A Model of Job and Worker Flows*

Nobuhiro Kiyotaki
Princeton University

Ricardo Lagos
New York University

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Abstract

We develop an equilibrium search model that incorporates job-to-job transitions, exhibits instances of replacement hiring, and conceptually distinguishes between job and worker flows. We propose a notion of competitive equilibrium for random matching environments and study the extent to which it achieves an efficient allocation of resources. The model can be used to study how the permanent incomes and employment states of individual workers evolve over time, as well as other features of labor markets, such as the amount of worker turnover in excess of job reallocation, the lengths of job tenures and unemployment durations, and the size and persistence of the changes in workers' incomes following displacements or job-to-job transitions. We also analyze the effects that labor market institutions and public policy have on the standard measures of job and worker flows.

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1 Introduction

Over the last fifteen years, a series of influential studies have documented that gross job flows—job creation and job destruction—are large, both during periods when aggregate employment is growing, as well as during periods when aggregate employment is declining. These findings have shown that the heterogeneous forces that cause employment to simultaneously expand in some establishments and contract in others are central in shaping labor market outcomes.[1]

Behind these large job flows, however, lie even larger worker flows. Recent studies have found that for most establishments, for most of the time, the sum of new hires, quits and displacements (worker turnover) is much larger than the change in the number of employees (job reallocation).[2] The finding that worker flows consistently exceed job flows at the establishment level exposes the importance of heterogeneity at the level of the employer-worker match—a layer of heterogeneity over and above the cross-establishment heterogeneity that can be inferred from the sheer size of the job flows alone.[3]

From an aggregate perspective, the amount of worker turnover in excess of job reallocation depends, not only on the amount of simultaneous hiring and firing at the establishment level, but also on the extent to which the market reallocates workers across establishments by means of job-to-job transitions. Recent studies have also documented sizable job-to-job flows.[4]

[1] Standard references include the seminal work of Davis and Haltiwanger (1992, 1999) as well as the comprehensive treatment in Davis, Haltiwanger and Schuh (1996).

[2] The empirical literature measures *gross job creation* as the sum of employment gains over all plants that expand or start up between dates $t-1$ and t; *gross job destruction* as the sum of employment losses over all plants that contract or shut down; and *gross job reallocation* as the sum of gross job creation and destruction. Estimates of worker flows are based either on establishment or on worker surveys. Empirical studies based on establishment data define *worker turnover* at establishment i as the sum of the number of accessions (new hires) and separations (quits and displacements) between dates $t-1$ and t, and aggregate worker turnover as the sum of worker turnover over establishments. Alternatively, empirical studies based on worker surveys define *worker reallocation* as the number of workers who change employment states (i.e., who change place of employment, find or lose a job, or enter or exit the labor force) between dates $t-1$ and t. Worker turnover measures the number of labor market transitions, while worker reallocation counts the number of workers who participate in those transitions.

[3] See Albæk and Sørensen (1998), Anderson and Meyer (1994), Burgess, Lane and Stevens (2000), or Hamermesh, Hassink and van Ours (1996).

[4] Fallick and Fleischman (2001), for instance, estimate that in the United States in 1999, on average 4 million workers changed employers from one month to the next (about 2.7% of employment)—more than twice the number who transited from employment to unemployment.

The image that emerges from the empirical studies on job and worker flows is one of a labor market which is continuously reallocating employment positions across establishments (job reallocation) and workers across existing employment positions (worker turnover). This grand reallocation process often does not force workers to go through unemployment in order to switch employers, and does not require employment positions to become vacant in order to replace workers. The study of the gross flows provides valuable insights into how the labor market carries out this continual reallocation of resources, and at the same time raises important questions: To what extent are market economies able to perform this reallocation process efficiently? How is this process affected by labor market policy? To us, the picture painted by the empirical literature suggests that in order to tackle such questions, we need to pry deeper into the nature of job flows, worker flows, replacement hiring, and job-to-job transitions.

In this paper we develop a canonical equilibrium search model that incorporates job-to-job transitions, exhibits instances of replacement hiring, and conceptually distinguishes between job and worker flows.[5] A situation that arises naturally whenever agents can continue to search while matched, is one in which a matched agent contacts a new potential partner (who may also be matched) and each must decide whether to form a new match with the new partner or to stay with the old one. In a labor-market context, the employer who is trying to recruit an employed worker may have to face competition from the worker's current employer, and in addition, the current employee of the recruiting employer may attempt to discourage the employer from replacing him with the new worker (e.g., by accepting a smaller share of the matching surplus). Natural as they may seem, these generic situations have not been systematically analyzed in the literature. One of the building blocks of our theory—and one of the contributions of this paper—is a simple and flexible noncooperative bargaining procedure that allows for competition among all parties taking part in such situations. The equilibrium of the bargaining game we

[5] Job and worker reallocation are one and the same by construction in the workhorse of much of the recent macro-labor literature, the matching model of Diamond (1982b), Mortensen and Pissarides (1994) or Pissarides (2000), and there is no room for replacement hiring in the influential on-the-job search model of Burdett and Mortensen (1998).

propose delivers the division of the gains from matching as well as privately efficient creation and destruction of matches.

Our second goal in this paper is to use the theory to shed light on various related features of labor markets. For example, what determines the amount of worker turnover in excess of job reallocation? Why is it that worker turnover in Europe is substantially smaller than in the United States, whereas—despite the differences in labor-market policy regimes—job reallocation is roughly the same? Why do displaced workers tend to experience a significant and persistent fall in income? Why do workers stay unemployed when on-the-job search is at least as effective as off-the-job search? Why are good jobs not only better paid, but often also more stable?

Our work is related to several strands of the search and matching literature. The problem we tackle was first studied by Diamond and Maskin (1979, 1981). The main difference with their work, is that their agents always split the match surplus symmetrically (e.g., according to the axiomatic Nash solution with the natural outside options), which results in privately inefficient separations and too many matches being endogenously destroyed in the *laissez faire* equilibrium. In contrast, through a more flexible bargaining procedure which determines the equilibrium outside options, our notion of equilibrium delivers privately efficient separations under *laissez faire*. Variants of this problem were also studied by Wolinsky (1987) and Burdett, Imai and Wright (2004), but under assumptions that rule out the multilateral breach situations that are an essential part of our analysis. From an applied standpoint, the model we develop is also related to search and matching models of the labor market with on-the-job search, such as Burdett and Mortensen (1998) and the extension of Postel-Vinay and Robin (2002).[6]

The rest of the paper is organized as follows. Section 2 lays out the environment and characterizes the efficient allocations. Section 3 introduces the notion of equilibrium and characterizes its salient features. Section 4 extends the model to allow for free entry of employers and shows how to incorporate employment protection policies. For a special case, Section 5 provides a fuller characterization of the equilibrium set, and shows how the model can help to rationalize

[6]We provide an extensive discussion of the related literature in Kiyotaki and Lagos (2006).

many of the properties of job and worker flows documented in the empirical literature. Appendix A contains all proofs and explains some properties of the bargaining procedure we propose, and Appendix B contains supplementary material.

2 The Model

Time is continuous and the horizon is infinite. The economy is populated by a continuum of equal and fixed numbers of employers and workers. We normalize the size of each population to unity. Employers and workers are infinitely-lived and risk-neutral. They discount future utility at rate $r > 0$, and are *ex ante* homogeneous in tastes and technology.[7]

A worker meets a randomly chosen employer according to a Poisson process with arrival rate α. An employer meets a random worker according to the same process.[8] Upon meeting, the employer-worker pair randomly draws a production opportunity of productivity y, which represents the flow net output each agent will produce while matched (i.e., the pair produces $2y$). The random variable y takes one of N distinct values: $y_1, y_2, \ldots, y_N$, where $0 < y_1 < y_2 < \ldots < y_N$, and $y = y_i$ with probability π_i for $i = 1, ..., N$, and $\sum_{i=1}^{N} \pi_i = 1$. The realization of the random variable y that an employer and a worker draw when they first meet is observed without delay. We assume y remains constant for the duration of the match.

Matched and unmatched agents meet potential partners at the same rate, so when an employer and a worker meet and draw a productive opportunity, each of them may already be matched with an old production partner. Each agent can be in no more than one productive partnership at any time. The productivity of the new potential match as well as the productivities of the existing matches are public information to all the agents involved, i.e., the employer

[7] Although our main interest here is in the labor market, our model is applicable to any other setting where bilateral partnerships are relevant, such as the interactions between spouses, or between a tenant and a landlord, or between a buyer and the supplier of a customized product.

[8] In general we can think of the total meeting rate as being equal to α·(population of employers)·(population of workers). Here, because the populations of employers and workers are both unity, the rate at which a worker meets a randomly chosen employer equals the rate at which an employer meets a randomly chosen worker, and both equal α, the total meeting rate. In this basic setup, employers and workers are completely symmetric. In Section 4, we analyze extensions where employers and workers meet each other at different rates and are treated asymmetrically by policy.

and the worker who draw the new production opportunity and their existing partners, if they have any. On the other hand, each agent's history is private information, except for what is revealed by the productivity of the current match.

When an employer and a worker draw a new production opportunity, the pair and their old partners (if they have any) determine whether or not the new match is formed (and consequently whether or not the existing matches are destroyed) as well as the once-and-for-all side payments that each party pays or receives, following a bargaining procedure which we will describe shortly. Utility is assumed to be transferable among all the agents involved in a meeting. There is no outside court to enforce any formal contract, so any effective contract must be self-enforcing among the parties involved. If the parties who made contact decide to form a new partnership, they leave their existing partners, who become unmatched. In addition to these endogenous separations, we assume any match is subject to exogenous separation according to a Poisson process with arrival rate δ.

Let n_{it} and n_{0t} denote the measures of matches of productivity y_i and of unmatched workers (or employers) at date t, respectively. Let τ^k_{ijt} be the probability that, having drawn an opportunity of productivity y_k at time t, a worker who is currently in a match of productivity y_i and an employer who is currently in a match of productivity y_j choose to leave their current partners to form a new match of productivity y_k. (Hereafter, we will suppress the time subindex when no confusion should arise). The measure of workers in each state evolves according to:

$$\dot{n}_i = \alpha\pi_i \sum_{j=0}^{N}\sum_{k=0}^{N} n_j n_k \tau^i_{jk} - \alpha n_i \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \tau^k_{ij} - \alpha n_i \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \tau^k_{ji} - \delta n_i \quad (1)$$

$$\dot{n}_0 = \alpha \sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} n_i n_j \pi_k \tau^k_{ij} + \delta \sum_{j=1}^{N} n_j - \alpha n_0 \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \tau^k_{0j}. \quad (2)$$

The first term on the right side of (1) is the flow of new matches of productivity y_i created by all types of workers and employers. The second term contains the total flow of matches of productivity y_i destroyed endogenously when the worker leaves to form a new match with another employer. The third term contains the total flow of matches of productivity y_i destroyed endogenously when the employer leaves to form a new match with another worker. The last term is the flow of matches dissolved exogenously. On the right side of equation (2), the first term is the flow of workers who become unmatched when their employers decide to break the current match to form a new match with another worker. The second term is the flow of workers who become unmatched due to the exogenous dissolution of matches. The third term is the flow of new matches created by employers and unemployed workers.

Before we describe the matching equilibrium with bargaining, we study the social planner's problem. The planner chooses $\tau_{ij}^k \in [0,1]$ to maximize the discounted value of aggregate output,

$$\int_0^\infty e^{-rt} \sum_{i=1}^N 2y_i n_i dt,$$

subject to the flow constraints (1) and (2), and initial conditions for n_i, for $i = 0, ..., N$. Let λ_0 be the shadow price of an unmatched employer-worker pair, and λ_i, for $i = 1, ..., N$, be the shadow price of a match of productivity y_i at date t. The planner's Hamiltonian is:

$$H = \sum_{i=1}^N 2y_i n_i - \delta \sum_{i=1}^N (\lambda_i - \lambda_0) n_i + \alpha \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^N n_i n_j \pi_k \tau_{ij}^k (\lambda_k + \lambda_0 - \lambda_i - \lambda_j).$$

The necessary conditions for optimality are

$$\tau_{ij}^k \begin{cases} = 1 & \text{if } \lambda_k + \lambda_0 > \lambda_i + \lambda_j \\ \in [0,1] & \text{if } \lambda_k + \lambda_0 = \lambda_i + \lambda_j \\ = 0 & \text{if } \lambda_k + \lambda_0 < \lambda_i + \lambda_j, \end{cases} \tag{3}$$

together with the Euler equations

$$\begin{aligned} r\lambda_i - \dot{\lambda}_i &= 2y_i - \delta(\lambda_i - \lambda_0) + \alpha \sum_{j=0}^N \sum_{k=1}^N n_j \pi_k (\tau_{ij}^k + \tau_{ji}^k)(\lambda_k + \lambda_0 - \lambda_i - \lambda_j), \\ r\lambda_0 - \dot{\lambda}_0 &= \alpha \sum_{j=0}^N \sum_{k=1}^N n_j \pi_k (\tau_{0j}^k + \tau_{j0}^k)(\lambda_k - \lambda_j), \end{aligned}$$

and (1) and (2). According to (3), to achieve the optimal allocation, the planner specifies that a type i worker and type j employer should form a new match of productivity y_k with certainty if and only if the sum of the shadow prices of the new match and the unmatched worker and employee (which the new match would generate) exceeds the sum of the shadow prices of the existing matches of productivity y_i and y_j. From (3) we also learn that $\tau_{ij}^k = \tau_{ji}^k$, except possibly for the case of randomized strategies. Intuitively, there is no inherent asymmetry between a worker and an employer, so the planner treats them symmetrically in the optimal allocation. These observations allow us to summarize the first-order necessary conditions as:

$$r\lambda_i - \dot{\lambda}_i = 2y_i - \delta\left(\lambda_i - \lambda_0\right) + 2\alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\tau_{ij}^k} \tau_{ij}^k \left(\lambda_k + \lambda_0 - \lambda_i - \lambda_j\right) \tag{4}$$

$$r\lambda_0 - \dot{\lambda}_0 = 2\alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\tau_{0j}^k} \tau_{0j}^k \left(\lambda_k - \lambda_j\right). \tag{5}$$

Equation (4) says that in the planner's problem, the flow return of a match of productivity y_i equals the capital gain associated with the change of the shadow price plus the sum of the three terms on the right side: the flow output generated by the match, minus the expected loss from an exogenous separation, plus the expected gain from the endogenous creation and destruction of matches that occurs when either of the agents in the match of productivity i meets an agent in a match of productivity j (which occurs at rate $2\alpha n_j$) and draws productivity y_k (with probability π_k). The flow return of a pair of unmatched agents in (5) is similar, but output is zero and there is no loss resulting from exogenous separation.

3 Competitive Matching Equilibrium

In order to analyze the behavior of the decentralized economy, we must specify the bargaining procedure through which gains from trade are divided among agents. The environment we have laid out requires a procedure that is flexible enough to handle an array of interactions, namely, (i) meetings between an unmatched employer and an unmatched worker, (ii) meetings between a matched agent and an unmatched agent, and (iii) meetings between a matched employer and a matched worker.[9] In each of these instances, the bargaining procedure must

[9] Since creating a new match entails destroying one existing match in situation (ii) and two existing matches in situation (iii), we follow Diamond and Maskin (1979), and refer to the former as a "single breach" and to the latter as a "double breach."

determine whether the new match is formed or the old match(es) continue, as well as how the gains from trade are allocated among the agents involved in the meeting. In addition, we deem it desirable that the bargaining procedure should afford enough flexibility in the negotiations between agents to be capable of delivering match destruction and preservation decisions that are efficient from the standpoint of the agents involved in the meeting, i.e., the two agents with the new production opportunity and their partners, if they have any.

The bulk of the existing random search and matching literature follows the seminal work of Diamond and Maskin (1979), Diamond (1982a), and Pissarides (1984) in assuming that agents matched in pairs share rents according to the axiomatic Nash solution. However, this approach is not well suited for environments with richer interactions among agents (such as double breaches), in which the agents' outside options are not self-evident, and must be determined along with the equilibrium match-formation decisions. In this context, another possibility would be to adopt a cooperative solution concept such as the Shapley value. The drawback to this approach is that the winning pair will generally leave some surplus to the losing partner(s), which is not rational for the winners since, in the *laissez faire* economy that we study, they can walk away from their current matches without penalty. For these reasons, we develop a notion of equilibrium where the gains from trade are apportioned and the prevailing matches are determined through the following bargaining procedure.

When a worker and an employer find a new production opportunity, a move by Nature first determines, with equal probability, whether the worker(s) or the employer(s) involved in the meeting have the bargaining power. Each agent with the bargaining power chooses to make a take-it-or-leave-it offer to her new potential partner (if she has one), to her old partner (if she has one), or to both. An offer consists of a proposal to produce together and a division of surplus to be implemented through spot side payments. Once the offers have been made, the recipient of multiple offers decides which offer to accept, if any.[10] Any other offer made by the

[10] In situations in which no agent has received multiple offers, if there is more than one recipient of a single offer, Nature selects with equal probability one these recipients to first decide which offer to accept.

proposer of an accepted offer is automatically withdrawn. The recipients of any offers which are still outstanding then decide whether to accept or reject, and the bargaining ends.[11] Bargaining does not take time: the entire process finishes instantaneously. We specify that matched agents split output equally as long as neither encounters a new production opportunity.[12] There is no outside court to enforce formal contracts, so anyone can walk away from a match at any time without penalty.[13]

A *competitive matching equilibrium* is a set of bargaining strategies specifying how much utility to offer bargaining partners in order to produce together, and whether to accept or reject a received offer, together with a population distribution of partnerships such that: *(a)* taking the population distribution and the bargaining strategies of the other agents as given, each agent chooses her bargaining strategies in order to maximize her expected discounted utility; and *(b)* given the agents' bargaining strategies, the population distribution satisfies (1) and (2). Since a worker and an employer who form a match are inherently symmetric, we will restrict attention to equilibria in which workers and employers are treated symmetrically and agents are distinguished only by the productivity of their current match.

Let V_i be the value of expected discounted utility of an agent who is in a match of productivity y_i, and let V_0 be the value of an unmatched agent. We begin by describing the equilibrium outcomes of the bargaining procedure for each of the three possible types of meetings, taking V_i and V_0 as given. (We will later describe how these values are determined in equilibrium.)

(i) AN UNMATCHED EMPLOYER AND AN UNMATCHED WORKER DRAW AN OPPORTUNITY

[11] In a different context, Felli and Roberts (2001) use a similar Bertrand-style procedure. There is also a strand of the random matching literature with on-the-job search that considers the special case of single breach in which two employers compete for a single worker and employers engage in Bertrand competition for the worker (e.g., Dey and Flinn (2005), Postel-Vinay and Robin (2002)). Although for this special case, the equilibrium outcome of our bargaining procedure coincides with the one implied by Bertrand competition, our procedure specifies how to resolve situations with richer interactions among agents, such as double breaches. We discuss alternative bargaining procedures in Appendix B.

[12] Equivalently, we can think of the matched pair without an outside production opportunity as being involved in continual negotiations by which the expected value of side payments nets out to be zero. (See the proof of Proposition 3 for more on this.)

[13] In Section 4, we introduce a government that imposes a tax on every employer who separates from a worker.

Suppose that an employer with a vacancy and an unemployed worker draw an opportunity of productivity y_k. Since both are unmatched, the outside option to each is V_0. This case is illustrated in Figure 1, where we have named the two agents A and B.

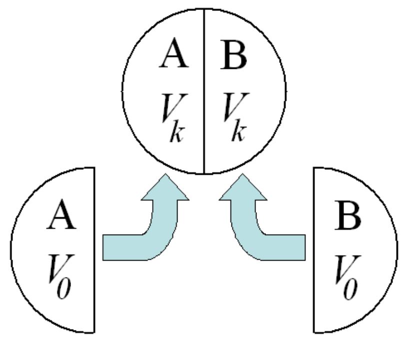

Figure 1: An unmatched employer meets an unmatched worker

Let X_{ij}^k be the value that agent i offers agent j in order to form (or preserve) a match of productivity y_k. X_{ij}^k includes the value of the match plus the net side payment that agent i pays to agent j. The bargaining unfolds as follows. With probability a half, A makes a take-it-or-leave-it offer X_{AB}^k to B. A will choose this offer in order to maximize her own utility, subject to the constraint that B will accept. Hence, A offers $X_{AB}^k = V_0$, and B accepts. With equal probability, B offers $X_{BA}^k = V_0$ to A, and A accepts. Let Π_i be the expected payoff to agent i and Γ_i be her expected gain. Then, $\Pi_A = \Pi_B = \frac{1}{2}V_0 + \frac{1}{2}(2V_k - V_0) = V_k$, and $\Gamma_A = \Gamma_B = V_k - V_0$. The expected value of the side payment is zero and both unmatched agents enjoy the same expected capital gains from forming the new match.

(ii) A MATCHED AGENT AND AN UNMATCHED AGENT DRAW AN OPPORTUNITY

Suppose that worker B, who is currently in a match of productivity y_i with employer A, meets employer C, who has a vacancy, and they draw a productive opportunity y_k. This situation is illustrated in Figure 2.

Depending upon Nature's draw, with equal probability, either worker B has bargaining

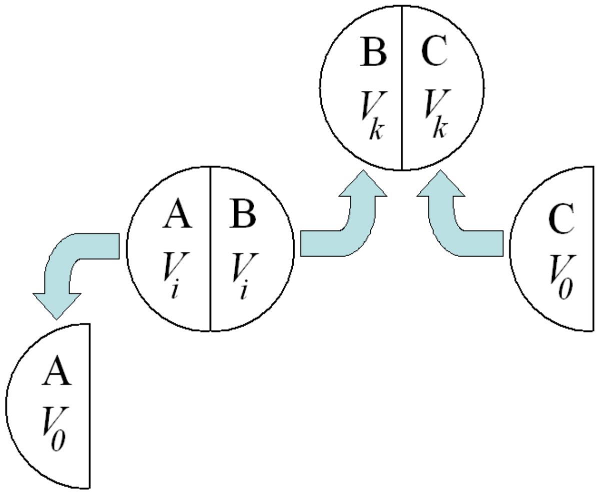

Figure 2: Single breach

power, or employers A and C have bargaining power. In the event that B has bargaining power, he makes a take-it-or-leave-it offer to A or C. Since A and C have no alternative matching opportunity, B would offer each of them their minimum acceptable payoff, V_0. Hence, B's payoff from continued production with A would be $2V_i - V_0$, while his payoff from forming a new match with C would be $2V_k - V_0$. Thus, if $V_i < V_k$, B offers C to produce together, she accepts, and the payoffs to A, B, and C are V_0, $2V_k - V_0$, and V_0, respectively. Conversely, if $V_k < V_i$, B offers A to continue to produce together, she accepts, and the payoffs to A, B, and C are V_0, $2V_i - V_0$, and V_0.

In the event that Nature draws employers A and C to have bargaining power, they simultaneously make offers to B. Because A's outside option is the value of being unmatched, V_0, the maximum A is willing to offer B to preserve their match is $2V_i - V_0$. Similarly, the maximum C is willing to offer B to form a new match is $2V_k - V_0$. Since A and C make their offers simultaneously, the competition becomes Bertrand; A offers $X^i_{AB} = \min(2V_i - V_0, 2V_k - V_0 + \varepsilon)$ and C offers $X^k_{CB} = \min(2V_k - V_0, 2V_i - V_0 + \varepsilon)$, where ε is an arbitrarily small positive number. If $V_i < V_k$, B accepts C's offer to form a new match and the payoffs to A, B, and C are V_0,

$2V_i - V_0$, and $2V_k - 2V_i + V_0$, respectively. If $V_k < V_i$, B accepts A's offer to continue the existing match and the payoffs to A, B, and C are $2V_i - 2V_k + V_0$, $2V_k - V_0$, and V_0.

Observe that regardless of whether the worker or the employers have bargaining power, the more valuable match prevails in the equilibrium. The sum of the payoffs of all agents is equal to $\Pi_A + \Pi_B + \Pi_C = V_0 + 2V_k$ when the new match is formed, and equal to $\Pi_A + \Pi_B + \Pi_C = 2V_i + V_0$ when the old match continues. The Coase Theorem holds *for given values of* V_j's: with fully transferable utility, the total sum of payoffs of all the agents involved in the meeting is maximized under the bargaining procedure of our competitive matching equilibrium.

Concerning how agents split the total sum of payoffs, when B and C form a new match, the expected capital gains to A, B, and C are $-(V_i - V_0)$, $V_k - V_0$, and $V_k - V_i$, respectively.[14] Interestingly, through the side payment of transferable utility, the expected gains to the agents who form the new match are equal to the capital gains of the new partner instead of their own capital gains: the gain to B is $V_k - V_0$ and the gain to C is $V_k - V_i$, so B, who is in a stronger bargaining position, enjoys a larger gain than C. On the other hand, A suffers a capital loss from becoming unmatched. Conversely, when B chooses to stay matched with A, the expected gains to A, B, and C are $-(V_k - V_0)$, $V_k - V_0$, and 0, respectively. Notice that although the current match is not destroyed, the old partner, A, has to buy out B's expected gains from matching with C (by making a utility side payment with expected value equal to $V_k - V_0$) in order to persuade B to stay in the current match.[15]

(*iii*) A MATCHED EMPLOYER AND A MATCHED WORKER DRAW AN OPPORTUNITY

Suppose that worker B and employer C meet and draw a productive opportunity y_k. The situation now is that B is currently in a match of productivity y_i with employer A, while C is in a match of productivity y_j with worker D. This is illustrated in Figure 3.

[14] The respective expected payoffs to A, B, and C are $[V_0, \frac{1}{2}(2V_k - V_0 + 2V_i - V_0), \frac{1}{2}(V_0 + 2V_k - 2V_i + V_0)] = [V_i, V_i, V_0] + [-(V_i - V_0), V_k - V_0, V_k - V_i]$. All this follows from the arguments in the previous three paragraphs.

[15] In the above analysis we assumed that the matched agent with the outside opportunity to form a new match was a worker; i.e., B was a worker, while A and C were employers. Since workers and employers are symmetric, the gains from trade will be the same if instead, A and C are workers and B is an employer.

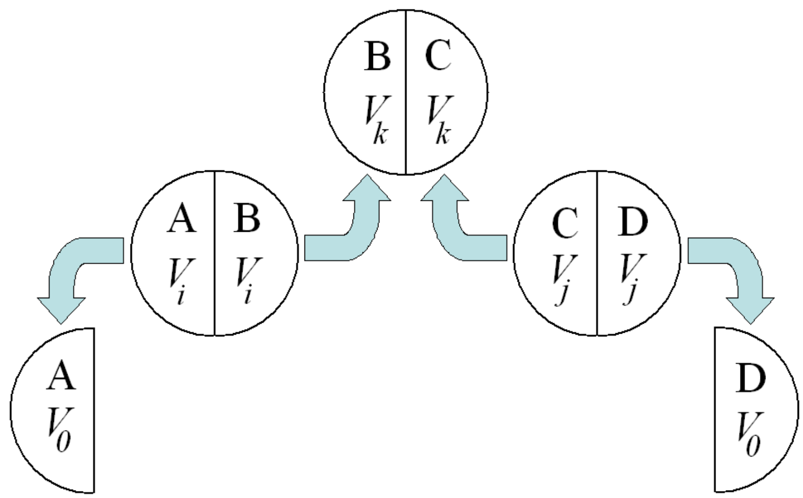

Figure 3: Double breach

With probability a half, the employers A and C simultaneously make take-it-or-leave-it offers to worker B. Employer C also makes a take-it-or-leave-it offer to his existing worker, D, and this offer is contingent on his offer to B being rejected. C makes the smallest acceptable offer to D, i.e., V_0, since D has no other productive opportunities. The resulting payoff to C from continuing to match with D is $2V_j - V_0$, which constitutes the opportunity cost for C to form a new match. Thus, the maximum C is willing to offer B is $2V_k - (2V_j - V_0)$. Because A's opportunity cost of continuing to match is the value of being unmatched, V_0, the maximum A is willing to offer B is $2V_i - V_0$. Since this valuation is positive, A will want to make sure that B finds her offer acceptable, and for this she must ensure that B's payoff is at least as large as V_0. Thus, A offers B a payoff $X^i_{AB} = \max\{V_0, \min[2V_i - V_0, 2V_k - (2V_j - V_0) + \varepsilon]\}$ and C offers B $X^k_{CB} = \min[2V_k - (2V_j - V_0), 2V_i - V_0 + \varepsilon]$, for an arbitrarily small positive ε. B will accept C's offer to form the new match if and only if $2V_i - V_0 < 2V_k - (2V_j - V_0)$, or $V_i + V_j < V_k + V_0$, i.e., if and only if the sum of the values of the new match and the unmatched exceeds the sum of the values of the existing matches. Conversely, if $V_k + V_0 < V_i + V_j$, A and B preserve their match and whether or not A may have to offer B a side payment depends on whether the new potential match between B and C is better or worse than C's current match. If the new potential match is better ($V_j < V_k$), then A has to bid C away by giving B an offer equal to C's offer, $2V_k - (2V_j - V_0) > V_0$. However, if $V_k < V_j$, C's offer poses no threat to A who only has to offer B utility V_0 to preserve their current match.

With probability a half, it is instead the workers B and D who have the bargaining power. The analysis is identical to that described in the previous paragraph up to a relabelling so we omit it. In the two possible sequences of bargaining, i.e., regardless of whether the employers or the workers have bargaining power, we find that B and C form a new match (and A and D become unmatched) with certainty if and only if the sum of the value of the new match and the unmatched exceeds the sum of two existing matches. Figure 4 summarizes the expected gains from trade in double-breach situations. (The figure assumes $V_i < V_j$, without loss of generality.)

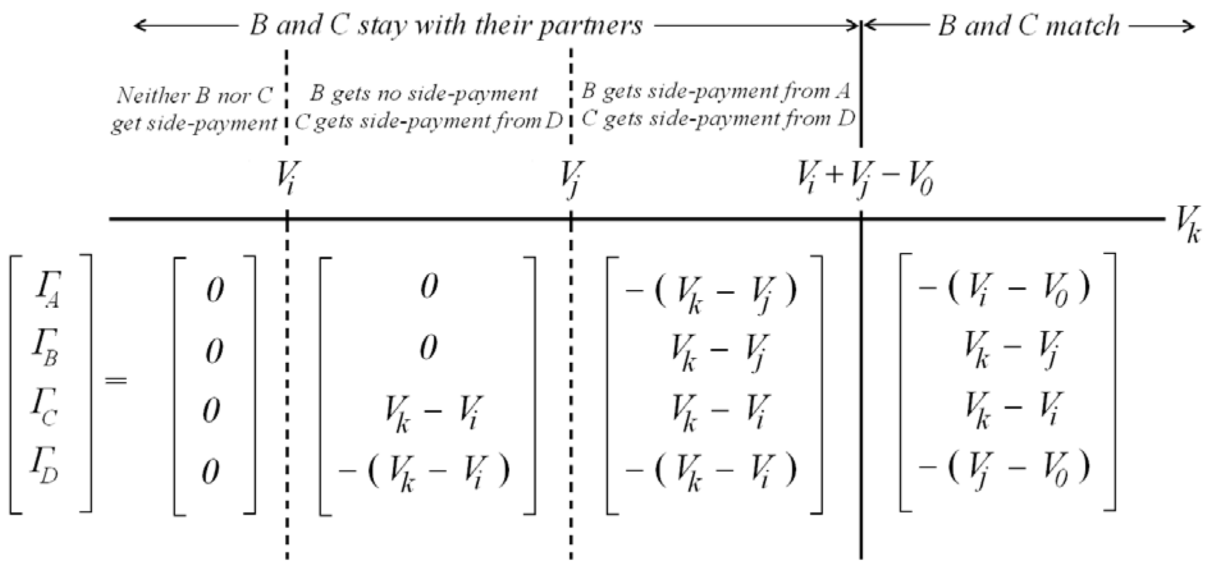

Figure 4: Double breach: expected capital gains

The figure shows that if $V_i + V_j - V_0 < V_k$, then B and C form a new match. The equilibrium expected side payment between B and C is such that the expected gain to each is equal to the capital gain to the new partner, instead of their own capital gain.[16] When $V_j < V_k < V_i + V_j - V_0$, although the existing matches continue, the old partner must, on average, pay his current partner her opportunity cost of giving up the option to form a new match. When $V_i < V_k < V_j$, because the value of the new potential match is not as large as the value of the existing match between C and D, A does not have to pay a side payment to B on average in order to persuade him to stay in the existing match. (Loosely speaking, A will ignore B's lousy opportunity to match with C.) When $V_k < V_i$, the value of the new potential match between B and C is so small, that on average, neither A nor D make side payments.

We summarize the main features of the bargaining outcomes in Proposition 1. The proof of parts *(a)* and *(b)* follows from the above discussion. Part *(c)* is proved in Appendix A, where also provide a graphical analysis of the bargaining procedure.

Proposition 1 *For given value functions, the matching decisions and side payments are uniquely determined in the symmetric competitive matching equilibrium. Moreover,*
(a) When two agents find an opportunity to form a new match, they form the new match and displace their existing partners (if they have any) if and only if the sum of the values of the new match and the unmatched exceeds the total value of the existing matches.
(b) Through the side payment, the expected net gain to the agent who forms a new match is equal to the capital gain of the new partner (instead of her own capital gain).
(c) For every meeting, the equilibrium outcomes (and expected outcomes) induced by the bargaining procedure lie in the core.

[16] The expected payoffs to B and C are $\left[\frac{1}{2}(2V_i - V_0 + 2V_k - 2V_j + V_0), \frac{1}{2}(2V_k - 2V_i + V_0 + 2V_j - V_0)\right] = [V_i, V_j] + [V_k - V_j, V_k - V_i]$. In the absence of side payments, if B and C were to form a new match, B would gain $V_k - V_i$ and C would gain $V_k - V_j$. Instead, the equilibrium side payments imply that these gains are swapped: B gains $V_k - V_j$ and C gains $V_k - V_i$, so when a new match is formed, the agent who is currently in the better match enjoys a larger capital gain.

In equilibrium, individual agents' expected payoffs satisfy the following Bellman equations:

$$\begin{aligned} rV_i - \dot{V}_i &= y_i + \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \left[\phi_{ij}^k (V_k + s_{ij}^k - V_i) + (1 - \phi_{ij}^k) z_{ij}^k \right] \\ & \quad - \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \left[\phi_{ji}^k (V_i - V_0) + (1 - \phi_{ji}^k) z_{ij}^k \right] - \delta (V_i - V_0) \end{aligned} \tag{6}$$

for $i = 1, ..., N$, and

$$rV_0 - \dot{V}_0 = \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} n_j \pi_k \phi_{0j}^k (V_k + s_{0j}^k - V_0). \tag{7}$$

Consider, for example, the most general situation illustrated in Figure 3. The return of agent B (who is in a match of type i with agent A) in equation (6) includes his share of the flow output generated by the match, y_i, plus the expected capital gain he experiences when meeting other potential partners, minus the expected capital loss he suffers when his partner meets other potential partners, or when the match is destroyed exogenously. The choice of a worker in a match of type i and an employer in a match of type j to form a new match of type k is represented by $\phi_{ij}^k \in [0, 1]$. Conditional on B having met a new partner, n_j is the probability that this new partner is in a match of type j, and π_k is the probability that the employer-worker pair draws an opportunity to form a new match of type k. If the new match is formed, then B gains $V_k - V_i$ plus s_{ij}^k, which denotes the expected side payment that B receives from the new partner to form the new match, as implied by the bargaining procedure we outlined earlier. (Thus, $s_{ij}^k = -s_{ji}^k$.) If the new match is not formed, which occurs with probability $1 - \phi_{ij}^k$, B receives an expected side payment, z_{ij}^k, from his current partner, A. The negative sum of terms on the right side of (6) are the expected losses that B experiences when A contacts other agents. For instance, if A meets another agent currently in a match of type j and they decide to form a new match of type k (which they do with probability ϕ_{ji}^k), B gets displaced and loses $V_i - V_0$. Alternatively, B may be able to persuade A to stay in the current match by paying her a side payment with expected value z_{ij}^k.

From part *(a)* of Proposition 1 we know that $\phi_{ij}^k = \phi_{ji}^k$, and $\phi_{ij}^k (V_k + V_0 - V_i - V_j) = \max_{\phi_{ij}^k} \phi_{ij}^k (V_k + V_0 - V_i - V_j)$, and from part *(b)* that $V_k + s_{ij}^k - V_i = V_k - V_j$, so (6) and (7) reduce to

$$
\begin{aligned}
rV_i - \dot{V}_i &= y_i - \delta\left(V_i - V_0\right) + \alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\phi_{ij}^k} \phi_{ij}^k \left(V_k + V_0 - V_i - V_j\right) \\
rV_0 - \dot{V}_0 &= \alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\phi_{0j}^k} \phi_{0j}^k \left(V_k - V_j\right).
\end{aligned}
$$

The value of an employer-worker pair, $\lambda_i^c = 2V_i$, satisfies

$$
r\lambda_i^c - \dot{\lambda}_i^c = 2y_i - \delta(\lambda_i^c - \lambda_0^c) + \alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\phi_{ij}^k} \phi_{ij}^k (\lambda_k^c + \lambda_0^c - \lambda_i^c - \lambda_j^c) \tag{8}
$$

$$
r\lambda_0^c - \dot{\lambda}_0^c = \alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\phi_{0j}^k} \phi_{0j}^k (\lambda_k^c - \lambda_j^c). \tag{9}
$$

With this notation, part *(a)* of Proposition 1 can be written as:

$$
\phi_{ij}^k \begin{cases} = 1 & \text{if } \lambda_k^c + \lambda_0^c > \lambda_i^c + \lambda_j^c \\ \in [0,1] & \text{if } \lambda_k^c + \lambda_0^c = \lambda_i^c + \lambda_j^c \\ = 0 & \text{if } \lambda_k^c + \lambda_0^c < \lambda_i^c + \lambda_j^c. \end{cases} \tag{10}
$$

The competitive matching equilibrium can now be summarized by a list $(\lambda_i^c, \phi_{ij}^k, n_i)$ for $i, j = 0, ..., N$ and $k = 1, ..., N$ that satisfies the Bellman equations (8) and (9), the match formation conditions (10) and the laws of motion (1) and (2).

Notice the similarity between the equilibrium conditions and the planner's optimality conditions. From (10) and (3) we see that the competitive matching equilibrium shares a key feature with the social optimum: match formation is *privately efficient*. That is, under the bargaining procedure we proposed, for given values of λ_i^c's, the individual agents' utility maximizing matching decisions imply that a new match is formed only in the cases when doing so is efficient from the point of view of all the agents involved in the meeting.

In order to illustrate how the match formation decisions would change if one were to use the surplus splitting rule implied by the axiomatic Nash solution instead of our bargaining procedure, we contrast our results with Diamond and Maskin's (1979, 1981). Consider a special case of the double breach illustrated in Figure 3, i.e., the symmetric case where C and D are each in a match of type i. Diamond and Maskin propose that B and C split their gains from matching by solving $\max_s (V_k - s - V_i)(V_k + s - V_i)$, where s denotes a side payment. Clearly, the solution has $s = 0$ and B and C's respective gains from matching equal $V_k - V_i$, so they choose to leave their partners to form a new match if and only if $V_i < V_k$. By doing this they generate total utility $2V_k + 2V_0$ and destroy $4V_i$, so there will be instances—specifically, those where $V_k + V_0 < 2V_i$—in which B and C form a new match but ought not to, from the point of view of the four agents involved in the meeting. These situations lead to too much breach in the equilibrium where agents are free to walk away from their partners.

Diamond and Maskin point out that private efficiency can be restored if agents are forced to pay what lawyers call *compensatory damages*, i.e., if in order to separate from their current partners, B and C are required to pay each of them $T = V_i - V_0$. Under this rule, the gains from trade are determined by solving $\max_s (V_k - s - V_i - T)(V_k + s - V_i - T)$, so the respective gains to B and C if they form a new match, equal $V_k + V_0 - 2V_i$. The payment of compensatory damages forces B and C to fully internalize the adverse effects that their match-formation decisions have on their current partners, and consequently the equilibrium generates privately efficient separations under this policy. Under the bargaining procedure that we have proposed, the fact that A is willing to pay B up to $2V_i - V_0$ (instead of the "natural" Nash bargaining threat point, V_i) to preserve their match, means that $2V_i - V_0$ effectively becomes B's opportunity cost of matching with C. In other words, A's ability to bid for B makes B internalize A's loss from being displaced, and consequently, our competitive matching equilibrium implements privately efficient separations without the need for government policy.

So far, our comparison of match formation decisions in the competitive matching equilibrium and the planner's problem has been for given value functions $\{V_i\}_{i=1}^N$ and shadow prices $\{\lambda_i\}_{i=1}^N$. The equilibrium values of the match to the pair satisfy very similar conditions to the ones that the shadow prices must satisfy in a social optimum: conditions (8) and (9) would be identical to (4) and (5), were it not for the fact that in the optimality conditions, there is a "2" in front of the contact rate α. This difference is due to a *composition externality* in match-formation: in the decentralized economy, an individual agent does not take into account the impact that her own decisions to form and destroy matches have on the distribution of agents across different types of matches, which in turn influences the matching opportunities of the other agents.

Consider an employer A and a worker B who are in a match of productivity y_i. According to (8), the equilibrium value of this match includes $(\phi_{ij}^k + \phi_{ji}^k)(V_k + V_0 - V_i - V_j) = \phi_{ij}^k(\lambda_k^c + \lambda_0^c - \lambda_i^c - \lambda_j^c)$, i.e., the expected capital gain to the A-B pair in the event that either of them forms a match of productivity y_k with an agent who is in a match of productivity y_j.[17] In contrast, according to (4), the planner's calculation of the expected return to having an additional match of type i, includes not only the expected gains to the A-B pair from matching with agents in matches of type j, but also the expected gains that having this match between A and B in the pool generates for all those other agents in matches of type j by increasing their contact rate with agents in matches of type i. Hence, the total expected gains that the planner assigns to a match of type i is $(\tau_{ij}^k + \tau_{ji}^k)(\lambda_k + \lambda_0 - \lambda_i - \lambda_j)$. The key observation is that, although the arrival rate of *any* new matching opportunity is constant and equal to α, random matching implies that the arrival rate of a new matching opportunity with an agent who is *in a match of type i* is αn_i—i.e., proportional to the measure of agents of that type. Since the decision to form a new match depends not only on the quality of the new potential match, but also on the types of the existing matches, the payoff-relevant meeting rate for an agent is in fact αn_i, not just α. Thus, with random matching and the heterogeneity induced by past matching decisions, the relevant meeting technology is *effectively* quadratic, since the total number of contacts between agents who are in matches of productivity y_i and agents in matches of productivity y_j is $\alpha n_i n_j$.[18]

[17] The expression $(\phi_{ij}^k + \phi_{ji}^k)(V_k + V_0 - V_i - V_j)$ is the sum of A and B's expected capital gains from future matching. The sum of the expected capital gains to the A-B pair in the event that B forms a new match is $\phi_{ij}^k[(V_k - V_j) - (V_i - V_0)]$, which includes B's expected gain from forming a match of type k with a new employer who is currently in a match of type j (the first term), as well as the loss to A from being displaced. Similarly, the sum of the expected capital gains to the A-B pair in the event that A forms a new match is $\phi_{ji}^k[(V_k - V_j) - (V_i - V_0)]$.

[18] Our use of the term "quadratic" here is slightly nonstandard since we are using it to describe meetings between two particular types of agents, while in random matching economies it is normally applied to the *aggregate* meeting technology, i.e., to meetings between *any* pair of agents. The externality we are emphasizing is related to Mortensen's (1982) observation that "mating models" in which an agent's decisions affect other agents' meeting probabilities typically fail to achieve the socially optimal allocation. Although the total number

The relationship between the equilibrium match values and the planner's shadow prices can be recast as follows. Define $\mu_i = \lambda_i - \lambda_0$ and $\mu_i^c = \lambda_i^c - \lambda_0^c$. From (4), (5), (8) and (9),

$$\begin{aligned}(r+\delta)\,\mu_i - \dot{\mu}_i &= 2y_i + 2\alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\tau_{ij}^k,\tau_{0j}^k} \left[\tau_{ij}^k\left(\mu_k - \mu_i - \mu_j\right) - \tau_{0j}^k\left(\mu_k - \mu_j\right)\right] \quad (11)\\ 2\,(r+\delta)\,\mu_i^c - 2\dot{\mu}_i^c &= 4y_i + 2\alpha \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\phi_{ij}^k,\phi_{0j}^k} \left[\phi_{ij}^k(\mu_k^c - \mu_i^c - \mu_j^c) - \phi_{0j}^k(\mu_k^c - \mu_j^c)\right]. \quad (12)\end{aligned}$$

Observe that if we focus on the steady state ($\dot{\mu}_i = \dot{\mu}_i^c = 0$) and modify the planner's problem by replacing r in (11) with $r' = 2r + \delta$, then the first order conditions of this modified planner's problem are identical to the equilibrium conditions for the competitive matching equilibrium, except that the flow outputs y_i all appear multiplied by one half for the planner. A proportional change of all output levels just induces a proportional change in the μ_i's and does not change the choices of $\{\tau_{ij}^k, \tau_{0j}^k\}$ nor the resulting distribution $\{n_i\}$. We summarize all this as follows.

Proposition 2 *There exists a steady-state competitive matching equilibrium. Moreover, all steady-state competitive matching equilibria satisfy the first order conditions of a modified social planner's problem, in which the subjective discount rate, r, is replaced by $r' = 2r + \delta$. A stationary allocation that satisfies the first order necessary conditions of the modified planner's problem can be decentralized as a steady-state competitive matching equilibrium.*

The existence proof is in Appendix A. In general, the steady-state equilibrium need not be unique.[19] If the modified planner's problem admits a stationary solution, then this allocation can be decentralized as a steady-state competitive matching equilibrium.[20]

of meetings is constant in our economy, the gains from forming a match of a given quality, and hence the agents' value functions $\{V_i\}_{i=1}^N$, depend on the endogenous distribution of match qualities, which is the source of the externality we have identified. This sort of composition externality is a generic feature of an economy with random matching (non-directed search) and heterogeneity; *ex post* match-specific heterogeneity and on-the-job search (e.g., Diamond and Maskin (1979, 1981)) and *ex ante* heterogeneity, even with no on-the-job search (e.g., Burdett and Coles (1997), Davis (2001), and Shimer and Smith (2000, 2001a)).

Having said this, our aggregate meeting technology *is* in fact quadratic (see footnote 8), but this is immaterial since the populations of employers and workers are being kept fixed. In Appendix B, we show that the composition externality is still present in the model with a constant-returns aggregate meeting technology.

[19] For the special case with $N = 2$, for instance, we have been able to prove that π_2 large enough is sufficient for uniqueness, and have constructed examples of multiplicity for low values of π_2 (e.g., $r = .04$, $\alpha = 1$, $\delta = .004$, $\pi_2 = .05$, $y_1 = 1$ and $y_2 = 1.382$).

[20] The subtle phrasing in the last part the proposition contemplates the possibility that the solution to the

4 Free Entry and Employment Protection

So far, we have assumed constant and equal populations of employers and workers, and have abstracted from any type of government policy that may affect match formation decisions. In this section we generalize the analysis by allowing free entry of employers and introducing employment protection policies. We will use this more general formulation in our labor market analysis of Section 5.

Let m_j be the number of employers in matches of productivity y_j. We still use n_i to denote number of workers employed in matches of productivity y_i. Since matching is one-to-one, $m_i = n_i$ for $i \geq 1$, but the number of unmatched employers, m_0, may be larger or smaller than the number of unemployed workers, n_0. We assume that a worker contacts an employer who is currently in a match of type j at rate αm_j, while an employer contacts a random worker in a match of type i at rate αn_i.[21] We assume that unmatched employers incur a total flow cost $C(m_0)$ of posting vacancies, with $C' \geq 0$ and $C'' > 0$ and $C'(0) = 0$. We begin by formulating the planner's problem. The planner chooses $\tau_{ij}^k \in [0,1]$ and $m_0 \geq 0$ to maximize the discounted value of aggregate output

$$\int e^{-rt}\left[\sum_{i=1}^{N} 2y_i n_i - C(m_0)\right] dt$$

subject to the flow constraints

$$\dot{n}_i = \alpha\pi_i \sum_{j=0}^{N}\sum_{k=0}^{N} n_j m_k \tau_{jk}^i - \alpha n_i \sum_{j=0}^{N}\sum_{k=1}^{N} m_j \pi_k \tau_{ij}^k - \alpha m_i \sum_{j=0}^{N}\sum_{k=1}^{N} n_j \pi_k \tau_{ji}^k - \delta n_i, \tag{13}$$

$$\dot{n}_0 = \alpha \sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} n_i m_j \pi_k \tau_{ij}^k + \delta \sum_{j=1}^{N} n_j - \alpha n_0 \sum_{j=0}^{N}\sum_{k=1}^{N} m_j \pi_k \tau_{0j}^k \tag{14}$$

and initial conditions for n_i, $i = 0, ..., N$. Let λ_i be the shadow price of a match with productiv-

modified planner's problem may be nonstationary. This possibility was emphasized by Shimer and Smith (2001b), who in the context of a random matching model with *ex ante* heterogeneity and no on-the-job search, constructed an example of an optimal policy that approaches a limit cycle.

[21] In other words, the total number of meetings is given by a quadratic matching technology, αmn, where m is the total number of employers and n the total number of workers. In our formulation, $n = 1$ and $m = 1 + m_0 - n_0$. In Appendix B we consider the case in which the aggregate meeting technology is instead given by a function $\xi(m,n)$ which is monotonic in both arguments and homogeneous of degree one.

ity y_i and let λ_0 be the sum of the shadow prices of an unmatched employer and an unemployed worker at date t. We summarize the planner's first order necessary conditions as follows.

Lemma 1 *Consider the economy with free entry of employers. The necessary conditions for optimality are (3) and*

$$C'(m_0) = \alpha \sum_{i=0}^{N} \sum_{k=1}^{N} n_i \pi_k \tau_{i0}^k (\lambda_k - \lambda_i), \tag{15}$$

together with the Euler equations

$$\begin{aligned} r\lambda_i - \dot{\lambda}_i &= 2y_i - \delta(\lambda_i - \lambda_0) + \alpha(m_0 + n_0) \sum_{k=1}^{N} \pi_k \max_{\tau_{i0}^k} \tau_{i0}^k (\lambda_k - \lambda_i) \\ &\quad + 2\alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{\tau_{ij}^k} \tau_{ij}^k (\lambda_k + \lambda_0 - \lambda_i - \lambda_j) \end{aligned} \tag{16}$$

$$\begin{aligned} r\lambda_0 - \dot{\lambda}_0 &= -C'(m_0) + \alpha(m_0 + n_0) \sum_{k=1}^{N} \pi_k \max_{\tau_{00}^k} \tau_{00}^k (\lambda_k - \lambda_0) \\ &\quad + 2\alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{\tau_{0j}^k} \tau_{0j}^k (\lambda_k - \lambda_j). \end{aligned} \tag{17}$$

The left side of condition (15) is the marginal cost of an unmatched employer (or the marginal cost of "maintaining a vacancy") and the right side is the expected return from having an additional unmatched employer: $\lambda_k - \lambda_i$ is the capital gain to the planner from creating a new match of productivity y_k by matching an unmatched employer to a worker employed in a match of productivity y_i, while $\alpha n_i \pi_k \tau_{i0}^k$ is the arrival rate of this capital gain. In the model with free entry we have an additional unknown at each date, and (15) provides the additional optimality condition. (Alternatively, we can drop (15) and treat the initial condition for m_0 parametrically to obtain the formulation with fixed but not necessarily equal populations of employers and workers). Conditions (16) and (17) are very similar to those for the model with constant and equal populations of employers and workers. In particular, note that they reduce to (4) and (5), respectively, if we set $C' = 0$ and $m_0 = n_0$.

Next, we characterize the competitive matching equilibrium using the bargaining procedure we described in Section 3. Each employer who posts a vacancy pays $c = C'(m_0)$, while matched employers do not have to pay anything (e.g., because production itself is free advertisement to attract workers).[22] We consider a class of employment protection policies that specify that when an employer breaks a match of type i, she must pay severance compensation $S_i \leq V_i - V_0$ to the worker she separates from, and a firing tax T_i to the government.[23] Note that the regulations apply to separations initiated by employers (dismissals) but not to those initiated by workers (quits), and therefore break the symmetry between employers and workers. For this reason we now let V_i be the value of a worker, J_i the value of an employer, and $M_i = J_i + V_i$ the value of a match of type i in the equilibrium. Since the employer and the worker in a match are no longer symmetric, we think of the matched pair without an outside production opportunity as being involved in continual negotiations, instead of simply splitting output equally[24]. Proposition 3, which we prove in Appendix A, summarizes the effects that employment protection policies have on the outcomes of the different bargaining situations, for given value functions.

Proposition 3 *(a) An unmatched employer and an unemployed worker who draw a new production opportunity y_k always choose to match, and the expected gain to each equals $\frac{1}{2}(M_k - M_0)$.*
(b) A worker in a match of productivity y_i and an unmatched employer who draw a new production opportunity y_k choose to form the new match if and only if $M_i < M_k$. The expected gains to the worker, the new employer, and the old employer equal $\frac{1}{2}\max(M_k - M_0 - S_i, 0)$, $\frac{1}{2}\max(M_k - M_i, 0)$, and $-\frac{1}{2}\{\max[\min(M_k, M_i) - M_0 - S_i, 0]\}$, respectively.
(c) An employer in a match of productivity y_i and an unemployed worker who draw a new production opportunity y_k choose to form the new match if and only if $M_i < M_k - T_i \equiv M'_k$. The expected gains to the employer, the new worker, and the old worker equal $\frac{1}{2}\max(M'_k - M_0 - S_i, 0)$, $\frac{1}{2}\max(M'_k - M_i, 0)$, and $-\frac{1}{2}\{\max[\min(M'_k, M_i) - M_0 - S_i, 0]\}$.
(d) An employer in a match of productivity y_i and a worker in a match of productivity y_j

[22] If $C(m_0)$ is strictly convex, profit $cm_0 - C(m_0)$ is distributed to the owners of the scarce factor in the vacancy-posting technology. This profit will not affect the labor market because the utility function is linear.

[23] In what follows, we will skirt the issue of exactly how a government may be able to collect taxes from agents in a random-matching economy, as well as why the same government is unable to facilitate the matching process.

[24] Recall footnote 12. The different population sizes of employers and workers also break the symmetry in their payoffs, since workers meet unmatched employers at a different rate than employers meet unmatched workers.

who draw a new production opportunity y_k choose to form the new match if and only if $M_i + M_j < M_k' + M_0$. The expected gains to the employer and the worker with the new matching opportunity equal $\frac{1}{2}\max\left(M_k' - M_j - S_i, 0\right)$ and $\frac{1}{2}\max\left(M_k' - M_i - S_j, 0\right)$. The expected gains to the old employer and the old worker are $-\frac{1}{2}\left\{\max\left[\min\left(M_k' - M_i, M_j - M_0\right) - S_j, 0\right]\right\}$ and $-\frac{1}{2}\left\{\max\left[\min\left(M_k' - M_j, M_i - M_0\right) - S_i, 0\right]\right\}$.

For given value functions, firing taxes T_i tend to make existing matches more stable. The reason is that individual creation and destruction decisions depend on the sum of the payoffs of all the agents involved in a meeting. Firing taxes force employers to pay resources to an outside party, which reduces the total surplus associated with double breaches and employer-initiated single breaches, so both become less likely. As another corollary to Proposition 3, note that—as one might expect from the Coase Theorem—forcing employers to make severance payments S_i to a worker upon dismissal has no effect on the match creation and destruction decisions, for given value functions. In principle, these policies may affect the value functions themselves, an issue which we turn to next.

As before, the choice of a worker in a match of type i and an employer in a match of type j to form a new match of type k is represented by $\phi_{ij}^k \in [0,1]$. Given the outcomes of the bargaining procedure, we have:

Lemma 2 *In an equilibrium with employment protection polices and possibly different population sizes of employers and workers, the value of a match of quality $i = 1, \ldots, N$ satisfies*

$$\begin{aligned} rM_i - \dot{M}_i \quad = \quad & 2y_i - \delta\left(M_i - M_0\right) + \alpha \sum_{k=1}^{N} \pi_k \max_{\phi_{i0}^k, \phi_{0i}^k} \left(m_0 \phi_{i0}^k \frac{M_k - M_i}{2} + n_0 \phi_{0i}^k \frac{M_k - M_i - T_i}{2} \right) \\ & + \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k \max_{\phi_{ij}^k, \phi_{ji}^k} \left(\phi_{ij}^k \frac{M_k + M_0 - M_i - M_j - T_j}{2} + \phi_{ji}^k \frac{M_k + M_0 - M_i - M_j - T_i}{2} \right) \end{aligned} \quad (18)$$

and the sum of the values of an unmatched employer and an unemployed worker satisfies

$$\begin{aligned} rM_0 - \dot{M}_0 &= -c + \alpha\left(m_0 + n_0\right)\sum_{k=1}^{N}\pi_k \tfrac{M_k - M_0}{2} \\ &\quad + \alpha\sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \max_{\phi_{0j}^k,\phi_{j0}^k}\left(\phi_{0j}^k \tfrac{M_k - M_j - T_j}{2} + \phi_{j0}^k \tfrac{M_k - M_j}{2}\right), \end{aligned} \tag{19}$$

where $c = C'(m_0)$.

Since there is free entry of employers, any equilibrium with a positive measure of unmatched employers must be such that the expected return to an unmatched employer is just enough to cover the entry cost:

$$c = \alpha\sum_{i=0}^{N}\sum_{k=1}^{N} n_i \pi_k \phi_{i0}^k \tfrac{M_k - M_i}{2}. \tag{20}$$

A competitive matching equilibrium with free entry can be summarized by a list $(M_i, \phi_{ij}^k, n_i, m_0)$ for $i, j = 0, ..., N$ and $k = 1, ..., N$ that satisfies the Bellman equations (18) and (19), the match formation conditions described in Proposition 3, the laws of motion (13) and (14), and the free-entry condition (20).[25]

Interestingly, the equilibrium is independent of severance pay, S_i. Thus, severance pay is neutral: it has no effect on the equilibrium allocations or the value of the match (even if it affects the way the worker and the employer split that value).[26] If we set $T_i = 0$ for all i and compare (18), (19) and (20) with (16), (17) and (15), we find that—just as in the case with fixed and equal populations of employers and workers—the planner's first-order conditions and the equilibrium conditions in the steady state differ only in that the planner imputes an "effective" contact rate equal to 2α instead of just α. Therefore, just as in Proposition 2, if we focus on steady states and replace the subjective discount rate of the social planner, r, with

[25] Note that if we set $T_i = 0$ for all i, the break-up conditions in Proposition 3 are equivalent to (10). If in addition, we set $c = 0$ and replace (20) with $m_0 = n_0$, we recover the *laissez faire* equilibrium with equal and fixed measures of employers and workers of Section 3.

[26] The idea that government-mandated transfers between the employer and the worker can be offset by private contracts between the parties goes back to Lazear (1990). Lazear also noted that severance pay effects are neutral only when the payment made by the employer is received by the worker, and not if third-party intermediaries receive or make any of the payments.

$r' = 2r + \delta$, the first order conditions for the modified planner's problem correspond to the equilibrium conditions of a competitive matching equilibrium.

5 Further Analysis and Labor Market Implications

In this section we consider a version of the model with two productivity levels ($N = 2$) and restrict our attention to the class of stationary equilibria. Our motivation is twofold. First, we want to illustrate some of the theoretical results derived in the previous sections, such as the inefficiency of the competitive matching equilibrium and the effects of labor market policy. Second, we want to show how the theory can help to conceptualize many of the empirical studies of gross job and worker flows.

5.1 Efficiency and Labor Market Policy

We begin by considering the model of Section 3, where there is no government policy and the numbers of employers and workers are fixed, equal, and normalized to unity. As long as the value function is increasing in the productivity of the current match ($V_0 < V_1 < V_2$)—which will be the case—we know that an unmatched agent and another agent always form a match with the higher productivity than the existing match ($\phi^1_{00} = \phi^2_{00} = \phi^2_{01} = 1$) and that the match with the highest productivity is not destroyed endogenously ($\phi^k_{i2} = 0$ for all i and k). Thus, the only non-trivial choice occurs when a worker employed in a match of productivity y_1 meets an employer who is in a match of productivity y_1, and they draw an opportunity of productivity y_2. To simplify notation, let $\phi = \phi^2_{11}$ and $\pi = \pi_2$. Lemma 3 in Appendix A establishes that a unique steady state distribution of workers $\{n_i\}^2_{i=0}$ exists for any given $\phi \in [0, 1]$. From Proposition 1 we know that $\phi = 1$ with certainty if and only if $V_2 + V_0 - 2V_1 > 0$. We can use the steady-state versions of the Bellman equations (8) and (9) to write this inequality as

$$\frac{y_2}{y_1} > 2 - \frac{\alpha \left[\pi n_1 + (1 - \pi) n_0\right]}{r + \delta + \alpha \left(n_0 + \pi n_1\right)}, \tag{21}$$

where n_0 and n_1 are the steady state numbers of matches characterized in Lemma 3 (see Corollary 1 in Appendix A for details). Since the right side of (21) is bounded, it is clear that $\phi = 1$ with certainty for y_2/y_1 large enough. In these cases, the agents involved will destroy two middle-productivity matches in order to form a single high-productivity match whenever the opportunity arises. Perhaps more surprisingly, there is always a range of the productivity differential $y_2/y_1 < 2$ for which two middle-productivity matches are destroyed to form a single high-productivity match even if this entails a reduction in current output.

To find a stationary equilibrium, let $n_i(\phi)$ denote the steady state number of matches of productivity y_i as characterized in Lemma 3. Define the best-response map $\Phi(\phi) = \frac{y_2}{y_1} + \frac{\alpha[\pi n_1(\phi)+(1-\pi)n_0(\phi)]}{r+\delta+\alpha[n_0(\phi)+\pi n_1(\phi)]} - 2$. From this we see that $\phi = 1$ is an equilibrium if $\Phi(1) > 0$, $\phi = 0$ is an equilibrium if $\Phi(0) < 0$, and $\phi^* \in [0,1]$ is an equilibrium if $\Phi(\phi^*) = 0$. The map Φ is continuous on $[0,1]$, so there always exists a stationary equilibrium.[27]

Given (21), Proposition 2 implies that the social planner chooses to destroy a pair of matches of productivity y_1 to create a single match of productivity y_2 if and only if

$$\frac{y_2}{y_1} > 2 - \frac{2\alpha\left[\pi n_1 + (1-\pi)\, n_0\right]}{r+\delta+2\alpha\,(n_0+\pi n_1)}, \tag{22}$$

with n_0 and n_1 again given by Lemma 3. From (21) and (22) we learn that the failure to internalize the composition externality makes atomistic agents less willing to destroy middle matches relative to the planner. For the planner, unmatched agents are a valued input in the matching process because the unmatched pair helps other agents climb the productivity ladder. Hence, for some parametrizations (e.g., y_2/y_1 slightly below 2), the planner may choose to reduce current output as a form of investment, in order to increase future output with a larger number of high productivity matches. Recall that the competitive matching equilibrium corresponds to a modified planner's economy with a higher discount rate, $r' = 2r + \delta$, so the modified planner is less willing to trade off current for future output than the real planner. Consequently, relative to the real planner, the modified planner is (agents in the competitive

[27] As we mentioned in Section 3, equilibrium need not be unique. For $N = 2$, we can show that , $\pi \geq 1/3$ is a sufficient condition for uniqueness. In what follows, we focus on the case of unique equilibrium.

matching equilibrium are) less willing to trade two matches of productivity y_1 for a match of productivity y_2 and two unmatched agents.[28]

To explore the effects of employment protection policies, we now turn to the more general formulation of Section 4. We introduce employment protection policies akin to those observed in many actual economies and allow for the number of unmatched employers (m_0) to differ from the number of unemployed workers (n_0). We now use T to denote the firing tax that an employer who breaks a match must pay to the government.[29] Let $\phi = \phi_{11}^2$ be the probability of a double breach, and $\psi = \phi_{01}^2$ denote the probability that an employer in a low productivity match initiates a single breach.[30] From Proposition 3, $\psi = 1$ if and only if $M_2 - M_1 - T > 0$. In Appendix A (Corollary 2) we show that this inequality holds if and only if

$$\frac{y_2}{y_1} > 1 + \frac{r + \delta + \frac{\alpha}{2}\pi m_0}{2y_1} T. \tag{23}$$

Also, from Proposition 3 we know that $\phi = 1$ if and only if $M_2 + M_0 - 2M_1 - T > 0$, and this inequality holds if and only if

$$\frac{y_2 + \frac{c}{2}}{y_1 + \frac{c}{2}} > 2 - \frac{\alpha\left[\pi n_1 + (1-\pi)\frac{m_0+n_0}{2}\right]}{r + \delta + \alpha\left(\frac{m_0+n_0}{2} + \pi n_1\right)} + \Omega T, \tag{24}$$

where Ω is a function of m_0, n_0, n_1 and parameters, reported in Appendix A (Corollary 2). Note that if we set $m_0 = n_0$ and $c = T = 0$, then (24) reduces to (21), the condition for double breaches to occur in the model with equal population sizes and no employment protection. Condition (24) is harder to satisfy for larger T: higher firing taxes make double breaches less likely. For an economy where (24) holds for $T = 0$, (23) and (24) define two cutoffs, T_ψ and T_ϕ, with $0 < T_\phi < T_\psi$, such that neither double breaches nor employer-initiated single breaches take place if $T_\psi < T$, single but no double breaches occur if $T_\phi < T < T_\psi$, and the match formation and destruction decisions are as in the model with no taxes if $T < T_\phi$.

[28] In Appendix B we show it is possible to design policies that bring the competitive matching equilibrium in line with the planner's first order conditions.

[29] We do not specify any severance payments here, because they have no effect on match formation and destruction decisions, which is what we focus on hereafter.

[30] These are the only nontrivial decisions in this setting with $N = 2$, since $\phi_{00}^k = 1$ for $k = 1, 2$, $\phi_{10}^2 = 1$, and $\phi_{ij}^k = 0$ if either $i \geq k$ or $j \geq k$.

5.2 Individual Employment Histories and Permanent Income Dynamics

The model has clear predictions for individual agents' employment histories, the various attributes of different types of jobs, and how they are affected by policy. For example, if we consider a job of productivity y_2 a "good job", and a job of productivity y_1 a "bad job". Then, good jobs are not only better paid, but also more stable. Good jobs are more stable than bad jobs in the sense that the expected time until a worker gets displaced is $\frac{1}{\delta}$ for the job of productivity y_2 and $\frac{1}{\delta+\alpha\pi(\psi n_0+\phi n_1)}$ for the job of productivity y_1.[31] Employment protection policies (e.g., $T > T_\psi$) could induce $\phi = \psi = 0$ and render jobs of productivity y_1 just as stable as jobs of productivity y_2. When displaced from a job of productivity y_i, the worker suffers a capital loss $V_i - V_0$, and it will typically take him some time to climb back up to a job of productivity equal to or higher than the one he was displaced from. So the theory is also consistent with the observation that displaced workers tend to suffer significant and persistent income losses.[32]

5.3 Job and Worker Flows

A large body of empirical work studies the properties of job and worker flows. One strand of this literature documents the sheer size of these flows, while another focuses on cross-country differences and tries to relate them to differences in labor market regulations.[33] Our model can help organize many of the empirical findings. To illustrate, we first carry out in the theory, the same types of accounting exercises as Davis and Haltiwanger have pioneered with actual data.

[31] Instead of measuring how stable a match is in terms of the expected time it takes the worker to get fired, we could measure it in terms of the expected time it takes the worker to *first find himself unemployed*, which is $\frac{\delta+\alpha\pi(m_0+\phi n_1)}{\delta[\delta+\alpha\pi(m_0+\psi n_0+2\phi n_1)]}$ for a worker in a match of productivity y_1 and $\frac{1}{\delta}$ for one in a match of productivity y_2.

[32] See Jacobson et al. (1993) or Violante (2002). For example, suppose a worker is displaced from a job of productivity y_1 (either because his match is hit by the exogenous destruction shock δ, or because his employer fires him in order to form a new match of productivity y_2 with another worker). The expected time it takes this worker to find himself in a job that pays at least as well as the one he lost is $\frac{1}{\alpha(m_0+\pi\psi n_1)}$. It takes a worker who gets displaced from a job of productivity y_2 even longer to see his income recover to the pre-displacement level; on average, precisely

$$\frac{\delta+\alpha[(1-\pi)m_0+\pi\psi n_0+2\pi\phi n_1]}{\alpha\pi\{\delta(m_0+\psi n_1)+\alpha[\pi(m_0+\psi n_1)(m_0+\psi n_0+2\phi n_1)+(1-\pi)(m_0+\phi n_1)m_0]\}}.$$

[33] The first strand stems from the work of Davis and Haltiwanger (1992), while examples of the second include Millard and Mortensen (1997), Bertola and Rogerson (1997), Ljungqvist and Sargent (1998), Mortensen and Pissarides (1999), Blanchard and Portugal (2001), and Pries and Rogerson (2005).

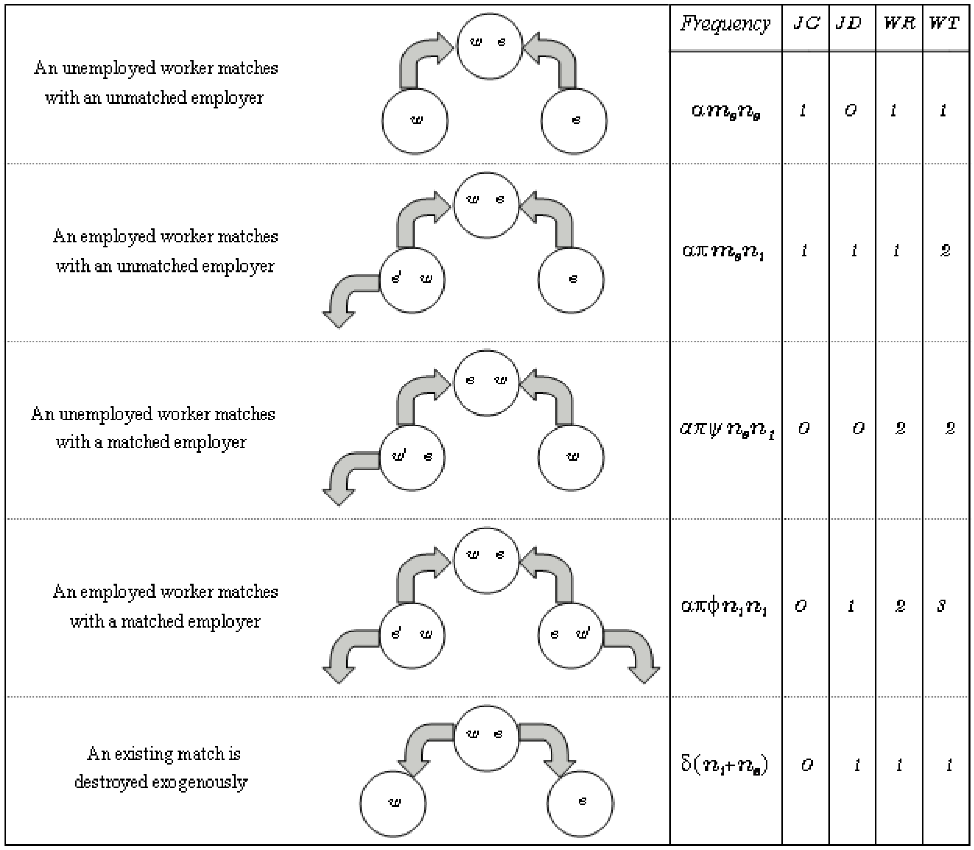

Figure 5: Job and worker flows accounting

Figure 5 displays the five types of transitions that an employer and a worker can find themselves in, and summarizes the basic calculations involved in computing the theoretical, "real-time" counterparts of the gross flows.[34] Let $JC^{(t)}$ and $JD^{(t)}$ denote gross job creation and gross job destruction over a period of length t, then

$$
\begin{aligned}
JC^{(t)} &= \alpha\left(m_0 n_0 + \pi m_0 n_1\right) t, \\
JD^{(t)} &= \left[\alpha\pi\left(m_0 n_1 + \phi n_1^2\right) + \delta\left(n_1 + n_2\right)\right] t,
\end{aligned}
$$

[34] See footnote 2 for the various definitions. The actual data used to construct the empirical job flows are collected rather infrequently, usually quarterly. Since our model economy is set in continuous time, we can compute all flows in "real time." We do this first, and address the issue of time-aggregation below. Also, empirical flows are usually normalized by a measure of the average employment level, but we omit this normalization to simplify the exposition.

and job reallocation is $JR^{(t)} = JC^{(t)} + JD^{(t)}$.

Let $H^{(t)}$ denote the number of hires, $L^{(t)}$ the number of layoffs (employer-initiated separations plus matches destroyed exogenously) and $Q^{(t)}$ the number of quits (worker-initiated separations) that occur over a period of length t. We have,

$$\begin{aligned} H^{(t)} &= \alpha \left[m_0 n_0 + \pi \left(m_0 n_1 + \psi n_1 n_0 + \phi n_1^2 \right) \right] t, \\ L^{(t)} &= \left[\alpha\pi \left(\psi n_1 n_0 + \phi n_1^2 \right) + \delta \left(n_1 + n_2 \right) \right] t, \\ Q^{(t)} &= \alpha\pi \left(m_0 n_1 + \phi n_1^2 \right) t, \end{aligned}$$

and total separations, $S^{(t)} = L^{(t)} + Q^{(t)}$. Empirical studies typically focus on worker turnover, $WT^{(t)} = H^{(t)} + S^{(t)}$, or on worker reallocation, $WR^{(t)}$, as measures of gross worker flows. According to the theory,

$$WR^{(t)} = \alpha \left[m_0 n_0 + \pi \left(m_0 n_1 + 2\psi n_1 n_0 + 2\phi n_1^2 \right) + \delta \left(n_1 + n_2 \right) \right] t.$$

In the model, workers only quit to take better jobs, so the number of job-to-job transitions over a period of length t is $JJ^{(t)} = Q^{(t)}$. Since employers can also search while matched, they can make "worker-to-worker" transitions, the natural counterpart to the workers' job-to-job transitions. In fact, as we discussed in the introduction, the available evidence suggests that those types of transitions, often referred to as *replacement hiring* in the empirical literature, may actually be quite common in real economies. In the model, aggregate replacement hiring over a period of length t is

$$RH^{(t)} = \alpha\pi \left(\psi n_0 n_1 + \phi n_1^2 \right) t. \tag{25}$$

Intuitively, $RH^{(t)}$ conveys how frequently employers churn their workers, while $JJ^{(t)}$ is a measure of how frequently workers churn their employers.[35]

[35] In fact, $JJ^{(t)} - RH^{(t)} = \alpha\pi n_1 \left(m_0 - \psi n_0 \right) t$, so in general, the difference depends on policy as well as on the relative numbers of unmatched employers and unemployed workers. For example, job-to-job transitions will exceed replacement hiring if $T > T_\psi$, and the magnitude of the difference is larger the greater the number of unmatched employers. Admittedly, capacity constraints are rather stark in our model (i.e., each employer can be matched with one worker), and perhaps overemphasize replacement hiring. We leave to future research the analysis of a formulation with more general decreasing returns-to-scale.

With the exact theoretical counterparts to all the empirical notions of job and worker flows in hand, it is easier to appreciate the relationships between the different measures used in the literature.[36] Job-to-job transitions account for the difference between worker turnover and worker reallocation: $WT^{(t)} - WR^{(t)} = JJ^{(t)}$. The difference between worker turnover and job reallocation,

$$WT^{(t)} - JR^{(t)} = 2RH^{(t)}, \tag{26}$$

is a common measure of the amount of worker flows in excess of job flows.[37] According to (26), in the model—as in the data—worker turnover is larger than gross job reallocation.[38] Instances of replacement hiring lie behind this discrepancy, since job creation and destruction are unchanged when a firm fires a worker to replace him with an unemployed one, yet these events add two transitions (one for each worker involved) to the worker turnover count.

Cross-country empirical studies have found that job reallocation turns out to be roughly similar across countries with very different labor market policies. Instead, the differences surface

[36] Davis and Haltiwanger (1999, p. 82) point out that some studies have failed to appreciate the conceptual differences between gross worker reallocation and total turnover.

[37] Burgess, Lane and Stevens (2000) refer to the difference between worker turnover and the net employment change at establishment j as *churning*, $\overline{C}_j^{(t)} = WT_j^{(t)} - |e_{jt} - e_{jt-1}|$, where $WT_j^{(t)}$ is worker turnover in establishment j between $t-1$ and t, and e_{jt} is employment in establishment j at the end of period t. $\overline{C}_j^{(t)}$ is a measure of the number of worker transitions in excess of the minimum needed to achieve the actual change in employment. Summing over establishments delivers an aggregate measure of churning, $\overline{C}^{(t)} = WT^{(t)} - \overline{JR}^{(t)}$, where $\overline{JR}^{(t)} = \sum_j |e_{jt} - e_{jt-1}|$ is gross job-reallocation measured using point-in-time employment data. The expression in (26) is the real-time analogue of this measure of churning. Below (in (28)), we provide the exact theoretical counterpart to Burgess, Lane and Stevens' measure.

[38] Burgess, Lane and Stevens (2000) use quarterly data from all private sector establishments in the state of Maryland and find that churning flows account for 70% of worker turnover in non-manufacturing and about 62% in manufacturing (job reallocation accounts for the rest). Similarly, based on data derived from the unemployment insurance systems of eight U.S. states, Anderson and Meyer (1994) report that gross job reallocation accounts for only 24% of quarterly worker turnover in manufacturing. Drawing from a data set covering the universe of Danish manufacturing plants, Albæk and Sørensen (1998) report a ratio of quarterly job reallocation to worker turnover of .42 and find that replacement hiring (defined as the sum of accessions minus job creation) is on average 16.5% of manufacturing employment. They also report interesting cross-establishment observations—for example, that 62% of all separations are accounted for by plants with employment growth rates in the interval $(-0.3, 0.1]$ and that plants with employment growth rates in the interval $(-0.1, 0.3]$ account for 56% of all hires. Hamermesh, Hassink and van Ours (1996) find that job reallocation accounts for only one-third of worker turnover in a random sample of establishments in the Netherlands. They also find that most mobility is into and out of existing positions, not to new or from destroyed ones; that a large fraction of all hires (separations) take place at firms where employment is declining (expanding); and that simultaneous hiring is mostly due to unobservable heterogeneity in the workforce.

in worker turnover.[39] The expressions (25) and (26) show how employment protection policies determine the amount of worker turnover in excess of job reallocation: for example, they will not differ much under very stringent regulations (e.g., T close to T_ψ). As can be seen in Figure 5, employment protection policies hinder precisely the kinds of transitions that generate worker flows in excess of job flows (those in the third and fourth rows in the figure). Along these lines, Blanchard and Portugal (2001) find that relative to the United States, Portugal exhibits a "sclerotic" labor market, i.e., one with longer unemployment durations and worker flows which are smaller even for given job flows. For instance, they estimate the flow of workers out of employment to be roughly between 1.5 and 2 times larger than job destruction in the United States, while in Portugal, that flow barely exceeds job destruction. In the theory, expected unemployment duration equals $\frac{1}{\alpha(m_0+\pi\psi n_1)}$, which is larger under more stringent regulations, as they make both m_0 and ψ small. The flow of workers out of employment in excess of job destruction is $L^{(t)} - JD^{(t)} = RH^{(t)} - JJ^{(t)} = \alpha\pi n_1 (\psi n_0 - m_0)\, t$. Policy influences this magnitude directly through its effect on the single breach decision ψ, and indirectly through the composition of the population, m_0, n_0, and n_1.

5.4 Time-aggregation

So far, our theoretical accounting exercise has implicitly assumed access to real-time data. However, many data are only available at a point in time, e.g., we know the employment level of a given establishment at dates t and $t+1$, but have no information on the path of employment during the time interval $(t, t+1)$. As another example, any attempt to measure employment-to-employment transitions faces this time-aggregation problem, as the data may count as a single job-to-job transition an employer change that in fact involved an intervening period of nonemployment or even multiple employers between survey dates. There is usually no way of

[39] Bertola and Rogerson (1997) were the first to note the fact that job reallocation rates do not vary much across countries with very different employment protection policies, such as Italy, Germany, France, the United Kingdom, and the United States. They also pointed out that average job tenures seem to be longer in regulated countries, suggesting that policy differences manifest themselves in worker turnover rates. See Pries and Rogerson (2005) for a summary of more recent evidence.

knowing the size or even the sign of this bias.[40]

In order to construct an exact mapping between the accounting carried out in the theory and that done for actual economies, next we will subject the theoretical accounting exercise to the same data limitations that empirical researchers face. Lemma 4 in Appendix A shows that the stochastic process that rules the evolution of a worker's employment state can be summarized by a matrix $[p_{ij}^{(t)}]$. Similarly, the stochastic process that rules the evolution of an employer's state can be summarized by a matrix $[\hat{p}_{ij}^{(t)}]$. For the case of $N = 2$ these are 3×3 matrices, and $p_{ij}^{(t)}$, for example, represents the probability that the worker finds himself in state j at time $t_0 + t$ given that he was in state i at time t_0. With the matrices $[p_{ij}^{(t)}]$ and $[\hat{p}_{ij}^{(t)}]$ in hand, and given our economy is set in continuous time, we can compute any variable of interest and subject it to an arbitrary degree of time-aggregation. For example, suppose that a period of length t elapses between worker surveys. Then, in the model, the time-aggregated number of job-to-job transitions is

$$\overline{JJ}^{(t)} = (p_{22}^{(t)} - e^{-\delta t})n_2 + (p_{11}^{(t)} - e^{-[\delta + \alpha\pi(m_0 + \psi n_0 + 2\phi n_1)]t})n_1 + p_{21}^{(t)} n_2 + p_{12}^{(t)} n_1, \qquad (27)$$

in contrast to the exact measure, $JJ^{(t)} = \alpha\pi \left(m_0 n_1 + \phi n_1^2\right) t$. The first term on the right of (27) is the flow of workers who transit from a match of productivity y_2 to another match of productivity y_2 during a time interval of length t (after subtracting those who never left the same match, with probability $e^{-\delta t}$). The second term is the similar worker flow from a match of productivity y_1 to another match of productivity y_1.

Consider the basic empirical finding that worker turnover is typically much larger than job reallocation. Since the former is calculated as the sum of all hires and separations over a given time interval, while job creation and job destruction are constructed from point-in-time employment data, one may wonder to what extent the amount of worker turnover in excess of job reallocation is a mere artifact of a time-aggregation bias. To address this question,

[40] Fallick and Fleischman (2001), for example, explicitly acknowledge this problem and choose to classify all workers who report different employers in the two months they were surveyed as having had a (single) job-to-job transition.

suppose that employment data is only available at intervals of length t, and compute the time-aggregated theoretical measures of job creation, job destruction, and job reallocation: $\overline{JC}^{(t)} = m_0[\hat{p}_{01}^{(t)} + \hat{p}_{02}^{(t)}]$, $\overline{JD}^{(t)} = n_1\hat{p}_{10}^{(t)} + n_2\hat{p}_{20}^{(t)}$, and $\overline{JR}^{(t)} = \overline{JC}^{(t)} + \overline{JD}^{(t)}$. Using the theory, the difference between *measured* worker turnover and job reallocation can be written as

$$WT^{(t)} - \overline{JR}^{(t)} = 2RH^{(t)} + [JR^{(t)} - \overline{JR}^{(t)}]. \tag{28}$$

The amount of worker turnover in excess of what would be needed to accommodate the reallocation of jobs across establishments is composed of two parts, a "genuine" churning component, and another component which is purely due to a time-aggregation bias in job flows. A parametrized version of the model could be used to assess how the relative sizes of the genuine churning component and the component due to time-aggregation depend upon the data frequency, say monthly versus quarterly—another example of how the theory can serve as a guide to understand the nature of job and worker flows.[41]

6 Concluding Remarks

We have developed a model of on-the-job search that has many of the stylized properties of actual labor markets. Worker flows exceed job flows, displaced agents suffer persistent reductions in permanent incomes, job-to-job transitions are common, and firms often engage in simultaneous hiring and firing. We have proposed and analyzed a notion of competitive equilibrium for random matching environments based on a particular bargaining procedure, and explored its efficiency properties.

[41] To be clear, the empirical observation that worker flows exceed job flows at the establishment level is not, per se, unquestionable evidence supporting the empirical relevance of the employer-initiated separations in our theory. That is, the transitions that account for the excess worker turnover may not be exclusively due to the fact that, in order to hire a new worker with whom he has a more productive opportunity, the employer must fire an existing employee due to capacity constraints. In principle, the observation is also consistent with separations that occur for other reasons (e.g., as captured by the δ-shock in the model) but are followed by a relatively quick replacement. Our view on this issue is that, from a theoretical standpoint, it is desirable to allow for the possibility that employers churn their workers (by engaging in replacement hiring), just as we allow workers to churn their employers (by engaging in job-to-job transitions). But also from an empirical standpoint, there are advantages to having a theory which exhibits *both* replacement hiring as well as separations followed by quick replacement. For one thing, as (28) suggests, such a theory can be used to extract more information from the available data in order to uncover what lies behind the excess worker reallocation at the micro level.

Several extensions seem worth pursuing. First, the model could be used to quantify the effects that employment protection policies have on the amount of worker reallocation in excess of job reallocation. According to our model, employers who are liable to pay taxes for firing their workers are discouraged from replacing their old workers with new workers, which reduces the amount of worker turnover in excess of job reallocation. Therefore, our theory suggests a simple explanation for why cross-country differences in employment protection policies would induce differences in measures of worker turnover rather than job turnover rates, as documented by Bertola and Rogerson (1997) and Blanchard and Portugal (2001). A quantitative assessment of these mechanisms awaits future work.

A related issue is that, given the empirical relevance of worker turnover in excess of job reallocation, an appropriate assessment of the welfare effects of employment protection policies calls for a model where both workers and employers can continue to search while matched, perhaps along the lines we have proposed here. Estimates based on models that do not allow for job-to-job transitions or replacement hiring are likely to understate the welfare losses from employment protection policies, since these policies affect the overall efficiency of job-worker matches through their effects on worker flows in addition to their effects on job flows.

At a deeper level, we would also like to understand why employment protection policies exist. In our framework with one-employer-to-one-worker matching and transferable utility, workers and employers are inherently symmetric (even if allowing for free entry of employers or certain policies introduces a slight asymmetry), and employment protection policies result in no efficiency gains. To explore the rationale behind employment protection policies, perhaps we have to introduce some fundamental asymmetry, such as that each worker works for one employer while each employer hires several workers. This extension would also be useful to study issues related to the size distribution of firms, including the relationship between firm size and job and worker flows.

A Proofs

Bargaining outcomes and the core. Before proving part *(c)* of Proposition 1 we introduce some notation and define the core in our model. Let I denote the set of agents who are directly or indirectly (i.e., through a partner) involved in a meeting. For example, $I = \{A, B, C, D\}$ in the situation illustrated in Figure 3. Within the context of a meeting, an *allocation* is a collection of partnerships. For example, there are two possible allocations for the meeting in Figure 3: $\langle (A, B), (C, D) \rangle$ and $\langle (B, C), (A, D) \rangle$. The first represents the case in which A remains matched to B while C remains matched to D. The second corresponds to the case in which B and C form a new match while A and D become unmatched (or become matched to each other but in state 0).[42] Let $\mathcal{A}_j$ denote the set of all possible allocations in a meeting that concerns j agents. Then, $\mathcal{A}_2 = \{\langle (A, B) \rangle, \langle (A), (B) \rangle\}$, $\mathcal{A}_3 = \{\langle (A, B), (C) \rangle, \langle (A), (B, C) \rangle\}$ and $\mathcal{A}_4 = \{\langle (A, B), (C, D) \rangle, \langle (B, C), (A, D) \rangle\}$. An allocation $a \in \mathcal{A}_j$ together with a payoff profile $\Pi \in \mathbb{R}^j$ constitute an *outcome* $[a, \Pi]$. For example, $[\langle (A), (B) \rangle, (\Pi_A, \Pi_B)]$ with $\Pi_A = \Pi_B = V_0$ is the outcome corresponding to a situation in which two unmatched agents meet and no match is formed. For any given meeting, a nonempty subset $S \subseteq I$ is called a *coalition.* Let v denote a function that assigns a real number to each coalition S. The number $v(S)$ is called the *worth* of coalition S. Since utility is fully transferable, $v(S)$ summarizes the utility possibility set of coalition S. Intuitively, $v(S)$ is the total utility available to the coalition, which can then be distributed among the coalition members in any way. An outcome $[a, \Pi]$ is *blocked* by a coalition S if there exists a payoff profile $\tilde{\Pi}$ with $\sum_{i \in S} \tilde{\Pi}_i \leq v(S)$ such that $\tilde{\Pi}_i > \Pi_i$ for all $i \in S$. With transferable utility, an outcome $[a, \Pi]$ is blocked by S if and only if $\sum_{i \in S} \Pi_i < v(S)$. An outcome $[a, \Pi]$ that is feasible for the grand coalition (i.e., such that $\sum_{i \in I} \Pi_i \leq v(I)$) is in the *core* if there is no coalition S that blocks this outcome. With transferable utility, an outcome $[a, \Pi]$ is in the core if and only if $\sum_{i \in S} \Pi_i \geq v(S)$ for all $S \subseteq I$ and $\sum_{i \in I} \Pi_i \leq v(I)$.[43]

[42] We ignore other feasible allocations such as $\langle (A, C), (B, D) \rangle$, which would correspond to "break up both matches without forming a new one" because they will play no role in the analysis that follows.

[43] Notice that in our random matching environment, the core is defined with respect to I, the set of agents who are involved in a meeting. (Since there is no central meeting place, the notion of a grand coalition of all the

Proof of part *(c)* of Proposition 1. The proof proceeds in three steps.

(Step 1). First consider the case illustrated in Figure 1, where an unemployed worker A and an unmatched employer B meet and have the opportunity to form a match of productivity $y_k > 0$. For this case we have $I = \{A, B\}$, and the list of all possible coalitions is $\{A, B\}$, $\{A\}$, $\{B\}$. The worth of the grand coalition is $v(I) = \max(2V_0, 2V_k) = 2V_k$, while $v(\{A\}) = v(\{B\}) = V_0$. A vector of payoffs (Π_A, Π_B) lies in the core if and only if *(i)* $\Pi_A + \Pi_B = 2V_k$; and *(ii)* $\Pi_j \geq V_0$ for $j = A, B$. We refer to the subgame that starts with A making a take-it-or-leave-it offer to B as "subgame 1." Figure 6 shows the core: it is the segment on the $\Pi_A + \Pi_B = 2V_k$ line that lies between the equilibrium payoffs of subgames 1 and 2 of the bilateral bargaining procedure. Both equilibrium payoffs as well as the expected payoff lie in the core.

(Step 2). Next consider the case illustrated in Figure 2: agent B, who is currently in a match of productivity y_i with agent A, meets unmatched agent C and they draw a productive opportunity y_k. Here $I = \{A, B, C\}$ and the list of all possible coalitions is $\{A, B, C\}$, $\{A, B\}$, $\{A, C\}$, $\{B, C\}$, $\{A\}$, $\{B\}$, $\{C\}$. The corresponding values are $v(I) = \max(2V_i + V_0, 2V_k + V_0)$, $v(\{A, B\}) = 2V_i$, $v(\{A, C\}) = 2V_0$, $v(\{B, C\}) = 2V_k$, $v(\{A\}) = v(\{B\}) = v(\{C\}) = V_0$. Hence, a payoff profile $\Pi = (\Pi_A, \Pi_B, \Pi_C)$ belongs to the core if and only if: *(i)* $\Pi_A + \Pi_B + \Pi_C = \max(2V_i + V_0, 2V_k + V_0)$; *(ii)* $\Pi_A + \Pi_B \geq 2V_i$; *(iii)* $\Pi_B + \Pi_C \geq 2V_k$; and *(iv)* $\Pi_j \geq V_0$ for $j = A, B, C$. If $V_k > V_i$, the four conditions can be rewritten as: *(1)* $\Pi_A = V_0$; *(2)* $\Pi_B \geq 2V_i - V_0$; *(3)* $\Pi_B + \Pi_C = 2V_k$; and *(4)* $\Pi_C \geq V_0$. The first panel of Figure 7 illustrates the core for this case; it consists of all the payoffs (V_0, Π_B, Π_C) such that (Π_B, Π_C) lie on the segment of the $\Pi_B + \Pi_C = 2V_k$ line between the equilibrium payoffs of subgames 1 and 2 of the bilateral bargaining procedure (subgame 1 is the one that obtains when B makes a take-it-or-leave-it offer to C, while subgame 2 is the one where C makes the offer to B). From the figure it is clear that the equilibrium payoffs of both subgames and the expected payoff all belong to the core. Conversely, if $V_k < V_i$, then the four conditions reduce to: *(1')* $\Pi_A \geq V_0$; *(2')* $\Pi_B \geq 2V_k - V_0$; *(3')* $\Pi_A + \Pi_B = 2V_i$; and *(4')* $\Pi_C = V_0$. The second panel of Figure 7 illustrates the core for

agents in the economy is meaningless here.)

this case; it consists of all the payoffs (Π_A, Π_B, V_0) such that (Π_A, Π_B) lie on the segment of the $\Pi_A + \Pi_B = 2V_i$ line between the equilibrium payoffs of subgames 1 and 2 of the bilateral bargaining procedure (here subgame 1 is the one that obtains when B makes a take-it-or-leave-it offer to A, while subgame 2 is the one where A makes the offer to B). From the figure it is again clear that the equilibrium payoffs of both subgames and the expected payoff all belong to the core.

(Step 3). Finally, consider the case illustrated in Figure 3: while A and B are in a match of productivity y_i and C and D are in a match of productivity y_j, agents B and C meet and draw a productive opportunity y_k. Here $I = \{A, B, C, D\}$ and the list of all possible coalitions is: $\{A, B, C, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{B, C, D\}$, $\{A, C, D\}$, $\{A, B\}$, $\{C, D\}$, $\{A, C\}$, $\{B, D\}$, $\{B, C\}$, $\{A, D\}$, $\{A\}$, $\{B\}$, $\{C\}$, $\{D\}$. The corresponding values are $v(I) = \max(2V_k + 2V_0, 2V_i + 2V_j)$, $v(\{A, B, C\}) = \max(2V_i + V_0, 2V_k + V_0)$, $v(\{A, B, D\}) = 2V_i + V_0$, $v(\{B, C, D\}) = \max(2V_j + V_0, 2V_k + V_0)$, $v(\{A, C, D\}) = 2V_j + V_0$, $v(\{A, B\}) = 2V_i$, $v(\{C, D\}) = 2V_j$, $v(\{A, C\}) = v\{\{B, D\}\} = v\{\{A, D\}\} = 2V_0$, $v(\{A\}) = v(\{B\}) = v(\{C\}) = v(\{D\}) = V_0$. A payoff profile $\Pi = (\Pi_A, \Pi_B, \Pi_C, \Pi_D)$ is in the core if and only if it satisfies the following inequalities: $\Pi_A + \Pi_B + \Pi_C + \Pi_D = \max(2V_k + 2V_0, 2V_i + 2V_j)$, $\Pi_A + \Pi_B + \Pi_C \geq \max(2V_i + V_0, 2V_k + V_0)$, $\Pi_B + \Pi_C + \Pi_D \geq \max(2V_j + V_0, 2V_k + V_0)$, $\Pi_A + \Pi_B + \Pi_D \geq 2V_i + V_0$, $\Pi_A + \Pi_C + \Pi_D \geq 2V_j + V_0$, $\Pi_A + \Pi_B \geq 2V_i$, $\Pi_C + \Pi_D \geq 2V_j$, $\Pi_B + \Pi_C \geq 2V_k$, $\Pi_A + \Pi_C \geq 2V_0$, $\Pi_B + \Pi_D \geq 2V_0$, $\Pi_A + \Pi_D \geq 2V_0$, $\Pi_j \geq V_0$ for $j = A, B, C, D$. It is straightforward to verify that the equilibrium and expected payoffs of the bilateral bargaining procedure satisfy these fifteen inequalities. ■

Proof of Proposition 2. Here, we prove existence of a symmetric steady-state competitive matching equilibrium. (The rest of the proposition was established in the text.) For a small enough length of time dt, approximate (1) for $i = 1, ..., N$, and (2) with

$$\begin{aligned} n_{it+dt} &= n_{it} + (\alpha dt)\,\pi_i \sum_{j=0}^{N}\sum_{k=0}^{N} n_{jt} n_{kt} \phi_{jk}^{i} - (\delta dt)\, n_{it} \\ &\quad - (\alpha dt)\, n_{it} \sum_{j=0}^{N}\sum_{k=1}^{N} n_{jt}\pi_k (\phi_{ij}^{k} + \phi_{ji}^{k}) + o_i\,(dt) \end{aligned} \tag{29}$$

$$\begin{aligned} n_{0t+dt} &= n_{0t} + (\alpha dt) \sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} n_{it} n_{jt} \pi_k \phi_{ij}^{k} + (\delta dt) \sum_{j=1}^{N} n_{jt} \\ &\quad - (\alpha dt)\, n_{0t} \sum_{j=0}^{N}\sum_{k=1}^{N} n_{jt}\pi_k \phi_{0j}^{k} + o\,(dt)\,, \end{aligned} \tag{30}$$

where $o\,(dt) = -\sum_{i=1}^{N} o_i\,(dt)$, and each $o_i\,(dt)$ satisfies $\lim_{dt \to 0} \frac{o_i(dt)}{dt} = 0$. Let $\mathbf{n}_t$ denote the vector $(n_{0t}, n_{1t}, n_{2t}, ..., n_{Nt})'$ and $\mathbf{1}$ denote the $1 \times (N+1)$ vector of ones. The right side of (29) and (30) defines a function F, so we write $\mathbf{n}_{t+dt} = F\,(\mathbf{n}_t)$. From (29) and (30), it is immediate that $\mathbf{1} \cdot \mathbf{n}_t = 1$ implies $\mathbf{1} \cdot F\,(\mathbf{n}_t) = 1$. Also, $\mathbf{n}_t \geq 0$ implies $F\,(\mathbf{n}_t) \geq 0$ provided $(2\alpha + \delta)\,dt < 1$. Thus, for dt small, $F : [0,1]^{N+1} \to [0,1]^{N+1}$. Since F is continuous, there exists $\bar{\mathbf{n}} \in [0,1]^{N+1}$ such that $\bar{\mathbf{n}} = F\,(\bar{\mathbf{n}})$ by Brouwer's Fixed Point Theorem (e.g., Corollary 6.6 in Border (1985)). Explicitly, $\bar{\mathbf{n}} = (\bar{n}_0, \bar{n}_1, ..., \bar{n}_N)'$ satisfies

$$\begin{aligned} \alpha\pi_i \sum_{j=0}^{N}\sum_{k=0}^{N} \bar{n}_j \bar{n}_k \phi_{jk}^{i} - \delta \bar{n}_i - \alpha \bar{n}_i \sum_{j=0}^{N}\sum_{k=1}^{N} \bar{n}_j \pi_k (\phi_{ij}^{k} + \phi_{ji}^{k}) + \frac{o_i\,(dt)}{dt} &= 0 \\ \alpha \sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} \bar{n}_i \bar{n}_j \pi_k \phi_{ij}^{k} + \delta \sum_{j=1}^{N} \bar{n}_j - \alpha \bar{n}_0 \sum_{j=0}^{N}\sum_{k=1}^{N} \bar{n}_j \pi_k \phi_{0j}^{k} + \frac{o\,(dt)}{dt} &= 0. \end{aligned}$$

Take the limit as dt approaches zero to find that $\bar{\mathbf{n}}$ satisfies the steady-state conditions for a given $\boldsymbol{\phi} = [(\phi_{ij}^{k})_{i,j=0}^{N}]_{k=1}^{N} \in [0,1]^{N(N+1)^2}$, namely

$$\begin{aligned} \alpha\pi_i \sum_{j=0}^{N}\sum_{k=0}^{N} \bar{n}_j \bar{n}_k \phi_{jk}^{i} - \delta \bar{n}_i - \alpha \bar{n}_i \sum_{j=0}^{N}\sum_{k=1}^{N} \bar{n}_j \pi_k (\phi_{ij}^{k} + \phi_{ji}^{k}) &= 0 \\ \alpha \sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} \bar{n}_i \bar{n}_j \pi_k \phi_{ij}^{k} + \delta \sum_{j=1}^{N} \bar{n}_j - \alpha \bar{n}_0 \sum_{j=0}^{N}\sum_{k=1}^{N} \bar{n}_j \pi_k \phi_{0j}^{k} &= 0. \end{aligned}$$

For each $i \in \{0, 1, ..., N\}$, these conditions allow us to write $\bar{n}_i = f_i\,(\boldsymbol{\phi})$, where f_i is a continuous function. Next, given $\bar{\boldsymbol{\phi}} = [(\bar{\phi}_{ij}^{k})_{i,j=0}^{N}]_{k=1}^{N} \in [0,1]^{N(N+1)^2}$, write the steady-state versions of the

Bellman equations (8) and (9) as

$$\begin{aligned} r\lambda_i^c &= 2y_i - \delta(\lambda_i^c - \lambda_0^c) + \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} f_j(\bar{\boldsymbol{\phi}}) \pi_k \max_{\phi_{ij}^k \in [0,1]} \phi_{ij}^k (\lambda_k^c + \lambda_0^c - \lambda_i^c - \lambda_j^c) \\ r\lambda_0^c &= \alpha \sum_{j=0}^{N} \sum_{k=1}^{N} f_j(\bar{\boldsymbol{\phi}}) \pi_k \max_{\phi_{0j}^k \in [0,1]} \phi_{0j}^k (\lambda_k^c - \lambda_j^c), \end{aligned}$$

which is a linear system that can be solved for $\lambda_i^c = \Lambda_i(\bar{\boldsymbol{\phi}})$ for $i = 0, 1, ..., N$, where Λ_i is a continuous function. Hence, for each $(i, j, k) \in \{0, 1, ...N\}^2 \times \{1, 2, ..., N\}$,

$$\Phi_{ij}^k(\bar{\boldsymbol{\phi}}) = \arg \max_{\phi_{ij}^k \in [0,1]} \phi_{ij}^k \left[\Lambda_k(\bar{\boldsymbol{\phi}}) + \Lambda_0(\bar{\boldsymbol{\phi}}) - \Lambda_i(\bar{\boldsymbol{\phi}}) - \Lambda_j(\bar{\boldsymbol{\phi}}) \right]$$

is a nonempty, compact-valued and upper-hemi continuous correspondence by the Theorem of the Maximum (e.g., Theorem 3.6 in Stokey and Lucas (1989)). Since the objective function is linear, it is clear that Φ_{ij}^k is also convex-valued. Let $\Phi(\bar{\boldsymbol{\phi}}) = \{[\Phi_{ij}^k(\bar{\boldsymbol{\phi}})]_{i,j=0}^N\}_{k=1}^N$, then by Kakutani's Fixed Point Theorem (e.g., Corollary 15.3 in Border (1985)), there exists $\boldsymbol{\phi}^* \in [0, 1]^{N(N+1)^2}$ such that $\boldsymbol{\phi}^* \in \Phi(\boldsymbol{\phi}^*)$, which constitutes a symmetric steady-state competitive matching equilibrium. ■

Proof of Lemma 1. The planner's Hamiltonian is

$$H = \sum_{i=1}^{N} 2y_i n_i - C(m_0) - \delta \sum_{i=1}^{N} n_i (\lambda_i - \lambda_0) + \alpha \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{N} n_i m_j \pi_k \tau_{ij}^k (\lambda_k + \lambda_0 - \lambda_i - \lambda_j).$$

The optimality conditions for τ_{ij}^k are still given by (3), while the choice of m_0 satisfies (15). The Euler equations are

$$\begin{aligned} r\lambda_i - \dot{\lambda}_i &= 2y_i - \delta(\lambda_i - \lambda_0) + \alpha m_0 \sum_{k=1}^{N} \pi_k \tau_{i0}^k (\lambda_k - \lambda_i) + \alpha n_0 \sum_{k=1}^{N} \pi_k \tau_{0i}^k (\lambda_k - \lambda_i) \\ &\quad + \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k (\tau_{ij}^k + \tau_{ji}^k)(\lambda_k + \lambda_0 - \lambda_i - \lambda_j) \end{aligned}$$

for $i = 1, \ldots, N$, and

$$r\lambda_0 - \dot{\lambda}_0 = \alpha m_0 \sum_{k=1}^{N} \pi_k \tau_{00}^k (\lambda_k - \lambda_0) + \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k \tau_{0j}^k (\lambda_k - \lambda_j).$$

Use the optimality conditions (3) and (15) to rewrite these Euler equations as (16) and (17). ■

Proof of Proposition 3. First consider the case of a continuing match when neither partner has an outside production opportunity. Every instant, with probability a half, the draw by Nature gives the bargaining power to the worker, who offers the employer J_0 (and keeps $M_i - J_0$). With the same probability Nature gives the bargaining power to the employer, who offers the worker his outside option including the severance payment, $V_0 + S_i$ (and keeps $M_i - V_0 - S_i$). As a result, utility is continuously divided among the partners in such a way so that the worker's and the employer's expected continuation payoffs are $V_i = V_0 + \frac{1}{2}(M_i - M_0 + S_i)$ and $J_i = J_0 + \frac{1}{2}(M_i - M_0 - S_i)$, respectively. Next, we consider each of the bargaining situations listed in the statement of the proposition.

(a). An unmatched employer and an unemployed worker draw a new opportunity y_k.

With probability a half the worker has bargaining power, offers J_0, and the employer accepts. With the same probability the employer offers the worker V_0, and the worker accepts. The new match is formed regardless of who makes the offer and the expected payoffs are $V_0 + \frac{1}{2}(M_k - M_0)$ for the worker and $J_0 + \frac{1}{2}(M_k - M_0)$ for the employer. The individual expected capital gains to the worker and the employer each equal $\frac{1}{2}(M_k - M_0)$.

(b). An unmatched employer and an employed worker draw a new opportunity.

Suppose that worker B, who is currently employed in a match of productivity y_i with employer A, draws a new productive opportunity y_k with an unmatched employer, C.

With probability a half, the worker, B, has bargaining power. If $M_k > M_i$, he chooses to offer the new employer, C, $X_{BC}^k = J_0$, and she accepts. Thus, B is able to appropriate $M_k - J_0$. If $M_i > M_k$, B chooses to offer the old employer, A, $X_{BA}^k = J_0$, and captures $M_i - J_0$.

With probability another half, the employers A and C have bargaining power, and they offer B payoffs $X_{AB}^k = \max[V_0 + S_i, \min(M_i - J_0, M_k - J_0 + \varepsilon)]$ and $X_{CB}^k = \min(M_k - J_0, M_i - J_0 + \varepsilon)$, respectively. Note that A's offer cannot be lower than B's outside option, $V_0 + S_i$, because A wants to stay matched with B. Then, B will accept C's offer to form the new match if

$M_k > M_i$, and stays matched with A if $M_i > M_k$.

Regardless of the bargaining power allocation, the new match is formed if $M_k > M_i$, and the expected payoffs to A, B and C, respectively, are

$$\begin{bmatrix} J_0 \\ \frac{1}{2}(M_k - J_0 + M_i - J_0) \\ \frac{1}{2}(J_0 + M_k - M_i + J_0) \end{bmatrix} = \begin{bmatrix} J_i \\ V_i \\ J_0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}(M_i - M_0 - S_i) \\ \frac{1}{2}(M_k - M_0 - S_i) \\ \frac{1}{2}(M_k - M_i) \end{bmatrix}.$$

If $M_i > M_k$, the old match continues, and the expected payoffs to A, B and C are

$$\begin{bmatrix} \frac{1}{2}[J_0 + M_i - \max(V_0 + S_i, M_k - J_0)] \\ \frac{1}{2}[M_i - J_0 + \max(V_0 + S_i, M_k - J_0)] \\ J_0 \end{bmatrix} = \begin{bmatrix} J_i \\ V_i \\ J_0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}\max(0, M_k - M_0 - S_i) \\ \frac{1}{2}\max(0, M_k - M_0 - S_i) \\ 0 \end{bmatrix}.$$

(c). A matched employer and an unemployed worker draw a new opportunity.

Suppose that employer B, who is currently in a match of productivity y_i with worker A, draws a new production opportunity y_k with an unemployed worker, C. In order for B and C to form the new match, they have to pay T_i to the government. Thus, the total value created by the new match (net of the tax) is $M'_k \equiv M_k - T_i$ instead of M_k. By the same reasoning used in *(b)* above, we obtain the following expected gains from trade:

$M_i < M'_k : (\Gamma_A, \Gamma_B, \Gamma_C) = \left[-\frac{1}{2}(M_i - M_0 - S_i), \frac{1}{2}(M'_k - M_0 - S_i), \frac{1}{2}(M'_k - M_i)\right].$

$M_i > M'_k : (\Gamma_A, \Gamma_B, \Gamma_C) = \left[-\frac{1}{2}\max(0, M'_k - M_0 - S_i), \frac{1}{2}\max(0, M'_k - M_0 - S_i), 0\right].$

(d). A matched employer and an employed worker draw a new opportunity.

Suppose that employer B and worker C meet and draw a productive opportunity y_k. The situation now is that B is currently in a match of productivity y_i with worker A, while C is currently in a match of productivity y_j with employer D.

With probability a half, workers A and C have bargaining power. Worker C offers employer D her outside option $X_{CD} = J_0$, which is contingent on C's offer to B being rejected. With this, C has an option to enjoy $M_j - J_0$, and thus the maximum payoff that C is willing to offer B is $M_k - T_i - S_i - (M_j - J_0) = M'_k - M_j + J_0 - S_i$ (after paying the firing tax and severance payment). Therefore, C offers B, $X_{CB} = \min(M'_k - M_j + J_0 - S_i, M_i - V_0 - S_i + \varepsilon)$, where the second term in the parenthesis is the maximum amount that A is willing to offer B, plus ε.

A offers B, $X_{AB} = \max[J_0, \min(M_i - V_0 - S_i, M'_k - M_j + J_0 - S_i + \varepsilon)]$. Here, A's offer cannot be less than B's outside option J_0, because A wants to preserve the current match with B.

With probability another half, employers B and D have bargaining power. By a similar argument, $X_{BA} = V_0 + S_i$, $X_{BC} = \min(M'_k - M_i + V_0, M_j - J_0 + \varepsilon)$, and $X_{DC} = \max[V_0 + S_j, \min(M_j - J_0, M'_k - M_i + V_0 + \varepsilon)]$.

Therefore, when $M_i + M_j - M_0 < M_k - T_i \equiv M'_k$, the new match is formed, and the expected payoffs to A, B, C and D are

$$\begin{bmatrix} V_0 + S_i \\ \frac{1}{2}(M'_k - M_j + J_0 - S_i + M_i - V_0 - S_i) \\ \frac{1}{2}(M'_k - M_i + V_0 + M_j - J_0) \\ J_0 \end{bmatrix} = \begin{bmatrix} V_i \\ J_i \\ V_j \\ J_j \end{bmatrix} + \begin{bmatrix} -(V_i - V_0 - S_i) \\ \frac{1}{2}(M'_k - M_j - S_i) \\ \frac{1}{2}(M'_k - M_i - S_j) \\ -(J_j - J_0) \end{bmatrix}.$$

Alternatively, the old matches prevail when $M_i + M_j - M_0 > M'_k$; the expected payoffs to A, B, C and D are

$$\begin{bmatrix} \frac{1}{2}[M_i - \max(J_0, M'_k - M_j + J_0 - S_i) + V_0 + S_i] \\ \frac{1}{2}[M_i - V_0 - S_i + \max(J_0, M'_k - M_j + J_0 - S_i)] \\ \frac{1}{2}[M_j - J_0 + \max(V_0 + S_j, M'_k - M_i + V_0)] \\ \frac{1}{2}[J_0 + M_j - \max(V_0 + S_j, M'_k - M_i + V_0)] \end{bmatrix} = \begin{bmatrix} V_i \\ J_i \\ V_j \\ J_j \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}\max(0, M'_k - M_j - S_i) \\ \frac{1}{2}\max(0, M'_k - M_j - S_i) \\ \frac{1}{2}\max(0, M'_k - M_i - S_j) \\ -\frac{1}{2}\max(0, M'_k - M_i - S_j) \end{bmatrix}$$

and this concludes the proof. ■

Proof of Lemma 2. A worker employed in a match of type i earns wage w_i and his expected payoff satisfies

$$\begin{aligned} rV_i - \dot{V}_i &= w_i - \delta(V_i - V_0) \\ &\quad + \alpha m_0 \sum_{k=1}^{N} \pi_k \left[\phi_{i0}^k \tfrac{M_k - M_0 - S_i}{2} + (1 - \phi_{i0}^k) \max\left(\tfrac{M_k - M_0 - S_i}{2}, 0 \right) \right] \\ &\quad - \alpha n_0 \sum_{k=1}^{N} \pi_k \left[\phi_{0i}^k \tfrac{M_i - M_0 - S_i}{2} + (1 - \phi_{0i}^k) \max\left(\tfrac{M_k - M_0 - S_i - T_i}{2}, 0 \right) \right] \\ &\quad + \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k \left[\phi_{ij}^k \tfrac{M_k - M_j - S_i - T_j}{2} + (1 - \phi_{ij}^k) \max\left(\tfrac{M_k - M_j - S_i - T_j}{2}, 0 \right) \right] \\ &\quad - \alpha \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k \left[\phi_{ji}^k \tfrac{M_i - M_0 - S_i}{2} + (1 - \phi_{ji}^k) \max\left(\tfrac{M_k - M_j - S_i - T_i}{2}, 0 \right) \right]. \end{aligned} \tag{31}$$

Similarly, the value of an employer in a match of productivity y_i is:

$$\begin{aligned}
rJ_i - \dot{J}_i &= 2y_i - w_i - \delta\left(J_i - J_0\right) \\
&+\alpha n_0 \sum_{k=1}^{N} \pi_k \left[\phi_{0i}^k \tfrac{M_k - M_0 - S_i - T_i}{2} + (1-\phi_{0i}^k) \max\left(\tfrac{M_k - M_0 - S_i - T_i}{2}, 0\right)\right] \\
&-\alpha m_0 \sum_{k=1}^{N} \pi_k \left[\phi_{i0}^k \tfrac{M_i - M_0 - S_i}{2} + (1-\phi_{i0}^k) \max\left(\tfrac{M_k - M_0 - S_i}{2}, 0\right)\right] \\
&+\alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \left[\phi_{ji}^k \tfrac{M_k - M_j - S_i - T_i}{2} + (1-\phi_{ji}^k) \max\left(\tfrac{M_k - M_j - S_i - T_i}{2}, 0\right)\right] \\
&-\alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \left[\phi_{ij}^k \tfrac{M_i - M_0 - S_i}{2} + (1-\phi_{ij}^k) \max\left(\tfrac{M_k - M_j - S_i - T_j}{2}, 0\right)\right]. \qquad (32)
\end{aligned}$$

The values of an unemployed worker and an unmatched employer solve

$$rV_0 - \dot{V}_0 = \alpha m_0 \sum_{k=1}^{N} \pi_k \phi_{00}^k \tfrac{M_k - M_0}{2} + \alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \phi_{0j}^k \tfrac{M_k - M_j - T_j}{2} \qquad (33)$$

$$rJ_0 - \dot{J}_0 = -c + \alpha n_0 \sum_{k=1}^{N} \pi_k \phi_{00}^k \tfrac{M_k - M_0}{2} + \alpha \sum_{j=1}^{N}\sum_{k=1}^{N} n_j \pi_k \phi_{j0}^k \tfrac{M_k - M_j}{2}. \qquad (34)$$

The expressions in the statement of the lemma result from using Proposition 3, and adding (31) to (32), and (33) to (34). ■

Lemma 3 *Consider the economy with $N = 2$ and population sizes of employers and workers which are fixed and equal to* 1*. A unique steady state distribution of workers exists for any given $\phi \in [0,1]$. The number of unemployed workers, n_0, solves*

$$\left[\alpha n_0^2 - \delta\left(1-n_0\right)\right]\left(\delta + 2\alpha\pi n_0\right)^2 - \phi\alpha\pi\left[2\delta\left(1-n_0\right) - \alpha\left(1+\pi\right)n_0^2\right]^2 = 0.$$

The number of workers employed in matches with productivity y_1 is $n_1 = \frac{2\delta(1-n_0)-\alpha(1+\pi)n_0^2}{\delta+2\alpha\pi n_0}$, and the number of workers employed in matches with productivity y_2 is $n_2 = 1 - n_0 - n_1$.

Proof of Lemma 3. For the case of $N = 2$ and population sizes of employers and workers which are fixed and equal to 1, (1) and (2) reduce to

$$\dot{n}_2 = \alpha\pi\left(n_0^2 + 2n_0n_1 + n_1^2\phi\right) - \delta n_2 \tag{35}$$

$$\dot{n}_1 = \alpha(1-\pi)n_0^2 - 2\alpha\pi n_0 n_1 - 2\alpha\pi n_1^2\phi - \delta n_1 \tag{36}$$

$$\dot{n}_0 = \delta(n_1 + n_2) + \alpha\pi n_1^2\phi - \alpha n_0^2. \tag{37}$$

Combine the $\dot{n}_2 = 0$ and $\dot{n}_0 = 0$ conditions, to find $n_1 = f(n_0)$, where $f(n_0) \equiv \frac{2\delta(1-n_0)-\alpha(1+\pi)n_0^2}{\delta+2\alpha\pi n_0}$. It can be shown that $f'(n_0) < 0$ on $[0,1]$, so to each $n_0 \in [0,1]$, corresponds a unique n_1. In addition, $f(n_0) \geq 0$ if $n_0 \leq \overline{\eta}_0$ and $f(n_0) \leq 1$ if $n_0 \geq \underline{\eta_0}$, where $\overline{\eta}_0 = \frac{\sqrt{\delta^2+2\alpha\delta(1+\pi)}-\delta}{\alpha(1+\pi)}$ and $\underline{\eta_0} = \frac{\sqrt{(\delta+\alpha\pi)^2+\alpha\delta(1+\pi)}-(\delta+\alpha\pi)}{\alpha(1+\pi)}$, with $0 < \underline{\eta_0} < \overline{\eta}_0 < 1$. Substitute $n_1 = f(n_0)$ back into the $\dot{n}_0 = 0$ condition to obtain a single equation in n_0 which can be written as $G(n_0;\phi) = 0$, where

$$G(n_0;\phi) \equiv \left[\alpha n_0^2 - \delta(1-n_0)\right](\delta + 2\alpha\pi n_0)^2 - \alpha\pi\phi\left[2\delta(1-n_0) - \alpha(1+\pi)n_0^2\right]^2.$$

Direct calculations reveal that $G(\overline{\eta}_0;\phi) = \alpha\overline{\eta}_0^2 - \delta(1-\overline{\eta}_0) > 0$ for all $\phi \in [0,1]$, and also that $G(\underline{\eta_0};\phi,\psi)$ has the same sign as $\alpha\underline{\eta_0}^2 - \delta(1-\underline{\eta_0}) - \alpha\pi\phi$. Note that an increase in ϕ causes G to shift down uniformly, so to ensure $G(\underline{\eta_0};\phi) < 0$ for all ϕ, it suffices to guarantee that $G(\underline{\eta_0};0) < 0$. This condition can be written as $\alpha\underline{\eta_0}^2 - \delta(1-\underline{\eta_0}) < 0$, a parametric restriction that is always satisfied. So a steady state exists. Finally, note that $\left.\frac{\partial G(n_0;\phi)}{\partial n_0}\right|_{G(n_0;\phi)=0} > 0$, which together with the fact that $f'(n_0) < 0$ implies that the steady state is unique. ■

Corollary 1 *For the case of $N = 2$ and population sizes of employers and workers which are fixed and equal to one, $\phi_{11}^2 = 1$ if and only if (21) holds.*

Proof of Corollary 1. In a stationary equilibrium the value functions satisfy:

$$\begin{aligned} rV_2 &= y_2 - \delta(V_2 - V_0) \\ rV_1 &= y_1 - \delta(V_1 - V_0) + \alpha n_0\pi(V_2 - V_1) + \alpha n_1\pi\phi(V_2 + V_0 - 2V_1) \\ rV_0 &= \alpha n_0\left[\pi(V_2 - V_0) + (1-\pi)(V_1 - V_0)\right] + \alpha n_1\pi(V_2 - V_1). \end{aligned}$$

From Proposition 1, we know that $\phi = 1$ with certainty if and only if $V_2 + V_0 - 2V_1 > 0$. We can use the Bellman equations to write $V_2 + V_0 - 2V_1 > 0$ as (21), where n_0 and n_1 are the steady state numbers of matches characterized in Lemma 3. ■

Corollary 2 *For the case of $N = 2$, employment protection policies and free entry of employers, $\phi_{01}^2 = 1$ if and only if (23) holds, and $\phi_{11}^2 = 1$ if and only if (24) holds.*

Proof of Corollary 2. The laws of motion for workers in each type of match are

$$\dot{n}_2 = \alpha\pi\left(m_0 n_0 + \psi n_0 n_1 + m_0 n_1 + \phi n_1^2\right) - \delta n_2 \tag{38}$$

$$\dot{n}_1 = \alpha(1-\pi) m_0 n_0 - \delta n_1 - \alpha\pi\left(\psi n_0 n_1 + m_0 n_1 + 2\phi n_1^2\right) \tag{39}$$

$$\dot{n}_0 = \alpha\pi\phi n_1^2 + \delta(n_1 + n_2) - \alpha m_0 n_0. \tag{40}$$

The values of a match to the pair in the stationary equilibrium satisfy the Bellman equations

$$\begin{aligned}
rM_2 &= 2y_2 - \delta(M_2 - M_0) \\
rM_1 &= 2y_1 - \delta(M_1 - M_0) + \alpha\pi\left[m_0 \tfrac{M_2 - M_1}{2} + n_0\psi \tfrac{M_2 - M_1 - T}{2} + n_1\phi(M_2 + M_0 - 2M_1 - T)\right] \\
rM_0 &= -c + \alpha(m_0 + n_0)\left[\pi\tfrac{M_2 - M_0}{2} + (1-\pi)\tfrac{M_1 - M_0}{2}\right] + \alpha n_1 \pi\left[\psi\tfrac{M_2 - M_1 - T}{2} + \tfrac{M_2 - M_1}{2}\right]
\end{aligned}$$

where $c = C'(m_0)$, and n_2, n_1 and n_0 solve (38)–(40) with $\dot{n}_2 = \dot{n}_1 = 0$. We can use the Bellman equations to write $M_2 - M_1 - T > 0$ as (23) and $M_2 + M_0 - 2M_1 - T > 0$ as (24), with

$$\Omega \equiv \frac{(r+\delta)^2 + (r+\delta)\frac{\alpha}{2}[m_0 + n_0 + \pi(m_0 - n_0 + n_1)] + \left(\frac{\alpha}{2}\right)^2 \pi[(m_0+n_0)(m_0 - \pi n_0) + \pi n_1(m_0 - n_0)]}{(2y_1 + c)\left[r + \delta + \frac{\alpha}{2}(m_0 + n_0 + 2\pi n_1)\right]}. \quad ■$$

Lemma 4 *The transition function for the stochastic process that rules a worker's state is*

$$[p_{ij}^{(t)}] = \sum_{n=0}^{\infty} \frac{(at)^n e^{-at}}{n!} K^n, \tag{41}$$

where $a \equiv 2\alpha + \delta$, and

$$K = \frac{1}{a}\begin{bmatrix} \delta + \alpha(2 - m_0 - \pi\psi n_1) & \alpha(1-\pi)m_0 & \alpha\pi(m_0 + \psi n_1) \\ \delta + \alpha\pi(\psi n_0 + \phi n_1) & \alpha[2 - \pi(m_0 + \psi n_0 + 2\phi n_1)] & \alpha\pi(m_0 + \phi n_1) \\ \delta & 0 & 2\alpha \end{bmatrix}.$$

The transition function for the stochastic process that rules an employer's state is

$$[\hat{p}_{ij}^{(t)}] = \sum_{n=0}^{\infty} \frac{(at)^n e^{-at}}{n!} \hat{K}^n, \tag{42}$$

where

$$\hat{K} = \frac{1}{a} \begin{bmatrix} \delta + \alpha (2 - n_0 - \pi n_1) & \alpha (1-\pi) n_0 & \alpha\pi (n_0 + n_1) \\ \delta + \alpha\pi (m_0 + \phi n_1) & \alpha [2 - \pi (m_0 + \psi n_0 + 2\phi n_1)] & \alpha\pi (\psi n_0 + \phi n_1) \\ \delta & 0 & 2\alpha \end{bmatrix}.$$

Proof of Lemma 4. Each match is subject to three independent Poisson processes: one with arrival rate δ (the exogenous destruction process), and two with arrival rate α (the process according to which the employer meets other workers and the one according to which the worker meets other employers). Conditional on the arrival of one of these Poisson events, the worker transits from state i to state j according to $K = \frac{\delta}{a} K_\delta + \frac{\alpha}{a} K_\alpha^e + \frac{\alpha}{a} K_\alpha^w$, where

$$\begin{aligned} K_\delta &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\ K_\alpha^e &= \begin{bmatrix} 1 & 0 & 0 \\ \pi (\psi n_0 + \phi n_1) & 1 - \pi (\psi n_0 + \phi n_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ K_\alpha^w &= \begin{bmatrix} 1 - m_0 - \pi\psi n_1 & (1-\pi) m_0 & \pi (m_0 + \psi n_1) \\ 0 & 1 - \pi (m_0 + \phi n_1) & \pi (m_0 + \phi n_1) \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

Then, since the number of arrivals over a period of length t follows a Poisson distribution with parameter at, (41) follows. See Cox and Miller (1965) for details. Similarly, conditional on the arrival of one of the three Poisson events that may hit a match, the employer transits from state i to state j according to $\hat{K} = \frac{\delta}{a} \hat{K}_\delta + \frac{\alpha}{a} \hat{K}_\alpha^e + \frac{\alpha}{a} \hat{K}_\alpha^w$, where $\hat{K}_\delta = K_\delta$,

$$\begin{aligned} \hat{K}_\alpha^e &= \begin{bmatrix} 1 - n_0 - \pi n_1 & (1-\pi) n_0 & \pi (n_0 + n_1) \\ 0 & 1 - \pi (\psi n_0 + \phi n_1) & \pi (\psi n_0 + \phi n_1) \\ 0 & 0 & 1 \end{bmatrix} \\ \hat{K}_\alpha^w &= \begin{bmatrix} 1 & 0 & 0 \\ \pi (m_0 + \phi n_1) & 1 - \pi (m_0 + \phi n_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

As for an employer, since the number of arrivals of shocks to the match over a period of length t follows a Poisson distribution with parameter at, we obtain (42). ■

B Supplementary Material

B.1 Alternative Bargaining Procedures

In previous versions of this paper, e.g., Kiyotaki and Lagos (2006), we considered the following alternative bargaining procedure. When a worker and an employer find a new production opportunity, a move by Nature first determines, with equal probability, whether the worker(s) or the employer(s) have the bargaining power. The agents with the new production opportunity then choose whether to bargain first with the new potential partner or with the old partner, if there is one.[44] All negotiations are bilateral, either between new potential partners or between old partners. Once the bargaining pairs have been decided, the agent with the bargaining power makes an offer which consists of a proposal to produce together and a division of surplus to be implemented through spot side payments. The recipient of the offer chooses whether to accept, reject, or continue to negotiate with the alternative partner (if there is one), withholding the received offer as her outside option. An outstanding offer is public information, and cannot be revised later. If the offer is rejected, then the agent who has an alternative partner will negotiate without the outside option.

The second round of bilateral bargaining, with or without outside option(s), is similar to the first round: The agent with the bargaining power makes an offer, and the recipient either accepts or rejects it. If an offer is rejected, the recipient then makes a final choice of whether to accept or reject the withheld offer (if she has one). The bargaining ends either when an offer is accepted, or when there is no alternative partner to bargain with after rejection (in which case there will be no match for production). The bargaining does not take time: the entire process finishes instantaneously. We also specify that as long as neither encounters a new production opportunity, matched agents split output equally. The equilibrium match-creation and destruction decisions and the expected payoffs induced by this sequential, bilateral

[44] If one of the agents with the option to form a new match chooses to bargain with her old partner first, while the other one chooses to bargain with his new partner first, then the latter will wait until the former comes to bargain with him.

bargaining procedure, turn out to be identical to those induced by the bargaining procedure we considered here, in Section 3.

We have also considered other bargaining procedures, including some that allow for the agent with the bargaining power to be selected independently by Nature in each bilateral negotiation. In these formulations we have typically found match formation and destruction conditions and equilibrium expected payoffs in the cases in which the new match is formed, that are identical to the ones in Proposition 1. For the meetings in which the old matches were not destroyed to form new ones, however, some of the bargaining procedures we considered delivered different expected side payments than those in Proposition 1. In any case, notice that expected side payments within a partnership net out to zero in a symmetric equilibrium (i.e., there are no side payments in (8) and (9) or the Bellman equations immediately above them), so our main results would have remained unchanged had we chosen to adopt any of these alternative formulations for the bargaining procedure.

B.2 Efficiency and Unemployment Benefits

In this section we show how an unemployment benefit system can be designed in order to restore efficiency in the economy we analyzed at the beginning of Section 5.1, with $N = 2$ and the numbers of employers and workers fixed and equal to unity. Suppose every agent receives a payoff $b > 0$ while unmatched, and that this transfer is paid for by levying a tax T_e from every matched agent. The balanced-budget condition is $bn_0 = T_e\,(n_1 + n_2)$. The Bellman equations for the competitive economy become

$$\begin{aligned}
r\hat{V}_2 &= y_2 - T_e - \delta(\hat{V}_2 - \hat{V}_0) \\
r\hat{V}_1 &= y_1 - T_e - \delta(\hat{V}_1 - \hat{V}_0) + \alpha n_0 \pi(\hat{V}_2 - \hat{V}_1) + \alpha n_1 \pi \phi(\hat{V}_2 + \hat{V}_0 - 2\hat{V}_1) \\
r\hat{V}_0 &= b + \alpha n_0 \left[\pi(\hat{V}_2 - \hat{V}_0) + (1-\pi)\,(\hat{V}_1 - \hat{V}_0)\right] + \alpha n_1 \pi(\hat{V}_2 - \hat{V}_1).
\end{aligned}$$

Notice that this policy can only affect the flow equations (35)–(37) indirectly through their effect on the separation decision ϕ. So for a given ϕ, the stationary distribution of agents across states is still described by Lemma 3. However, now $\phi = 1$ with certainty if and only if $\hat{V}_2 + \hat{V}_0 - 2\hat{V}_1 > 0$, which, using the government budget constraint, can be written as

$$\frac{y_2 - \frac{T_e}{n_0}}{y_1 - \frac{T_e}{n_0}} > 2 - \frac{\alpha\left[\pi n_1 + (1-\pi) n_0\right]}{r + \delta + \alpha\left(n_0 + \pi n_1\right)}. \tag{43}$$

Observe that if we let $T_e = T_e^*$, where

$$T_e^* = \frac{\alpha n_0 (r+\delta)\left[\pi n_1 + (1-\pi) n_0\right]}{\left[r + \delta + 2\alpha\left(n_0 + \pi n_1\right)\right]\left(r + \delta + \alpha \pi n_0\right)} y_1,$$

then (43) coincides with (22). In other words, the compensation $b^* = \frac{n_1+n_2}{n_0} T_e^*$ makes agents internalize the composition externality in the competitive matching equilibrium and implements the planner's match creation and destruction decisions.[45]

B.3 Free Entry with Constant Returns Meeting Technology

Suppose the aggregate meeting technology is given by a function $\xi(m,n)$, which is monotonic in both arguments and homogeneous of degree one. Since $n = 1$, in this alternative formulation an employer contacts a random worker at rate $\alpha(m) \equiv \xi(1, 1/m)$ and a worker contacts a random employer at rate $m\alpha(m)$. So the probability an employer meets a worker who is employed in a match of type i is $\alpha(m) n_i$, and the probability a worker meets an employer who is in a match of type i is $\alpha(m) m_j$. Therefore, if we replace α with $\alpha(m)$, the Hamiltonian, the flow equations (13) and (14), and the optimality conditions for τ_{ij}^k are all unchanged, while since $m = 1 - n_0 + m_0$, condition (15) becomes

$$\begin{aligned} C'(m_0) &= \alpha(m) \sum_{i=0}^{N} \sum_{k=1}^{N} n_i \pi_k \tau_{i0}^k \left(\lambda_k - \lambda_i\right) \\ &\quad + \alpha'(m) \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{N} n_i m_j \pi_k \tau_{ij}^k \left(\lambda_k + \lambda_0 - \lambda_i - \lambda_j\right). \end{aligned} \tag{44}$$

[45] The subsidy to the unmatched agents is socially beneficial because it internalizes search externalities and not because it provides insurance. A single subsidy and tax rate are enough to achieve the efficient allocation in this case with $N = 2$, but this need not be the case with more heterogeneity of matches ($N > 2$).

The Euler equations associated with n_i for $i = 1, ..., n$ are as in Section 5 (again, after replacing α with $\alpha(m)$), but since $\alpha(m) = \alpha(1 - n_0 + m_0)$, the one associated with n_0 is now

$$r\lambda_0 - \dot{\lambda}_0 = \alpha(m) \sum_{j=0}^{N} \sum_{k=1}^{N} m_j \pi_k \tau_{0j}^k (\lambda_k - \lambda_j) - \alpha'(m) \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{N} n_i m_j \pi_k \tau_{ij}^k (\lambda_k + \lambda_0 - \lambda_i - \lambda_j).$$

Using (44), which holds as long as $m_0 > 0$, and collecting terms, this last condition becomes

$$\begin{aligned} r\lambda_0 - \dot{\lambda}_0 \quad = \quad & -C'(m_0) + \alpha(m)(m_0 + n_0) \sum_{k=1}^{N} \pi_k \tau_{00}^k (\lambda_k - \lambda_0) \\ & + \alpha(m) \sum_{j=1}^{N} \sum_{k=1}^{N} n_j \pi_k (\tau_{0j}^k + \tau_{j0}^k)(\lambda_k - \lambda_j). \end{aligned} \tag{45}$$

Summarizing, the optimality conditions for the economy with a constant returns meeting technology are (3) and (44), together with (16) and (17), but with $\alpha = \alpha(m)$. Comparing these optimal conditions with the conditions for the competitive matching equilibrium: (13), (14), (18), (19), and (20) without firing taxes—which remain unchanged—we learn that the constant-returns-to-scale aggregate matching function fails to solve the inefficiency associated with the composition externality we discussed in Section 3.

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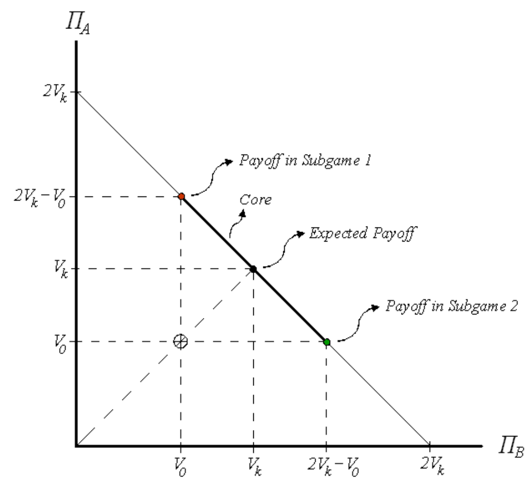

Figure 6: Core payoffs for a meeting involving two agents.

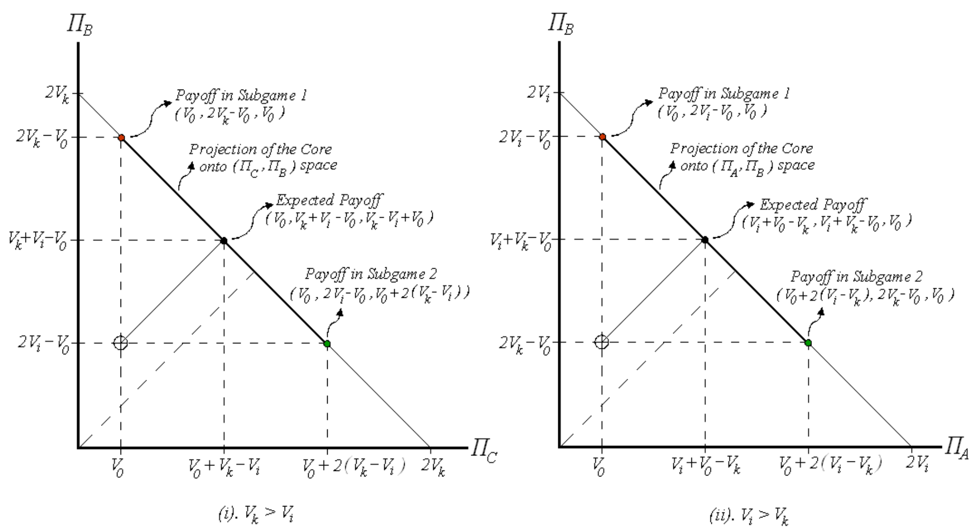

Figure 7: Core payoffs for a meeting involving three agents.