuentes, et al. (2005), Gorton and Huang (2003) and Schnabel and Shin (2004)), where the price repercussions of asset sales have important adverse welfare consequences. Similarly, the inefficient liquidation of long assets in Diamond and Rajan (2005) has an analogous effect. The shortage of aggregate liquidity that such liquidations bring about can generate contagious failures in the banking system.

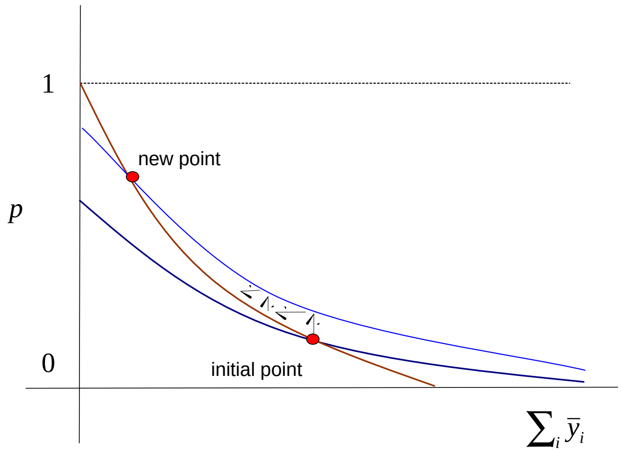

Figure 6: Housing Bust

5 Concluding Remarks

The focus of this paper has been on the liquidity of the financial system as a whole, where "liquidity" refers to the funding conditions for current and potential borrowers. For existing borrowers, rising asset prices strengthen their balance sheets making them more creditworthy. For potential borrowers, the stronger balance sheets of financial intermediaries play to their advantage, since these financial intermediaries are more willing to extend new credit, and extend credit on easier terms. The simplicity of the framework holds some promise for several lines of research.

- The framework is easily extended to deal with claims of differing seniority classes, and is well-suited to pricing complex debt instruments such as collateralized debt obligations (CDOs), since CDOs are obligations that are backed by claims on others.

- Our framework is well-suited to quantitative analysis of risks (such as value at risk calculations) that take account of endogenous changes in asset prices and the feedback effects that result.

There are also broader conceptual issues. A number of trends have served to sharpen the effects outlined in the paper. Rajan (2005) notes how financial development has been made possible by greater use of "arms length" relationships in financial contracts, but such arms length relationships also imply greater delegation. The shortened decision horizons entailed by delegation increase the potency of market prices in causing feedback (Shin (2005)).

The key effect is the feedback between strength of balance sheets (as implied by x) and the level of debt (as given by $\bar{x}$). Strong balance sheets induce banks to increase their lending. In turn, increased lending raises asset prices, leading to stronger balance sheets.

The reason why banks would increase their lending in the face of stronger balance sheets would be intimately tied to the short-term incentives facing the banks' management. Stronger balance sheets imply a larger marked-to-market equity for the bank. The more conscious is a bank's management to shareholder returns, the greater will be the incentive to react to the erosion of leverage by trying to restore leverage. The trend in recent years towards improved corporate governance through greater transparency, greater accountability to shareholders and greater use of incentive schemes tied to the share price will all strengthen the motives of the management to restore leverage.

The feedback from increased debt (given by $\bar{x}$) to stronger balance sheets (given by x) has to do with how quickly the increased debt translates into higher asset prices and how quickly the increase in asset prices is reflected in visibly stronger balance sheets. Here, marking to market is key. For the United States, the prevalence of mortgage-backed instruments as the prime source of finance for the property sector means that this pre-condition has been in place for some time. For those economies that rely on bank lending, the accounting regime will be important. When assets and liabilities are marked to market continuously, the accounting numbers mirror the underlying market prices immediately.

Accounting numbers serve an important certification role in financial markets. They are audited numbers that carry quasi-legal connotations in bringing the management to account. As such, accounting numbers serve as a justification for actions. If decisions are made not only because you believe that the underlying fundamentals are right, but because the accounts give you the external validation to take such decisions, then the accounting numbers take on great significance. Plantin, Sapra and Shin (2005) discuss these and other dimensions of the tradeoff between mark-to-market accounting and historical cost accounting.

It can be argued that mark-to-market accounting has already had a far-reaching impact on the conduct of market participants through those institutions that deal mainly with tradeable securities, such as hedge funds and the proprietary trading desks of investment banks. However, even these developments will pale into insignificance to the potential impact of the marking to market of loans and other previously illiquid assets.

Accounting numbers, such as Return on Equity (ROE) have traditionally made reference to book equity (accumulated value of past profit) rather than the market price of equity claims. However, this distinction is becoming increasingly less relevant. The recent trend (as prescribed by the new accounting standards) is to feed any capital gains to the profit and loss account (the income statement) so that capital gains and losses will be reflected immediately on the balance sheet. The potential for feedback is thus enhanced, and the effects outlined in the paper take on greater significance.

References

Adrian, T. and H. S. Shin (2007) "Liquidity and Leverage" working paper, Federal Reserve Bank of New York and Princeton University

Allen, F. and D. Gale (2004) "Financial Intermediaries and Markets," *Econometrica* 72, 1023-1061.

Acharya, Viral and Lasse Pedersen (2005) "Asset Pricing with Liquidity Risk" *Journal of Financial Economics* 77, 375-410.

Bernanke, B. and A. Blinder (1988) "Credit, Money and Aggregate Demand" *American Economic Review*, 78, 435-39.

Bernanke, B. and M. Gertler (1989) "Agency Costs, Net Worth, and Business Fluctuations" *American Economic Review*, 79, 14 - 31.

Boss, M., M. Summer and S. Thurner (2004) "Contagion Flow through Banking Networks" Lecture Notes in Computer Science, Volume 3038, Computational Science - ICCS 2004 4th International Conference, Krakw, Poland, Proceedings, Part III , Editors: Marian Bubak, Geert Dick van Albada, Peter M. A. Sloot, et al. 2004, Springer-Verlag. Heidelberg.

Breeden, Douglas and Robert Litzenberger (1978) "Prices of State-Contingent Claims Implicit in Option Prices" Journal of Business, 51, 621-651.

Brunnermeier, Markus and Lasse Heje Pedersen (2005a) "Predatory Trading", *Journal of Finance*, 60, 1825-1863

Brunnermeier, Markus and Lasse Heje Pedersen (2005b) "Market Liquidity and Funding Liquidity", working paper, Princeton University and NYU Stern School.

Cifuentes, R., G. Ferrucci and H. S. Shin (2005) "Liquidity Risk and Contagion" Bank of England working paper 264,
http://www.bankofengland.co.uk/publications/workingpapers/wp264.pdf

Diamond, Douglas and Raghuram Rajan (2005) "Liquidity Shortages and Banking Crises" *Journal of Finance*, 60, pp. 615

Eisenberg, L. and T. H. Noe (2001) "Systemic Risk in Financial Systems" *Management Science*, 47, 236-249.

Geanakoplos, J. (2003) "Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium" *Advances in Economics and Econometrics: Theory and Applications, Eighth World Conference, Volume II,* Cambridge University Press

Gennotte, Gerard and Hayne Leland (1990) "Hedging and Crashes", *American Economic Review*, 999–1021.

Gorton, Gary and Lixin Huang (2004) "Liquidity and Efficiency" *American Economic Review* 94(3), 455-483

Kashyap, A. and J. Stein (2000) "What Do a Million Observations on Banks Say about the Transmission of Monetary Policy?" *American Economic Review*, 90, 407-428.

Kiyotaki, N. and J. Moore (1998) "Credit Chains" LSE working paper, http://econ.lse.ac.uk/staff/kiyotaki/creditchains.pdf.

Kiyotaki, N. and J. Moore (2001) "Liquidity and Asset Prices" LSE working paper, http://econ.lse.ac.uk/staff/kiyotaki/ liquidityandassetprices.pdf.

Merton, R. (1974) "On the Princing of Corporate Debt: the Risk Structure of Interest Rates" *Journal of Finance*, 29, 449-470.

Milgrom, P. and J. Roberts (1990) "Rationalizability, Learning and Equilibrium in Games with Strategic Complementarities" *Econometrica*, 58, 1255-78.

Milgrom, P. and J. Roberts (1994) "Comparing Equilibria" *American Economic Review,* 84, 441-459.

Morris, S. and H. S. Shin (2004) "Liquidity Black Holes" *Review of Finance*, 8, 1-18.

Plantin, G., H. Sapra and H. S. Shin (2004) "Marking to Market: Panacea or Pandora's Box" forthcoming in the *Journal of Accounting Research*.

Rajan, R. (2005) "Has Financial Development Made the World Riskier?" paper presented at the Federal Reserve Bank of Kansas City Economic Symposium at Jackson Hole,
http://www.kc.frb.org/publicat/sympos/2005/sym05prg.htm

Schnabel, I. and H. S. Shin (2004) "Liquidity and Contagion: the Crisis of 1763" *Journal of the European Economic Association*, 2(6), 929-968.

Shin, H. S. (2005) "Discussion of Rajan "Has Financial Development Made the World Riskier?"" presented at the Federal Reserve Bank of Kansas City Economic Symposium at Jackson Hole,
http://www.kc.frb.org/publicat/sympos/2005/sym05prg.htm

Tarski, A. (1955) "A Lattice-Theoretical Fix-point Theorem and Its Applications" *Pacific Journal of Mathematics*, 5, 229-31.

Topkis, D. M. (1978) "Minimizing a Submodular Function on a Lattice" *Operations Research*, 26, 305-21.

Van den Heuvel, S. (2002) "The Bank Capital Channel of Monetary Policy", working paper, Wharton School, University of Pennsylvania,
http://finance.wharton.upenn.edu/~vdheuvel/BCC.pdf