Crisis Management in Canada: Analyzing Default Risk and Liquidity Demand during Financial Stress

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Using detailed information from the Canadian interbank payments system and from liquidity-providing facilities, we find that despite sustained increases in market-rate spreads, the increase in banks' willingness-to-pay for liquidity during the 2008/2009 financial crisis was short-lived. Our study suggests that high-frequency distress indicators based on demand for liquidity offered by central banks can be complementary, and perhaps even superior, to market-based indicators, especially during times and in markets where uncertainty in the economic environment may lead to lack of meaningful information in prices due to absence of trading.

This paper uses transactions-level data from the Canadian unsecured interbank market and term liquidity auctions to evaluate the extent and dynamics of counterparty risk and to estimate market participants' willingness-to-pay for liquidity during the global financial crisis of 2008/2009. The estimated willingness-to-pay for liquidity remains at low levels throughout almost the entire crisis and there appears to be little change in perceived counterparty risk. While market-based measures of default and liquidity risk increased, transactions data provides little support for the narrative that banks faced increasing costs of short-term borrowing. As the size of the COVID-19 pandemic became apparent, and market-based measures of default and liquidity risk once again spiked, central banks injected unprecedented amounts of liquidity—super-sized versions of the facilities offered in 2008/2009. We highlight lessons learned from Canada during the global financial crisis and the value of transactions-based measures of financial stress.

In periods of financial distress, bank supervisors and central banks are at the front lines. These institutions, like all market participants, attempt to determine which institutions are under distress as well as understand the systemic implications of the event. Important questions are whether to provide cheap liq-

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uidity, how to provide it, and for how long.[1] During these periods, market prices (or sometimes only the available dealer quotes) such as those on credit default swaps (CDS) or interbank spreads (e.g. LIBOR/EURIBOR-OIS, or CDOR-OIS in Canada), are used as indicators of financial market stress.[2,3] This is because financial markets should be able to gather large amounts of information efficiently and reveal it through prices. Overall, this appears to be the case. Take for example CDS prices. Following a surge in the prices of CDS contracts on General Motors' bonds, GM needed government intervention. In addition, some American and European banks with skyrocketing CDS prices indeed ended up facing severe difficulties (e.g. Merrill Lynch, RBS, or Dexia). At the same time, however, the insurance cost (prices of CDS contracts) for many banks' bonds increased substantially, and often stayed at high levels, without any (ex post) evidence of financial difficulties. This was the case in Canada.

Given the illiquidity of the Canadian CDS market, this might not be surprising. Small movements in beliefs can have large impacts in illiquid markets. However, as CDS prices increased so did the CDOR-OIS one-month and three-month interbank spreads, also indicating an increase in (market-wide) counterparty and/or liquidity risk.[4] The high prices of CDS contracts suggested markets expected banks to potentially face solvency issues. In addition, interbank spreads for shorter maturities suggested more immediate liquidity risks. As highlighted in Bank of Canada documents, e.g. Zorn, Wilkins and Engert (2009) and Enenajor, Sebastian and Witmer (2010), the increase in interbank spreads was especially concerning. Our findings using bank-level transactions, however, reveal that in the short-term there was very little change in perceptions of counterparty risk or in the willingness to pay for short-term (3 month or less) liquidity.

Knowing the true levels of bank default risk (over various time horizons) and

[1]Financial markets are concerned with three main sources of liquidity: (i) market liquidity, which refers to an ability-to-trade with little price impact, (ii) funding liquidity, which refers to the ability of solvent institutions to obtain immediate means of payment to meet obligations, and (iii) central bank liquidity, i.e., access to money from the central bank. Our focus is on funding and central bank liquidity.

[2]Similarly to LIBOR and EURIBOR, CDOR, or Canadian Dollar Offered Rate, is determined daily from a survey of bid-side rates provided by seven principal market-makers, including the major Canadian banks. It is administered and calculated by Thomson Reuters. One important difference, however, is that CDOR represents a bank's lending rate whereas LIBOR represents the bank's borrowing rate. OIS, an overnight index swap, is an over-the-counter derivative in which two parties agree to exchange, or swap, for an agreed period, a fixed interest rate determined at the time of the trade for a floating rate that will vary over time. Given that the OIS market is highly liquid and there is no exchange of principal, the market is almost all about expectations of future rates and not credit or illiquidity. Finally, a CDS contract is a financial agreement where the seller of the CDS compensates the buyer in the event of default.

[3]Christensen et al. (2015) review the Bank of Canada's approach to assessing market vulnerabilities. In addition to using real-time market rates, the Bank of Canada uses balance sheet measures of liquidity. Post-crisis, central banks have developed numerous early warning indicator models. See for example Christensen and Li (2015), MacDonald and v. Oort (2017), and Duprey and Roberts (2017).

[4]A number of other measures (such as e.g., CoVaR) also pointed to bank-stress. Figure 2 defines and plots an index of five such financial measures for Canada, US, UK, and Euro area. These market measures suggest that bank resiliency fell substantially in all areas. CDOR, however, plays a key role in pricing consumer and business loans as well as tens of billions of dollars in derivatives trading. It is therefore the most widely used indicator for stresses in the banking industry.

their demand for liquidity is important because it affects the policy response. The response in Canada to the widening of market-based spreads was large interventions. On December 12, 2007, for example, the Bank of Canada put out a joint press release with the Federal Reserve, Bank of England, Swiss National Bank, and European Central Bank saying that they were introducing measures to alleviate pressures in short-term funding markets.[5] The first ever term cash auction was the following day. Auction activity further increased in the Spring of 2008 following the collapse of Bear Stearns. The federal government also introduced new rules/programs to relax collateral requirements and therefore free up marketable securities for secondary market liquidity. Given the movements in the CDS prices and in the interbank market rates, these moves all appear to be well warranted.

Figure 1, for example, depicts the spread between unsecured interest rates and swap rates in Canada between 2006-2009. The spread between the Canadian Dealer Offered Rate (CDOR) and overnight index swap (OIS) rate can be thought of as a measure of tension in the Canadian interbank money market. The pattern was similar in the U.S. and EURO markets: immediately after August 2007 the spreads in money markets substantially widened and culminated after the collapse of Lehman Brothers. Spreads, however, remained above their pre-crisis levels throughout 2009, with banks willingness-to-pay for longer maturity loans somewhat higher than shorter maturities. The informational content of these spreads is, however, rather questionable as the unsecured rates (such as CDOR for Canada, EURIBOR for Euro-zone or LIBOR for UK) are based on a limited survey of banks' hypothetical ability to transact. Furthermore, several recent antitrust actions were brought against these banks accusing them of manipulating these rates (c.f. Snider and Youle (2014)).[6]

We argue in this paper that indicators for Canadian banks' willingness-to-pay for liquidity (for terms ranging from overnight to three months) and the amount of counterparty risk incorporated in the overnight unsecured interbank market, paint a somewhat different picture than market rates. Such a divergence can arise when the CDS market is rather illiquid, in which case CDS prices are often substituted with quotes without evidence that there indeed is any market at those quotes. In addition, the CDOR-OIS spread is an aggregate measure and might thus not capture the heterogeneity in the willingness-to-pay for liquidity among

[5]See Chailloux et al. (2008) and Lavoie, Sebastian and Traclet (2011) for a review of central bank responses to the crisis.

[6]There is already a broad literature studying the impact of the financial crisis on term lending in the United States. This is likely due to the problems being most obvious in this market. The Federal Reserve introduced three main facilities during the financial crisis: (i) Term Auction Facility, (ii) Term Securities Lending Facility, (iii) Primary Dealer Credit Facility. The majority of studies focus on the effectiveness of these facilities in bringing down spreads in the interbank market for term loans. The large spreads were caused by two factors: counterparty risk and liquidity demand. Taylor and Williams (2009) focus on counterparty risk and find that TAF had no effect on the interbank market. Wu (2008) separates out both risks and claims that TAF was effective in satisfying banks liquidity demand, which did reduce the stresses of the interbank market.

individual banks; and as mentioned above, it suffers from being a survey-based measure.

The lesson from our analysis, therefore, which is true beyond Canada, is that there can be instances when market prices (or quotes) hide important details and policy makers should try to use other available information to measure their response. Ideally, this additional information should be at the bank-level and directly related to bank's payoffs. In Canada, the transactions data suggest that bank liquidity was more benign than public measures of bank risk and less intervention may have perhaps been justified. In other countries this type of information could also be valuable in determining the appropriate response of regulators and central banks. This is important given that the costs of an overly aggressive response, in addition to the pure cost of providing the liquidity ($114 billion of liquidity support was provided in Canada), include distorting bargaining power in the interbank market to favor some institutions over others (see Allen et al. (2016)), as well as potentially exacerbating too-big-to-fail. For example, banks may take correlated risks in anticipation of future liquidity support.[7]

We begin by analyzing the demand for liquidity on an individual bank level by using data from the Bank of Canada's Receiver General auctions for very short-term liquidity (typically 1-3 days) as well as term purchase and resale agreement auctions (PRA). These latter auctions provided banks with cheap short-term liquidity (typically 1-3 months) during the crisis in exchange for collateral. Our analysis of liquidity demand focuses on heterogeneity in risk and bids in the Bank of Canada's liquidity facilities. We document that following the collapse of Lehman Brothers in September 2008, there was a short period of distress, in which Canadian banks offered a substantial spread over the reference interest rate (OIS) in order to get a short-term loan from the Bank of Canada. As illustrated in Figure 3, Canadian banks bid aggressively for liquidity provided by the Bank of Canada only in October and November 2008. We find the same pattern in auctions of liquidity with 1-day, 1-month and 3-months maturities. This higher offered spread was a consequence of a temporary increase in the willingness-to-pay for liquidity, which was likely caused by more difficult (or more expensive) access to liquidity on the interbank market. This period of distress, however, lasted less than two months, after which the banks returned to their pre-Lehman collapse behavior. This differs markedly from the situation in Europe – Cassola, Hortaçsu and Kastl (2013) analyze the liquidity auctions of the European Central Bank during 2007 and document a persistent change in bidding behavior by market participants. In Europe, banks were willing to pay rates even in the primary market (the liquidity auctions) that were much higher than OIS.[8]

[7]There is a long line of research on strategic complementarities. See for example Farhi and Tirole (2012) and Cooper (1999) for a review of the macroeconomic literature on strategic complementarities.

[8]Evidence in Eisenschmidt and Tapking (2009) suggests this continued throughout 2008 until rule changes in October led to a single-price auction where anybody could borrow as much as they like at the posted rate. The rule change was due to the increased demand for liquidity by European banks.

The situation in Canada was remarkably different. While interbank interest rate spreads in the Canadian system followed a similar pattern as their European counterparts, the Canadian banking system showed very limited signs of stress (or increase in liquidity demand) in 2007 and in the first half of 2008. In particular, the Canadian banks were much less willing to pay a premium above OIS to obtain liquidity from the Bank of Canada. Figure 3 depicts the aggregate bidding functions in each auction that the Bank of Canada conducted before September 2008. It suggests that virtually all banks judged that even if they would not have their demands in these auctions satisfied, they could secure the liquidity elsewhere and hence were bidding at, or very close to, the reference overnight swap rate.

In addition, using transactions level data on the overnight market, we find that the overnight market remained quite active throughout the crisis – total loans transacted stayed virtually unchanged while prices actually fell. This indicates that participants did not believe that there were significant liquidity or counterparty risks. Similarly, Afonso, Kovner and Schoar (2011) find that the overnight Feds Fund market remained active following the collapse of Lehman Brothers, although they do find evidence of increased counterparty risk. We argue that the impact of various liquidity-providing actions undertaken by the cental bank and federal government during 2008 might have led to a surplus of liquidity in the market, resulting in overnight unsecured loans transacting even below the target rate.

We are further able to explore the question of counterparty risk using data on intraday credit lines granted between market participants. This is the major advantage of using Canadian data relative to U.S. or European data. We take advantage of the fact that the payment system is one where banks provide each other intraday credit lines (rather than participants receiving intraday credit from the Federal Reserve, for example in Fedwire), and therefore face manageable default risk. We can analyze changes in these credit lines to measure change in counterparty risk. We find that there are no significant changes in the credit lines during the crisis, even after the Bank of Canada introduced changes to the payment system that encouraged financial institutions to lower credit limits to riskier banks. This provides further evidence that there was no increase in counterparty risk during the financial crisis for Canadian financial institutions.

The rest of the paper proceeds as follows. In section I we describe the Canadian markets for and present some summary statistics related to counterparty risk and the demand for liquidity. Section II outlines our stress-measure. In section III we go through the results of our analysis and section IV concludes.

I. Canadian Markets

Our goal is to provide evidence that, in the Canadian banking sector, there seems to have been very little perceived counterparty risk even though the prices of

CDS contracts and interest rate spreads signalled otherwise. In order to contrast the riskiness of Canadian financial institutions implied by public signals to private signals, we now describe in detail the various data sources that we employ in our analysis. In particular, we link together detailed data on banks' bilateral trades with other banks, banks' behavior in various types of liquidity facilities, information about their balance sheets, and the prices of their CDS contracts.

A. *Canadian Banking Sector Background*

The Canadian banking sector is highly concentrated, with 90 per cent of the industry's assets held by the six largest banks, worth a total of $2,605 billion as of March 2010.[9] The Canadian banking landscape also includes hundreds of small and large cooperatives/credit unions, foreign bank branches, and trust companies. Larger Canadian financial institutions have an informational advantage over smaller institutions stemming from the tiering of the payments system and securities dealings. Tiering in a payments network refers to an organizational structure whereby some financial institutions participate directly ("direct" clearers) while other financial institutions participate indirectly. Afonso, Kovner and Schoar (2011) suggest that tiering might be the reason why they do not find liquidity hoarding in Fedwire during the financial crisis, while Acharya and Merrouche (2012) do find liquidity hoarding in the British Pound money market.

B. *Large Value Transfer System (LVTS)*

The Bank of Canada implements monetary policy not through reserve requirements, but instead through the Large Value Transfer System (LVTS). LVTS is a payment and settlement system operated by the Canadian Payments Association through which all interbank trades have to be settled by the end of the trading day - and any potential short or long positions must be settled by the appropriate trade with the Bank of Canada at rather unfavorable interest rates. During our period of study LVTS included 14 direct participants plus the central bank.[10]

Like real-time gross settlement systems, which are used in almost all countries, finality of payment sent through LVTS is in real-time. If payments are sent in Tranche 1 (T1), settlement is also in real time. However, if payments are sent via Tranche 2 (T2), settlement occurs on a multilateral net basis at the end of the

[9]These banks are the Bank of Montreal, Bank of Nova Scotia, Banque Nationale, Canadian Imperial Bank of Commerce, Royal Bank Financial Group, and TD Bank Financial Group.

[10]The direct participants include the Big 6 banks, Laurentian Bank, foreign banks with branches in Canada (State Street Bank, Bank of America, BNP Paribas, HSBC), the largest co-operative movement in Canada (Caisse Desjardins) and a provincially owned deposit-taking institution (Alberta Treasury Branches) as well as a credit union consortium (Central 1 Credit Union). Any deposit-taking institution and member of the Canadian Payments Association (CPA) can be a member of LVTS so long as they maintain an account with the Bank of Canada and have the facilities to pledge collateral for LVTS purposes. Deposit-taking institutions that are not members of LVTS must send (or receive) their payments through one of the direct participants.

day. In other words, positions are netted at the end of the day across multiple counterparties. In normal times, participants prefer to send payments through T2 because the collateral requirements are much weaker than in T1. T2 operates by banks extending bilateral credit lines (BCL), which may be adjusted at any time. Providing credit lines, however, is not costless. The banks have to post collateral proportional to the extended credit lines with the Bank of Canada. The size of the credit lines extended towards a potentially failing bank determine a bank's exposure in case of a failure of that particular bank. Therefore, T2 is a payment system where the survivor pays (whereas the defaulter pays in T1).

Our data consist of daily credit lines and T1 and T2 interbank transactions, covering the time period between 03/01/2004 and 08/31/2009. As expected, when the need arises, credit lines are adjusted upwards: conditional on an adjustment, the mean credit line is \$841 million (with standard deviation of \$757 million) (henceforth, all figures are in Canadian dollars). A striking feature of this data is reciprocity: a bank very often extends to and obtains from another bank the exact same credit line. While the amount of the credit line varies depending on the pair of banks, reciprocity is regular. The mean credit line is \$400 million with a standard deviation of \$484 million. To illustrate the level of reciprocity, the mean credit line (over the whole time period) extended by bank labeled S in our sample to bank N is \$758.7 million, while the mean credit line from N to S is \$758.9 million. The standard deviations are \$62 million and \$67 million, respectively. The credit lines between these two banks are adjusted during the day in about 2% of cases. This description is representative of the other pairs in our data.

C. Counterparty Risk Implied by CDS Prices

Credit default swap (CDS) contracts rose to prominence during the financial crisis with the U.S. Treasury's bail-out of AIG. These types of contracts had been traded since the early 1990s and publicly listed prices of CDS contract on a bank's debt, for example, could be used to measure counterparty risk.[11]

A CDS contract is essentially an insurance contract: the buyer (who may or may not actually hold a bond issued by the bank whose CDS contract he/she purchases) is protected against default of this bank over the duration of the contract. In the event of default (or other triggering event), the seller of the insurance contract has to pay either the face value of the bond in exchange for it or the difference between the face value and the recovery value of the bond. Since 2009, CDS contracts have been standardized to require the seller of the insurance to pay the difference between the face value of the bond and its market price as determined in an auction run post-default (or other qualifying event) occurs.

[11]Even before the collapse of AIG some research had started to focus on the counterparty risk of the insurer in the CDS market. See for example Thompson (2010) and Arora, Gandhi and Longstaff (2012).

Using the standard formula (see e.g., Hull (2007)):

$$Pr(default)_T = 1 - e^{-\frac{CDS \times T}{1-R}}$$

we can recover the risk-neutral default probabilities implied by the CDS spread to OIS for $T = 5$ years and constant recovery rate, R. Assuming that the time series indeed records prices (and not just quotes), Figure 4 shows the default probabilities of 5-year credit default swap contracts for 6 banks in our sample, which combines data from Bloomberg and Markit. As elsewhere, the market perception of default risk of Canadian banks captured in the default probabilities of the CDS contracts started increasing in the second half of 2007 and peaked in the last quarter of 2008 following the collapse of Lehman Brothers in September 2008. For the largest institutions the implied default probabilities (assuming 40% recovery rates[12]) went from close to zero to over 15%. Given the substantial increase in default risk implied by the CDS contracts on the banks in our sample (i.e., the risk of a bank becoming insolvent in the next 5 years), we might expect an increase in liquidity hoarding during the crisis and also an increased perception of counterparty risk even over shorter horizons. In fact, one could argue that at the onset of the crisis and the unavailability of the usual sources of liquidity, the short term default probability might have been perhaps of an order of magnitude similar to that implied by the CDS.

D. Liquidity

Financial institutions manage their daily liquidity needs through market operations and their interactions with the Bank of Canada. The Bank of Canada's primary objective in providing liquidity intraday and overnight to the banking system is to reinforce the target rate. This target rate, as well as the one-month and three-month overnight index swap rates, are plotted in Figure 5. The main facility through which this is achieved, is LVTS, occasionally supplemented by open market operations. In addition, the Bank provides liquidity to LVTS participants facing shortfalls in their end-of-day settlement balances at the Bank Rate, and in rare cases financial institutions can request Emergency Lending Assistance if they are facing serious liquidity problems. The Bank of Canada also manages the federal government's cash balances by holding twice-daily 'Receiver-General' auctions. Financial institutions can access short-term loans in these auctions. We document that during the crisis financial institutions accessed these facilities in addition to new ones created during the crisis. In particular, as term funding costs appeared to be increasing, the Bank of Canada created term liquidity facilities. We therefore present the different options for financial institutions searching for

[12]Mora (2012) documents that on average in the U.S. recovery rates are 39.3%. In banking, more specifically in the category 'FIRE' (banks, insurance real estate), the historical average is 24.6%. By using 40% we are being conservative.

liquidity, with terms ranging from overnight to three months.

Receiver General Auctions. — The Bank of Canada is the Canadian government's fiscal agent and as such manages the Receiver General account from which the balances required for day-to-day operations are drawn. To manage its cash balances, the Bank of Canada holds twice-daily multi-unit sealed bid discriminatory price auctions through which it auctions off short-term loans, whereby it secures short-term interest income on its revenues. All auctions are held at 9:30am and at 4:30pm. The morning auctions can be both secured or unsecured and the terms can be more than one day. We focus solely on the unsecured morning auctions. The maturity is between 1 and 21 days, with 93% of offers in our sample being for loans with less than 7 days. Our data set included bids (size, rate) from the morning Receiver General auctions between 01/02/2007 and 04/20/2009. These auctions have been analyzed in Chapman, McAdams and Paarsch (2007) who find that bidders' behavior is reasonably approximated by a Bayesian Nash Equilibrium. Summary statistics for the auctions are provided in Table 1. The average number of bidders is 7 and the number of submitted bids averages less than 1.5. The average amount offered in an RG auction was \$1.07 billion and the yield bid (expressed as a spread over OIS) is seventeen basis points, but with a large range of bids. We use these bids to estimate willingness-to-pay (WTP) for very short-term liquidity.

Table 1—: Data Summary for Receiver General Auctions

Summary statistics are from daily morning RG auctions between January 2, 2007 and April 20, 2009.

	Mean	Min	Max	StdDev
Active bidders in an auction	7.4	1	17	3.24
Number of submitted bids	1.41	1	4	0.82
Yield bid (normalized by OIS)	0.17	-4.25	0.51	0.27
Quantity Bids (as a share of supply)	0.38	0.00	1.00	0.34
OIS rate	2.86	0.25	4.25	1.38
Offered Deposits (billion CAD)	1.07	0.03	3.70	0.55

Term Purchase and Resale Agreement Auctions (PRA). — As the financial crisis unfolded the Bank of Canada introduced cash auctions to alleviate potential stresses. The Bank conducted two 1-month repo auctions in December 2007.[13] These and all future PRA auctions were multiple-yield competitive auctions (i.e.

[13]The broad principle that underlined the liquidity facility was to maintain liquidity in the financial system while minimizing distortions (Selody and Wilkins (2010)).

discriminatory auctions). The amounts of the initial auctions were relatively modest (just \$2 billion). Initially, only primary dealers could participate in the auctions.[14] The majority of primary dealers are the securities subsidiaries of a larger parent company. RBC Dominion Securities, for example, is the securities subsidiary of Royal Bank Financial Group. The Big 6 banks and the largest credit union, Caisse Desjardins all participated in the term PRA auctions via their securities dealers.[15] Almost all of the primary dealers are members of LVTS (via the parent company), but some members of LVTS are not primary dealers. The set of eligible collateral was of high quality assets – it included securities issued or guaranteed by the Government of Canada and of provincial governments as well as bankers' acceptances and bearer deposit notes with remaining maturities of less than 180 days.

After the two December 2007 term auctions, problems in funding markets quickly faded and the Bank of Canada stopped providing term liquidity. Following the collapse of Bear Stearns in March 2008, however, the Bank reintroduced term PRAs, this time on a biweekly basis but with the same auction rules. Throughout the spring and summer the Bank auctioned off \$1 to \$2 billion of cash at 1-month terms. The Bank also expanded the set of eligible collateral in LVTS so that participants could more easily secure funding in the term PRA auctions as well as in other markets.

Following the collapse of Lehman in September 2008, the Bank of Canada added repo auctions with 3-months maturity and held them on a weekly basis. These auctions had the same format as previous ones. For monetary policy reasons the Bank of Canada also introduced 6- and 12-month loan-maturity starting in April 2009. The supply of cash in each auction was either \$2 or \$4 billion. The Bank of Canada once again expanded the list of eligible collateral[16] By accepting a wider range of assets as collateral the Bank of Canada in effect reduced the illiquidity premium of those assets.

Consistent with the G7 Action Plan, on October 14th the Bank modified its liquidity facilities further so as to provide exceptional liquidity to the financial system. First, the Bank substantially increased the size of the term auctions to as much as \$12 billion. At its peak the amount of liquidity the Bank of Canada auctioned was 2% of the value of total bank assets. There were 71 term PRA auctions in total. In the U.S. the peak was about 7% and in the Euro area the peak was 5%, again of total banking system assets.

[14] This includes RBC Dominion Securities, CIBC World Markets, Scotia Capital, BMO Nesbitt Burns, Toronto-Dominion Bank, Banque Nationale Financière, Valeurs Mobilières Laurentienne, Desjardins Valuers Mobilières, HSBC Securities Canada, Casgrain & Co., Deutsche Bank Securities, and Merrill Lynch Canada.

[15] The majority of the primary dealers are large players. We do not know the assets of Casgrain because it is a private company, but for the other players Laurentienne is the smallest with \$22.6 billion in assets in 2010; RBC is the largest Canadian bank with \$655 billion in assets in 2010.

[16] For a full list see `http://www.bankofcanada.ca/en/notices_fmd/2008/not230908.html`.

Second, LVTS members that were not primary dealers became eligible to participate in the repo auctions. This includes ATB, Credit Union Central of Canada, Bank of America, BNP Paribas, and State Street Bank. Thirdly, the Bank announced on October 17, 2008 that (eligible) LVTS participants would be temporarily allowed to pledge their non-mortgage loan portfolio (NMLP) as collateral for LVTS and Standing Liquidity Facility purposes. This measure was intended to free-up more marketable securities to support borrowing or be used in the term PRA.[17]

Table 2 offers some summary statistics of our sample of the bids in the term PRA auctions. The average number of bidders is 9 and the number of submitted bids averages less than 2. The average amount issued in PRAs was $4.76 billion and the yield bid (expressed as a spread over OIS) is quite low, four basis points, with a minimum of negative 58 basis points and a maximum of 92 basis points. We use these bids to estimate banks' WTP for liquidity during the crisis. The reason is that since the WTP for liquidity obtained from the Bank of Canada should be equal to the opportunity cost of obtaining liquidity in the interbank market, studying the dynamics of the WTP allows us to study counterparty risk. Banks that are perceived by their counterparties as riskier would be required, in the interbank market, to pay a premium for any loan that is not backed by a risk-free collateral, and thus would be willing to pay more.

We now move on to estimate the WTP for liquidity using the bids in the liquidity auctions described above. We then combine this data with transactions from the interbank market and CDS market to analyze the mutual relationship between rates in these markets. Finally, we present evidence that the credit lines extended in the LVTS exhibit very little movement during the crisis, even for those banks with CDS contracts trading at significantly high premia.

II. Measuring stress using banks' bids for central bank-provided liquidity

Like many central banks during the crisis, an important question for the Bank of Canada was how to quantify the demand for term liquidity. To get at this, we use data on bids in auctions of liquidity with maturity of 1-3 days administered by the Bank of Canada on behalf of the Receiver General and of 1- or 3-month term PRA operations. As is standard in the empirical auction literature, to recover banks' true WTP from the observed bids, we assume that they employ bidding strategies that are consistent with a Bayesian Nash Equilibrium. For interested

[17] Chailloux et al. (2008) discuss cross-country differences in collateral policies pre-crisis and how this affected central bank responses during the crisis. Canada, like the U.S. had a much narrow set of eligible collateral for its open market operations and standing lending facilities than the ECB. During the crisis changes to collateral requirements was one way the Bank of Canada could free of liquidity. The fact that the central bank was taking on more credit risk – bad collateral was clearly driving out good collateral (a form of Gresham's Law) – was less of a concern that getting liquidity to the market. Central banks would point out, however, that they took significant haircuts were applied to this new collateral.

Table 2—: Data Summary for Term PRA Auctions

The first Term PRA auction was held on December 12, 2007. Our data stops on April 21, 2009, when the facility was used for monetary policy in addition to liquidity reasons and a minimum bid rate equal to the target rate of 25 basis points was set.

	Mean	Min	Max	StdDev
Active bidders in an auction	9.3	5	12	1.86
Number of submitted bids	1.87	1	5	0.88
Yield bid (normalized by OIS)	0.04	-0.58	0.92	0.15
Quantity Bids (as a share of supply)	0.14	0.0025	0.25	0.09
OIS rate	1.78	0.24	4.26	1.26
Issued Amount (billion CAD)	4.76	1	12	2.85

readers, Appendix A lays out the formal model and our estimation approach.[18]

In the context of European short-term loan auctions, Cassola, Hortaçsu and Kastl (2013) show that accounting for the strategic component of bids is empirically important, since many banks increase their bids during the crisis not necessarily because of their increased WTP (for example due to a higher need for liquidity), but rather as a best response to more aggressive behavior by some of their rivals. Using the same method as Cassola, Hortaçsu and Kastl (2013), we recover the (opportunity) cost of funding (or WTP) from the observed bid from the equilibrium relationship:

$$v\left(q_k, \theta_i\right) = b_k + \frac{\Pr\left(b_{k+1} \geq P^c\right)}{\Pr\left(b_k > P^c > b_{k+1}\right)}\left(b_k - b_{k+1}\right), \tag{1}$$

where P^c is the market clearing price (random from the perspective of each bidder); q_k is the quantity demanded at step k, and b_k is the associated bid for that quantity. θ_i is a random variable summarizing the private information held by bank i before bidding. This relationship basically says that in equilibrium of a discriminatory auction, bidders will shade their bids so as to trade-off the effect of saving due to shading against the decreased probability of winning. The second term on the RHS of (1) is the strategic component of bids due to the discriminatory auction structure. This mark-up arises due to the uncertainty about the market clearing price. This is captured by the ratio of the two probabilities involving this random variable (similarly to the shading factor in a first price auction, which involves a hazard ratio of the first-order statistic of rivals' bids).

[18]In the main text we omit details of the approach and underlying assumptions because they are standard. For a recent discussion see Kastl (2017). The key assumption is that bidders have private valuation for liquidity and conditional on public information the private signals received by participants are independent and ex-ante identically distributed.

To proceed with our analysis, we use the resampling procedure described in Appendix A.A2 to estimate the distribution of P^c from the perspective of every bidder and use (1) to recover each bidder's willingness-to-pay for liquidity rationalizing the observed bids. We pool together 3 neighboring auctions for resampling to strike a reasonable compromise between potential unobserved heterogeneity across auctions and sampling error due to the size of the data set available for resampling. The resulting estimates are quite similar for alternative choices of auction groupings. This suggests that the unobserved heterogeneity is most likely less of an issue in our application, which is most likely due to the OIS, the swap on the overnight rate targeted by the central bank by which all bids are normalized, capturing a lot of this heterogeneity already.

III. Results

During the financial crisis 2008/2009, central banks and financial regulators appeared very concerned with illiquidity and potential subsequent insolvency of important financial institutions. Given that our measure of WTP for liquidity obtained at the central bank corresponds to the opportunity cost of procuring this liquidity on the interbank market, the dynamics in WTP can be used as a signal of both short- and long-term stress facing a financial institution. For example, a high WTP for liquidity, as revealed by the bids at auction, that persistently exceeds that of other market participants, could be viewed as a bad signal for solvency. We begin the discussion of our results by analyzing the liquidity positions taken by Canada's main financial institutions and test how closely these relate to the financial market's perceptions of the riskiness of these institutions.

Figure 6 depicts the aggregate WTP curve corresponding to the bids for the 1-month term PRA. This picture suggests that the Canadian banks did not seem to have been hard pressed for liquidity except for the turbulent period in October 2008. During that period, some banks were willing to pay up to a 140 basis points premium over the reference interest rate to obtain a 1-month loan. This is perhaps surprising since at the same time, banks could borrow overnight from the central bank at the premium of 50 basis points. This suggests that the temporal distinction between obtaining a loan for 1-month versus rolling over overnight loans might have been an important trade-off.

Turning our attention to the Receiver General Auctions, perhaps surprisingly, given the very short-maturity structure of these loans, we find the same general story. On the top panel of Figure 7 we plot aggregate WTP curves for a subset of randomly chosen auctions for very short-term loans during 2008. It is evident that banks were initially not willing to pay more than the reference interest rate, and in most cases were offering yields significantly below this rate - in fact, bidding up to 25 basis points lower. Following the collapse of Lehman Brothers, banks' WTP increased temporarily as banks started offering a premium over the reference interest rate in order to obtain liquidity. Only a few auctions cleared below the reference interest rate - and only minimally so. Many auctions, on the other hand, cleared at prices about 5 basis points higher than the reference interest rate. After December 2008, bidding again returned to its pre-Lehman state: virtually no bid exceeded the reference interest rate thereafter. It should be noted that most of the premium offered was concentrated on the longer-term PRA auctions, in which banks could secure liquidity for up to 3 months. It is nevertheless interesting that an effect of the increased uncertainty in the environment shows up even in these very short-term loan auctions. Reassuringly, the bottom panel of Figure 7 shows that the same pattern can be detected in the data during the ABCP crisis, before the PRA facility was active.[19] This verifies that even WTP for loans with very short-term maturities are sensitive to the uncertain economic environment.

We begin our regression analysis by studying the relationship between the spread (of both the quantity-weighted bids and quantity-weighted WTP) over the index swap rate (OIS) that an individual bank is offering to pay to obtain a repo-loan from the Bank of Canada in one of the liquidity auctions and key determinants of risk.[20] Table 3 presents summary statistics of the risk variables used in the regressions. All balance-sheet measures are relative to total assets. On average, 16% of a banks' balance sheet is considered liquid. On the funding side, 25.1% of assets are retail deposits while a larger fraction are wholesale. In terms of distance-to-default, the average probability of banks' assets failing below it's liabilities is 2.6%. The average CDS spread is 118.3 bp or 1.183% – meaning to insure \$100 of bank debt it costs \$1.183 per year, on average, for the firms in our data. The one month CDOR-OIS spread is on average 32 basis points and the three month spread is 51 basis points.

Table 4 summarizes the regression results when the dependent variable is the BID-OIS spread (columns (1) and (2)) and when it is the WTP-OIS spread (Columns (3)-(6)). Comparing columns (1) and (3) to (2) and (4) we see that the relationships between the BID-OIS spread to the balance sheet measures and market-based risk measures are similar to the WTP-OIS spread, even though the bid reflects strategic considerations in addition to the WTP. What the results do clearly show is that controlling for bank heterogeneity using bank fixed effects is important; otherwise several variables would have a counterintuitive sign. For example, the larger the share of conventional mortgages, i.e. low loan-to-value mortgages, to total assets, the lower the spread. The results depict a convincing picture that the WTP is strongly related to the share of retail deposits, share of conventional mortgages and to Merton (1974) measure of the distance to default. The distance to default measures the market value of a financial institutions assets relative to the book value of its liabilities. Smaller the distance to default the

[19]In the fall of 2007 the \$32 billion market for ABCP froze due to concerns that the paper was backed by U.S. subprime mortgage market.

[20]Given the competitive outcome of the term PRA auctions it turns out that there is very little difference between using bids and willingness-to-pay. This is very different than the ECB auctions analyzed in Cassola, Hortaçsu and Kastl (2013) where bidders perceived much more uncertainty about the market clearing price despite the large number of participants.

Table 3—: Summary of Bank Characteristics used in the Regression Analysis

The sample is from the first Term PRA auction on December 12, 2007 to April 21, 2009. Most variables are relative to total assets (TA). For confidentiality we cannot report the minimum and maximum values of the Overnight Loan Amount/TA.

Variable	Mean	S.D.	Min	Max
	Table 4 variables			
Quantity-Weighted Bid-OIS Spread	0.148	0.276	-0.07	1.46
Quantity-Weighted WTP-OIS Spread	0.252	0.376	-0.169	1.71
Liquid assets/TA	0.160	0.058	0.071	0.293
Retail deposits/TA	0.251	0.039	0.184	0.368
Wholesale deposits/TA	0.380	0.048	0.304	0.478
Conventional mortgages/TA	0.086	0.044	0.033	0.166
Insured residential mortgages/TA	0.084	0.037	0.037	0.171
Allowances for losses/TA	0.004	0.001	0.003	0.006
Loans outstanding/TA	0.522	0.055	0.432	0.627
Loans obtained via TAF (mio)	7.85	6.90	0	16.59
1-month CDOR-OIS spread	0.320	0.177	0.149	0.869
3-month CDOR-OIS spread	0.511	0.240	0.206	1.206
Distance to default	2.575	1.685	1.6685	6.800
5 year CDS spread	118.30	53.52	13.26	308.5
	Table 5 variables			
Spread to Target (F)	-0.011	0.063	-0.497	0.453
Overnight Loan Amount/TA	0.013	0.035	-	-

larger the probability the bank will default on its debt. A negative and significant coefficient on the distance to default during the peak of the crisis, therefore, is consistent with financial institutions that are further away from default willing to pay less for liquidity in the term PRA auctions than financial institutions closer to default. Unlike the correlation between distance to default and WTP we do not find a similar correlation between CDS prices and WTP at the peak of the crisis. This suggests that banks' own opportunity cost of funding is not correlated with CDS prices. Finally, both bids and WTP are positively correlated with the 1-month CDOR-OIS spread (and 3-month spread, unreported); meaning that our WTP measure does partly capture movements in aggregate risk.

In addition to Canadian banks' access to Canadian markets, the large Canadian banks had access to the U.S. Federal Reserve's Term Auction Facility (TAF). We do not find a significant relationship between banks' bids (or their WTP for liquidity) and their share of liquid assets to total assets or the amount of liquidity they obtained through TAF. The TAF was the United States Federal Reserve's extraordinary liquidity facility implemented in December 2007 to extend collateralized term loans to deposit-taking institutions. Canadian banks with operations in the U.S. could bid at the auctions for 1 and 3 month loans. Unlike the Bank of Canada's Term PRA facility the TAF was a single-price auction.[21] TAF was another way Canadian banks could access liquidity, but more importantly a way for Canadian banks to access U.S. dollar liquidity. The 5 major banks accessed TAF dollars, largely in the period immediately following the collapse of Lehman Brothers, but the amount accessed was not correlated with their bids in the Term PRA.[22]

A. *Dynamics of overnight lending*

Using the Furfine algorithm we construct a series of (unsecured) overnight lending on the interbank market between January 2007 and July 2009.[23] While neither the total transacted volume nor the average size of an individual loan changed significantly during the period of our data, the interest rates varied substantially. Figure 8 plots prices and quantities obtained from the algorithm. In particular, the data suggest that there was excess liquidity in the market and banks were willing to lend cash overnight for less than the target rate throughout the crisis, with the exception of the fall of 2007 during the Asset-Backed Commercial Paper (ABCP) crisis. At the same time, the variance of rates increased substantially during the crisis. Together, we view these facts as suggestive evidence that in the face of increased uncertainty banks took conservative liquidity positions and thus were willing to lend cash at rates below the target to reliable counterparties, which was not observed before the crisis.

For a deeper understanding of overnight lending patterns, we now broadly replicate the basic regression exercise presented in Afonso, Kovner and Schoar (2011) while also taking into account the events of the summer of 2007 as in Acharya

[21]On November 12th, 2008 the Bank of Canada announced it would provide exceptional liquidity to financial institutions under a Term Loan Facility (TLF). The TLF was a single price auction and the primary goal was to further increase flexibility in providing 1-month term liquidity since participants could use non-mortgage loan portfolios as eligible collateral. One effect of TLF would be to remove the stigma associated with the Emergency Lending Assistance program (see Armantier et al. (2015) for the effects of stigma in the Fed's discount window and its effects on bidding in TAF by American banks during the crisis). There was very little demand, however, for TLF.

[22]In addition to the variables presented in Table 4 we also looked at bank issuance of equity and domestic and foreign debt using the SDC database and found insignificant effects.

[23]The idea behind the algorithm (Furfine (1999)), as we apply it, is to look at flows in the payment system and find a payment π from bank A to bank B in the late afternoon at date t and a return payment in the morning of date $t+1$ of size $\pi(1+r_i)$ and classify these payments as loans. The price of the loan is r_i. The main issue with the Furfine algorithm is that it has the potential to identify some transactions as loans when they are indeed payments. This is particularly true in the case of using the algorithm early in the day or for small transactions. Unlike Fedwire, which processes Euro-dollar transactions, tri-party repo legs and bank to non-Fedwire institution transactions, which may or may not be considered loans, LVTS does not process these types of transactions. Therefore using the Furfine algorithm in LVTS is less likely to lead to misclassification of transactions. In addition we only examine transactions late in the day making misclassification even less likely. Some recent papers discuss potential issues with algorithm, see e.g., Armantier and Copeland (2015) or Kovner and Skeie (2013).

Table 4—: Explaining BID-OIS and WTP-OIS Spreads

The sample is from the first Term PRA auction on December 12, 2007 to April 21, 2009 – when the facility was used for monetary policy in addition to liquidity reasons and a minimum bid rate equal to the target rate of 25 basis points was set. The dependent variable in columns (1) and (2) is log(1+BID minus OIS) and in columns (3)-(6) it is log(1+WTP minus OIS). $I(peak)$ is an indicator variable equal to 1 during the period August 24, 2008 and November 10, 2008 and 0 otherwise. TAF is the U.S. Term Auction Facility run by the New York Federal Reserve. All balance sheet data are monthly and reported as a fraction of total assets. "Mort" is short for mortgage. Distance-to-default (DD) is calculated using Merton (1974)'s model. The prices of the 5-year CDS contracts are based on the CDS of senior debt. Constant term not reported. Standard errors clustered at bank level. *,**,*** denotes significance at 10, 5, 1% level, respectively.

VARIABLES	BID-OIS		WTP-OIS			
I(peak)	0.76	0.72	1.10	1.02	1.20	1.07
	(0.16)***	(0.15)***	(0.18)***	(0.17)***	(0.53)**	(0.53)**
log(Liquid Assets)	0.004	-.08	-.05	-.11	0.14	-.11
	(0.01)	(0.09)	(0.03)	(0.21)	(0.04)***	(0.16)
log(Retail Deposits)	0.11	-.52	-.02	-1.48	-.19	-1.86
	(0.03)***	(0.22)**	(0.1)	(0.67)**	(0.08)**	(0.45)***
log(Conventional Mort.)	-.04	0.18	-.11	0.22	-.14	0.18
	(0.03)	(0.05)***	(0.05)**	(0.13)*	(0.05)***	(0.16)
log(Insured Mort.)	-.02	0.08	-.004	0.18	-.04	0.16
	(0.02)	(0.09)	(0.06)	(0.21)	(0.06)	(0.22)
log(Loss Allowances)	-.02	0.04	0.02	0.14	-.23	0.14
	(0.09)	(0.09)	(0.19)	(0.19)	(0.17)	(0.15)
log(Loans Outstanding)	0.3	-.04	0.42	-.26	0.81	0.31
	(0.25)	(0.26)	(0.55)	(0.58)	(0.59)	(0.71)
log(1+TAF loans)		-.003		0.02	0.01	0.01
		(0.009)		(0.01)	(0.01)	(0.01)
1M CDOR-OIS	0.28	0.25	0.3	0.24	0.29	0.24
	(0.09)***	(0.09)***	(0.09)***	(0.09)***	(0.09)***	(0.1)**
DD	-.007	-.03	0.01	-.004	0.02	-.0006
	(0.01)	(0.007)***	(0.03)	(0.03)	(0.03)	(0.02)
I(peak) x DD	-.17	-.15	-.27	-.23	-.29	-.24
	(0.05)***	(0.05)***	(0.06)***	(0.06)***	(0.09)***	(0.09)***
log(CDS)					0.05	0.08
					(0.04)	(0.04)*
I(Peak) x log(CDS)					-.02	-.01
					(0.06)	(0.06)
Bank fixed effects	NO	YES	NO	YES	NO	YES
Obs.	162	162	162	162	144	144
R^2	0.74	0.75	0.61	0.64	0.61	0.65

and Merrouche (2012). The main focus in Afonso, Kovner and Schoar (2011) is the impact of the Lehman collapse on liquidity hoarding and counterparty risk in the U.S. overnight interbank market. Their primary specification is as follows:

$$F_{b,t} = \beta(Date) + \theta \log\left(\frac{amount_{b,t}}{assets_{b,t-1}}\right) + \alpha_b + \epsilon_{b,t}, \tag{2}$$

where $F_{b,t}$ is spread to target or log of loan size for bank (borrower/lender) b at time t. The mean spread to target, reported in Table 3 is -1.1 basis points, with a standard deviation of 6.3 basis points. $loan$ is the amount borrowed/lent overnight in the spread regression and α_b is a borrower/lender fixed effect. The dates of interest are $I(date = Sep12/2008)$, $I(date = Sep15/2008)$, and $I(date = Sep16/2008)$, corresponding to the Friday before the Lehman collapse, the Monday of the Lehman collapse, and the Tuesday after. The key result is that aggregate trading volumes in the Feds Funds market did not change during the crisis but that the composition of the participants and the price and volumes at which they traded did change.

Acharya and Merrouche (2012) instead focus on the ABCP crisis in the summer of 2007 and its impact on the demand for liquidity in the U.K. overnight interbank market. They are interested in measuring the impact of the news of BNP Paribas' exposure to the U.S. subprime crisis on August 9, 2007 on overnight lending in the U.K. interbank market.[24] The key result is that there was precautionary liquidity hoarding in the summer of 2007 and this increased interbank rates, regardless of counterparty risk.

In Table 5 we report results combining the date indicator variables from each period and measure the impact of both set of dates on interbank lending in the Canadian overnight market. Focussing first on the summer of 2007, as in Acharya and Merrouche (2012), we find the effect of BNP Paribas' acknowledgement of the crisis on August 9th led to an increase in overall spreads as well as borrower- and lender-specific spreads.[25] The amount traded, however, did not change on August 9th and 10th. At the end of day on August 13th, there was an announcement of disruption in the Canadian non-bank ABCP market, and this shows up in the coefficient on $I(date = Aug13-14/2017)$. Again, spreads increased. Throughout the remaining part of summer and fall of 2007 as banks brought back ABCP onto their balance sheet, spreads on the interbank market went higher than their level during the rest of the sample, as did the amounts traded. Interestingly, the coefficients on the amount borrowed (lent) with and without fixed effects are nearly identical. This indicates that the result of the crisis was an upward shift in

[24]Given the reserve requirement framework in the U.K. the level of liquidity demanded (supplied) by any particular bank is not strictly exogenously given by the idiosyncratic payments flows throughout a day. Therefore Acharya and Merrouche (2012) require an instrumental variable approach.

[25]We pool the effects of August 9th and 10th, although this is simply for brevity since the coefficients for both days are of the same sign and significance. We do the same for August 13th and 14th.

demand spread across the same borrowers pre-crisis and met by the same lenders pre-crisis, but at a higher rate. On December 12th, 2007 the Bank of Canada implemented its first term PRA auction. At this point spreads in the overnight market fell back to their normal levels.

Focusing on the post-Lehman period, we find strong evidence that Canadian banks were not hoarding liquidity in the overnight interbank market. Neither prices nor loan amounts appear to be affected by the events surrounding Monday September 15th. This is true on average (regressions without bank fixed effects) and at the bank level (regressions with bank fixed effects). Only on September 16th do we see a significant drop in spreads, although there was no change in amount borrowed or lent. Note that in the run-up to the Lehman crisis spreads and average loan sizes were falling. In addition, post-AIG spreads fall dramatically. We interpret these coefficients as evidence that liquidity was in excess and therefore relative to pre-crisis levels borrowing was done at lower spreads.

Afonso, Kovner and Schoar (2011) report a substantial change in borrower composition related to counterparty risk in the Feds Funds market. Larger, worst-performing banks lost access to the overnight market, while smaller, better performing banks increased their access to the market. Unlike Angelini, Nobili and Picillo (2009) who find that the most liquid lenders charge the highest rates (on the Italian interbank deposit market), which suggests banks were hoarding liquidity, Afonso, Kovner and Schoar (2011) do not find any such evidence. We do not find a change in the average price or loan size or a change in the composition of borrowers and/or lenders during the Lehman crisis. This suggests that Canadian banks did not hoard liquidity or that there was not an observable change in counterparty risk in the overnight interbank market.[26]

B. Repo market

Gorton and Metrick (2012) argue that the financial crisis was centered around the repo market. Repo transactions are short-term transactions between dealers of government securities. For example, a dealer sells a Treasury bill to another dealer and buys it back the next day. As counterparty risk increased, concerns about the value and liquidity of collateral suffered a run due to higher required haircuts. That is, the loan size as a percentage of the securities market value fell. Similarly, Brunnermeier (2009) argues that repo lenders required more collateral, which further drained the system of liquidity. In Figure 9 we depict the end of month outstanding balances of repo transactions of the 12 largest Canadian banks. Similar to our analysis of the overnight interbank market, there seems to be little evidence of any persistent impact of the crisis on the Canadian repo

[26]Given our sample period is longer than what is presented in Afonso, Kovner and Schoar (2011) one might be concerned that our results might be driven by differences in the baseline effect. In results not reported here we redo the estimation using the shorter sample period and do not find any difference in results.

Table 5—: Overnight Rate Regressions

	borrower				lender			
VARIABLES	spread		log(loan)		spread		log(loan)	
I(date=Aug9-10/2007)	0.0677**	0.0689**	0.0452	-0.0711	0.0527***	0.0548***	-0.0740	-0.145
	(0.0301)	(0.0305)	(0.226)	(0.211)	(0.0114)	(0.0111)	(0.291)	(0.279)
I(date=Aug13-14/2007)	0.0393***	0.0361***	0.508**	0.504**	0.0338***	0.0338***	0.237	0.337
	(0.00695)	(0.00759)	(0.230)	(0.203)	(0.00664)	(0.00585)	(0.206)	(0.218)
I(date=Aug15-Dec11/2007)	0.0370***	0.0365***	0.218**	0.189*	0.0307***	0.0311***	0.164	0.177*
	(0.00799)	(0.00832)	(0.0927)	(0.0915)	(0.00902)	(0.00875)	(0.0962)	(0.0877)
I(date=Dec12-31/2007)	0.00241	0.00177	0.161	0.144	-0.00310	-0.00320	0.133	0.187
	(0.00468)	(0.00476)	(0.128)	(0.116)	(0.0133)	(0.0135)	(0.137)	(0.121)
I(date=Sep5-11/2008)	-0.0194***	-0.0157***	-0.511**	-0.620***	-0.00635	-0.0129	-0.398**	-0.402*
	(0.00306)	(0.00341)	(0.175)	(0.163)	(0.0109)	(0.00937)	(0.175)	(0.200)
I(date=Sep12/2008)	-0.0323*	-0.0296*	-0.609	-0.450	-0.0228	-0.0203	0.452	0.248
	(0.0156)	(0.0157)	(0.392)	(0.297)	(0.0218)	(0.0191)	(0.256)	(0.272)
I(date=Sep15/2008)	-0.00908	-0.0125	0.656**	0.598***	-0.00713	-0.0114	0.396	0.445*
	(0.00731)	(0.00998)	(0.262)	(0.127)	(0.00770)	(0.00659)	(0.288)	(0.238)
I(date=Sep16/2008)	-0.0325***	-0.0327***	-0.278	-0.123	-0.0583**	-0.0593**	-0.354	-0.121
	(0.00606)	(0.00651)	(0.324)	(0.282)	(0.0196)	(0.0198)	(0.222)	(0.235)
I(date=Sep17/2008+)	-0.0454***	-0.0439***	-0.0459	-0.0774	-0.0421***	-0.0445***	0.00839	0.0276
	(0.00555)	(0.00517)	(0.0704)	(0.0647)	(0.00992)	(0.00940)	(0.119)	(0.118)
log(overnight loan/TA)	-0.000226	0.00333			0.00847**	0.00377		
	(0.00266)	(0.00330)			(0.00386)	(0.00323)		
Constant	0.0136	0.0435	19.30***	19.20***	0.0606**	0.0116	19.27***	19.11***
	(0.0172)	(0.0258)	(0.167)	(0.0161)	(0.0200)	(0.0248)	(0.297)	(0.0275)
Bank fixed effects	NO	YES	NO	YES	NO	YES	NO	YES
Observations	4673	4673	4673	4673	3923	3923	3923	3923
R^2	0.158	0.197	0.011	0.351	0.119	0.185	0.004	0.506

The sample is from Jan 2, 2007 to April 20, 2009. The dependent variable *spread* is the difference between the transaction rate and the Bank of Canada target rate. Coefficients of interest are those on the indicator variables which are equal to 1 for specific dates. TA is total assets. Robust standard errors in parentheses. *,**,*** denotes significance at 10, 5, 1% level, respectively.

market. Part of the repo transactions shifted from trade between private banks to trade with the Bank of Canada via the purchase and resale agreement auctions, but the overall amount seems to be fairly constant over the period of our study. Of course, this figure needs to be interpreted cautiously since the structure of the outstanding collateral might have changed.

C. *Liquidity risk or counterparty risk?*

So far we have analyzed the demand for liquidity by Canada's main financial institutions. Results from the term PRA and Receiver General Auctions suggest that banks' hoarding of liquidity was largely isolated to an immediate period following the collapse of Lehman Brothers. The dynamics of overnight lending do not suggest liquidity hoarding. Next, we highlight that in the intraday interbank market there was very little change in perceived counterparty risk across members.

During the financial crisis, the Bank of Canada substantially expanded the collateral deemed acceptable in the payment system. In particular, most participants were allowed to pledge their whole non-mortgage loan portfolio as collateral. This in turn meant that virtually every participant in the LVTS had an abundance of unused collateral in the system. Since as described in section I.B, the LVTS has two tranches: a fully collateralized tranche, T1, and a partially collateralized one, T2 – it would thus seem natural that if the opportunity cost of collateral vanishes and default probabilities of counterparties increase, banks would be reluctant to deal with their counterparties through T2 (and thus potentially bear some default risk). First, we see from Figure 10 that payments did not in fact move to T1. The volume of transactions in T1 is below 1%, and the dollar value of transactions was about 9% during the crisis compared to 7% pre-crisis. In addition we can look at bilateral credit lines. In Figure 11 we plot the time series of credit lines extended to and from three of the most volatile financial institutions.[27] Each line is a relationship. Bank A, for example, in the top left corner of the figure, is extending different-sized credit limits to the other participants in the payment system. Only twice, in 2007, however, do we see any movement–in both case a credit extension (the first large and the second small). To the immediate right of this, we observe credit limits extended to bank A. We only observe one change in the credit limit extended to bank A, towards the end of 2005. Despite looking at the most volatile BCL limits, in all cases, and despite different CDS price experiences, there is little to no change in credit limits being increased or decreased to or from each participant.[28]

[27]Note that there are no prices or non-price terms attached to BCLs – only quantities matter.

[28]In unreported regressions, available upon request, we also regress changes in BCLs on 5 year CDS spreads. Not surprisingly, given that BCLs barely change over our sample period, we do not find any significant correlation between CDS spreads and the change in credit limits.

D. Putting Everything Together

In the previous sections we offered pieces of a puzzle. We showed that CDS prices and aggregated interbank spreads suggested a considerable increase in the default probability at least for some Canadian banks and a substantial increase in the cost of liquidity. Despite this high degree of heterogeneity in implied default probability, banks' behavior vis-à-vis each other was virtually unaffected. Even though all banks pledged excessive collateral into the payment system, they did not require the "riskier" banks to send more payments through the fully collateralized channel or reduced the credit lines extended to them. Moreover, the distribution of the WTP for liquidity across the different banks across different facilities, suggests that there is no significant correlation with the CDS-implied default probability: banks with high CDS prices do not seem to have higher value for liquidity obtained from the central bank, corroborating the evidence that other banks' behavior towards these "riskier" banks did not change and hence it was probably not impossible to secure liquidity through other channels than the auctions administered by the Bank of Canada. The persistently high prices of CDS contracts and spreads may suggest that virtually all Canadian banks may face solvency issues over the life of the contract, or at least very high cost to funding. Nevertheless, we did not find any bank having persistently high WTP for liquidity, which should be a necessary condition for insolvency.

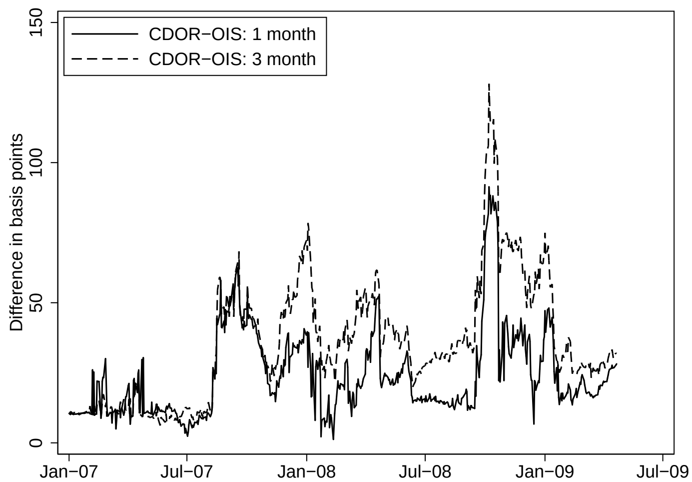
IV. Conclusion

As in most developed countries, the Canadian banking sector came under substantial stress during the 2008/2009 financial crisis. In this paper we document that, surprisingly, this stress lasted only for a few months immediately following the demise of Lehman Brothers. By the end of 2008, the behavior of Canadian banks was virtually indistinguishable from their behavior prior to the crisis, in contrast to European banks. This is despite public signals of similar distress. Our results also suggest that although the Bank of Canada's response to public signals of the financial crisis may have been warranted, in light of the financial institutions' behavior in the payment system and their willingness-to-pay for liquidity, it may have been too strong. Given that providing liquidity has costs, including creating moral hazard, determining the right level of central bank interventional is crucial.

We argue that examining banks' behavior in liquidity auctions organized by the central bank might provide the Bank of Canada with a better picture of individual bank-stress. The spread over the index swap rate that each bank is offering to pay for a repo-loan, when looked at through the lens of an auction model, is suggestive of how much it is pressed for liquidity, which it might not be able to secure elsewhere due to its perceived riskiness. Using an equilibrium model of bidding, one can recover the willingness-to-pay from the bids, which is indicative of the opportunity cost of obtaining funding from other sources (interbank markets). We show that in our application these spreads are indeed related to various measures of bank's solvency and riskiness from the balance sheets. The bids thus may act as an aggregated snapshot of a bank's situation.

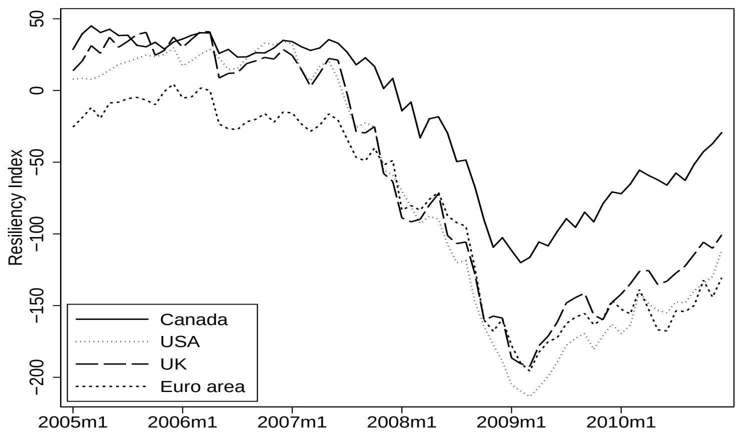
A number of caveats. One concern with our approach is that by explicitly monitoring bank-bids, the central bank could change behavior. This is of course true for all monitoring tools, which have been widely adopted by central banks post-financial crisis. Our approach could be modified that maintains incentive compatibility, i.e., banks would truthfully reveal their type at auction. Second, the tiering structure of the Canadian banking system means we only observe bids at auction from dealers. Smaller institutions must bid through their dealers. The lack of small-bank participation in the allocation of central bank liquidity is an important policy consideration. Separate access to liquidity facilities might provide better information about where stress in a system is originating than in a tiered system.

Figure 1. : Interest Rate Spreads in Canada



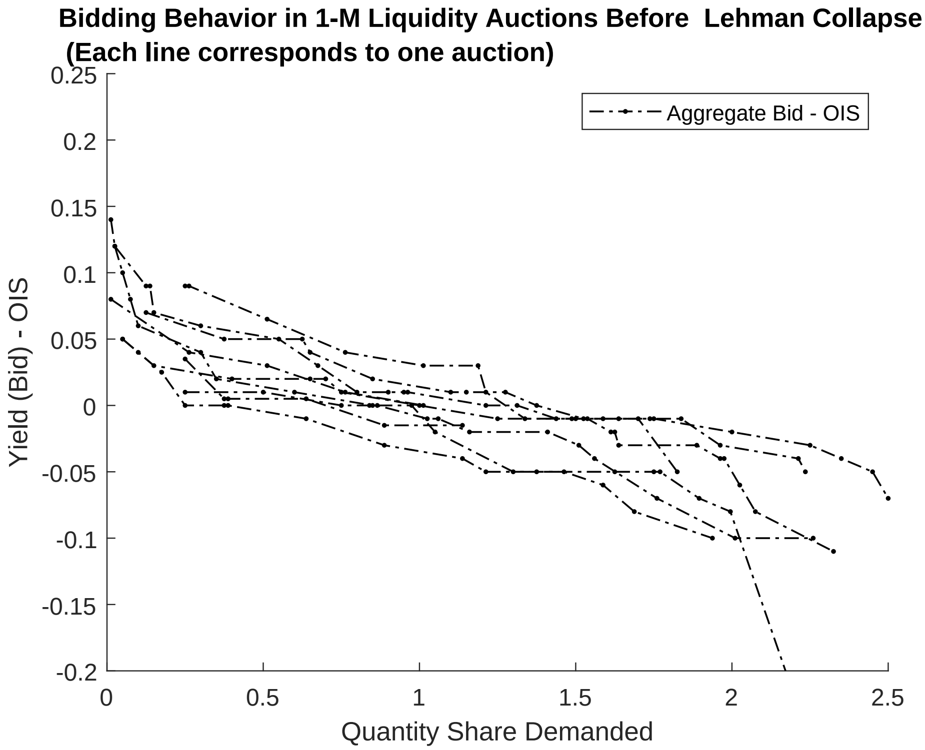
Source: Bank of Canada: Figure 1 shows the daily CDOR and OIS rates. CDOR is the Canadian Dealer Offered Rate and is similar to LIBOR/EURIBOR. It is the average rate for Canadian bankers' acceptances for specific terms-to-maturity, determined daily from a survey on bid-side rates provided by the principal market-makers, including the major Canadian banks. OIS is the Overnight Index Swap rate. It is an over-the-counter derivative primarily used for hedging short-term funding costs or exposures to short-term interest rate movements. OIS rates are calculated in reference to the CORRA, or Canadian Overnight Repo Rate Average. In the U.S. the reference is the daily Fed Funds rate.

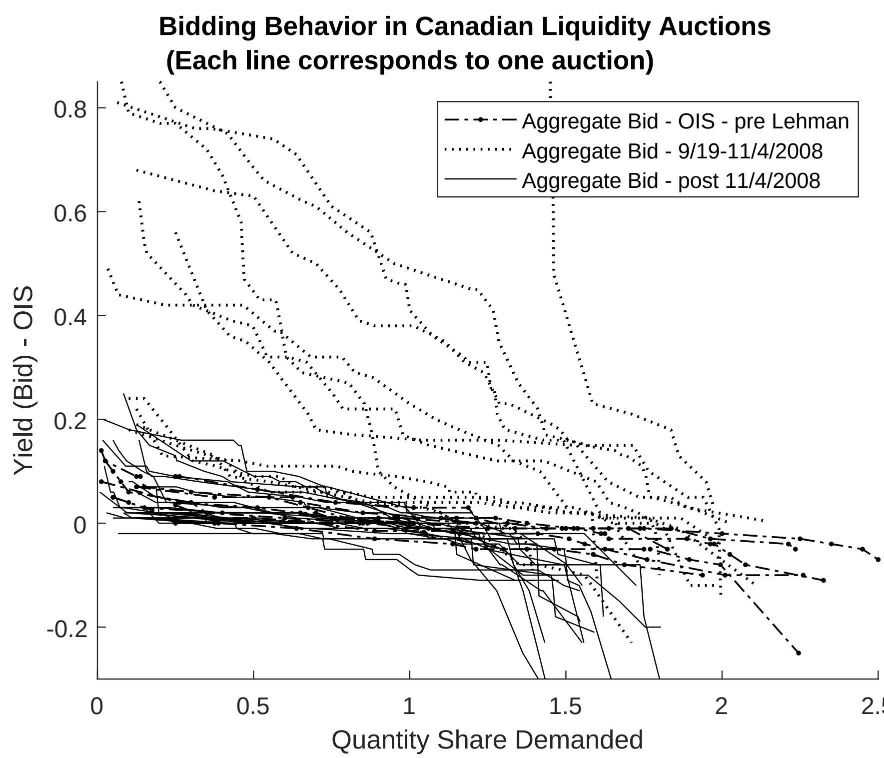
Figure 2. : Measuring Resiliency of Banking Systems



Source: Bank of Canada: Country-specific measures of resiliency are taken from MacDonald and v. Oort (2017). A negative number means less resiliency. The individual measures included in the index are (i) long-run marginal expected shortfall (LRMES), (ii) marginal expected shortfall (MES), (iii) exposure ΔCoVaR, (iv) distance to default (DD), and (v) market-based capital ratio (MBCR). LRMSE and MES measure the expected losses given an adverse shock. The MES is based on one-day losses and LRMES are expected cumulative losses of market value over a longer period. The measure exposure ΔCoVaR measures difference in value-at-risk conditional on a system-wide shock (Adrian and Brunnermeier (2016)). In addition, distance to default measures the market value of a financial institutions assets relative to the book value of its liabilities. Finally, MBCR is defined as the market value of common equity as a percentage of the market value of assets.

Figure 3. : Aggregate demand curves in Canadian liquidity auctions





Note: Since Figure 5 shows that OIS rates capture expectations of the financial markets with respect to the target rate well, we use them to express all bids as spreads over the expected target rate.

Figure 4. : Probability of default (over 5-years) implied by the CDS prices

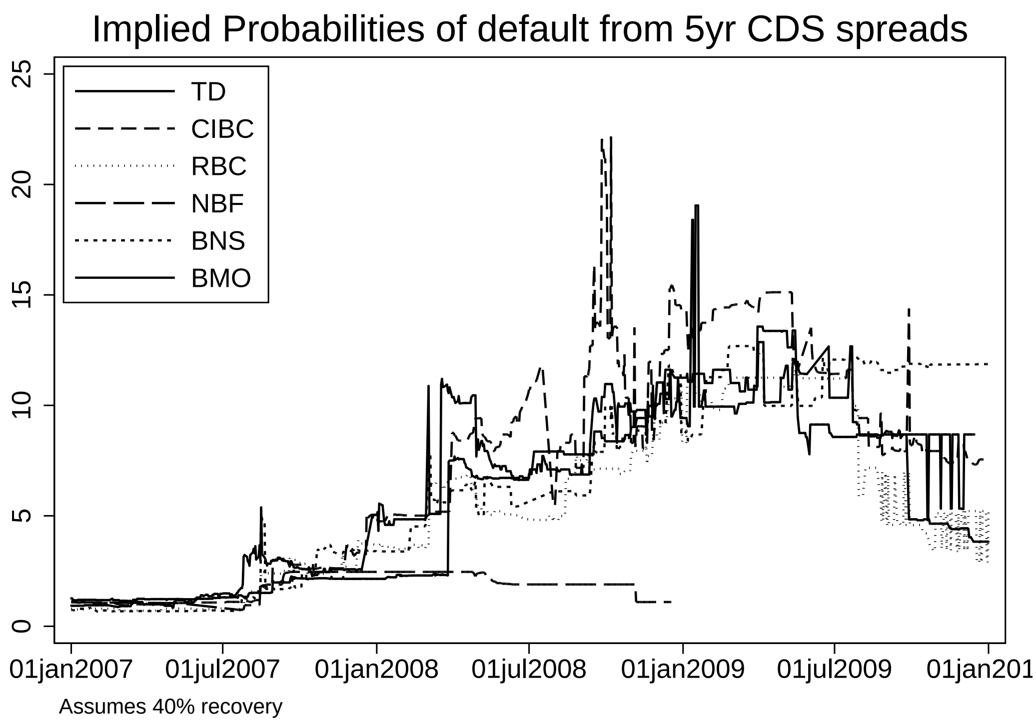
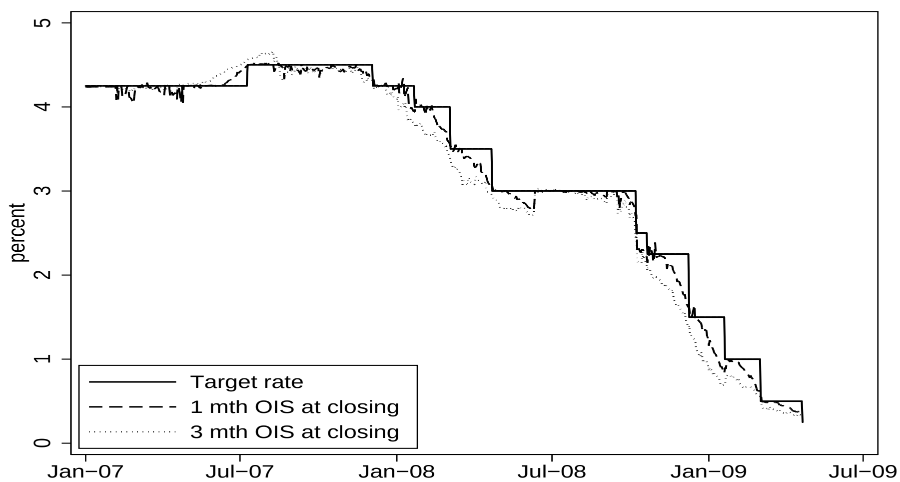


Figure 5. : Overnight Index Swap Rates and the Target Rate



Source: Bank of Canada: Figure 5 shows the daily values of 1-month and 3-month OIS rates and the Target Rate of the Bank of Canada. Even though in Canada OIS rates are calculated in reference to the CORRA, or Canadian Overnight Repo Rate Average, they capture expectations of the financial markets with respect to the target rate well.

Figure 6. : Willingness-to-pay for 1-m and 3-m liquidity

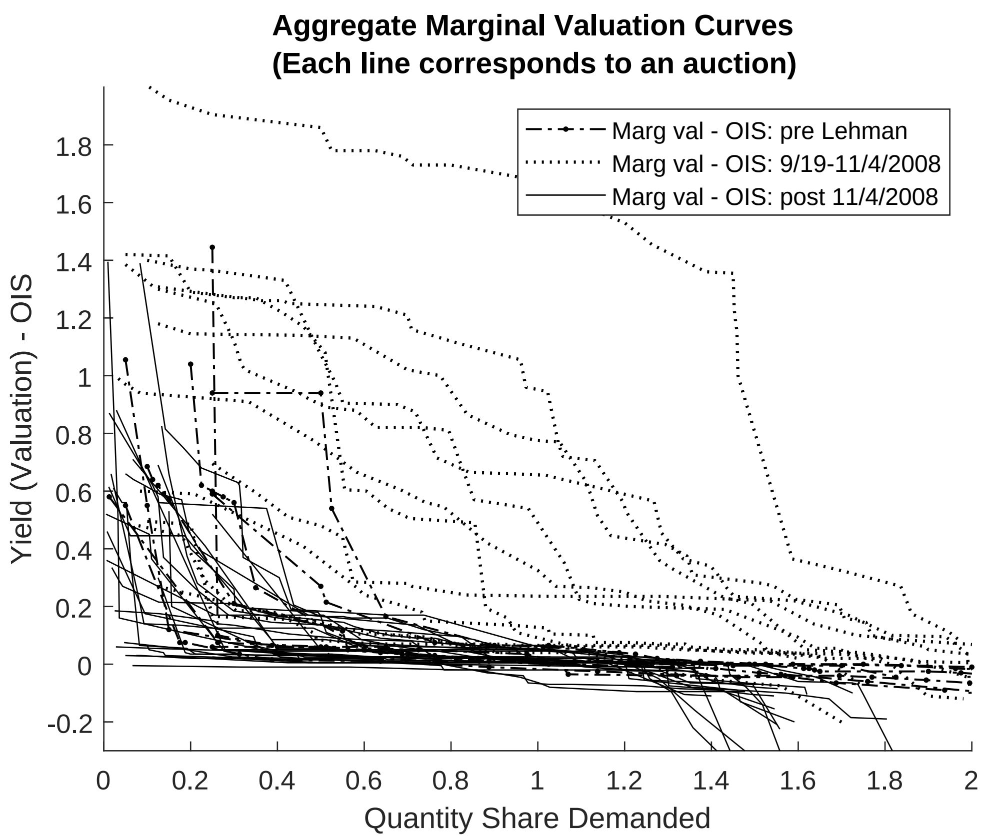


Figure 7. : Bidding for liquidity in Receiver General Auctions: Lehman vs ABCP Crises

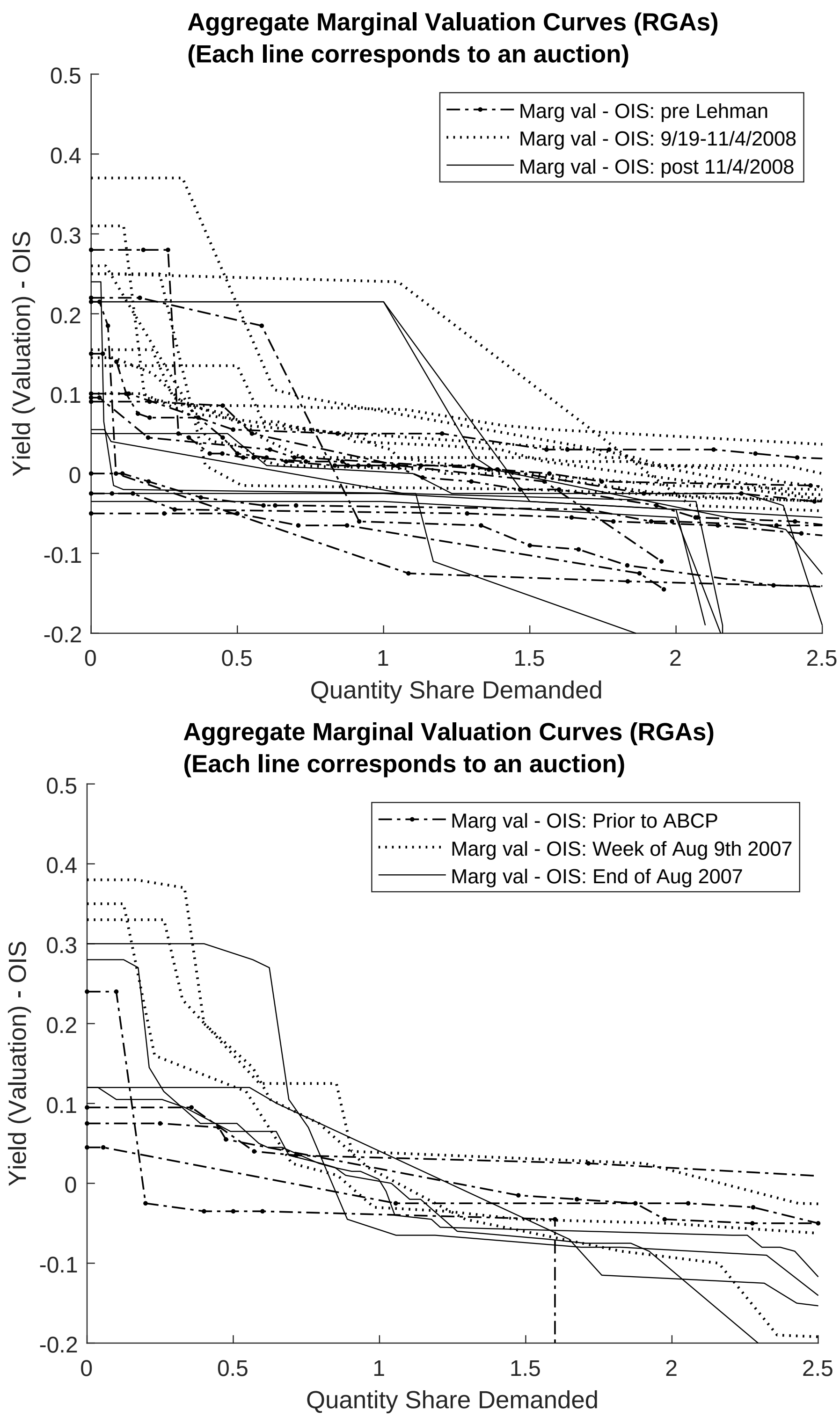
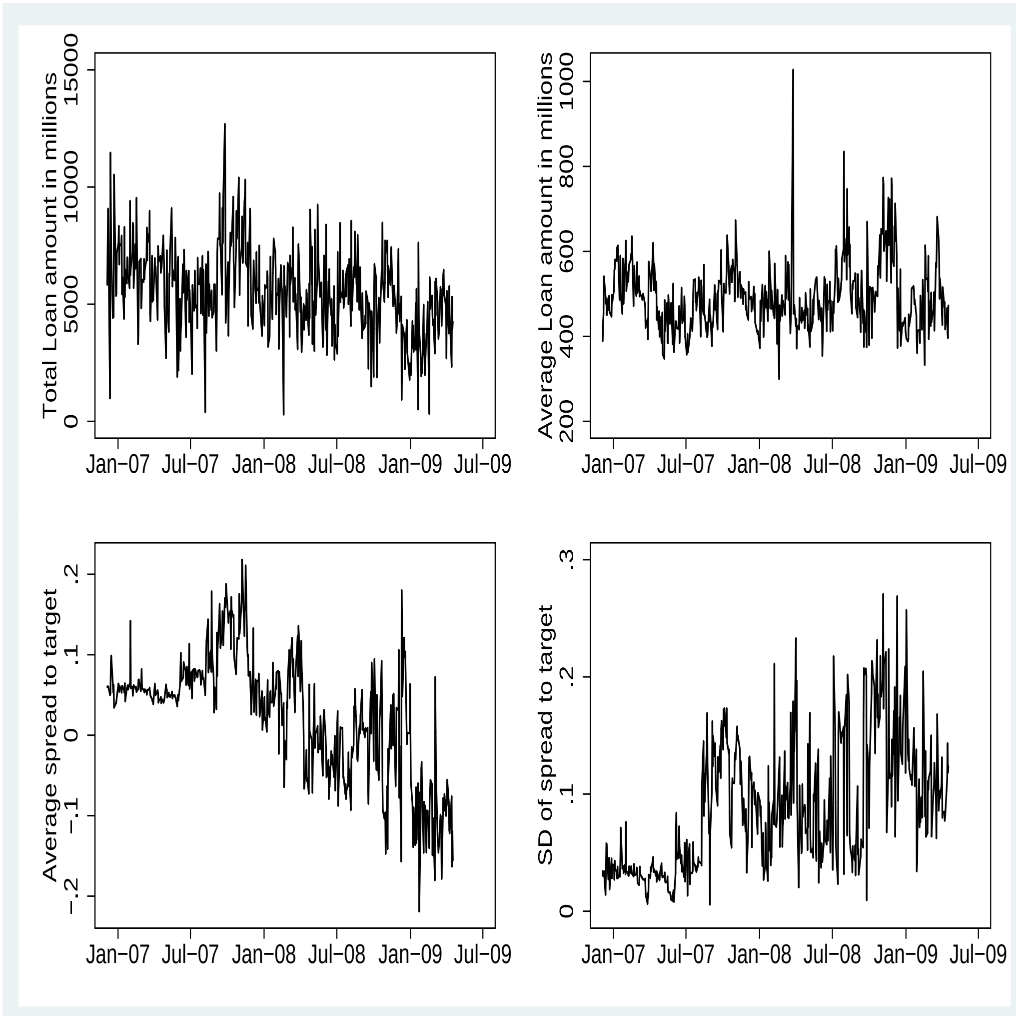
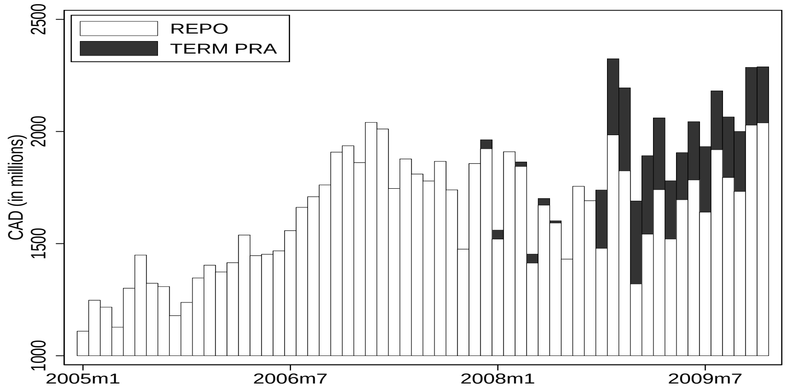


Figure 8. : Volumes and Rates in Canada's Interbank Market



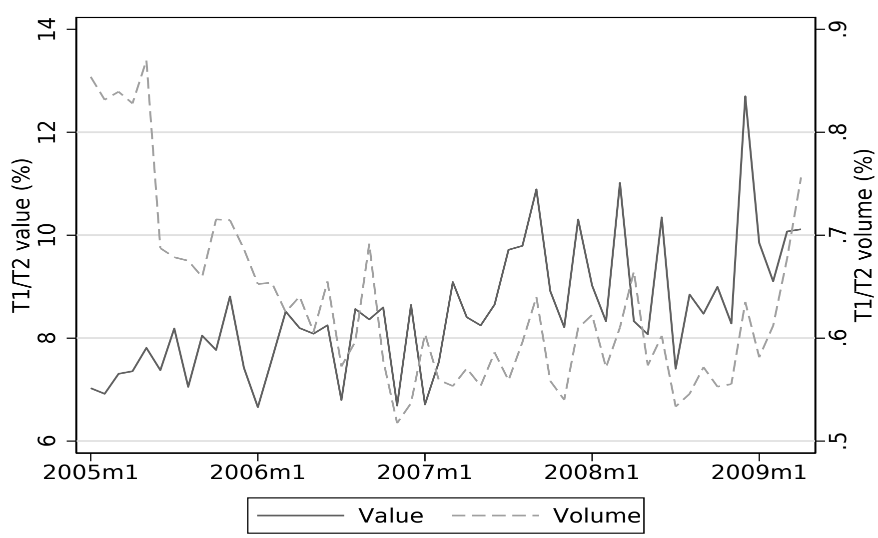
Source: Bank of Canada: Figure 8 shows the aggregate amount borrowed in the Canadian unsecured interbank market (in millions of Canadian dollars) between January 2007 and April 20, 2009, average amount borrowed, the average spread between the rate paid for a loan and the Bank of Canada target rate in the Canadian unsecured interbank market and the standard deviation in the same spread. Source: Bank of Canada.

Figure 9. : End of Month Outstanding Balance of Repo Transactions of 12 largest Canadian Banks



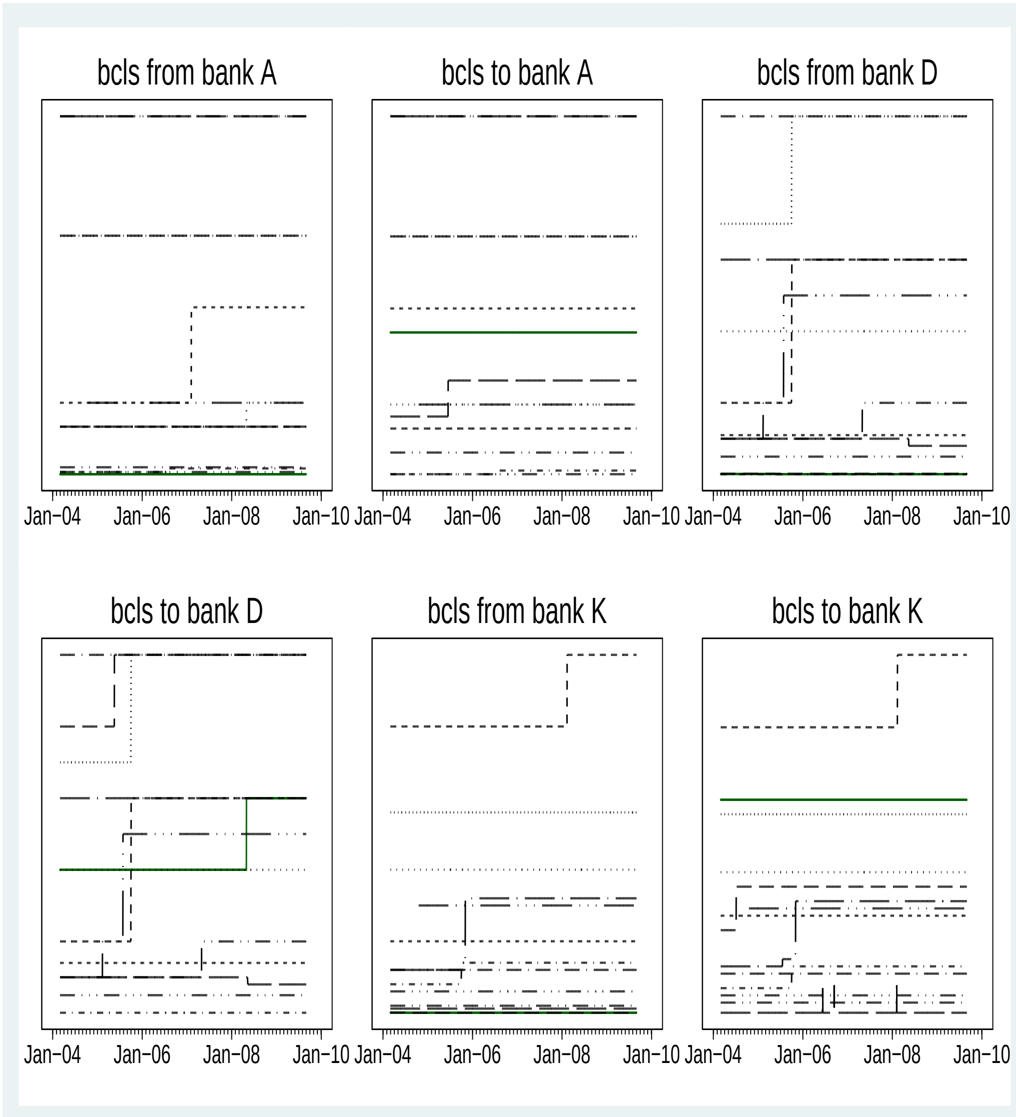
Source: Bank of Canada: Figure 9 depicts the end of month outstanding balances of repo transactions of the 12 largest Canadian banks. Source: Bank of Canada.

Figure 10. : Ratio of Tranche 1 to Tranche 2 Transactions – Value and Volume



Source: Bank of Canada: Figure 10 shows the percentage of transactions in dollar value and volume sent in Canada's LVTS via T1 versus T2 between January 2005 to April 2009. Source: Bank of Canada.

Figure 11. : Bilateral Credit Lines in LVTS



Source: Bank of Canada: Figure 11 shows the bilateral credit lines in LVTS to and from all participants to financial institution i where i is one of three different institutions, from April 2004 to April 2009. For confidentiality reasons we cannot show the names of the financial institutions or the limits themselves, only that there is very little change over time. Source: Bank of Canada.

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Appendix

A1. Model of a Discriminatory Share Auction

The auctions model we use in the paper is based on a model of the share auction proposed by Wilson (1979). Kastl (2012) extends this model to setting, in which bidders are restricted to employ step-function strategies. Let N be the number of potential bidders and S_i be an $D \geq 1$ dimensional private signal that i observes. This signal affects the underlying value for the auctioned good. The description below is largely borrowed from Kastl (2017).

ASSUMPTION 1: *Bidders' signals, $S_1, ..., S_N$, are drawn from a common support $[0,1]^D$ according to an atomless joint d.f. $F(\cdot)$ with density f.*

The following assumption guarantees smoothness of the distribution of residual supplies, which is needed for the characterization result below.

ASSUMPTION 2: *Supply Q is a random variable distributed on $\left[\underline{Q}, \overline{Q}\right]$ with strictly positive density conditional on S_i $\forall i$.*

Obtaining a share q of the supply Q is valued according to a marginal valuation function $v_i(q, S_i)$. Assume that this function satisfies some regularity conditions.

ASSUMPTION 3: *$v_i(q, S_i)$ is non-negative, bounded, strictly increasing in each component of S_i $\forall q$, and weakly decreasing and continuous in q $\forall S_i$.*

Note that under Assumption 3, values are assumed to be private since $v_i(\cdot)$ does not depend on private information of the rivals. Similarly to Cassola, Hortaçsu and Kastl (2013) we are studying auctions for liquidity, for which private value assumption seems fine since the willingness-to-pay for liquidity depends mostly on the structure of the balance sheet (and availability of good collateral), investment opportunities and opportunity cost of the collateral. The banks then simply decide based on their particular situation whether to borrow from the central bank, or from a private counterparty.

Furthermore, to respect the rules governing the actual auction markets, we restrict the strategy set available to each bidder to step functions with at most K steps. Let the index k denote the place of a particular bidpoint in the vector of bids that are ordered to be increasing in the quantity dimension and decreasing in price. A bidpoint (q_k, b_k) together with the preceding bidpoint (q_{k-1}, b_{k-1}) thus specify the marginal quantity, $q_k - q_{k-1}$, that a bidder is bidding b_k for. In addition, we allow a bid l, which is sure to lose, and hence basically corresponds to not participating in the auction.

ASSUMPTION 4: *Each player $i = 1, ..., N$ has an action set:*

$$A_i = \left\{ \begin{array}{c} \left(\vec{b}, \vec{q}, K_i\right) : \dim\left(\vec{b}\right) = \dim(\vec{q}) = K_i \in \{1, ..., K\}, \\ b_{ik} \in B = \{l\} \cup \left[0, \bar{b}\right], \ q_{ik} \in [0,1], \ b_{ik} > b_{ik+1}, q_{ik} < q_{ik+1} \end{array} \right\}.$$

If demand exceeds supply at the market clearing price, the marginal demand is rationed proportionally.

Equilibrium Characterization. — Let $V_i(q, S_i)$ denote the gross utility: $V_i(q, S_i) = \int_0^q v_i(u, S_i)\, du$. The expected utility of a bidder i of type s_i employing a strategy $y_i(\cdot|s_i)$ can be written as:

$$EU(s_i) = \sum_{k=1}^{K_i} \left[\Pr(b_{ik} > P^c > b_{ik+1}|s_i)\, V(q_{ik}, s_i) - \Pr(b_{ik} > P^c|s_i)\, b_{ik}(q_{ik} - q_{ik-1})\right] \tag{A-1}$$
$$+ \sum_{k=1}^{K_i} \Pr(b_{ik} = P^c|s_i)\, E_{Q,s_{-i}|s_i}\left[V\left(Q_i^c(Q, \mathbf{S}, \mathbf{y}(\cdot|S)), s_i\right) - b_{ik}\left(Q_i^c(Q, \mathbf{S}, \mathbf{y}(\cdot|S)) - q_{ik-1}\right)|b_{ik} = P^c\right]$$

where $q_{i0} = b_{iK_i+1} = 0$. The random variable $Q_i^c(Q, \mathbf{S}, \mathbf{y}(\cdot|S))$ is the (market clearing) quantity bidder i obtains if the state (bidders' private information and the supply quantity) is $(Q, \mathbf{S})$ and bidders submit bids specified in the vector $\mathbf{y}(\cdot|S) = [y_1(\cdot|S_1), ..., y_N(\cdot|S_N)]$. Since bidders have private information and since the supply is random, the market clearing price is a random variable, denoted by P^c.

A Bayesian Nash Equilibrium in this setting is a collection of functions such that (almost) every type s_i of bidder i is choosing his bid function so as to maximize her expected utility: $y_i(\cdot|s_i) \in \arg\max EU_i(s_i)$ for a.e. s_i and all bidders i. The system of necessary conditions implicitly characterizing such a BNE is the link between the observables and unobservables that we seek to establish. Kastl (2012) derives the following necessary conditions that the quantity requested in any step of a pure strategy that is a part of a Bayesian Nash Equilibrium has to satisfy.

PROPOSITION 1: *Under assumptions 1–4 in any K-step Equilibrium of a discriminatory auction, for almost all s_i, every step $k < K_i$ in the equilibrium bid function $y_i(\cdot|s_i)$ has to satisfy*

$$\Pr(b_{ik} > P^c > b_{ik+1}|s_i)\left[v(q_{ik}, s_i) - b_{ik}\right] = \Pr(b_{ik+1} \geq P^c|s_i)(b_{ik} - b_{ik+1}) \tag{A-2}$$

and at the last step K_i it has to satisfy $v(\overline{q}, s_i) = b_{iK_i}$ where $\overline{q} = \sup_{q,s_{-i}} Q_i^c(q, s_{-i}, s_i, \mathbf{y}(\cdot|S))$.

Note that this condition is simply a multi-unit counterpart of the equilibrium condition for bidding in a first-price auction: $g(b)(v - b) = G(b)$, where $G(b)$ is the CDF (and $g(b)$ is the PDF) of the distribution of the first-order statistic of rival bids (i.e., of the highest of rival bids) Guerre, Perrigne and Vuong's (2000). The trade-off in the multi-unit environment is identical. The bidder is simply trading off the expected surplus on the marginal (infinitesimal) unit versus the probability of winning it.

A2. Estimation

The estimation approach relies on the fact that the observable (bids) and object of interest (willingness-to-pay) is given by an equilibrium relationship of an explicit economic model given by (A-2). This relationship involves uncertainty about the market clearing price. To account for the uncertainty we assume that all participants agree on the respective distributions of all random variables (private information and supply) ex-ante and hence any differences in their observed strategies ex-post are due to differences in realizations of their private information. We can then estimate the relevant probability distribution of market clearing price by "resampling," i.e., by following a bootstrap-like procedure. By drawing repeated samples of strategies (by sampling with replacement from the observed data) one simulates different possible states of the world and thus eventually obtains an estimate of the distribution of the random variable(s) of interest. This idea appeared originally in Hortaçsu (2002) and was later applied in Hortaçsu and McAdams (2010) and Kastl (2011). It should be intuitive to see that as one constructs more and more samples (as the data set gets larger and thus the observed strategies span more and more of the type space), one obtains more and more simulated states of the world, where the probability of a type profile being in some subset of the type space $[0,1]^M \times \left[\underline{Q}, \overline{Q}\right]$ corresponds to the probability of that subset implied by the population distribution functions given in Assumptions 1 and 2.

Estimating the Distribution of the Market Clearing Price. — Formally, equation (A-2) reveals that to get at the object of interest, $v(\cdot)$, we need to estimate the distribution of P^c, $H(X) = \Pr(P^c \leq X)$. With such an estimate in hand, one can simply plug-in to (A-2) and obtain an estimate of $v(\cdot)$. $H(X)$ is defined as

$$H(X) \equiv \Pr(X \geq P^c | s_i) = E_{\left\{Q, S_{j \neq i}\right\}} I\left(Q - \sum_{j \neq i} y(X|S_j) \geq y(X|s_i)\right) \tag{A-3}$$

where $I(\cdot)$ is the indicator function and $E_{\{\cdot\}}$ is an expectation over the random supply and other bidders' private information. The market clearing price will be lower than X whenever the realized types and the realization of the supply are such that the implied realized aggregate residual supply (i.e., total supply minus demands of everyone but i) falls short of i's demand. Hortaçsu and Kastl (2012) describes how to estimate $H(\cdot)$. The approach begins by defining an indicator of excess supply at price X (given bid functions $\{y_j(X|s_j)\}_{j \neq i}$ and i's own bid $y_i(X|s_i)$) as follows:

$$\Phi\left(\{y_j(X|s_j)\}_{j \neq i}; X\right) = I\left(Q - \sum_{j \neq i} y_j(X|s_j) \geq y_i(X|s_i)\right).$$

One estimator of $H(X)$ can be derived as a V-statistic:

$$\xi\left(\hat{F};X,h_T\right)=\frac{1}{(NT)^{(N-1)}}\sum_{\alpha_1=(1,1)}^{(T,N)}...\sum_{\alpha_{N-1}=(1,1)}^{(T,N)}\Phi\left(y_{\alpha_1},...,y_{\alpha_{N-1}},X\right)$$

where $\alpha_i\in\{(1,1),(1,2),...(1,N),...,(T,N)\}$ is the index of the bid in the subsample and $\hat{F}$ is the empirical distribution of bids, i.e., the empirical probability distribution over points in 2K-dimensional space.

It is straightforward to see that an estimator defined as a simulator of $\xi\left(\hat{F};X,h_T\right)$ by drawing only M subsamples rather than all $(NT)^{(N-1)}$ is consistent as $T\to\infty$ (and under appropriate conditions on the rate at which the number of simulations, M, increases). Cassola, Hortaçsu and Kastl (2013) establish that it is also consistent as $N\to\infty$ provided some further technical conditions are satisfied (one of which is that there is a non-vanishing uncertainty about supply per bidder in the limit).