Offshoring in a Knowledge Economy

Pol Antràs, Luis Garicano and Esteban Rossi-Hansberg*

January 17, 2005

Abstract

How does the formation of cross-country teams affect the organization of work and the structure of wages? To study this question we propose a theory of the assignment of heterogeneous agents into hierarchical teams, where less skilled agents specialize in production and more skilled agents specialize in problem solving. We first analyze the properties of the competitive equilibrium of the model in a closed economy, and show that the model has a unique and efficient solution. We then study the equilibrium of a two-country model (North and South), where countries differ in their distributions of ability, and in which agents in different countries can join together in teams. We refer to this type of integration as globalization. Globalization leads to better matches for all southern workers but only for the best northern workers. As a result, we show that globalization increases wage inequality in the South but not necessarily in the North. We also study how globalization affects the size distribution of firms and the patterns of consumption and trade in the global economy.

*Harvard University, University of Chicago, and Stanford University, respectively. A previous version of this paper was circulated under the title “Outsourcing in a Knowledge Economy.” We thank several colleagues and seminar participants at various institutions for very useful comments.
1 Introduction

A number of recent technological\textsuperscript{1} and institutional\textsuperscript{2} developments have blurred the borders between national labor markets and have allowed for the formation of international teams. These developments have altered what teams of agents can do at a distance. Some tasks such as data entry in consumer banking, software upgrades and maintenance, low-level customer handling in call centers, or standardized manufacturing processes, are now frequently done offshore. Other, more knowledge intensive, tasks (such as data manipulation, software development, higher-end sales and service, and R&D and product design in manufacturing industries) continue to be undertaken domestically.\textsuperscript{3} Broadly, routine tasks are offshored, while more complex tasks are done domestically. Thus, the traditional vertical division of labor within a team, whereby some low skill agents (workers) undertake routine tasks and some high skill agents (managers) specialize in knowledge intensive tasks, can now take place across countries.

In our view, what is important about this new division of labor is that it alters the feasible matches between agents’ skill types. High skilled agents in more developed countries can leverage their knowledge at lower cost by working with cheaper labor on routine tasks, and the better workers in less developed countries are able to become part of international high value added teams.

In this paper, we present a simple framework that puts agent skill heterogeneity and matching at the center of the analysis. By allowing us to analyze changes in matching and in the supporting earnings functions, our framework allows us to examine the impact of offshoring tasks on wages, on occupational choice (production versus knowledge jobs) and (as matches are ‘many-to-one’) on the distribution of firm sizes.

We model an economy in which production requires physical inputs and knowledge, and where a continuum of agents with heterogenous abilities sort into teams competitively. Agents of different skill levels form teams. Less skilled agents (‘workers’) specialize in production work and deal with routine tasks; while more skilled agents specialize in knowledge intensive tasks (‘managers’). Relative to less skilled managers, better managers are able to increase the productivity of all the workers in their team, as they are able to solve a wider range of the problems their team confronts in production. Better production workers allow individuals to manage larger teams, as workers can solve more problems by themselves and require less help. This results in a complementarity between manager and worker ability that determines the identity of agents working and managing different teams. It also determines, through

\textsuperscript{1}Improvements in information technology have reduced the cost of international data and voice transfer from prohibitively expensive to levels that are virtually identical to within country communication costs.

\textsuperscript{2}Recent political and economic reforms in China, India and Eastern Europe have substantially liberalized economic activity. Meanwhile, the world-wide deregulation and competition in the telecommunications industry has contributed substantially to the drop in communication costs.

\textsuperscript{3}For an unsystematic but useful data survey documenting these patterns see \textit{The Economist}, November 13, 2004. “A world of work: A survey of outsourcing.”
comparative advantage, the occupational choice of agents. More able agents, although more productive as production workers, want to set up their own firms and manage their own teams of workers, instead of working for other managers. The distribution of skills in the population then determines the types of teams that are formed and the wages that different agents command.

To study the impact of the formation of international teams in this economy, we study a simple one-sector, two-country model in which countries differ only in their skill distributions. In particular, one country, the North, has a distribution of skills with a relatively high mean, while the other country, the South, has a distribution of skills with a relatively low mean. In our model, the ‘skill overlap’ implied by these skill differences is captured by a single parameter, which plays a crucial role in the analysis. The other key parameter in our model is the cost of communicating knowledge within teams (i.e., the state of communication technology), which determines the extent to which managers can leverage their knowledge via larger teams.

We initially study the case in which cross-country teams are prohibitively expensive, so that the equilibria in the North and in the South correspond to those of two closed economies. We then compare these equilibria to that of a perfectly integrated international economy, where cross-border teams are as expensive as local ones. We refer to this type of integration as ‘globalization.’

We first show that globalization leads to the formation of international teams in which Northern managers supervise teams of Southern workers: offshoring. Offshoring thus allows for the geographic separation of production and problem solving, and the delocation of physical production towards the South. It leads to the creation of production jobs and an increase in production in the South, and to the creation of knowledge intensive jobs and a decrease in production in the North. This implies that the pattern of trade is such that the South is a net exporter of manufacturing goods, while the North is a net exporter of knowledge services. If international trade statistics do not appropriately record the managerial services inherent in the flow of knowledge within and across multinational firms, our model is consistent with the emergence of trade imbalances between the North (e.g., the United States) and the South (e.g., China).

Globalization also affects the level and structure of earnings of individuals, both in the North and in the South. We first show that our model is consistent with the empirical regularity that ‘Southern’ workers employed in multinational firms receive wages that are on average higher than those received by workers employed in domestic firms (see Aitken et al. (1996) for empirical evidence). We next analyze how globalization affects income inequality within each of the two countries. We show that globalization leads to an increase in within-worker wage inequality in the South. This prediction is consistent with the findings of several empirical studies (e.g., Feenstra and Hanson (1997), Anderton et. al. (2002), and Marin (2004) among many others). These findings have received considerable attention in the
international trade literature since they cannot be easily rationalized with standard factor proportions trade frameworks. Our theory predicts an increase in within-worker inequality in the South as a result of changes in matching: globalization improves the quality of the managers with whom southern workers are matched, thus raising the productivity of these workers, and thereby leading to an increase in their marginal return to skill. This effect is reinforced by an occupational choice effect: more agents become workers, hence increasing the range of abilities in the worker skill distribution.

The effect on wage inequality in the North is more complicated. On the one hand, low-skilled workers in the North face increased competition from Southern workers and this tends to reduce their marginal return to skill. On the other hand, our model highlights a new force leading to an increase in wage inequality in the North. When more low skilled agents are available, the time of high skilled managers becomes more scarce, and workers who are better able to economize on this time become relatively more valuable. As a result, the value of more skilled workers relative to less skilled ones increases, as does the difference between the ability of the managers they are matched with. When either communication costs or the skill overlap are sufficiently low, so that high skill managers are particularly valuable and scarce, this last effect dominates and globalization increases wage inequality not only in the South but also in the North. Conversely, when communication costs and the skill overlap are sufficiently large, the former effect dominates and offshoring is associated with lower wage inequality in the North. This may help rationalize the findings of Feenstra and Hanson (1996b, 1999) that offshoring raised wage inequality in the United States in the 1980s but not in the 1970s. Our theory suggests that these findings can be explained by lower communication costs and deeper trade integration with less developed countries in the 1980s than in the 1970s.4

Which firms engage in offshoring? We show that the answer depends on the two main parameters in the model: the level of communication costs and the size of the skill overlap. When the skill overlap is large and communication costs are high, only the most productive, large firms will engage in offshoring; while, when the skill overlap is small and communication costs are low, the firms that engage in offshoring will actually be the least productive firms, those controlled by the lowest skilled managers. More generally, we show that the ‘quality’ of offshoring, as measured by the average skill level of the workers that form international teams relative to the skill level of all Southern workers, is weakly increasing in both the skill overlap and communication costs. At the same time, we show that the ‘quantity’ of offshoring, as measured by the proportion of Southern workers that work for Northern managers, is instead weakly decreasing in both communication costs and the skill overlap, and converges to zero as the skill distributions completely overlap. We also study how occupational choices, the size distribution of firms, and wage inequality are affected by these same parameters.5

4 The ability of our model to deliver the level of income for all agents in the economy also allows us to identify the winners and losers from globalization. In particular, In Section 4.3 we show that there is always a subset of workers that are hurt by globalization.

5 For instance, we show that our model is consistent with an increase in the relative endowment of skilled
One of the advantages of our approach is that offshoring is not only the result of the relative aggregate supply of skills, but rather follows from the competitive sorting of agents with different skill levels into teams. Paraphrasing Sattinger (1993) wages in the economy play an allocative role rather than simply being rewards for the possession of particular characteristics. This allows us to derive conclusions on the characteristics of offshoring firms as well as on the distribution of wages. Most other efforts to understand offshoring do not have this feature. Feenstra and Hanson (1996a, 1997 and 2003), for example, assume factor endowments of skilled and unskilled workers in the North and South, and a production function that uses these inputs either to produce intermediate or final goods. In these models offshoring is the result of foreign direct investment and leads to changes in wage inequality as a result of changes in the sectoral composition of production. Their work is important in that it determines the changes in wages due to these sectoral (in inputs or output) compositional changes. In general, however, it is silent about changes in wage inequality within narrowly-defined sectors as well as on the cross-sectional characteristics of offshoring firms. Other papers have developed frameworks with similar characteristics that also abstract from the dimensions that we focus on, in particular Acemoglu (2003), Bernard et al. (2004), Zhu and Trefler (2004) and Verhoogen (2004).

Our paper is closely related to the work of Grossman and Maggi (2000) and Kremer and Maskin (2003), in that they also study the relationship between patterns of trade and patterns of matching between the skill of different workers. Grossman and Maggi (2000) consider the consequences of different types of production functions involving substitutability or complementarity in skills for the patterns of specialization and trade. A maintained assumption in their analysis is that international teams are not allowed to form. In this respect, our work is more closely related to Kremer and Maskin (2003), who study the patterns of trade and wages that result from production functions that are characterized by complementarity between inputs and imperfect substitutability between them.

Consistent with any production function that may hope to address within-worker wage inequality, the production function we study involves skill complementarity, imperfect substitutability between workers skill, and differential sensitivity to the skill of different workers (see Kremer and Maskin 1997). Our model, however, is novel in four key dimensions. First, fol-agents in the South leading to increased within-worker inequality in that country. This is consistent with the findings of Zhu and Trefler (2004) and is not easily rationalizable with standard general equilibrium models.

Feenstra and Hanson (1997) assume that the supply of skill and unskilled labor does respond to relative wages, but they do not model the occupational choice decision or the sorting of agents into production teams.

Feenstra and Hanson (2003) stress that these changes in the sectoral composition of production may occur within industries and may therefore not be picked up by industry-level price indices (c.f., Lawrence and Slaughter (1993)).

Nocke and Yeaple (2004) present an assignment model of FDI, but focus on the matching between brands of different quality and entrepreneurs of heterogenous ability.

Our paper is also related to a branch in the literature that has stressed the importance of heterogeneity in understanding the differential impact that globalization may have on different types of firms or workers (e.g. Manesse and Turrini (2001), Melitz (2003), Helpman et al. (2004), Antrás and Helpman (2004), and Yeaple (2004)).
ollowing Garicano and Rossi-Hansberg’s (2003, 2004), it is the only one to involve hierarchical one-to-many matching (rather than one-to-one–matching), where a manager is endogenously matched with a potentially large number of workers, and can potentially raise the output of all of them. Second, the identity of managers and workers is endogenous and is the result of an occupational choice decision. Third, the actual team production function results naturally and endogenously from a production process which does not assume skill complementarities, but rather derives them from the specialization of agents in different aspects of the process – production and knowledge. Fourth, the relation between the skill of the manager and that of the worker is mediated by communication technology – that is, the state of communication technology determines the extent to which a manager can leverage his knowledge by communicating it to many or few production workers. As a result of these differences, we are able to move beyond previous contributions in formally analyzing how the process of globalization interacts with the state of communication technologies in determining the worldwide organization of production and the structure of rewards that support it.10

Our paper differs from Garicano and Rossi-Hansberg’s (2003) in several key aspects. Given our focus on offshoring, we simplify the analysis in two directions to be able to analyze the impact of matching across distributions. We take the skill level of agents as exogenous, and we limit team sizes to two layers, while in Garicano and Rossi-Hansberg (2003) knowledge and the number of layers are endogenous. The payoff for this simplification is that we obtain a number of new results. First, we are able to show that the equilibrium of the model is unique. Second, we provide a closed-form solution to the model which permits a more detailed characterization of the equilibrium. Third and most importantly, we are able to study the relationship between matching and wage inequality across countries.

The rest of the paper is organized as follows. Section 2 presents the basic framework for a closed economy and shows existence, uniqueness, and optimality. Section 3 constructs an equilibrium in the integrated economy and discusses its basic properties. Section 4 discusses the effects of economic integration or globalization. Section 5 presents comparative statics with respect to communication costs and the skill overlap, and Section 6 concludes. All the proofs in the paper are relegated to Appendices A and B.

2 The Model

Agents are endowed with one unit of time and a skill level $z$. The distribution of skills in the population is given by a cumulative distribution function $G(z)$, with density $g(z)$, that for the moment we will assume has support in $[0, \bar{z}]$ with $\bar{z} \leq 1$. Agents rank consumption according to a linear utility function, so they are income maximizers, given that we normalize the price of the only good in the economy to one.

10For previous equilibrium models of the allocation of heterogeneous agents to hierarchical teams, but which do not involve matching between workers and managers, see Lucas (1978), Rosen (1982), and Waldman (1984).
Our theory of the organization of production follows Garicano (2000). Production is done by teams with one manager and production workers. While producing, workers face problems that have to be solved. If a worker knows the solution to his problem, he solves it and uses his time to produce one unit of output. If he does not know the solution, he can ask his manager. If the manager knows the solution to the problem, the manager solves it immediately, communicates the solution to the worker, and the worker then uses his time to produce. The manager spends \( 0 < h < 1 \) units of time communicating what he knows to the worker no matter if he knows the solution to the problem or not. The skill level of an agent determines the set of problems he can solve. An agent with skill \( z \) can solve all problems that require knowledge between 0 and \( z \). We normalize the set of problems so that the skill level \( z \) is also the proportion of problems an agent can solve.\(^{11}\) Hence, a manager in a team with \( n \) workers of skill \( z_p \) faces the following time constraint

\[
h \left(1 - z_p \right) n = 1,
\]

and so can deal with \( n \left(z_p \right) = 1 / \left[h \left(1 - z_p \right) \right] \) workers.\(^{12}\) Production in a team formed by a manager with skill level \( z_m \) and workers with skill \( z_p \) is therefore given by

\[
z_m n \left(z_p \right) = \frac{z_m}{h \left(1 - z_p \right)}.
\]

Given wages, managers choose the ability of their workers to maximize rents,

\[
R \left(z_m \right) = \max_{z_p} \frac{z_m - w \left(z_p \right)}{h \left(1 - z_p \right)}, \quad (1)
\]

The first order condition of this problem is given by

\[
w' \left(z_p \right) = \frac{z_m - w \left(z_p \right)}{1 - z_p}.
\]

Agents choose whether to become managers or workers so as to maximize their utility, that is, their income. Hence given their ability \( z \) they solve

\[
\max \left\{ R \left(z \right), w \left(z \right) \right\}.
\]

This implies that the earnings function, the envelope of the wage and rent functions, given

\(^{11}\)The upper bound \( \bar{z} \) thus represents the fraction of problems that the most-skilled agent in the economy can solve.

\(^{12}\)In principle, the interpretation of our technology given in the text requires us to address the stochastic element in the arrival of problems, which could result in congestion and queuing. Doing so would not, we believe, add to the economics of the question at hand. An alternative interpretation, that circumvents the need to address these issues, is that each worker draws a continuum of problems of measure one. Workers then solve the problems that they can, given their skill level, and asks managers for help on the measure of problems that they do not know how to solve. Then, \( h \) would be interpreted as the time cost for a manager of helping on a measure one of problems.
occupational choices, has to be continuous.

In equilibrium labor markets clear. Namely, at the equilibrium wages and earnings, the supply and demand of production workers equalize at all skill levels. Let \( w(\cdot) \) be an equilibrium wage function, and let the equilibrium occupational choice decision be such that agents with skill levels in \([0, z^*] \) become workers and agents in \([z^*, \bar{z}] \) become managers. Agents with knowledge \( z^* \) are indifferent. This restriction turns out to be without loss of generality, as Theorem 1 below shows. Let \( m(z) \) be the skill level of the manager of a worker with ability \( z \). We prove in Theorem 1 that an equilibrium allocation of this economy has to satisfy positive sorting, and therefore that \( m \) is invertible. Then, labor market clearing implies that

\[
\int_0^{z_p} g(z) \, dz = \int_{m(0)}^{m(z_p)} n\left(m^{-1}(z)\right) g(z) \, dz \quad \text{all } z_p \leq z^*,
\]

where \( m^{-1}(z) \) is the ability of the workers hired by a manager of ability \( z \). The left-hand-side of this equation is the supply of workers between 0 and \( z_p \). The right-hand-side is the demand for workers by managers between \( m(0) = z^* \) and \( m(z_p) \). Market clearing is guaranteed when supply equals demand for every skill level of workers \( z_p < z^* \). Substituting for \( n \) and deriving with respect to \( z_p \) we obtain that, as long as \( z < z^* \) and \( m(z) \) is increasing (positive sorting),

\[
m'(z) = h(1-z) \frac{g(z)}{g(m(z))}. \tag{3}
\]

Notice that in this economy positive sorting is always guaranteed because of the complementarity between workers' and managers' talent. Hence, better workers always work for better managers, a property we will exploit intensively below. This differential equation, together with the two boundary conditions \( m(0) = z^* \) and \( m(z^*) = \bar{z} \), determines the equilibrium assignment function. Notice that the equilibrium assignment of workers to managers is independent of wages and rents once positive sorting is imposed.\(^1\) The reason is that the span of control of managers is a technological restriction of the problem. Managers add agents to their teams until they do not have any time left. If agents could acquire skill, or could work by themselves, this helpful property of our economy would be lost and the analysis would be much more complicated. Garicano and Rossi-Hansberg (2003) and (2004) present closed economy frameworks that incorporate these dimensions.

A competitive equilibrium in our economy is therefore given by a wage function \( w \), a rent function \( R \), an assignment function \( m \), and occupational choice decisions (summarized by \( z^* \)), such that managers maximize rents ((2) is satisfied and \( w'(z^*) < R'(z^*) \))\(^1\), agents maximize utility \( (w(z^*) = R(z^*)) \), and labor markets clear ((3) is satisfied together with \( m(0) = z^* \)).

\(^1\)Of course, equilibrium wages and rents sustain the assignment as an equilibrium allocation, but we do not need to know them to compute the assignment function.

\(^1\)The second condition is needed to guarantee that managers at \( \bar{z} \) do not profit from hiring agents with abilities slightly above \( z^* \). The condition is necessary given that (2) only holds for \( z \in (0, z^*) \) but not for \( z^* \). Garicano and Rossi-Hansberg (2003) show that this condition would always be satisfied if we were to allow agents to produce individually as well as in teams.
and \( m(z^*) = \tilde{z} \). The following theorem shows that an equilibrium of this economy exists as long as \( h \) is lower than a threshold \( h^* \). It also shows that if an equilibrium exists, it is unique, efficient, exhibits positive sorting, and can be characterized by a threshold \( z^* \) as we have done so far. On top of this, we can show in general that the earnings function \( \max \{ R(z), w(z) \} \) is strictly convex.

**Theorem 1** There exists a threshold \( h^* > 0 \) such that if \( h \in [0, h^*] \) there exists a unique competitive equilibrium of this economy. In equilibrium the set of managers and the set of workers are connected, the equilibrium exhibits positive sorting, and the earnings function is strictly convex. Furthermore, the equilibrium allocation is efficient.

In the rest of the paper we will analyze the case in which we specify the distribution of abilities to be a piecewise uniform density. In this case we can show that \( h^* > 0.85 \), and so for the rest of the paper we will assume that communication costs are such that \( h \in [0, 0.85] \). The reason that we need to restrict \( h \) for an equilibrium to exists, is that we are not allowing agents to be self-employed. If we where to allow them to work on their own, we could guarantee existence also for communication costs between 0.85 and 1 (see Garicano and Rossi-Hansberg (2004)). However, this would come at the cost of a more complicated analysis of the model, which we believe would obscure the main message of the paper. Finally, note that we have assumed that a manager with ability \( z_m \) hires workers of homogeneous ability \( z_p \). In Appendix B, we generalize the technology and show that this assumption is without loss of generality.

### 2.1 Equilibrium in the Closed Economy

Consider a world formed by two independent economies where agents can only form teams with other agents in the same economy. The first one, that we call the North, is exactly as described before but with a uniform distribution of skills in the population, \( G_N(z) = z \) for \( z \in [0, 1] \), with density \( g_N(z) = 1 \). In the North, the best agents of the economy can therefore solve all the problems that arise in production. The second economy, that we call the South, also has a uniform distribution of skills, but the support of the distribution is the interval \( [0, \alpha] \) for \( \alpha < 1 \), with \( G_S(z) = z/\alpha \) for \( z \in [0, \alpha] \), and density \( g_S(z) = 1/\alpha \). The best agents in the South can thus solve only a fraction \( \alpha \) of the problems that they face while producing. The North is, therefore, relatively better endowed with skilled agents, but both countries are identical in all other respects, including population size. We will often refer to the parameter \( \alpha \) as the skill overlap. The uniform assumption for the distribution of skills in both countries serves two purposes. First it implies that the skewness in the wage distribution implied by our framework is the result of the economic forces we consider and not the distribution of skills we assume. Second, it allows us to solve the whole model analytically.

The northern economy is just a special case of the southern economy when \( \alpha \) is equal to 1. Hence, we start by describing an equilibrium in the South. All the expressions are identical
for the North if we substitute $\alpha = 1$. Using (3) and the boundary condition $m_S(0) = z_S^*$, we obtain that

$$m_S(z) = z_S^* + h z \left(1 - \frac{1}{2} z \right),$$

(4)

and using $m_S(z_S^*) = \alpha$, we can solve for the threshold ability $z_S^*$:

$$z_S^* = \frac{1 + h - \sqrt{1 + h^2 + 2h(1 - \alpha)}}{h}.$$  

(5)

That is, all agents with skill between 0 and $z_S^*$ become workers and all agents with skill between $z_S^*$ and $\alpha$ become managers. It is easy to show that $z_S^*$ increases as communication technology improves, that is, as $h$ declines. An increase in $z_S^*$ implies that more agents become workers, which is the result of an increase in the span of control of managers. All managers can have larger teams, so in equilibrium there are less managers and more workers.

In an economy with more skilled agents, larger $\alpha$, $z_S^*$ is higher. There are two forces that determine this effect. First, as $\alpha$ increases and therefore the density $1/\alpha$ decreases, we are reducing the mass of low skilled agents and increasing the mass of high skilled agents, which implies that, given the size of teams, agents with higher skill decide to become workers. Second, as we increase the skill level of the best workers in the economy, the best agents manage larger teams which reduces the set of managers and increases the set of workers.

An economy with higher $\alpha$ or lower $h$ is an economy in which the skill levels of the agents that become workers is more dispersed. Given wages, this higher skill dispersion will lead to higher measured wage inequality. We call this effect the occupational choice effect. Of course, there is also an equilibrium response of wages that needs to be combined with this effect to obtain conclusions on the implications of changes in $h$ and $\alpha$ on the distribution of wages. We study these general equilibrium effects below.

A characteristic of this equilibrium is that, because of positive sorting, more skilled managers lead teams with more skilled workers. Since the size of a firm is uniquely determined by the skill levels of its workers and by an economy-wide parameter $h$, higher skilled agents work in larger firms. Because managers of these firms have more skill, they solve a larger proportion of the problems they face, and so these firms are more productive. The average product of labor is thus higher in these teams. As we will now see, this will result in both managers and workers in these teams earning more per unit of skill: The wage and rent functions will be convex in the level of skill (see also Theorem 1 above).

We now turn to a formal analysis of the determination of earnings in this economy. Equation (2), together with $w_S(z_S^*) = R_S(z_S^*)$, implies that the equilibrium wage function is given by

$$w_S(z) = z_S^* - \sigma_S (1 - z) + \frac{1}{2} h z^2,$$  

(6)

where

$$\sigma_S = \frac{hz_S^* (1 + \frac{1}{2} h z_S^*)}{1 + h - h z_S^*}.$$  

(7)
The slope of the wage function, the marginal return to skill for workers, is thus given by

\[ w'_S(z) = \sigma_S + hz. \]

Hence the wage function is convex: the marginal return to skill increases with the skill level. This force is captured by the convexity term \( hz \), and it reflects the imperfect substitutability between workers of different skill — the skill price per unit of skill varies with the skill level. Throughout the paper, we refer to this force as the \textit{complementarity} effect. There is a second determinant of the marginal return to skill, the one given by \( \sigma_S \), which is determined by the supply and demand of worker’s skill in equilibrium: the \textit{competition} effect.

The marginal return to skill can be shown to be an increasing function of \( h \). That is, it decreases with an improvement in communication technology. There are two interacting equilibrium effects. As communication costs decrease, given the threshold \( z^*_S \), team size increases. Since the difference between the skill levels of the managers of two different workers will be smaller the larger are team sizes, complementarity between worker and manager skills implies a decrease in the marginal return to skill: a decrease in the complementarity effect. The second effect is the result of the need for more workers’ skills which increases their baseline price: an increase in the competition effect. Overall, one can show that the second effect dominates, so an improvement in communication technology increases the returns to skill of workers.

The marginal return to skill is also increasing in \( \alpha \), since \( \sigma_S \) is an increasing function of \( z^*_S \) which in turn increases with \( \alpha \). In this case the complementarity effect is unchanged. In contrast, the competition effect increases: since agents are more skilled, there are too few workers per manager at the old threshold, which requires raising workers’ return to skill in equilibrium. Again, workers are matched with better managers and this increases the returns to their own skill.

After solving for the distribution of wages, we turn next to the analysis of managerial rents. From equation (1) managerial rents are given by

\[ R_S(z) = \frac{z - w_S(m_S^{-1}(z))}{h(1 - m_S^{-1}(z))}. \]

Using the envelope condition, the marginal return to skill for managers is given by

\[ R'_S(z) = \frac{1}{h(1 - m_S^{-1}(z))}. \]

Given that the assignment function is increasing (positive sorting), the rent function is convex: The marginal return to skill for managers increases with their skill level (see Theorem 1 above). Note also that the marginal return to skill for managers is equal to the number of workers in their team. Hence, every time we derive conclusions about firm size the same
applies for the marginal return to skill of managers.

A worker of ability \( z \) works for a manager with ability \( m(z) \). This means that the total output produced by this worker is given by \( m(z) \). Total production in the South is therefore given by\(^{15}\)

\[
Y_S = \int_0^{\frac{1}{hS}} m_S(z) g_S(z) \, dz = \frac{1}{6\alpha} z_S^2 (6 + 3h - h z_S^2) .
\]  
(8)

It is easy to verify that \( Y_S \) decreases with \( h \) and increases with \( \alpha \). The reasoning is simple: the larger is \( h \), the higher are communication costs, the less can managers leverage their knowledge, and the lower is the implied average productivity. As \( \alpha \) increases, the average skill level in the economy increases, which also leads to larger output.

3 Equilibrium in the World Economy

Consider a world economy formed by the two countries described above, North and South. In the world equilibrium, agents can form production teams with agents in their own country or with agents in the other country. We assume that the cost of communicating the solution to a problem, \( h \), is the same whether communication happens between agents in the same or in different countries. We could add an extra cost of communicating with agents in another country. However, this extra cost would then influence the formation of international teams directly and would open a wedge between wages in different countries, thereby greatly complicating the analysis of the economic forces in the equilibrium of our setup. Furthermore, this added complexity would be associated with relatively small gains in terms of new results or economic insights, unless we allowed for multiple layers of management within a firm. We leave these potentially interesting extensions for future research.

The equilibrium in the world economy is very similar to an equilibrium in the individual countries once we adjust the distribution of talent in the population. The distribution of skills in the world population is given by the sum of the distribution of skills in the South and in the North (and so it is not a probability distribution since it integrates to 2), namely,

\[
G_W(z) = \begin{cases} \frac{1+\alpha}{\alpha} z & \text{if } 0 < z < \alpha \\ z & \text{if } \alpha < z < 1 \end{cases} .
\]

The construction of an equilibrium in this economy parallels the one for a closed economy with one caveat. Since the density of skills in the world is not continuous, the derivative of the assignment function is not continuous. However, an equilibrium allocation must be such that the earnings function is continuous and differentiable for all \( z \) except at the threshold that divides workers and managers, at which it is not differentiable, just as in the closed economy.

If this condition is not satisfied, some managers and workers would have incentives to form

\(^{15}\)Equivalently, output may be calculated as the integral over managerial skill of the production function, \( n(m^{-1}(z))z \). Both expressions yield the same result, as one results from a change of the variable of integration in the other.
new teams. This characteristic of an equilibrium allocation implies that the assignment function is continuous. Hence, Theorem 1 applies for the world economy as well.

Depending on the value of $h$ and $\alpha$, we can show that there are two types of equilibria in the world. The first one is an equilibrium in which all agents in the South are workers. Since there are no managers in the South, all of them work for northern managers. That is, they all work in international teams. Positive sorting implies that, because they are the lowest quality workers in the world (there is an identical set of workers in the North plus some more skilled ones), they work for the worst managers in the North. Hence, international teams are associated with the worst managers in the world. We call this the *Low Quality Offshoring Equilibrium (LQE)*.

The second type of equilibrium is one in which some of the agents in the South are managers. This equilibrium features the less skilled workers in the South working for southern managers, and the more skilled ones working for the best managers in the North. We call this the *High Quality Offshoring Equilibrium (HQE)*. All our results are derived under the assumption that international teams are formed only if managers strictly prefer to hire foreign workers than domestic ones.\textsuperscript{16}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{types_of_equilibria.png}
\caption{Types of Equilibria}
\end{figure}

The type of equilibrium in the world depends on the values of $h$ and $\alpha$. Given $\alpha$, a low communication cost $h$ implies that teams are large and so the set of agents that choose to become managers in the world is small. Since the top managers are always in the North, by positive sorting, all agents in the South are workers. Thus, we will be in a LQE. Conversely, a high communication cost $h$ implies that teams will be small, the set of managers will be small.

\textsuperscript{16}We are effectively selecting the equilibrium with the least amount of offshoring. This is analogous to the approach in Helpman (1984).
large, so some agents in the South will be managers, and we will be in a HQE. Given \( h \), if \( \alpha \) is small all agents in the South will have relatively low skill, and so they will be workers: A LQE. If \( \alpha \) is large, some agents in the South have relatively high ability, and some of them will be managers: A HQE. In general, the set of parameter values that determines the boundary between these two equilibria is a non-linear function of \( h \) and \( \alpha \) that we will determine below, and which we plot in Figure 1. We analyze each equilibrium in turn.

### 3.1 Low Quality Offshoring Equilibrium

Denote by \( z_{WL}^* \) the threshold that separates the ability of the agents that choose to be workers or managers in a LQE. In order for the world equilibrium to be a LQE it must be the case that \( \alpha < z_{WL}^* \) (i.e., that all agents in the South are workers). For an assignment to satisfy the world labor market equilibrium condition it has to satisfy (3) or, in this case,

\[
m' (z) = \begin{cases} 
\frac{1+\alpha}{\alpha} h (1-z) & \text{if } 0 < z < \alpha \\
\frac{1}{h (1-z)} & \text{if } \alpha < z < z_{WL}^* 
\end{cases}.
\]

Equilibrium in the labor market also implies that \( m (0) = z_{WL}^* \) and \( m (z_{WL}^*) = 1 \). In order for the wage function to be differentiable (see (2)), the assignment function has to be continuous at all \( z < z_{WL}^* \), and in particular at \( \alpha \). This characteristic of the equilibrium allocation provides another boundary condition of the problem. Using the two differential equations and the three boundary conditions we can solve for the assignment function

\[
m_{WL} (z) = \begin{cases} 
z_{WL}^* + \frac{1+\alpha}{\alpha} h z (1-\frac{1}{z}) & \text{if } 0 < z < \alpha \\
z_{WL}^* + h (1-\frac{1}{\alpha}) + h z (1-\frac{1}{z}) & \text{if } \alpha < z < z_{WL}^* 
\end{cases},
\]  

as well as for the threshold

\[
z_{WL}^* = \frac{1 + h - \sqrt{1 + h^2 (3 - \alpha)}}{h}.
\]

Again, simple differentiation verifies that \( z_{WL}^* \), the set of workers in the world, decreases with \( h \) and increases with \( \alpha \), where the intuition is similar to the one for the closed economy. Note that the assignment function is continuous, but not differentiable, at \( \alpha \).

In order for the world to be in a LQE we need to guarantee that \( z_{WL}^* > \alpha \) or

\[
h < \frac{2 (1 - \alpha)}{2 + \alpha - \alpha^2}.
\]

The right-hand-side of the inequality is decreasing in \( \alpha \) and is equal to zero for \( \alpha = 1 \) and equal to one for \( \alpha = 0 \). This condition, with equality, is the curve that separates the parameter set where we obtain each equilibrium and that was plotted in Figure 1.

Maximization of rents by managers implies that wages have to satisfy (2). Furthermore, in order for agents not to have incentives to join other firms in the economy, that would
be willing to hire them, we also know that the earnings function has to be continuous. In particular, the wage function has to be continuous at $\alpha$ and wages and rents have to be equal at $z^*_{WL}$. The latter condition is given by $w_{WL}(z^*_{WL}) = R_{WL}(z^*_{WL})$. Combining all these conditions we obtain

$$w_{WL}(z) = \begin{cases} 
  z^*_{WL} - \sigma_1 L (1-z) + \frac{1}{2} \left( \frac{1+z}{\alpha} \right) h z^2 & \text{if } 0 < z < \alpha \\
  z^*_{WL} + h \left( 1 - \frac{1}{2} \alpha \right) - \sigma_2 L (1-z) + \frac{1}{2} h z^2 & \text{if } \alpha < z < z^*_{WL}, 
\end{cases}$$  

(12)

where

$$\sigma_1 L = \frac{hz^*_{WL} \left( 1 + h \frac{1}{2} h z^*_{WL} \right) - \frac{1}{2} h^2 \alpha}{1 + h - h z^*_{WL}}$$

(13)

and

$$\sigma_2 L = \sigma_1 L + h.$$

Note that at $z^*_{WL}$,

$$w'_{WL}(z^*_{WL}) = \frac{1 - w_{WL}(z^*_{WL})}{1 - z^*_{WL}} \leq \frac{1}{h} = R'_W L(z^*_{WL}),$$

for $h < h^*$ (see the proof of Theorem 1). Hence, the earnings function has a kink, a non-differentiability, at $z^*_{WL}$. This implies that, given that the wage and rent functions are convex, the marginal return to skill is larger for managers than for workers.

Figure 2: Low Quality Offshoring Equilibrium

Figure 2 summarizes what we have discussed about a LQE. Agents with skill in $[0, \alpha]$ in the South and North work for northern managers with skill in $[z^*_{WL}, m_{WL}(\alpha)]$. Agents in
the North with skill in $[\alpha, z^*_W]$ work for managers in the North with skill in $[m_W L (\alpha), 1]$. The wage function of all workers in the world, and the rent function of northern managers, is a continuous and differentiable function of skill. The marginal return to skill of managers is larger than that of workers.

It is easy to draw an initial conclusion on the effect of globalization (moving from autarky to a world with international offshoring). If the skill level of agents in the South is low and communication costs are also relatively low we will be in a LQE. Then, after globalization, some agents switch occupation and become workers in the South and managers in the North. These occupational choice implications of globalization will have multiple consequences for wages, rents, output, consumption, as well as the cross-sectional differences between national and international teams. We will study those in Section 4 in detail. First we turn to the analysis of the second type of world equilibria: the HQE.

### 3.2 High Quality Offshoring Equilibrium

A HQE is such that the highest skilled agents in the South decide to become managers. If we denote by $z^*_W$ the threshold such that agents with lower skill decide to become workers and agents with higher skill decide to become managers, in a HQE it must be the case that $z^*_W < \alpha$. Positive sorting implies that, since the managers in the South are part of the group of the lowest skilled managers in the world, they are matched with the lowest skilled agents. In particular, agents in the set $[0, z_\alpha]$. Agents with skill lower than $z_\alpha$ work for managers in their own country (since we focus on the equilibrium with the least amount of offshoring), and workers with skill greater than $z_\alpha$ work in international teams. The threshold $z_\alpha$ is defined by the worker type that works for the best agent in the South, namely, $m (z_\alpha) = \alpha$. Then, labor market clearing implies that

$$m'_W (z) = \begin{cases} h (1 - z) & \text{if } 0 < z < z_\alpha \\ \frac{1 + \alpha}{\alpha} h (1 - z) & \text{if } z_\alpha < z < z^*_W \end{cases},$$

which restates condition (3) for this case, together with the same boundary conditions as in the LQE: $m (0) = z^*_W$ and $m (z^*_W) = 1$. On top of this we have to guarantee again that the equilibrium assignment function is continuous, in particular at $z_\alpha$, in order for the wage function to be differentiable. These conditions then result in an equilibrium assignment function given by

$$m_W (z) = \begin{cases} z^*_W + h z (1 - \frac{1}{2} z) & \text{if } 0 < z < z_\alpha \\ z^*_W - \frac{1}{\alpha} h z_\alpha (1 - \frac{1}{2} z_\alpha) + \frac{1 + \alpha}{\alpha} h z (1 - \frac{1}{2} z) & \text{if } z_\alpha < z < z^*_W \end{cases},$$

and a threshold

$$z^*_W = \frac{1 + h - \sqrt{1 + h^2 + \left(\frac{1 - \alpha}{1 + \alpha}\right) 2h}}{h}.$$
One can verify again that $z_{WH}^*$ is decreasing in $h$ and increasing in $\alpha$. Using the definition of $z_\alpha$ we also obtain that

$$z_\alpha = 1 - \sqrt{1 - 2 \left( \frac{\alpha - z_{WH}^*}{h} \right)}.$$  \hspace{1cm} (16)

It is straightforward to show that the condition that ensures that this world equilibrium is a HQE (i.e., $z_{WH}^* < \alpha$) is the reciprocal of condition (11).

Again, maximization of rents implies that condition (2) has to be satisfied, together with $w_{WH}(z_{WH}^*) = R_{WH}(z_{WH}^*)$ and continuity of wages at $z_\alpha$. Solving the two differential equations we obtain that

$$w_{WH}(z) = \begin{cases} 
    z_{WH}^* - \frac{1}{\alpha} h z_\alpha (1 - \frac{1}{2} z_\alpha), & \text{if } 0 < z < z_\alpha \\
    z_{WH}^* - \frac{1}{\alpha} \frac{h z_\alpha}{1 + h - h z_{WH}^* (1 - z)} + \frac{1}{2} \left( \frac{1 + \alpha}{\alpha} \right) h z^2 & \text{if } z_\alpha < z < z_{WH}^*
\end{cases}$$

where

$$\sigma_1 = \frac{h z_{WH}^* (1 - \frac{1}{\alpha} h z_\alpha + \frac{1 + \alpha}{2 \alpha} h z_{WH}^* h z_\alpha) + \frac{1}{2} (h z_\alpha)^2}{1 + h - h z_{WH}^* (1 - z)}$$

and

$$\sigma_2 = \sigma_1 - \frac{h}{\alpha} z_\alpha.$$  \hspace{1cm} (17)

As in the LQE, it is easy to show that the marginal return to skill is increasing in the level of skill and is larger for managers than for workers.
Let us summarize what we have shown for the HQE using Figure 3. Agents with skill in \([0, z_\alpha]\) work in national firms for managers with skill in \([z_{WH}^*, \alpha]\). Agents with skill between \(z_\alpha = m^{-1}(\alpha)\) and \(z_{WH}^*\) work for northern managers with skill in \([\alpha, 1]\). This set of managers includes the ones that manage international teams. As before, the earnings function is continuous and differentiable everywhere except for \(z_{WH}^*\) in which its slope increases discreetly.

4 Effects of Globalization

We study here the impact of an exogenous policy or technological change, that we call globalization, and that allows for the formation of international teams. We analyze its effects on the composition of teams, occupational choices, and the rewards structure of the economy. To do so, we compare the world equilibrium of Section 3 with the autarkic equilibria in the North and South that we described in Section 2.

4.1 Matching, Occupational Choice and Firm Characteristics

How does globalization affect occupational choices? To compare the open and autarkic equilibria we use Figure 4. The figure presents the matching functions in autarky and the two type of world equilibria. The blending of the two skill distributions produces a rearrangement of the matches for both northern and southern workers. Independently of the equilibrium we
are studying, it is evident from the figure that all the workers in the South strictly improve their match, that is, the presence of better managers in the international market improves their matches. This is the case even for southern workers that do not match with international managers, since some southern managers become workers in international teams, and the absence of these managers increases the quality of the match of every worker. Agents who were managers before globalization may either become workers (as there is a supply of higher quality managers that can do better their problem solving job) or they may remain managers. In the latter case, they are matched with lower skilled workers, precisely because some of the southern managers who were previously managing low quality workers have become workers, and the remaining managers are left to hire lower quality agents. In other words, while workers always benefit from the higher quality managers available for matching, managers’ matches suffer from the increasing competition of better international managers.

The picture is considerably different for workers in the North. The key change is in the opportunities of the middle-skilled agents in the North. Previously, they were not ‘good enough’ to be team managers. After globalization, their opportunities have greatly expanded, since there is a set of low-skilled agents who need managing. As a result, some of these marginal workers become managers of low skilled agents. This implies that matches of northern agents with sufficiently low skill necessarily become worse. However, the highly skilled workers in the North have now less competition, since some of their highly skilled competitors, particularly the ones who were previously matched with the best northern managers, have become managers. Hence, as we show formally in Proposition 1 and illustrate in Figure 4, there is a skill level $\zeta$ below which workers have worse matches, while above it northern workers improve their matches. The following proposition formalizes these results.\footnote{When the distinction between LQE and HQE is not relevant we denote variables in the world economy with a subscript $W$. We follow this notation for all variables and functions.}

**Proposition 1** Globalization has the following effects on team formation:

(i) The mass of southern workers and the mass of northern managers both increase, i.e., $z^*_S < z^*_W < z^*_N$;

(ii) (a) Southern workers that were already workers are matched with a better manager;
    (b) Southern managers that remain managers are matched with worse workers;
    (c) Southern managers that become workers are matched with a northern manager;

(iii) (a) There exists a unique threshold $\zeta$ such that all northern workers that remain workers with $z < \zeta$ are matched with a worse manager, while those with $z > \zeta$ are matched with a better manager;
    (b) All northern managers that were already managers with $z < m_W(\zeta)$ are matched with a better worker, while those with $z > m_W(\zeta)$ are matched with a worse worker.
Part (i) of Proposition 1 has implications for the effects of globalization on job creation/destruction and firm creation/destruction in each of the two countries. In particular, if we associate the number of production jobs with the measure of production workers, a corollary of the proposition is that globalization leads to production job creation in the South and to production job destruction in the North. Similarly, if firms are identified by the managers that run them, we can conclude that globalization leads to firm destruction in the South and to firm creation in the North.

Parts (ii) and (iii) of Proposition 1 also have implications for the size distribution of firms in each country. Since the best workers in the North are now matched with a better manager, the size of the largest firms in the North decreases. However, the size of the smallest firms increases since those managers are matched with better workers. Hence, the range of firm sizes in the North decreases. In contrast, all businesses that remain alive in the South shrink, since all southern managers are matched with worst workers. Therefore, the size dispersion of firms in the South also decreases. Summarizing, measured from the southern perspective, globalization leads to firm destruction and to downsizing of the surviving businesses. At the end of this section we will show that this structural change, which at first glance may seem to be damaging for the South, actually results in welfare gains for both countries.

**Corollary 1** Globalization leads to production job creation and firm destruction in the South, and to production job destruction and firm creation in the North. Furthermore, it compresses the size distribution of firms in both countries and reduces the size of all surviving southern firms.

Proposition 1 also implies that the best workers in the South are in international teams and thus work for the most productive and larger firms doing business in the South. This is also evident in Figure 4: international matches (those that obtain for southern workers with managers of quality higher than \( \alpha \)) always involve the higher skilled among the southern workers. This sorting may provide a rationale for the often-found evidence that ‘southern’ workers employed in multinational firms receive wages that are on average higher than those received by workers employed in domestic firms (see, for instance, Aitken, Harrison and Lipsey (1996) and Lipsey and Sjoholm (2004)). More specifically, a rationale for the regression result is simply that those that hold offshored jobs are unobservably more skilled than those that do not, and so they are matched with better managers. Controlling linearly for the skill of workers is unlikely to solve this problem, as earnings are the result, as we showed above, of the interaction between the skill of the worker and that of the (higher quality) international manager. In particular, Aitken et. al. (1996) only distinguish between skilled and unskilled workers and Lipsey and Sjoholm (2004) control linearly for educational attainment of workers. None of these controls eliminates the relationship between wages and multinationals generated by our framework. In sum:

**Corollary 2** The best workers in the South work for northern managers and receive higher
wages than southern workers that are employed by southern managers.

4.2 Wage Inequality and Returns to Skill

The previous subsection focused on the implications on quantities of our theory. Corresponding to those quantities there are equilibrium effects of globalization on prices. That is, workers wages and managerial earnings must be such that the matches are rearranged in the way we have described. In particular, recall that equilibrium requires that earnings of the marginal worker be equal to those of the marginal manager; and that the slope of wages is such that managers do not want to change the workers they hire.

We first need to propose a set of measures that will help us characterize the effect of globalization on the distribution of wages, and in particular wage inequality. One potential measure of wage inequality is the ratio of the wage of the highest skilled agent and the wage of the lowest skilled agent. That is, \( \frac{w(z^*_W)}{w(0)} \), when we are referring to wage inequality in the world. The problem with this measure is that it combines the level and slope effects on the wage distribution in a way that is not always straightforward to disentangle. In particular, important level effects may imply reductions in inequality in cases where the slope of the wage function is strictly increasing. To avoid this problem we focus on changes in the absolute difference between the wage of the highest skilled workers and that of the lowest skilled ones. That is, for the particular case illustrated before, \( w_W(z^*_W) - w_W(0) \). We will use this measure consistently every time we talk about wage inequality.

An alternative measure of changes in wage inequality in the context of our model is the change in the non-linear (quadratic) term in the wage equation. This term, which we refer to as the complementarity effect, measures the premium that a worker receives for possessing a particular skill level, in excess of what several separate workers would receive for possessing the same aggregate amount of skill. In other words, the term reflects the extent to which workers with different skill levels are imperfect substitutes in production, which is one fundamental measure of the extent to which they are unequal in production. This narrower measure may be more appealing from the perspective of recent empirical research in labor economics, which controls for changes in inequality driven by simple increases in the price of a unit of skill. Whatever measure of inequality is used, an increase in the complementarity effect will always unambiguously push the wage schedule towards higher inequality.

The model allows us to study the changes in the return to skill driven by the changes in matching that globalization brings about. The analysis follows quite directly from the changes in matching. First, inequality within southern workers unambiguously increases. The marginal value of workers’ skill is driven by the skill of the manager with whom they are matched, which increases for all southern workers. Thus, the sum of the complementarity and the competition effects unambiguously leads to higher returns to skill in this case. Moreover, measured within worker inequality will increase even more, since the mass of workers in the South unambiguously increases (occupational choice effect).
Proposition 2  Globalization increases within-worker wage inequality in the South. Furthermore, it increases the marginal return to skill for southern workers at all skill levels.

In the discussion above, the three effects that determine southern wage inequality go in the same direction. In general, however, the complementarity effect depends on the specification of the distributions of ability in both countries. Hence, a natural question is how robust is the direction of this effect to other assumptions on the distribution of ability. For general distributions of abilities, one can show that the complementarity effect increases with globalization as long as: 1) the distribution of abilities in the South is first order stochastically dominated by the distribution of abilities in the world (once the mass in both is normalized to one); 2) both densities are nonincreasing in ability; and 3) the range of abilities in the South is a subset of weakly smaller abilities than the range of abilities in the North. Under these conditions one can show that $z^*_{S} < z^*_W < z_N$, which is implied by first order stochastic dominance and the labor market equilibrium condition. Given this, under the stated conditions, $m'_S (z) < m'_W (z)$ and so $m_S (z) < m_W (z)$ for all $z \in [0, z^*_S]$. Hence,

$$w''_S (z) = h \frac{g_S (z)}{g_S (m_S (z))} < h \frac{g_W (z)}{g_W (m_W (z))} = w''_W (z), \text{ all } z \in [0, z^*_S].$$

Note that these conditions are only sufficient not necessary, and that they guarantee that the complementarity effect increases with integration for all ability levels in the South. If any of these assumptions on the distributions does not hold, it may be the case that the complementarity effect decreases with integration in the South. To illustrate this consider the case where the distributions in the North and South are both uniform, but the range of abilities in the South is given by $[0, 1]$ and in the North by $[\beta, 1]$ (so condition 3 is not satisfied). In this case there exists a $\beta$ such that for some $z \in [0, z^*_S]$,

$$w''_W (z) = \frac{g_W (z)}{g_W (m_W (z))} = h \frac{1}{1 + \beta} < h = w''_S (z).$$

That is, integration leads to a decrease in the complementarity effect. Clearly, the particular assumptions we have imposed in this paper guarantee the three conditions above, and so integration leads to an increase in the complementarity effect for all $\alpha$ and $h$.

Consider next the effects of globalization on northern wage inequality. Globalization decreases the quality of the match of those northern workers who are relatively unskilled and increases it for the more skilled among them. As we could expect, given that the marginal return to skill of all workers is a function of the quality of the match, the returns to skill for relatively low skilled northern workers go down, and the returns to skill for the more skilled
ones go up. The change in the marginal return to skill can be written as

\[
\begin{align*}
\sigma_{1W} - \sigma_N + \frac{h}{\alpha} z & \quad \text{if } 0 < z_p < \alpha \text{ and } h < 2(1 - \alpha) / (2 + \alpha - \alpha^2) \\
\sigma_{1W} - \sigma_N + h & \quad \text{if } \alpha < z_p < z_{WL}^* \text{ and } h < 2(1 - \alpha) / (2 + \alpha - \alpha^2) \\
\sigma_{1W} - \sigma_N & \quad \text{if } 0 < z_p < z_\alpha \text{ and } h > 2(1 - \alpha) / (2 + \alpha - \alpha^2) \\
\sigma_{1W} - \sigma_N + \frac{h}{\alpha} (z - z_\alpha) & \quad \text{if } z_\alpha < z_p < z_{WH}^* \text{ and } h > 2(1 - \alpha) / (2 + \alpha - \alpha^2)
\end{align*}
\]

The equilibrium effect on the marginal return to skill can again be decomposed in two. First, because now there is more competition from workers in the South, the baseline return per unit of skill always goes down \((\sigma_{1W} - \sigma_N < 0)\). The competition effect thus lowers the marginal return to skill. Figure 5 plots \(\sigma_{1W} - \sigma_N\) confirming numerically that in fact the term is negative for all \(h\) and \(\alpha\).

Second, since there are relatively more workers with low skill in the world than in the North, an increase in the skill level of workers increases the quality of their managers more after globalization. Thus the complementarity effect, captured by the convexity term in the marginal returns equation (i.e., the term of the form \(\text{constant} \times z\)), tends to increase the marginal return to skill. The latter effect is more valuable the higher the ability of the workers, since they are part of larger teams, as long as southern and northern workers compete for the same manager. In fact, for workers without skill, \(z = 0\), this effect is not present so the first effect has to dominate and the marginal return to skill decreases. Numerically, we can show that the second effect dominates for high skilled workers and so the marginal return to skill.
increases for them. The threshold of ability at which both effects are identical is a function of the parameters \( h \) and \( \alpha \). The lower \( \alpha \) the more southern agents are being added at each skill level where workers in both countries compete, and so the larger the set of abilities in which the complementarity effect dominates. The lower \( h \), the smaller the competition effect, and so again the threshold of abilities decreases.

In order to understand the effects on wage inequality, we need to combine this reasoning with the occupational choice effect. In particular the fact that after globalization less agents in the North become workers, which reduces wage disparity. Numerically we can conclude that wage inequality in the North increases when \( h \) and \( \alpha \) are small. Figure 6 illustrates this claim with and without the occupational choice effect. The figure plots the points for which the change in wage inequality after globalization is zero. Wage inequality in the North increases for all parameter combinations to the left of these curves. Notice how, for wage inequality in the North to increase, the main requirement is for communication costs to be sufficiently low. As mentioned in the introduction, this prediction is consistent with the findings of Feenstra and Hanson (1996b, 1999), who reported a significant positive effect of offshoring on U.S. wage inequality in the 1980s but not in the 1970s. We summarize these results below.

**Summary 1** Globalization increases within-worker wage inequality in the North if \( h \) and \( \alpha \) are sufficiently low, but it decreases it if \( h \) and \( \alpha \) are sufficiently high. Furthermore, globalization decreases the marginal return to skill of all northern workers with knowledge \( z \) below a threshold but increases the marginal return to skill of all northern workers above this threshold.

Our model also allows us to derive some conclusions on wage inequality among managers in both countries. In particular, remember that the marginal return to skill of managers is given by the size of their team. From Proposition 1 we know that all managers in the South will have smaller teams and so the marginal return to skill for them decreases. Since there are also fewer of them, within-manager income inequality in the South decreases. Again, for the North the analysis is more complicated. First, from Proposition 1 we know that the lowest skilled managers, that were in managerial positions before globalization, will have larger teams, but the best managers will have smaller teams. This implies that the return to skill of low ability managers increases and of high ability managers decreases. Second, there are more managers in the North, so the occupational choice effect leads to more income inequality among managers. This reasoning leads to the following corollary of Proposition 1.

**Corollary 3** Globalization has the following effects on within-manager income inequality and on the marginal return to skill of managers:

(i) Globalization decreases within-manager income inequality and the marginal return to skill of southern managers.

\(^{18}\)In our two parameter model it is straightforward to analyze numerically different equilibrium values for a tight grid of the whole parameter space.
(ii) Globalization increases the marginal return to skill for northern managers with knowledge $z$ below a threshold but decreases it for the rest.

### 4.3 Winners and Losers

As the previous sections have shown, globalization leads to an improvement in the matches of all southern workers, thus raising the marginal product of all of them. At the same time, it increases the competition among low skilled agents, since low skilled southern workers must now compete with northern workers for good matches. Which effect dominates the balance depends on communication costs and on the skill overlap between the two countries. If the South is relatively very unskilled ($\alpha$ is low) the gain from the new available matches is very high and the competition effect is dominated by the improvement in matching. Moreover, if $h$ is low, all agents are matched with the top agents available in the distribution, and thus opening the borders raises substantially the quality of the match. In fact, we can show numerically that for low enough $h$ and $\alpha$, the gain in the quality of the match is sufficiently high that all southern workers with skill $z = 0$ are better off. Conversely, if $h$ and $\alpha$ are sufficiently high southern workers are worse off. The change in $w(0)$ is shown in Figure 7, where the areas to the left of the curve are the parameter combinations where the change is positive. In the previous section we showed that the marginal return to skill in the South increases for all workers, hence for $h$ and $\alpha$ low an increase in $w(0)$ in the South implies that all workers (that were workers before globalization) are better off.

Northern workers suffer a direct damage from the increase in competition from the South; this is the standard market effect that would appear in any general equilibrium model, even without matching. However, as we saw in the previous subsection, the fact that the demand for northern managers increases with globalization means that some workers become managers and this raises the wage schedule for workers around them, who are matched with better managers. Thus, indirectly (through the drop in the number of northern production workers and the increase in managers) globalization increases the value of northern workers and may actually help them. What ultimately determines the balance of these two effects is, as before, the skill overlap and the costs of communication. If the country with which the offshoring is undertaken is highly skilled (high $\alpha$) this effectively limits the competitive effect. If communication costs are also high some of the southern agents will be managers, thereby decreasing further the competition effect. Thus, for high $h$ and $\alpha$, low skilled northern workers are better off after globalization. The change in the wage of northern workers with no skill, $w(0)$, is shown in Figure 8 where the areas to the right of the dark line represent parameter combinations where the change is positive. Notice that Figure 7 and 8 combined imply that there are always losers from globalization, as illustrated in Figure 8 with the light curve. We summarize below this discussion and our numerical results.
Summary 2  Globalization has the following effects on wages:

(i) Increases the wages of low skilled southern workers for low $h$ and $\alpha$, but decreases them for high $h$ and $\alpha$;
(ii) Decreases the wages of low skilled northern workers for low $h$ and $\alpha$, but increases them for high $h$ and $\alpha$;
(iii) It decreases the wage of at least some low skilled agents.

4.4 Production, Consumption, and Trade

As argued above, Theorem 1 applies also to the equilibrium of the world economy, and therefore this equilibrium is unique and efficient. As a result, since in the world economy we could always replicate the equilibrium in the closed economies of the North and the South, we know that in our framework there are always welfare gains from international offshoring. The following corollary summarizes this conclusion.

Corollary 4  Globalization increases total production in the world economy. That is, there are gains from trade.

How are these gains distributed between the countries? We want to derive conclusions on the effect of globalization on manufacturing production and consumption in both countries and, as an implication, on the balance of trade in goods. We define a country’s manufacturing output as the quantity of goods that are produced by its workers, since they are the ones that combine labor and knowledge to produce. Production in the South and North in the
world economy are thus given by

\[
Y_{WS} = \begin{cases} 
\int_{0}^{\alpha} m_W(z) \frac{1}{\alpha} dz & \text{if } h > \frac{2(1-\alpha)}{2+\alpha-\alpha^2}, \\
\int_{0}^{\ast_{WH}} m_W(z) \frac{1}{\alpha} dz = \frac{1}{1+\alpha} Y_W & \text{if } h < \frac{2(1-\alpha)}{2+\alpha-\alpha^2},
\end{cases}
\]

and

\[
Y_{WN} = \begin{cases} 
\int_{0}^{z_{WL}} m_W(z) dz & \text{if } h > \frac{2(1-\alpha)}{2+\alpha-\alpha^2}, \\
\int_{0}^{\ast_{WH}} m_W(z) dz = \frac{\alpha}{1+\alpha} Y_W & \text{if } h < \frac{2(1-\alpha)}{2+\alpha-\alpha^2}.
\end{cases}
\]

From these definitions, we know that in a HQE the South’s share of production is always \(1/(1+\alpha)\) and the North’s share \(\alpha/(1+\alpha)\), since in this equilibrium the sets of workers overlap perfectly.

In Proposition 1 (part (i)) we concluded that the number of workers increases in the South and falls in the North with globalization. Other things equal, this will in turn increase manufacturing production in the South and decrease it in the North. Other forces, however, come into play as well. In particular the other two parts of Proposition 1. Southern workers are matched with a better manager after globalization and so each of them produces more. Low skilled northern workers are matched with worse managers and produce less, while high skilled northern workers are matched with better managers. The implication for northern manufacturing production resulting from this effect is, thus, ambiguous. Overall, it is clear that manufacturing output will increase in the South, and it is also easy to show numerically that, for all combinations of \(h\) and \(\alpha\), manufacturing production actually decreases in the North (see Figure 9). Hence, in terms of manufacturing value added, the winners of globalization are the southern countries. This conclusion refers, of course, only to manufacturing production and not consumption or welfare to which we turn next.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9.png}
\caption{Change in northern manufacturing output after globalization}
\end{figure}
Consumption in the South and North in the world economy is given by

\[
C_{WS} = \begin{cases} 
\int_0^h w(z) \frac{1}{\alpha} dz & \text{if } h > \frac{2(1-\alpha)}{2+\alpha-\alpha^2} \\
\int_0^{z_{\alpha}} m_W(z) \frac{1}{\alpha} dz + \int_{z_{\alpha}}^h w(z) \frac{1}{\alpha} dz & \text{if } h < \frac{2(1-\alpha)}{2+\alpha-\alpha^2}
\end{cases},
\]

and

\[
C_{WN} = Y_W - C_{WS}.
\]

The reason why the North produces less after globalization is that manufacturing production does not take into account that managers’ rents have to be repatriated. Managers consume in their own country and they receive – as compensation for the time spent helping and communicating with workers abroad – part of the production of these workers. These rents can be substantial and in fact imply that once we take them into account consumption in both countries increases. This can be shown numerically, for all values of \( h \) and \( \alpha \), as Figures 10 and 11 show.

Figures 10 and 11 show that both countries are better off in terms of total consumption and, since utility is linear, in terms of welfare. In general, as countries globalize their economies, northern countries will be disappointed in terms of manufacturing output, but pleased in terms of consumption. Southern countries will gain under both criteria, but their consumption increase will be smaller than what their manufacturing output statistics may suggest. This difference in consumption and production outcomes has to be reflected in the trade balance of these countries. In particular, the South features net exports of manufacturing goods, while the North features net exports of knowledge services.\(^{19}\) Furthermore, if knowledge transactions are not registered as imports for the South and exports for the North, the trade balance of the northern country will be in deficit and that of the southern country

\(^{19}\)Indeed, with the expressions above and using \( m_W(z) > w(z) \), it is straightforward to show that \( Y_{WS} - C_{WS} > 0 \), which of course implies \( Y_{WN} - C_{WN} < 0 \).
in surplus. This deficit or surplus is, however, not evidence of an imbalance, but just the
result of the miss-recording of knowledge transactions. This reasoning suggests that some of
these forces may be at play when we look at the trade balance of the US with some of its
Asian trade partners, like China. We summarize these conclusions below.

Summary 3  Globalization has the following effects on manufacturing production, consumption,
and the trade balance:

(i) It increases manufacturing production in the South and decreases manufacturing pro-
duction in the North;
(ii) It increases consumption (and thus welfare) in both countries;
(iii) The pattern of trade is such that the South exports manufactures and the North exports
knowledge services;
(iv) If knowledge transactions are not reported, globalization generates a trade surplus in the
South and a trade deficit in the North.

5  Comparative Statics in the World Economy

In this section we analyze the effect of changes in communication costs ($h$) and the skill overlap
($\alpha$) on the equilibrium outcome of the integrated economy. We first study the impact of these
changes on the occupational choice decision and on the implied measures of workers and
managers in each of the two countries. We next analyze the effects on the matching between
managers and workers, which in turn determine the impact on the characteristics (quantity
and quality) of international offshoring, as well as the impact on the size distribution and
productivity of firms. Finally, we study the implications for the structure of earnings in the
world economy. In this latter respect, we show that the effect of changes in both parameters
on wage inequality can again be decomposed into the three effects we have discussed so far.

5.1 Communication Costs

5.1.1 Occupational Choice

As $h$ decreases managers can deal with larger teams. This implies that firms will be larger
in equilibrium, and therefore, for a given set of managers, the demand for workers increases.
This will lead to higher wages for the best workers and will incentivate the worst managers
to become workers. Since managers can leverage their knowledge more, given the lower
communication costs, only the most able agents in the world become managers. In a LQE all
agents in the South are workers and the decrease in communication costs implies that all of
them will remain workers. In contrast, in the North, as the set of workers increases, the set
of managers decreases. In a HQE the decrease in $h$ implies that the set of workers in both
countries increases and the set of managers decreases, even if the decline in communication
costs implies that we move from a HQE to a LQE. We formalize this result in the next proposition.

**Proposition 3** The skill of the world’s most-skilled worker \((z^*_W)\) is decreasing in communication costs \((h)\). Hence, the number of workers in the South weakly decreases with \(h\) and the number of workers in the North decreases with \(h\). The number of managers in the South weakly increases with \(h\) and the number of managers in the North increases with \(h\).

From the previous proposition we can conclude that as communication costs in the world decrease, we will see firm destruction in the North and, if communication costs and the skill level in the South are high, we will see it in the South as well. In contrast, an improvement in communication technology will lead to production job creation in the North and, if \(h\) and \(\alpha\) are large, to production job creation in the South as well.

**5.1.2 The Quantity and Quality of Offshoring**

To analyze the quantity of offshoring we need a measure that captures the extent to which firms in the economy are formed by national versus international teams. Therefore, we define the quantity of offshoring as the proportion of southern workers that work for international teams. In a LQE, all agents in the South are workers in international teams, so it follows that our measure of offshoring equals one for all \(h\) and \(\alpha\) such that \(h < 2(1 - \alpha)/ (2 + \alpha - \alpha^2)\). Conversely, in a HQE, the quantity of offshoring is given by

\[
O_W = \frac{z^*_W - z^*_0}{z^*_W}.
\]

The proportion of workers in international teams is always less than one in a HQE, but converges to one as we change parameters to approach the boundary between both type of equilibria. Hence, in a LQE there is always more offshoring than in a HQE.

In our framework it is also interesting to define what we call the quality of offshoring. This will help us analyze the characteristics of the firms that engage in offshoring. We measure the quality of offshoring as the average skill level of the workers that form international teams relative to the skill level of all southern workers. Again, in a LQE the quality of offshoring is always equal to one. Instead, in a HQE the quality of offshoring is given by

\[
Q_W = \frac{z^*_W + z^*_0}{z^*_W}.
\]

Hence, in a HQE the quality of offshoring is always larger than one and again converges to 1 as we change parameter values in a way that approaches the boundary between equilibria. It thus follows that in a LQE the quality of offshoring is always lower than in a HQE, thus justifying the names that we chose for the two types of world equilibria in our setup. Understanding the quality of offshoring is informative about the cross-sectional characteristics
of the firms that engage in offshoring. In a HQE the firms that engage in offshoring are on average larger than national firms and, therefore, are on average more productive in terms of output per capita. These firms will also pay on average higher wages than national firms both in absolute terms and in units of skill. In contrast, in a LQE the firms that engage in offshoring are, on average, smaller, less productive, and pay lower absolute and per unit of skill wages.

An improvement in communication technology implies that the quantity of offshoring increases. To see this, note that from the previous proposition \( z^*_W \) decreases with \( h \), which increases the quantity of offshoring given \( z_\alpha \) (the skill level of the best worker that works for a national firm). The reason is that, given \( z_\alpha \), more agents decide to become workers and they work for international firms. Of course, \( z_\alpha \) will change as well. As communication technology improves, there is a direct and an indirect effect on \( z_\alpha \). The direct effect increases \( z_\alpha \) since a decrease in \( h \) increases the team sizes of national firms and therefore the number of workers that work for national managers. However, an improvement in communication technology increases \( z^*_W \) which has a negative effect on \( z_\alpha \): less agents in the South decide to become managers so they hire less workers. Hence the effect of \( h \) on \( z_\alpha \) is, in principle, ambiguous. The relationship that determines the increase or decrease in the quantity of offshoring is the proportional change of \( z_\alpha \) versus the proportional change in \( z^*_W \). The next proposition shows that both the proportion and the total number of workers in international firms increases with an improvement in communication technology.

**Proposition 4** The quantity of international offshoring is weakly decreasing in communication costs \((h)\). That is, the quantity of offshoring increases with improvements in communication technology. The number of workers engaged in offshoring also decreases with \( h \).

Proposition 4 has an immediate corollary on the quality of offshoring. As the quantity of offshoring increases, the average skill level of workers in international teams weakly decreases. The reason is that a reduction in \( h \) leads to an improvement in the average quality of the workers in both national and international firms. However, the increase in the quantity of offshoring implies that the increase is larger for national than for international teams: international teams need to hire more workers and so they get workers that have lower skill than other workers in international firms.

**Corollary 5** The quality of offshoring is weakly increasing in communication costs \((h)\). That is, the quality of offshoring decreases with improvements in communication technology.

### 5.1.3 Matching and Firm Characteristics

We now turn to the effect of improvements in communication technology on the characteristics of firms. As we have discussed in the previous propositions, a decline in \( h \) implies that more
agents in the world become workers. Hence, a fall in \( h \) improves the skill level of the less skilled agent that decides to become a manager. Since these managers are always matched with workers without skill, \( z = 0 \), the match of these workers improves. In contrast, the best worker before the technological improvement was matched with the best manager in the world, \( z = 1 \), and now is matched with a worse manager since some agents became workers after the fall in \( h \). Hence, the match of these workers worsens. Even though some of the managers in the world economy are matched with less skilled workers after the improvement in communication technology, we can show that the direct effect of the technological improvement on firm size dominates other effects, thereby leading to an increase in the size of all firms in the economy. We formalize these conclusions in the next proposition.

**Proposition 5** A decrease in communication costs (\( h \)) has the following effects on matching and firm size:

(i) It improves the match for workers below a threshold skill level \( \varphi \), while it worsens the match for workers (that were already workers) above \( \varphi \);

(ii) It increases the size of all firms.

### 5.1.4 Wage Inequality and Returns to Skill

So far we have analyzed improvements in communication technology on allocations, namely, occupational choice, offshoring, and firm characteristics. These changes are, of course, the result of agents’ reactions to changes in equilibrium prices. The whole distribution of wages and rents varies with an increase in \( h \).

Concerning managerial rents, the implication of a technological change follows as a straightforward corollary of the previous proposition. Since all firm sizes increase, and the marginal return to skill of managers is equal to firm size, the marginal return to skill for managers increases with improvements in communication technology.

**Corollary 6** A decrease in communication costs (\( h \)) increases the marginal return to skill of managers.

The effect on the returns to worker skill is more complex. There are three components determining the effect of changes in \( h \) on the marginal return to skill of workers: the complementarity effect, the competition effect, and the occupational choice effect.

First, the complementarity effect is the effect on the dispersion in the price of each unit of skill. It is easy to see that the degree of convexity of the wage function in (12) and (17) increases with \( h \). Hence, as \( h \) declines, the increasing returns associated with possessing many units of skill decrease. The reason is that as \( h \) decreases, team sizes increases and, therefore, having more skill will not imply matching with a much better manager. This effect decreases the marginal return to skill of workers via a decrease in the dispersion in the price of units.
of skill. Hence, in our model this measure declines unambiguously with an improvement in communication technology.

Second, consider the change in the baseline return to a unit of ability, as captured by $\sigma_{1L}$ and $\sigma_{1H}$: the competition effect. As $h$ decreases, all managers run larger firms and so the demand for workers increases. Thus their units of skill are more valuable, which increases the average wage per unit of skill. However, the total sign of the effect on the $\sigma$’s is ambiguous, since there is also a direct effect of $h$ on the $\sigma$’s that goes in the opposite direction.

Third, the set of workers in the world unambiguously increases. As a result, this occupational choice effect increases the dispersion in the return to skill. The following corollary summarizes these results:

**Summary 4** A decline in communication costs ($h$) has the following effects on the wage structure:

(i) It decreases the dispersion of the marginal return to skill of workers;
(ii) It has an ambiguous effect on the baseline return to skill;
(iii) It increases the variance in worker skill, leading, everything else equal, to increases in wage inequality.

### 5.2 The Skill Overlap

#### 5.2.1 Occupational Choice

As $\alpha$ increases agents in the South become relatively more skilled. In a LQE an increase in $\alpha$ decreases the supply of low skilled workers and increases the supply of high skilled ones. This in turn tends to increase the size of firms (firms hiring more able workers are larger) which increases the demand for workers’ skills and decreases the demand for managers’ skills. Hence, the set of skills of agents that become workers increases and the set of agents that become managers falls: an increase in $z^*_W$. After the increase in $\alpha$, only the best agents become managers. In a LQE, an increase in $\alpha$ also leads to an increase in the number of workers in the North (all agents are workers in the South) and to a reduction in the number of managers, or firm destruction. In a HQE, an increase in $\alpha$ has similar implications but due to different reasons. After an increase in $\alpha$, the relative ability of the agents that become workers does not change given occupational choices. The mass of workers decreases and the mass of managers increases which implies that, to restore equilibrium, wages will adjust so that more able agents decide to become workers: an increase in $z^*_W$. This change leads again to an increase in the number of workers in the North. Conversely, the number of workers in the South can be shown to decrease with $\alpha$. The next proposition formalizes the result:

**Proposition 6** The skill of the world’s most-skilled worker ($z^*_W$) is increasing in the skill overlap ($\alpha$). The number of workers in the South weakly decreases with $\alpha$, whereas the number of workers in the North increases with $\alpha$. The number of managers in the South weakly increases with $\alpha$, whereas the number of managers in the North decreases with $\alpha$. 
5.2.2 The Quantity and Quality of Offshoring

In a LQE all agents in the South work for foreign managers and so the quantity of offshoring, as defined in Section 5.1.2, is always one. However, in a HQE, an increase in $\alpha$ leads to a decrease in the quantity of offshoring. The proportion of agents working in international teams decreases as agents in the South become more skilled. Intuitively, as agents in the South become more skilled, a larger proportion decide to become managers of national firms. Their higher skill level allows them to leverage their knowledge enough to earn more as managers of national firms than as workers of international firms. Of course, this has general equilibrium consequences, there is competition for national workers which raises the wages of low skilled workers thereby offsetting some of the direct effects. However, after taking all these effects into account, this simple logic still holds as we show in the next proposition.

**Proposition 7** The quantity of international offshoring is weakly decreasing in the skill overlap ($\alpha$). That is, the quantity of offshoring decreases as southern agents become more skilled. The number of workers engaged in offshoring also decreases with $\alpha$.

The previous proposition has an immediate corollary on the effect of $\alpha$ on the quality of offshoring. As the southern agents become more skilled the quality of offshoring increases: More productive and larger firms engage in offshoring.

**Corollary 7** The quality of offshoring is weakly increasing in the skill overlap ($\alpha$). That is, the quality of offshoring increases as southern agents become more skilled.

5.2.3 Matching and Firm Characteristics

Changes in the skill distribution of southern agents have distinct effects on small versus large firms. In particular, as $\alpha$ increases workers with low skill levels will now be matched with better managers, and will thus work for more productive firms. In contrast, workers with high skill levels will be matched with worse managers and so the productivity of the firms for which they work will decrease. As $\alpha$ increases, in a LQE, the ability distribution of workers increases. This leads to larger teams, which in turn implies that the number of firms in the world decreases, as discussed before. Only the most productive firms survive, or in other words, only the agents with the largest amount of skill decide to become managers. On the one hand, this implies that the worst workers in the world now work for better managers. On the other hand, the best workers before the change, that used to work for managers with $z = 1$, now work for worse managers since they are not the best workers anymore. In a HQE, an increase in $\alpha$ implies that, given wages, there is an excess supply of managers, which reduces rents and increases wages, therefore leading to more agents choosing to become workers.

These results on matching imply that the managers that ran small firms, and are still managers after the change in $\alpha$, hire worse workers. Therefore, their firms become smaller.
Conversely, the managers that used to run the largest firms now hire the workers that used to be managers. Since these workers are more skilled, these firms become larger. The same reasoning leads to the expansion of large offshoring firms and a contraction of small offshoring firms. The next proposition formalizes these claims.

**Proposition 8** *An increase in the skill overlap (α) has the following effects on matching and firm size:*

(i) It improves the match for workers below a threshold skill level, while it worsens the match for workers with skill above this threshold;

(ii) It increases the size of the largest firms and decreases the size of the smallest firms;

(iii) It increases the size of the largest offshorers and decreases the size of the smallest offshorers;

(iv) It increases the size of all non-offshorers in a LQE, but decreases the size of all non-offshorers in a HQE.

### 5.2.4 Wage Inequality and Returns to Skill

Proposition 8 shows that the effect of α on firm size is not the same for all firms. Since the size of firms is identical to the marginal return to skill of managers this implies that we cannot draw a uniform conclusion on the effect of α on the marginal return to skill of managers (as we did for communication costs). We can, however, conclude that the marginal return to skill of the worst managers decreases and the marginal return to skill of the best managers increases.

As in the case of communication costs, as the skill overlap changes, we can observe three changes in the structure of wages. First, there is the complementarity effect. Again, this effect is captured by the quadratic term in both (12) and (17), which both weakly decrease with α. The reason is that α decreases the population density of all skill levels in the South, and skill levels of workers in the North that face competition from workers in the South. This implies that a slightly better worker will now match with only a slightly better manager. The difference in the skill level of the managers that these agents work for decreases with α, since there are less workers at each skill level. Hence, an increase in α implies a drop in the dispersion in the price of units of skill. This effect is, as it was for h, unambiguous and may be what empirical studies capture when they control for different measures of ability or skill. By itself it implies a decrease in wage inequality as a result of increases in α.

Second, consider the competition effect. We can show that an increase in α increases both $\sigma_{1L}$ and $\sigma_{1H}$. The baseline return to the skill of workers in international or national firms needs to go up, since workers became more skilled in a LQE, or workers are more scarce in a HQE.

Third, there is an occupational choice effect. In a LQE an increase in α has a direct effect on the dispersion in the skill level of agents in the South, and from Proposition 6 we know
that in a HQE an increase in $\alpha$ also increases the dispersion in the skill of workers. Hence, the occupational choice effect always leads to an increase in wage inequality in the South. In the North, Proposition 6 guarantees that this effect increases wage inequality in both equilibria. We summarize this discussion below:\textsuperscript{20}

**Summary 5** An increase in the skill overlap ($\alpha$) has the following three effects on the wage structure:

\begin{enumerate}
\item It decreases the dispersion of the marginal return to skill of workers;
\item It increases the baseline return to skill;
\item It increases the variance of worker skill, leading, everything else equal, to increases in wage inequality.
\end{enumerate}

6 Conclusions

We have developed a theory of offshoring in which agents with heterogeneous abilities sort into teams competitively. In our model, the distribution of skills in the population determines both the types of teams that are formed and the wages that different agents command. We have interpreted globalization as a process that enables the formation of international teams and that thereby affects the distribution of agents available to form teams. From the point of view of the North, globalization increases the mass of agents with relatively low ability, while from the point of view of the South, globalization increases the mass of agents with relatively high ability. We have analyzed the effects of globalization on the organization of work, the size distribution of firms, and the structure of earnings of individuals, and we have illustrated how these outcomes in turn determine the patterns of production, consumption and international trade in the global economy.

We have shown that the effects of globalization interact in nontrivial ways with the state of communication technologies. For example, in our model globalization always increases within-worker wage inequality in the South, but it increases within-worker inequality in the North only if the costs of communicating knowledge are relatively low. Similarly, we have shown that the characteristics of international offshoring also depend very much on the state of communication technologies: the lower are communication costs, the higher is the amount of international offshoring, but the lower is its quality.

In order to highlight the main forces in the model, our theoretical framework has abstracted from certain aspects that are central in shaping the international organization of production. First, we have imposed that production is undertaken by two-layer teams consisting of a manager and a set of workers. It would be interesting to incorporate the possibility of both self-employment and multiple layers in our model. This would open the door for a

\textsuperscript{20}If the complementarity effect is dominated by the other two effects, our model may provide a rationale for the puzzling empirical results of Zhu and Trefler (2004), who found that, in less-developed countries, an increase in the supply of skills is associated with higher wage inequality.
study of how globalization affects the incentives to offshore or not to different countries, as well as the way it affects the hierarchical structure of firms. Second, we have present a purely technological theory of the international organization of production. A caveat of this approach is that we can explain why a northern manager might have an incentive to form a team with a group of southern workers, but we have less to say about why this international exchange of knowledge will occur within firm boundaries (i.e., within multinationals), rather than through arm’s length subcontracting or licensing. It would be interesting to incorporate contractual frictions in our setup in order to obtain a more well-defined trade-off between in-house versus arm’s-length offshoring.\(^{21}\)

Beyond providing a range of testable hypothesis concerning the relation between firm size, inequality, and technology, our model points to some new avenues for empirical research. In particular, it suggests that empirical analysis of the labor market must focus on three separate channels through which the formation of international teams affects the wage structure and economic organization. First, what labor economists have normally called compositional effect, which is equivalent to the occupational choice effect in our analysis – globalization affects the proportion of managers in the economy of each country – it increases it in the North and decreases it in the South. Second, the competition effect, typical of any equilibrium model – globalization increases the relative supply of low skilled workers in the North (and that of high skilled workers in the South), leading as a result to a drop in the unit skill prices in the North and a raise in the South. Third, the complementarity effect – globalization increases the difference in ability between the manager assigned to a low and a high skill worker, and as a result raises the difference between the marginal productivities of their skill.

Empirical studies of wage inequality, which focus on the average price of a year of education or experience, miss this third effect, which is novel in our theory. The effect could, in principle, be captured empirically using data that includes organizational variables (such as matched employee-employer data).\(^{22}\) These type of studies may contribute to illuminate empirically the question that this paper addresses: to what extent the formation of cross-country teams affects the matches between workers and managers, changing the organization of work and the returns to skill in the world.


\(^{22}\) See also the empirical strategy in Juhn, Murphy and Pierce (1993)
References


Appendix A

Proof of Theorem 1: We first show that an equilibrium of this economy exhibits positive sorting. Let $\Pi(z_m, z_p)$ denote the rents of a manager of ability $z_m$ and hires workers with ability $z_p$. From our definitions above, we know that $\Pi(m(z), z) = R(m(z))$ if $m(\cdot)$ is the equilibrium assignment function. In equilibrium we know that managers choose the ability of their workers optimally so

$$\frac{\partial \Pi(z_m, z_p)}{\partial z_p} = 0.$$ 

Totally differentiating this expression we obtain that

$$\frac{\partial z_m}{\partial z_p} = -\frac{\partial^2 \Pi(z_m, z_p)}{\partial z_p \partial z_m}.$$ 

The numerator has to be negative since managers are maximizing rents in equilibrium. To show that the denominator is positive, notice that

$$\frac{\partial^2 \Pi(z_m, z_p)}{\partial z_p \partial z_m} = \frac{\partial}{\partial z_p} \left( \frac{1}{h(1 - z_p)} \right) = \frac{1}{h(1 - z_p)^2} > 0.$$ 

Hence,

$$\frac{\partial z_m}{\partial z_p} > 0.$$ 

Since the argument is valid for all workers, we conclude that in an equilibrium allocation $m'(z) > 0$ for all workers with ability $z$.

Let $w(\cdot)$ be an equilibrium wage function, and let $W(w)$ and $M(w)$ be the equilibrium set of agents that become workers and managers respectively. Let $m(z)$ be the skill level of the manager of a worker with ability $z$ then

$$h \int_{W \cap [0, z_p]} (1 - z) g(z) \, dz = \int_{M \cap [m(0), m(z_p)]} g(z) \, dz \text{ all } z_p \in W.$$ 

Deriving with respect to $z_p$ we obtain that, as long as $z_p$ is in the interior of $W$ and $m(z)$ is increasing, as we showed above,

$$m'(z_p) = h (1 - z_p) \frac{g(z_p)}{g(m(z_p))}.$$ 

We want to prove that an equilibrium in this economy implies that $W(w)$ is a connected interval. Suppose it is not. In particular, suppose $W = [a_1, a_2] \cup [a_3, a_4]$ and $M = [a_2, a_3] \cup [a_4, a_5]$. Then, given $a_1$ and $a_3$ we know that $m(a_1) = a_2$ and $m(a_2) = a_3$. Combining these conditions with the differential equation for wages (and continuity of wages at $a_2$) and the expression above that have to hold in the interior of $W$– we can solve for a wage function $w_{13}$, a rent function $R_{13}$ and a threshold $a_2$. Similarly, given $a_3$ and $a_5$ we can solve for a wage function $w_{35}$, a rent function $R_{35}$ and a threshold $a_4$ that satisfy all the equilibrium conditions in the interval $[a_3, a_5]$. In order for $W$ and $M$ to represent equilibrium occupational choices, we have to guarantee that agents in the interval $[a_3, a_5]$ do not want
to form teams with agents in the interval \([a_1, a_3]\). The first necessary condition is that

\[ R_{13}(a_3) = w_{35}(a_3). \]

Since, if \( R_{13}(a_3) > w_{35}(a_3) \) agents with skill above but arbitrarily close to \( a_3 \) would like to become managers. If \( R_{13}(a_3) < w_{35}(a_3) \) agents with skill marginally below \( a_3 \) would like to become workers and agents at \( a_4 \) would like to hire them at a wage marginally larger than \( R_{13}(a_3) \). The next condition is that

\[ \lim_{z \to a_3} \frac{\partial R_{13}(z)}{\partial z} > \lim_{z \to a_3} \frac{\partial w_{35}(z)}{\partial z}. \] (A.1)

To show that this is the case, notice that

\[ \frac{\partial R_{13}(a_3)}{\partial z} = \frac{1}{h(1-a_2)} > 1 \]

by the envelope theorem. We will prove that the inequality above has to hold in equilibrium in two distinct cases: for the case when \( w_{13}(a_2) \geq a_2 \) and for the case when \( w_{13}(a_2) < a_2 \). Suppose that \( w_{13}(a_2) \geq a_2 \), then, since \( \partial R_{13}(z)/\partial z > 1 \) for all \( z \in [a_2, a_3] \), we know that \( R_{13}(a_3) > a_3 \) and since \( R_{13}(a_3) = w_{35}(a_3), w_{35}(a_3) > a_3. \) Then since \( h < 1 \) and \( a_2 < a_3 < a_4 < \bar{z} \leq 1 \), we can conclude that

\[ \frac{\partial R_{13}(a_3)}{\partial z} = \frac{1}{h(1-a_2)} > a_4 - w_{35}(a_3) \quad \frac{1-a_3}{a_4 - w_{35}(a_3)} = \frac{\partial w_{35}(a_3)}{\partial z}, \]

which proves condition (A.1) if \( w_{13}(a_2) \geq a_2 \). Now suppose that \( w_{13}(a_2) < a_2 \), then, since \( w_{35}(a_3) = R_{13}(a_3) \), we can rewrite the right-hand-side of the inequality as

\[ \frac{\partial w_{35}(a_3)}{\partial z} = \frac{a_4 - w_{35}(a_3)}{1-a_3} = \frac{a_4 h(1-a_2) - a_3 + w_{13}(a_2)}{(1-a_3) h(1-a_2)}. \]

Proving that condition (A.1) holds then amounts to prove that

\[ a_4 h(1-a_2) + w_{13}(a_2) < 1 \]

or

\[ a_4 < \frac{1 - w_{13}(a_2)}{h(1-a_2)}. \]

But this is trivially satisfied given that \( a_4 < \bar{z} \leq 1 \), and \( w_{13}(a_2) < a_2 \).

We have established that condition (A.1) has to hold in equilibrium, but then \( a_4 \) would like to hire \( a_3 - \varepsilon \) at a better wage than what he makes as a manager, and \( a_3 - \varepsilon \) would accept the offer. To show this, consider the rents that \( a_4 \) would get from hiring \( a_3 - \varepsilon \) at wage \( R_{13}(a_3 - \varepsilon) \),

\[ \Pi(a_4, a_3 - \varepsilon) = \frac{a_4 - R_{13}(a_3 - \varepsilon)}{h(1-(a_3 - \varepsilon))}, \]

and note that

\[ \lim_{\varepsilon \to 0} \frac{\partial \Pi(a_4, a_3 - \varepsilon)}{\partial \varepsilon} = \frac{R'_{13}(a_3) - w'_{35}(a_3)}{h(1-a_3)} > 0, \]

where the inequality comes from the result above. Hence, an allocation where \( W \) is not connected implies that there are incentives for agents to form different teams. This implies that an equilibrium requires that \( W \) be a connected interval of the form \([0, z^*]\). Hence, in equilibrium \( m(0) = z^* \) and
\( m(z^*) = \bar{z} \). An allocation that (i) satisfies the two differential equations above, (ii) satisfies the previous boundary conditions for assignment function \( m \), and (iii) yields a continuous earnings function, exists and is unique (see Garicano and Rossi-Hansberg (2003), but basically there always exists a solution and we have the same number of equations as variables to determine).

The final step is to prove that such an allocation is in fact an equilibrium. For this we need to prove that there exists an \( h^* > 0 \) such that the allocation guaranteed to exist by the above reasoning is such that \( R'(z^*) > w'(z^*) \). To show this we use a similar argument to the one above. Consider the incentives of a manager with ability \( \bar{z} \) to hire a worker with ability \( z^* + \varepsilon \). Her profits are given by

\[
\Pi(\bar{z}, z^* + \varepsilon) = \frac{\bar{z} - R(z^* + \varepsilon)}{h(1 - (z^* + \varepsilon))}
\]

so

\[
\lim_{\varepsilon \to 0} \frac{\partial \Pi(\bar{z}, z^* + \varepsilon)}{\partial \varepsilon} = \frac{w(z^*) - R'(z^*)}{h(1 - z^*)},
\]

since \( R(z^*) = w(z^*) \). Hence, in equilibrium it has to be the case that \( R'(z^*) > w'(z^*) \) in order for this term to be negative. But notice that, since \( R(z^*) = w(z^*) \),

\[
w'(z^*) = \frac{\bar{z} - w(z^*)}{1 - z^*} = \frac{h\bar{z} - z^* + w(0)}{(1 - z^*)h},
\]

which is smaller than \( 1/h \) if \( h\bar{z} + w(0) < 1 \). Hence, since \( w(0) < 1 \) (if not rents of all managers would be negative), this implies that there exists an \( h^* > 0 \) such that for all \( h < h^* \),

\[
w'(z^*) < \frac{1}{h} = R'(z^*).
\]

Hence, for all \( h < h^* \), there exists a unique competitive equilibrium in this economy. Given that markets are complete and competitive, this implies that the equilibrium allocation in the economy is Pareto Optimal.

To show that the earnings function is convex first notice that from Equation (3)

\[
w''(z_p) = h \frac{g'(z_p)}{g(m(z_p))} > 0,
\]

while the rent function is such that

\[
R''(z_m) = \frac{(m^{-1})'(z_m)}{h(1 - m^{-1}(z_m))^2} > 0,
\]

where the last inequality follows from positive sorting.

**Proof of Proposition 1:** (i) We first show that the mass of workers increases in the South. This is obviously the case in a LQE, because \( z_S^* < \alpha < z_{WL}^* \).\(^{23}\) On the other hand, for the case of a HQE, it suffices to show that \( z_S^* < z_{WH}^* \), but this follows from simple inspection of the formulas for these thresholds. That the mass of workers decreases in the North follows from \( z_{WH}^* < z_N^* \) and \( z_{WL}^* < z_N^* \), which are both clearly true from the expressions for these thresholds.

(ii) For the first statement we want to show that both \( m_S(z) < m_{WL}(z) \) and \( m_S(z) < m_{WH}(z) \)

\(^{23}\)There are parameter values for which \( z_S^* > z_{WL}^* \), but these are inconsistent with the existence of a LQE.
for all \( z \leq z_S^* \). The first inequality follows directly from \( z_{WL}^* > z_S^* \) in a LQE. Similarly, \( z_{WH}^* > z_S^* \) immediately implies that \( m_S(z) < m_{WH}(z) \) for \( z \leq z_\alpha \). For the interval \( z_\alpha < z < z_{WH}^* \), it is useful to rewrite \( m_{WH}(z) \) as \( m_{WH}(z) = h z (1 - \frac{1}{2} z) + z_{WH}^* + \frac{1}{\alpha} h \left[ z (1 - \frac{1}{2} z) - z_\alpha \left(1 - \frac{1}{2} z_\alpha \right)\right] \). The inequality then follows from \( z_{WH}^* > z_S^* \) and the fact that \( h (1 - \frac{1}{2} x) \) is non-decreasing in \( x \) for \( x \in [0, 1] \). The second statement is an immediate corollary of this first result. For the third statement, it is sufficient to show that \( m_{WH}(z_S^*) > \alpha \) for all \( z_S^* < z < z_{WH}^* \). But notice that with a couple of substitutions, \( m_{WH}(z_S^*) = \alpha + \frac{1}{\alpha} (z_{WH}^* - z_S^*) > \alpha \), and the result follows from the monotonicity of \( m_{WH}(.) \).

(iii) To prove the first part, we simply write \( m_W(z) - m_N(z) \) for each of the two equilibria. For the LQE one, this equals

\[
m_{WL}(z) - m_N(z) = \begin{cases} 
  z_{WL}^* - z_N^* + \frac{1}{\alpha} \left(1 - \frac{1}{2} z\right) & \text{if } 0 < z < \alpha \\
  z_{WL}^* - z_N^* + h \left(1 - \frac{1}{2} \alpha\right) & \text{if } \alpha < z < z_{WL}^*,
\end{cases}
\]

which is non-decreasing in \( z \), is negative for low enough \( z \) and is positive for high enough \( z \) (notice that \( z_{WL}^* - z_N^* + h \left(1 - \frac{1}{2} \alpha\right) > 0 \) is implied by \( m_N(z_{WL}^*) < 1 \)). For the HQE case, this equals

\[
m_{WH}(z) - m_N(z) = \begin{cases} 
  z_{WH}^* - z_N^* & \text{if } 0 < z < z_\alpha \\
  z_{WH}^* - z_N^* + \frac{1}{\alpha} h z_\alpha \left(1 - \frac{1}{2} z_\alpha\right) - \frac{1}{\alpha} h z_\alpha \left(1 - \frac{1}{2} \alpha\right) & \text{if } z_\alpha < z < z_{WH}^*,
\end{cases}
\]

which is again non-decreasing in \( z \), is negative for low enough \( z \) and is positive for high enough \( z \) (the latter is implied by \( m_N(z_{WH}^*) < 1 \)). The second part follows immediately, since the matching functions are monotonic and thus invertible. That is, at the same point at which workers are matched with better managers, managers are matched with worse workers. See Figure 4 for an illustration.

Proof of Proposition 2: Let us start with the last claim. The difference in the marginal return to skill in the South with and without globalization is given by

\[
\begin{align*}
\sigma_{1L} - \sigma_S + \frac{1}{\alpha} z & \quad \text{if } 0 < z < \alpha \text{ and } h < \frac{2(1-z)}{2(1-\alpha)}, \\
\sigma_{1L} - \sigma_S + h & \quad \text{if } \alpha < z < z_S^* \text{ and } h < \frac{2(1-z)}{2(1-\alpha)}, \\
\sigma_{1H} - \sigma_S & \quad \text{if } 0 < z < z_\alpha \text{ and } h > \frac{2(1-z)}{2(1-\alpha)}, \\
\sigma_{1H} - \sigma_S + \frac{h}{\alpha} (z - z_\alpha) & \quad \text{if } z_\alpha < z < z_S^* \text{ and } h > \frac{2(1-z)}{2(1-\alpha)}.
\end{align*}
\]

It is thus sufficient to show that \( \sigma_{1L} > \sigma_S \) and \( \sigma_{1H} > \sigma_S \). That \( \sigma_{1L} > \sigma_S \) follows directly from the expressions after realizing that \( z_{WL}^* > z_S^* \) and \( z_{WL}^* > \frac{1}{2} \alpha \) in a LQE. For \( \sigma_{1H} > \sigma_S \), rewrite (18) as

\[
\sigma_{1H} = \frac{h z_{WH}^* \left(1 + \frac{1}{\alpha} h z_{WH}^*\right)}{1 + h - h z_{WH}^*} + \frac{\frac{1}{\alpha} \left(z_{WH}^* - z_\alpha\right)^2}{1 + h - h z_{WH}^*} > \sigma_S
\]

where the inequality follows since the first term is increasing in \( z_{WH}^* \) and \( z_{WH}^* > z_S^* \), and the second term is positive. This result, combined with \( z_{WH}^* > z_S^* \), implies that wage inequality in the South increases with globalization. \( \blacksquare \)

\(^{24}\)Notice that \( m_{WH}(z_S^*) = h z_S^* \left(1 - \frac{1}{2} z_S^*\right) + z_{WH}^* + \frac{1}{\alpha} h \left[z_S^* \left(1 - \frac{1}{2} z_S^*\right) - z_\alpha \left(1 - \frac{1}{2} z_\alpha\right)\right] \). The two substitutions are \( h z_S^* \left(1 - \frac{1}{2} z_S^*\right) + z_S^* = \alpha \) and \( h z_\alpha \left(1 - \frac{1}{2} z_\alpha\right) + z_{WH}^* = \alpha \).
**Proof of Proposition 3:** Simple differentiation of (10) yields
\[
\frac{\partial z_{WL}^*}{\partial h} = -\frac{\sqrt{1 + h^2 (3 - \alpha)} - 1}{h^2 \sqrt{1 + h^2 (3 - \alpha)}} < 0,
\]
and differentiation of (15) results in
\[
\frac{\partial z_{WH}^*}{\partial h} = \frac{1 + h + \alpha - h\alpha - (1 + \alpha) \sqrt{1 + h^2 + \left(\frac{1 - \alpha}{1 + \alpha}\right) 2h}}{(\alpha + 1) h^2 \sqrt{1 + h^2 + \left(\frac{1 - \alpha}{1 + \alpha}\right) 2h}} < 0
\]
where the sign follows from
\[
(1 + h + \alpha - h\alpha)^2 - (1 + \alpha)^2 \left(1 + h^2 + \left(\frac{1 - \alpha}{1 + \alpha}\right) 2h\right) = -4\alpha h^2.
\]
The conclusions on the set of workers and managers follow directly from this result and the definitions of a LQE and HQE. ■

**Proof of Proposition 4:** The measure of the quantity of offshoring is given by
\[
O_W = \begin{cases} 
1 & \text{if } h < \frac{2(1 - \alpha)}{z_{WH}^* - z\alpha}, \\
1 - \frac{z\alpha}{z_{WH}^*} & \text{if } h > \frac{2(1 - \alpha)}{z_{WH}^* - z\alpha}.
\end{cases}
\]
That is, the quantity of offshoring is the proportion of southern workers in international teams. The quantity of workers engaged in offshoring is in turn given by \(\alpha\) in a LQE and by \(z_{WH}^* - z\alpha\) in a HQE.

We first prove the first statement of the Proposition, namely, that \(z_{WH}^* - z\alpha\) is a decreasing function of \(h\). Towards a contradiction, suppose that \(z_{WH}^* - z\alpha\) is a weakly increasing function of \(h\). Then, the number of workers hired by northern managers in \([\alpha, 1]\) weakly increases with \(h\). But as we show below in the proof of Proposition 5, firm size (given by \(1/h \left(1 - m_{WH}^{-1} (z)\right)\)) is decreasing in \(h\) for any skill level \(z_m\) of the manager. Hence, since the number of managers in \([\alpha, 1]\) has not changed, the number of workers in their firms must have gone down: A contradiction. Hence \(z_{WH}^* - z\alpha\) decreases with \(h\) or
\[
\frac{\partial z_{WH}^*}{\partial h} - \frac{\partial z\alpha}{\partial h} < 0.
\]

Moving to the first statement of the Proposition, notice that since \(z\alpha < z_{WH}^*\), the above inequality implies
\[
\frac{\partial z_{WH}^*}{\partial h} z\alpha < \frac{\partial z\alpha}{\partial h} z_{WH}^*
\]
and thus \(z\alpha/z_{WH}^*\) is increasing in \(h\). This in turn implies that the quantity of offshoring is weakly decreasing in \(h\). ■

**Proof of Proposition 5:** From Proposition (3), a decrease in \(h\) increases \(z_{WH}^*\), say from \(z_{WH}^*_{0}\) to \(z_{WH}^*_{1}\). From the boundary condition \(m_W (0) = z_{WH}^*\), it follows that the worst agent is matched with a better manager. Similarly, the boundary condition \(m_W (z_{WH}^*) = 1\), implies that the match for workers with \(z_p = z_{WH}^*\) worsens. It remains to show that the change in the match is a monotonic function of the skill of the worker. But simple inspection of the formulas for \(m_{WL} (z_p)\) and \(m_{WH} (z_p)\) reveals that \(\partial^2 m_{WL} (z_p) / \partial h \partial z_p > 0\) because \(z_p (1 - \frac{1}{2} z_p)\) is increasing in \(z_p\). To prove the second claim we
need to show that \( h (1 - m_W^{-1} (z_m)) \) increases in \( h \) for all \( z_m \) in \([z_W^*, 1]\). This amounts to computing these partial derivatives for each segment of each equilibrium and showing they are positive. Simple but tedious derivation confirms this claim. ■

**Proof of Proposition 6**: The proposition comes directly from differentiating \( z_W^* \) with respect to \( \alpha \), namely

\[
\frac{\partial z_W^*}{\partial \alpha} = \frac{h}{2\sqrt{1 + h^2 (3 - \alpha)}} > 0
\]

and

\[
\frac{\partial z_W^*}{\partial \alpha} = \frac{2}{(1 + \alpha)^2 \sqrt{(1+\alpha)(1+h^2)+2h(1-\alpha)}} > 0.
\]

The last two statements follow from this result as well as from the fact that \( z_W^*/\alpha \) is decreasing in \( \alpha \) (i.e., \( \partial z_W^*/\partial \alpha < z_W^*/\alpha \)), as shown in the proof of Proposition 7. ■

**Proof of Proposition 7**: Simple differentiation and equation (14) imply that

\[
\frac{\partial z_W^*}{\partial \alpha} = \frac{1}{(1 - z_\alpha) h z_W^* \alpha} \left( z_W^* \left( 1 - \frac{\partial z_W^*}{\partial \alpha} \right) - h z_\alpha (1 - z_\alpha) \frac{\partial z_W^*}{\partial \alpha} \right)
\]

\[
> \frac{1}{(1 - z_\alpha) h z_W^* \alpha} \left( z_W^* \left( 1 - \frac{\partial z_W^*}{\partial \alpha} \right) - h z_\alpha \frac{\partial z_W^*}{\partial \alpha} \right),
\]

and so \( \partial (z_\alpha/z_W^*) / \partial \alpha > 0 \) if \( z_W^* > \alpha \partial z_W^*/\partial \alpha \). But simple algebra delivers

\[
z_W^* - \left( \frac{\partial z_W^*}{\partial \alpha} \right) \alpha = \frac{v_1 (h, \alpha) - v_2 (h, \alpha)}{h (\alpha + 1)^2 \left( \sqrt{\frac{1}{\alpha+1} (2h + \alpha - 2h\alpha + h^2 + h^2\alpha + 1)} \right)}
\]

where

\[
v_1 (h, \alpha) = (\alpha + 1)^2 (h + 1) \left( \sqrt{\frac{1}{\alpha+1} (2h + \alpha - 2h\alpha + h^2 + h^2\alpha + 1)} \right)
\]

\[
v_2 (h, \alpha) = (2h + 2\alpha + 2h\alpha + h^2 + \alpha^2 - 2h\alpha^2 + 2h^2\alpha + h^2\alpha^2 + 1).
\]

Now note that \( v_1 (h, \alpha) > v_2 (h, \alpha) \) if and only if

\[
(v_1 (h, \alpha))^2 - (v_2 (h, \alpha))^2 = 4\alpha^2 h (h + 2\alpha + h^2 + \alpha^2 - 2h\alpha^2 + 2h^2\alpha + h^2\alpha^2 + 1) > 0,
\]

which is clearly true. Hence,

\[
\frac{\partial z_W^*/z_W^*}{\partial \alpha} > 0,
\]

which implies from the definition of \( O_W \) that the quantity of offshoreing is strictly decreasing in \( \alpha \).

Finally, we next want to show that the measure of workers engaged in offshoreing also decreases with \( \alpha \), that is,

\[
\frac{\partial (z_W^*/z_W^*)}{\partial \alpha} < 0.
\]
To see this, notice that

\[- \frac{\partial (z_{WH} - z_\alpha)}{\partial \alpha} = \frac{1}{2} \left( \frac{z_{WH}^* z_{WH}^\alpha}{h (1 - z_\alpha)} + \frac{\alpha}{h (1 - z_\alpha)} \right) \]

\[> \frac{1}{\alpha^2 (1 - z_\alpha)} \left( \alpha \left( 1 - \frac{\partial z_{WH}^*}{\partial \alpha} \right) - h z_\alpha (1 - z_\alpha) \right) \]

\[> \frac{1}{\alpha^2 (1 - z_\alpha)} \left( \alpha \left( 1 - \frac{\partial z_{WH}^*}{\partial \alpha} \right) - h z_\alpha (1 - \frac{1}{2} z_\alpha) \right) \]

\[= \frac{1}{\alpha^2 (1 - z_\alpha)} \left( \alpha \left( 1 - \frac{\partial z_{WH}^*}{\partial \alpha} \right) - (\alpha - z_{WH}^*) \right) > 0 \]

where we have used twice that \( z_{WH}^* > \alpha \frac{\partial z_{WH}^*}{\partial \alpha} \) as well as \( m_{WH} (z_\alpha) = \alpha \). ■

**Proof of Proposition 8:** Again, making use of the boundary conditions as well \( \partial z_{WH}^*/\partial \alpha > 0 \), it is straightforward to see that the least-skilled worker is matched with a better manager, while the ex-ante most-skilled worker is matched with worse manager. For claim (i), it remains to show that the change in the match is a monotonic function of the skill of the worker, i.e., \( \partial^2 m_W (z_p) / \partial \alpha \partial z \leq 0 \). Again this is clear from inspection of the formulas for \( m_{WL} (z) \) and \( m_{WH} (z) \) because \( z (1 - \frac{1}{2} z) \) is increasing in \( z_p \). Hence, for each equilibrium, there exist a thresholds \( \varphi_i < z_{WH}^j, j = H, L \) such that all workers with \( z < \varphi_j \) are matched with a better manager, while all workers with \( z > \varphi_j \) are matched with a worse manager.

A corollary of this result is that all managers with skill below \( m_{Wj} (\varphi_j) \) are matched with lower-skilled workers, while all managers with skill above \( m_{Wj} (\varphi_j) \) are matched with higher-skilled workers. This immediately delivers claim (ii) because remember that firm size is the inverse of \( h (1 - m_{Wj}^{-1} (z_m)) \).

For claims (iii) and (iv), notice that \( \partial^2 m_{WL} (z) / \partial \alpha \partial z = 0 \) for \( \alpha < z < z_{WH}^* \) which implies (given the effect on the boundary agents) that \( 0 < \varphi_L < \alpha \). Similarly, \( \partial^2 m_{WH} (z) / \partial \alpha \partial z = 0 \) for \( 0 < z_p < z_\alpha \) implies that \( z_\alpha < \varphi_H < z_{WH}^* \). By the monotonicity of \( m_W (z) \), these inequalities in turn imply \( z_{WL}^* < m_{WL} (\varphi_L) < m_{WL} (\alpha) \) and \( \alpha < m_{WH} (\varphi_H) < 1 \). To see that this is sufficient for claims (iii) and (iv) simply remember that the interval of managers that offshore in each equilibrium is \( (z_{WL}^*, m_{WL} (\alpha)) \) and \( (\alpha, 1) \), respectively; while the interval of non-offshorers is \( (m_{WL} (\alpha), 1) \) and \( (z_{WH}^*, \alpha) \), respectively. ■

**Appendix B**

In this appendix we formally prove that the assumption that managers hire only one type of worker is without loss of generality.

**Lemma B.1** A particular manager cannot be matched with an interval of workers.

**Proof:** Suppose instead that the assignment function is such that for some interval with positive Lebesgue measure \( F, M' (z_p) = 0, z_p \in F \). Then, from the labor market equilibrium condition,

\[\int_F g(z) \, dz = 0\]

a contradiction. Note that this is simply a restriction imposed by measurability. ■
Thus it follows that the problem of a manager who hires the set of workers $A = \{z_1, z_2, \ldots, z_n\}$ can be written as:

$$R = \max_{n,z} \sum_{i=1}^{m} n_i (z_m - w(z_i))$$

subject to $1 = h \sum_{i=1}^{m} (1 - z_i) n_i$

The following first order conditions must hold for all $i$:

$$\frac{z_m - w(z_i)}{h(1 - z_i)} = \lambda,$$
$$w'(z_i) = \lambda h.$$

Moreover, we can obtain $\lambda$ quite simply from the time constraint:

$$n_i (z_m - w(z_i)) = \lambda n_i (h(1 - z_i))$$

and taking sums on both sides and using $1 = h \sum_{i=1}^{m} (1 - z_i) n_i$, we have that:

$$\sum_{i=1}^{m} n_i (z_m - w(z_i)) = \lambda.$$

Thus $\lambda = R(z_m)$.

The first order condition says that for all worker types, the output provided by that worker type must be identical. The second condition says that the marginal value of a particular worker type must be equal to the marginal cost for the manager to not want to hire that particular worker type. These conditions immediately imply that a particular worker type may not be matched with several different managers.

**Lemma B.2** A particular worker type may not be matched with several different managers.

**Proof**: Suppose that a particular worker type maybe matched with several alternative manager types. In that case $M(z_p) = F$, for some set with positive Lebesgue measure. Take $z_m < z'_m \in F$. Then

$$\frac{\partial w(z_p)}{\partial z_p} = \frac{z_m - w(z_p)}{1 - z_p}$$

and

$$\frac{\partial w(z_p)}{\partial z_p} = \frac{z'_m - w(z_p)}{1 - z_p},$$

but

$$\frac{z_m - w(z_p)}{1 - z_p} < \frac{z'_m - w(z_p)}{1 - z_p},$$

a contradiction. ■

Intuitively, for a worker type to be matched with several different managers, it is necessary that the slope and the level of the wage function at that particular spot is identical; but, by complementarity, the marginal value of the worker matched with the better manager type is higher, so the marginal
cost (the wage slope); which yields the contradiction.

Thus the correspondence $M(z)$ is in fact a single valued function, that delivers a manager $z_m$ for each worker type $z_i$. As the next lemma shows, this function is also continuous.

**Lemma B.3** The matching function $M(z)$ is continuous.

**Proof:** The solution to (B.1) exists for all $z_m$, since it is the maximization of a continuous function over a compact set. Thus the maximizers $z_i$ and $n_i$ also vary continuously with $z_m$. $lacksquare$

**Lemma B.4** $M(z)$ is one to one — that is each manager hires strictly one worker type.

**Proof:** Suppose that it does not. In particular, suppose it hires two worker types, $z_1$ and $z_2$. The first order condition of the manager problem implies that

$$w(z_1) - w(z_2) = \lambda h.$$ 

Hence the manager is indifferent in hiring any worker $z \in [z_1, z_2]$ if

$$\hat{w}(z) = w(z_1) + \lambda h (z - z_1).$$

And, since the manager is hiring $z_1$ and $z_2$, we know that

$$w(z_2) = \hat{w}(z_2) = w(z_1) + \lambda h (z_2 - z_1).$$

Since the manager is hiring only these two types of workers, we know that the wage function in between has to be larger than $\hat{w}(z)$ (if it was smaller the manager would prefer to hire any of these workers, and if it was equal it would hire them as well). Hence the wage function has to be such that

$$w(z) > \hat{w}(z) \text{ all } z \in [z_1, z_2].$$

Since $w(z_2) = \hat{w}(z_2)$, the slope of the wage function has to be higher than $\lambda h$ for some interval $H$ and lower for some other interval $L$. Now take a manager $z_m^H$ hiring workers in $H$, and let the slope of the wage function at the ability of the workers he hires ($z_m^H$) be given by $s > h\lambda$. Then, $z_m^H$ is indifferent in hiring workers with different abilities if wages are such that

$$\hat{w}^H(z) = w(z_m^H) + s (z - z_m^H).$$

But notice that this implies that

$$\hat{w}^H(z_2) > w(z_2) = \hat{w}(z_2),$$

since $w(z_m^H) > \hat{w}(z_m^H)$. Hence $z_m^H$ will have an incentive to hire the workers of type $z_2$, and offer them a wage of $w(z_2) + \epsilon$ for $\epsilon$ small enough. A contradiction with the assertion that $z_m$ hires workers of type $z_2$. Hence since the first lemma above prevents managers from hiring an interval of workers, and the argument goes through for arbitrary $z_1$ and $z_2$, managers hire only one type of workers. $lacksquare$