Housing, Distribution and Welfare

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Abstract

Housing is a long-lived asset whose value is sensitive to variations in long-term growth and interest rates. When a large fraction of the population is leveraged, housing price fluctuations cause large-scale redistribution and consumption volatility. We examine policies to mitigate the impact of housing fluctuations on vulnerable households. We find that the most practical way to insure the young and the poor from the housing cycle is through a well-functioning rental market. In practice, home-ownership subsidies keep the rental market small and the housing cycle affects aggregate consumption. Removing home-ownership subsidies hurts older home-owners, while leverage limits hurt younger home-owners.

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1 Introduction

Housing is the most important non-human asset for a large fraction of households. Booms and busts in housing prices have been associated with many financial crises throughout recent history (Jorda, Schularick and Taylor (2015)), especially when associated with real estate lending booms (Jorda et. al. (2016)). Many academics and policy makers are concerned about the vulnerability of household balance sheet conditions, calling for the housing finance market to be reformed so as to make it more stable.

In this paper we build a tractable housing credit cycles model and use it to examine the causes and consequences of housing market volatility and evaluate a number of housing-related policies. We argue that long run growth rates and interest rates are the main fundamentals that determine housing values and show that these can generate substantial volatility in housing prices and consumption. The paper then examines removing home ownership subsidies and restricting loan-to-value (LTV) ratio as alternative policies that can help to reduce the economy's volatility in response to changes in fundamentals. Both policies are found to hurt home owners significantly, even though they benefit renters. The removal of home ownership subsidies would hurt old home owners, while LTV caps would hurt young leveraged home owners. As older generations tend to be more politically active, our results may explain why reform of the system of home ownership subsidies has been so difficult to implement.

This paper is related to a growing literature examining the importance of housing collateral for macroeconomic fluctuations (Iacoviello (2005), Iacoviello and Neri (2010) Liu, Wang and Zha (2013), and Kaplan, Mittman and Violante (2017) to name but a few). Our emphasis on the vulnerability of leveraged home-owners to income and wealth shocks receives empirical support from the work of Campbell and Cocco (2007), Cloyne and Surico (2016) and Cloyne, Ferreira and Surico (2015), among others.

We make a modelling contribution by building a tractable overlapping generations (OLG) model which aggregates nicely into a representative borrower household (whose income is growing) and a representative saver household (whose income is declining).[1] This does not require any assumptions about heterogeneous impatience, instead using the well documented facts about the life-cycle earnings profile (Gourinchas and Parker, 2002) to calibrate the model's parameters and to motivate borrowing and lending.

To generate the kinds of large fluctuations in housing prices we have observed in recent years, we model housing as "land" in common with Davis and Heathcote (2005), Kiyotaki, Michaelides and Nikolov (2011), and Liu, Wang and Zha (2013). Like Mankiw and Weil (1989), Kiyotaki, Michaelides and Nikolov (2011) and Miles and Sefton (2018), we stress the importance of long term interest rates and growth rates in explaining the fluctuations of the price of housing through the importance of land.[2] Relatedly, recent empirical evidence from 14 advanced economies supports the role of land prices, rather than construction costs, in explaining housing price booms (Knoll, Schularick and Steger (2017)).

Despite the linear period utility in our framework, households care about housing market risks because households' marginal utility of wealth is negatively correlated with the rate of return on home ownership. Households value wealth more in states when consumption is low temporarily and the user cost of housing is cheap with the arrival of an adverse shock. This is why both households and policy makers in the model are concerned with housing market volatility.

Our main interest is to examine what policies and institutions can help mitigate the problems originating from the housing market. Since the key missing market in the model is the market for state contingent mortgage debt, any policy or insti-

[1] The OLG structure we are using relies on the model with recursive preference developed by Gertler (1999). These preferences are a variant of Epstein-Zin-Weil preferences and feature risk neutrality across states of nature but finite intertemporal elasticity of substitution across time periods.

[2] In addition, in a robustness exercise left to the Appendix, we solve the model under uncertainty, and we are also able to replicate qualitatively the finding in Favilukis, Ludvigson and van Niuewerburgh (2016) that changes in real interest rate generate partially offsetting changes in housing risk premia. We end up with a model that captures most of the key channels identified in the literature but in a smaller and more tractable model.

tution that replicates its insurance properties would be welfare improving. Equity mortgage contracts would be a slightly inferior, though close alternative, that has been proposed as a way to replace traditional mortgage debt (Shiller (1998)), but a housing equity market has not developed so far. The one practical form of housing tenure that delivers net worth insurance (because house prices are substantially more volatile than rents) is renting. For renters, net worth and consumption are just as well shielded from housing price fluctuations, as for home owners under state contingent debt markets. Therefore, any policy that increases the size of the rental market, makes the consumption of young households more stable by transferring the risk of housing price fluctuations to the old. In this respect, a well-functioning rental market becomes a good substitute for state contingent mortgage debt.

In a stochastic version of the model, we also show that, when households recognize the risk involved in home ownership, they self insure by taking on less debt. This significantly moderates the level of housing prices relative to rents and the prevalence of home ownership. However, the generous home ownership subsidies in the US and other advanced economies push in the opposite direction. Such subsidies make home ownership more attractive, boosting housing prices and squeezing the rental market. This, in turn, moves the economy further away from the full insurance benchmark.

Two main housing market finance policies have been proposed or used in different countries to mitigate households' vulnerability to housing price fluctuations. We contribute a novel analysis of the preferences of different households (old, young, renters and owners) over two such sets of policies.[3] Specifically, we examine a policy that reduces home ownership subsidies and a policy that imposes LTV limits on borrowers. Reducing home ownership subsidies diminishes the incentive of households to take on too much leverage through housing ownership, while LTV limits directly impose constraints on the leverage households can take.

[3] Mendicino, Lambertini and Punzi (2013) and Mendicino, Nikolov, Supera and Suarez (2016) stress the importance of agent heterogeneity (different degree of impatience) in generating winners and losers from macroprudential policy.

We find that some groups in society are significantly hurt by each of these types of policies. Young home owners are hurt the most by LTV caps, because they reduce the housing consumption of leveraged house buyers. In contrast, older home owners are hurt the most by the home ownership subsidy removal. This analysis illustrates that policies to reduce the economy's vulnerability to housing price fluctuations have substantial redistributive effects. Moreover, since they create prominent losers, such policies would be politically difficult to change.

The paper is structured as follows. Section 2 shows some motivating empirical evidence and presents a simple example to build intuition on the main mechanism we use to generate large housing price fluctuations. Section 3 introduces the baseline economic environment, while Section 4 derives the main equilibrium conditions. Section 5 describes the calibration and Section 6 examines the way different shocks affect the welfare of heterogeneous households in the model. Section 7 then evaluates the various policies currently used to manage (or mis-manage) the housing cycle.

2 Long-term Housing Fundamentals and Housing Prices

2.1 Evidence

We start with some observations to motivate our emphasis in the rest of the paper. Figure 1 shows a time series of the nominal 30-year mortgage rate minus the realized inflation rate (from a year before) in the United States. The series has been smoothed by taking a 5 year moving average and it represents a very simple and crude measure of the long term real mortgage interest rate. Despite the shortcomings of the measure we use, the graph clearly shows that long term US real interest rates have changed substantially over time. The real interest rate sharply increased from the mid-1970s to the mid-1980s. This is perhaps related to the high inflation in 1970s, the dramatic monetary policy tightening that started in 1979, and the gradual decline of inflation expectations since then. From the mid-1980s, the US long-term real interest rates have declined substantially as many authors emphasized. The long term real mortgage rates fell sharply from 9% in the mid-1980s to 5% in the early 1990s, and declined gradually after to levels of 2-2.5% right now.

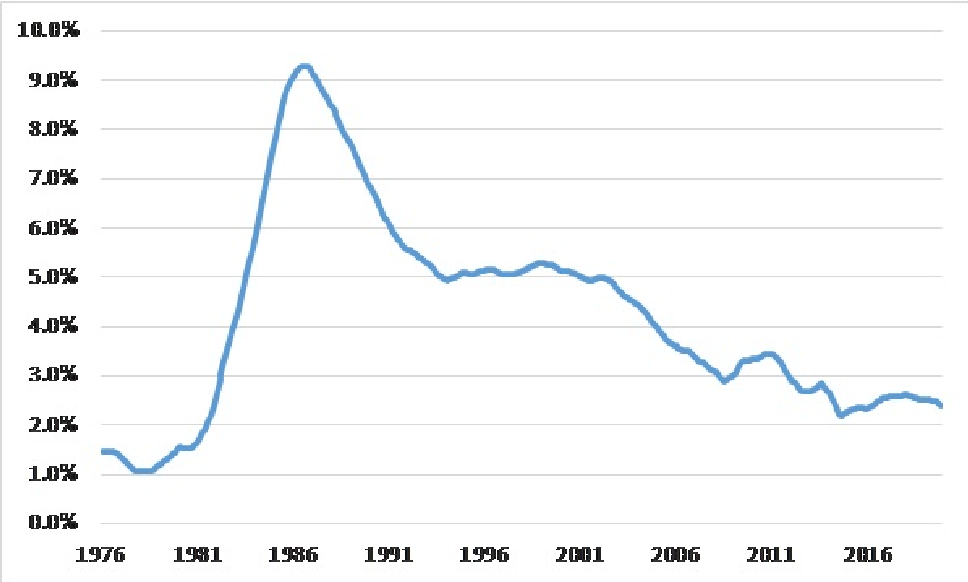

Figure 1: US Real Mortgage Rate

(30-year mortgage rate - realized annual inflation: 5 year moving average)

Source: See Appendix *A*

Figure 2 shows data from the Budget Office that estimates the trend growth of the US economy over different time periods. It shows that the long term growth rate for the US economy has also declined from 3.2% in the 1980s and 1990s to 2.5% in the 2000s and to levels of 1.5-2% projected for the future.

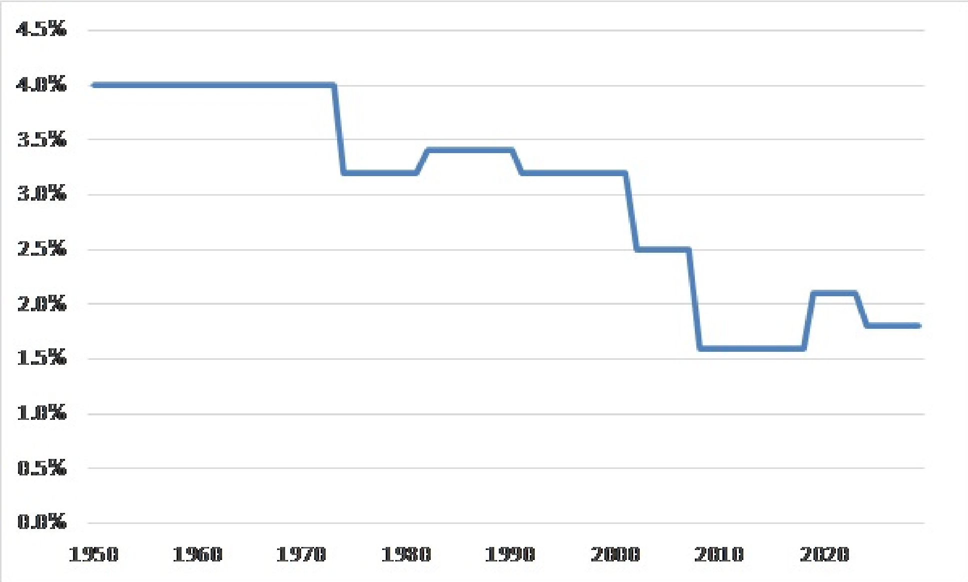

Figure 2: US trend annual GDP growth rate (CBO Estimate - 2019)

The decline in real interest rates since the mid-1980s is most likely partly driven by falling growth. However, a closer examination of the data reveals that, over time, the gap between the long term real interest rate and the economy's trend growth rate has been declining since the early 1980s. It was approximately 2% on average in the late 1980s and in the 1990s before falling to 0.5% in the 2000s and to zero since the 2007-2009 crisis.

Figure 3 below plots the evolution over the same time period of the housing rental yield (inverse of the price-to-rent ratio) for the US housing market.[4] Two key facts stand out from the figure. First, the housing rental yield has been declining over time, falling from around 5.5% in the 1960s and 1970s to 5% in the 1990s and to 4% in more recent years. Second, it has been volatile recently, going down to a record low of 3.5% during the boom, before rising back up towards 5% during the bust. More recently, the rental yield has been on its way down again, declining to approximately 4%.

[4]The data are from the website of the Lincoln Institute of Land Policy. It was constructed by Davis, Lehnert and Martin (2008).

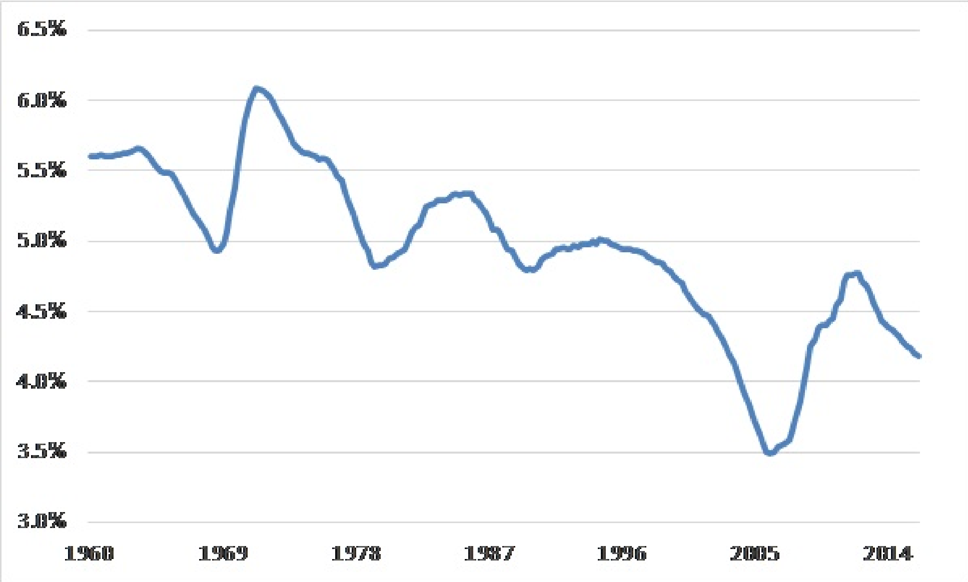

Figure 3: US residential housing rental yield: 1960-2016

(Annual gross residential housing rent divided by purchase price)

Source: Lincoln Institute of Land Policy and Davis, Lehnert and Martin (2008)

2.2 A simple example

To build up intuition, we start with a simple dividend discount model under perfect foresight.[5] Suppose that housing does not depreciate and its rental price r_t grows by a factor G_t. We will later develop a model with land constraints in which the growth rate G_t is determined in an endowment economy; for a production economy setting, see, for example, Kiyotaki et. al. (2011). Let R_t be the gross real interest rate. Then, the housing price P_t is given by the following dividend discount formula.

$$P_t = r_t + \frac{P_{t+1}}{R_t}. \tag{1}$$

[5] See Mankiw and Weil (1989) for a similar partial equilibrium model of housing.

If we define $\widehat{P}_t \equiv \frac{P_t}{r_t}$ as the price-to-rent ratio, then we can solve for the evolution of $\widehat{P}_t$ as follows:

$$\begin{aligned}\widehat{P}_t &= 1 + \frac{r_{t+1}}{r_t}\frac{\widehat{P}_{t+1}}{R_t} \\ &= 1 + \frac{G_{t+1}}{R_t}\widehat{P}_{t+1}.\end{aligned}$$

In the long run, the price-to-rent ratio is given by:

$$\widehat{P} = \frac{R}{R-G}, \tag{2}$$

where the variables without time subscripts denote long-term steady state values. The housing rental yield is given by the inverse of $\widehat{P}$.

The simple dividend discount model presented here has several key implications for housing prices which are broadly consistent with the observations in the previous subsection. First, a lower value of $R - G$ leads to a higher value of the price-to-rent ratio $\widehat{P}$ and to a lower housing rental yield which is consistent with the evidence in the previous subsection. Second, the smaller the gap between the long term real interest rate and growth rate, the larger is the impact of permanent change of the interest rate and growth rate on the price-to-rent ratio. Loosely speaking, news about long term interest rates or growth rates have a disproportionately larger impact on housing prices when the gap between interest rates and growth rates is small. This is consistent with the higher volatility of the US housing rental yield in recent years.

3 OLG Model of Housing Credit Cycles

In the previous section we argued that the influence of a single factor, the difference between the long run real interest rate and growth rate $(R-G)$, is broadly consistent with the evolution of the housing price-to-rent ratio over the last 30 years. We now integrate the simple housing market example outlined above into a general equilibrium framework that is suitable for assessing the aggregate and distributional consequences of housing price fluctuations.

Our aim is to analyze the macroeconomic impact of shocks to real interest rates and trend growth rates as well as the effect on the welfare of different groups in society. After laying out the model and its calibration in Sections 3 to 5, we will show in Section 6 that these shocks cause large-scale redistribution which hurts leveraged home owners in particular. We argue that the size of these redistributive effects are likely to have grown in a world of low $R - G$. This is leading to policy analysis in Section 7 where we examine what kinds of risk-sharing institutions and mechanisms would work best in protecting vulnerable households.

3.1 Endowment Economy and Demographics

We build an endowment economy model consisting of two types of households: 'young' and 'old' households. There are two main differences between the different types of households: (i) the age-related labor productivity of the young household grows, while that of the old household declines; (ii) the old household faces mortality risk, while the young household faces the risk of becoming old. Here we briefly review the model's main implications, while the full derivations are left to Appendix B.

Households are born as young with zero assets and they remain young with probability γ and become old with probability $1 - \gamma$ in the following period. When old, agents survive with probability σ and die with probability $1-\sigma$. The population of young households grow at a rate G_N, which we assume to be constant. The ratio of the population of young (N_t^y) and old (N_t^o) households is constant at

$$N_t^y : N_t^o = G_N - \sigma : 1 - \gamma.$$

We normalize the efficiency unit labor of newborns to be unity as $x_t = 1$. Let g_t^y and g_t^o be the age-related labor productivity growth rate (plus one) of young and old agents at date t:

$$\begin{aligned}\frac{x_t}{x_{t-1}} &= g_t^y > 1, \text{ when young at date } t-1 \\ \frac{x_t}{x_{t-1}} &= g_t^o < 1, \text{ when old at } t-1 \text{ and survives at } t.\end{aligned}$$

Let X_t^y and X_t^o be the aggregate age-related labor of young and old agents. If the age-related labor productivity growth rates and population growth rate are constant, then the age-related labor of young and old grow at the same rate with population growth rate, $\frac{X_t^y}{X_{t-1}^y} = \frac{X_t^o}{X_{t-1}^o} = G_N$, and the average age-related labor productivity of young and old agents is constant as

$$x^y = \frac{X_t^y}{N_t^y} = \frac{G_N - \gamma}{G_N - \gamma g^y} > 1,$$

and

$$x^o = \frac{G_N - \sigma}{G_N - \sigma g^o} g^y x^y < g^y x^y.$$

Because labor productivity increases with age while young, the average age-related labor productivity of young households is larger than that of a new-born, which is unity. For old households, labor productivity decreases with age and hence the average age-related labor productivity is smaller than that of a newly old, $g^y x^y$.

Aggregate efficiency unit labor is

$$X_t = X_t^y + X_t^o,$$

and aggregate income is

$$Y_t = A_t X_t,$$

where A_t is the aggregate labor productivity which is equal to the productivity of the new born and grows at

$$\frac{A_t}{A_{t-1}} = G_{At}.$$

3.2 Households

The preferences of young and old agents ($i = y, o$) are given by

$$V_t^y = \left\{ (u_t^y)^{\frac{\eta-1}{\eta}} + \beta[\gamma E_t V_{t+1}^y + (1-\gamma) E_t V_{t+1}^o]^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}, \tag{3}$$

and

$$V_t^o = \left[(u_t^o)^{\frac{\eta-1}{\eta}} + \beta\sigma \left(E_t V_{t+1}^o \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{4}$$

where β is the discount factor. This utility implies that agents are risk-neutral but have finite intertemporal elasticity of substitution of $\eta \in (0, \infty)$.

Period utility is given by

$$u_t^i = \left(\frac{c_t^i}{\phi} \right)^{\phi} \left(\frac{h_t^i}{1-\phi} \right)^{1-\phi},$$

if the household is a home owner and by

$$u_t^i = \left(\frac{c_t^i}{\phi} \right)^{\phi} \left(\frac{\chi^i h_t^i}{1-\phi} \right)^{1-\phi},$$

if the household is a renter. Here c_t^i and h_t^i are consumption of goods and housing by type i agent at date t. χ^i is an individual specific parameter that represents the preference for renting over owning. The household with $\chi^i > 1$ prefers to rent, and that with $\chi^i < 1$ prefers to own. We assume that, for young households, χ^y is uniformly distributed on $[0, \overline{\chi}]$. For old households, $\chi^o = 0$ meaning that all old households want to own housing. This is a simple and tractable way of capturing the fact that home ownership is increasing over the life cycle, and that retirees do not downsize, perhaps due to "place attachment" (see Cocco and Lopes (2020)).

3.2.1 Constraints for young home owners

The budget constraint of young home owners is given by

$$c_t + Q_t (1 + \mu_t - \upsilon_t) h_t + b_{t-1} = (1 - \tau_t) w_t x_t + Q_t h_{t-1} + \frac{b_t}{R_t},$$

where w_t, Q_t and R_t are the wage rate, housing price and gross interest rate, all in terms of consumption goods, while x_t and b_t are age-related labor productivity and debt at date t. The variable μ_t is the maintenance cost of housing in terms of goods, which the residents have to pay in proportion to the value of housing.[6] The variable υ_t is a home ownership subsidy, while τ_t is a wage tax imposed on all households in order to pay for the subsidy.

Each young home owner faces the borrowing constraint

$$E_t Q_{t+1} h_t - b_t \geqslant \omega \left(Q_t h_{t-1} - b_{t-1} \right),$$

for $\omega \in (0, 1)$. Defining the non-human net worth of agent at date t (i.e. not including human capital) as

$$a_t = Q_t h_{t-1} - b_{t-1},$$

we can rewrite the budget constraint as

$$c_t + r_t^h h_t + \frac{E_t a_{t+1}}{R_t} = (1 - \tau_t) \, w_t x_t + a_t,$$

where

$$r_t^h = (1 + \mu - \upsilon_t) \, Q_t - \frac{E_t Q_{t+1}}{R_t} \tag{5}$$

is the imputed rent (or user cost) of housing for home owners. The borrowing constraint becomes

$$E_t a_{t+1} \geq \omega a_t. \tag{6}$$

This borrowing constraint implies that, in the long run, $a_t = 0$ and the agent cannot borrow more than the housing value. In the short run, if unanticipated aggregate shocks drive the household into negative equity, the constraint allows the

[6] We think of 'maintenance costs' as including a number of costs of owning property: property taxes and the cost of maintaining the structures. We assume the total maintenance cost is proportional to the housing value for simplicity. We will calibrate the cost in order to match the average US housing price-to-rent ratio in the 1980-2015 period.

negative equity position to be closed gradually over time when $\omega > 0$. This is a tractable way to capture the effects of long term debt in the model.

3.2.2 Budget constraint of young renters

The budget constraint for a young renter is given by:

$$c_t + r_t^r h_t + b_{t-1} = (1 - \tau_t)\, w_t x_t + \frac{b_t}{R_t},$$

where r_t^r is the rental price of housing. The borrowing constraint is given by:

$$b_t \leq 0,$$

meaning that renters cannot borrow.

3.2.3 Budget constraint of the old

The old agents purchase housing both for own use (h_t) and as an investment (s_t) as landlord. The costs of the two investments differ because owner occupied housing receives subsidies υ_t. In addition, they typically save ($b_t < 0$) by lending to young home-owners.

$$c_t + Q_t (1 + \mu_t - \upsilon_t)\, h_t + [Q_t (1 + \mu_t) - r_t^r]\, s_t - \frac{b_t}{R_t} = (1 - \tau_t)\, w_t x_t + \frac{Q_t (s_{t-1} + h_{t-1}) - b_{t-1}}{\sigma}. \tag{7}$$

When agents are old at date $t-1$, we assume a perfect annuity market in which surviving old agents share the assets of dying agents proportionally, which is why the return on assets for survivors is multiplied by $1/\sigma$.

The indifference condition between saving through rental housing and bonds gives us the rental price of housing:

$$r_t^r = (1 + \mu_t)\, Q_t - \frac{E_t Q_{t+1}}{R_t}. \tag{8}$$

It differs from the imputed rental cost of owner-occupied housing due to home ownership subsidies.

Defining

$$a_t = \frac{1}{\sigma}\left[Q_t\left(s_{t-1} + h_{t-1}\right) - b_{t-1}\right],$$

we can rewrite the budget constraint as

$$c_t + r_t^h h_t + \frac{\sigma E_t a_{t+1}}{R_t} = \left(1 - \tau_t\right) w_t x_t + a_t, \tag{9}$$

where r_t^h is the user cost for home owners in (5).

4 Perfect Foresight Equilibrium

This section describes the general equilibrium of the small open economy with the exogenous real interest rate under perfect foresight. The equilibrium under uncertainty is explained in Appendix D.

4.1 The Young Home Owner

The young households expect an upward sloping earnings profile, while they are likely to remain young for a long time. Since they start life with no assets, $(a_t = 0)$, we expect the young households to be borrowing constrained, $a_{t+1} = \omega a_t$, which we will verify later. This implies that the household exhausts its entire available resources in paying for non-durables and for its housing downpayment:

$$c_t + r_t^h h_t = \left(1 - \tau_t\right) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t.$$

It is easy to show that, due to the Cobb-Douglas period utility and due to the absence of uncertainty, the usual consumption and housing demands obtain.[7]

$$\begin{aligned} c_t &= \phi \left[(1-\tau_t)\, w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t \right] \\ h_t &= \frac{1-\phi}{r_t^h} \left[(1-\tau_t)\, w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t \right]. \end{aligned}$$

In the steady state $a_{t+1} = 0$ and the household simply consumes its post tax wage.

We guess and verify that the value function is proportional to period utility:

$$V_t^h = \Delta_t^h u_t^h.$$

Then the ratio of the value function to period utility is a recursive function that depends on the growth rates of utility over time:

$$\Delta_t^h = \left\{ 1 + \beta \left[\gamma \Delta_{t+1}^h G_{t+1}^{hh} + (1-\gamma)\, \Delta_{t+1}^o G_{t+1}^{ho} \right]^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}, \tag{10}$$

where

$$G_{t+1}^{hh} = \frac{G_{t+1}^A g^y \left(1 - \tau_{t+1}\right) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) \omega a_t}{(1-\tau_t) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t} \left(\frac{r_t^h}{r_{t+1}^h} \right)^{1-\phi}$$

is the utility growth rate conditional on remaining young, while

$$G_{t+1}^{ho} = \frac{\nu_{t+1}^o (W_{t+1}^{ho} + \omega a_t)}{(1-\tau_t) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t} \left(\frac{r_t^h}{r_{t+1}^o} \right)^{1-\phi}$$

is the utility growth when the household switches from youth to old age. The variable ν_{t+1}^o is the marginal propensity to consume out of human capital W_{t+1}^{ho} and non-human wealth when old, which we will explain shortly.

In steady state when $a_{t+1} = 0$, the expenditure of the young home owner follows his income while her utility grows at the growth rate of expenditure adjusted by the housing inflation rate caused by the upward trend in the user cost of housing r_t^h.

[7] Once we introduce uncertainty, the expenditure shares will depart from the Cobb-Douglas utility weights of housing and consumption. See Appendix B for derivations on this case.

Once the household becomes old and its income starts to fall, it loses some of its human wealth but it gains the ability to smooth consumption over time as reflected in the fall in the marginal propensity to consume out of total wealth from unity to ν^o_{t+1}.

4.2 The Young Renter

Due to the upward sloping path of income, young renters also do not wish to save. Borrowing constraints prevent them from borrowing hence they choose $a_{t+1} = 0$. They spend their entire budget on consumption and rent

$$\begin{aligned} c_t &= \phi \left(1 - \tau_t\right) w_t x_t \\ h_t &= \frac{1-\phi}{r^r_t} \left(1 - \tau_t\right) w_t x_t. \end{aligned}$$

Yet again, we can guess and verify that the young renter will have a value function which is proportional to period utility

$$V^r_t = \Delta^r_t u^r_t,$$

where Δ^r_t is given below

$$\Delta^r_t = \left\{ 1 + \beta \left[\gamma \Delta^r_{t+1} G^{rr}_{t+1} + (1-\gamma) \Delta^o_{t+1} G^{yo}_{t+1} \right]^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}. \tag{11}$$

Here

$$G^{rr}_{t+1} = \frac{1-\tau_{t+1}}{1-\tau_t} G^A_{t+1} g^y \left(\frac{r^r_t}{r^r_{t+1}} \right)^{1-\phi}$$

is the utility growth rate while the household remains a young renter and

$$G^{ro}_{t+1} = \nu^o_{t+1} \frac{W^{ho}_{t+1} g^y}{(1-\tau_t) w_t} \left(\frac{r^r_t}{\chi r^o_{t+1}} \right)^{1-\phi}$$

is the utility growth rate in the period when the household becomes an old home-owner.

4.3 Housing tenure choice

Households decide whether to rent or own by comparing the utility of the two types of housing tenure. Since switching tenure is assumed to be costless, households compare period utility rather than value functions. In a perfect foresight equilibrium, the period utility from owning for a household with zero net worth is given by

$$u_t^h = \frac{(1-\tau_t)\, w_t x_t}{\left(r_t^h\right)^{1-\phi}},$$

where r_t^h is the imputed rent of home owners. The utility of renting is given by

$$u_t^r = \frac{(1-\tau_t)\, w_t x_t}{\left(r_t^r/\chi\right)^{1-\phi}},$$

where r_t^r is the rental price. The household chooses to own rather than rent when the following condition is satisfied:

$$r_t^h < \frac{r_t^r}{\chi}. \tag{12}$$

If the user cost of owning is cheaper than the rental price taking into account of preference for renting over owning (χ), the household chooses to own. Since χ is uniformly distributed on $[0, \overline{\chi}]$ at the individual level, the above utility comparison allows us to solve for the indifferent household for whom equation (12) holds with equality. Then we can compute the aggregate home ownership rate of young households as:

$$\xi_t = \frac{1}{\overline{\chi}} \left(\frac{r_t^r}{r_t^h} \right). \tag{13}$$

4.4 The Old

The old agent chooses consumption and saving to maximize the utility (4) subject to the budget constraint (9) . Since the old agent faces the downward sloping earning profile, we expect that he is not borrowing constrained, which we verify later.

Optimal expenditure implies that

$$
\begin{aligned}
c_t &= \phi e_t \\
h_t &= \frac{1-\phi}{r_t^h} e_t,
\end{aligned}
$$

where

$$
e_t = c_t + r_t^h h_t
$$

is total expenditure.

The budget constraint for the old household is given by

$$
e_t + \frac{\sigma a_{t+1}}{R_t} = (1-\tau_t)\, w_t x_t + a_t. \tag{14}
$$

Then the first order condition for savings a_{t+1} implies that the growth rate of consumption basket (period utility) is given by

$$
\frac{u_{t+1}}{u_t} = (\beta R_t^u)^{\eta}, \tag{15}
$$

where

$$
R_t^u = \left(\frac{r_t^h}{r_{t+1}^h} \right)^{1-\phi} R_t. \tag{16}
$$

We can think of R_t^u as the real interest rate in terms of utility. For full derivations, see Appendix C.

Using Gertler (1999), we guess that expenditure is proportional to total wealth

$$
e_t = \nu_t^o W_t^o, \tag{17}
$$

where total wealth (W_t^o) is sum of non-human (a_t) and human wealth ($W_t^{ho} x_t$) as

$$
\begin{aligned}
W_t^o &= a_t + W_t^{ho} x_t \\
W_t^{ho} &= (1-\tau_t)\, w_t + \frac{\sigma}{R_t} g^o W_{t+1}^{ho}.
\end{aligned}
$$

Then we can show that:

$$\frac{1}{\nu_t^o} = 1 + \sigma\beta^\eta \left(R_t^u\right)^{\eta-1} \frac{1}{\nu_{t+1}^o}. \tag{18}$$

This verifies the guess that consumption is proportional to total wealth in (17).[8]

We also follow Gertler (1999) to guess $V_t^o = \Delta_t^o u_t^o$. Then from (4, 15) , we get[9]

$$\left(\Delta_t^o\right)^{\frac{\eta-1}{\eta}} = 1 + \sigma\beta^\eta \left(R_t^u\right)^{\eta-1} \left(\Delta_{t+1}^o\right)^{\frac{\eta-1}{\eta}}.$$

Thus from (18) , we verify the guess by setting

$$\Delta_t^o = \left(\nu_t^o\right)^{\frac{\eta}{1-\eta}}. \tag{19}$$

The old agent is not constrained in the borrowing if $a_{t+1} > \omega a_t$, and the most likely old agent who becomes constrained is the old agent who was young in the previous period so that there is no initial non human asset. Thus the old households are not constrained if

$$\left(1-\tau_t\right) w_t x_t > \nu_t^o W_t^{ho} = \nu_t^o \left[\left(1-\tau_t\right) w_t x_t + \frac{\sigma}{R_t} W_{t+1}^{ho}\right].$$

We will check that this inequality holds for our parameters in the neighborhood of the steady state later.

4.5 Aggregate Equilibrium

In our simple open economy, housing supply is constant at $\overline{H}$ but requires maintenance which is proportional to the value of housing. Aggregate output is sum of

[8] See Appendix C for full derivations.

[9] The full derivations are provided in Appendix C.

aggregate consumption C_t, housing maintenance $\mu_t Q_t \overline{H}$ and net exports NX_t as[10]

$$Y_t = C_t + NX_t + \mu_t Q_t \overline{H}.$$

Aggregate demand for consumption and housing by young households are

$$\begin{aligned}
C_t^y &= C_t^h + C_t^r = \phi\left[(1-\tau_t)\,w_t X_t^y + \left(1-\frac{\omega}{R_t}\right)A_t^h\right] && (20)\\
H_t^h &= \frac{1-\phi}{r_t^h}\left[\xi_t\,(1-\tau_t)\,w_t X_t^y + \left(1-\frac{\omega}{R_t}\right)A_t^h\right] && (21)\\
H_t^r &= \frac{1-\phi}{r_t^r}\,(1-\xi_t)\,(1-\tau_t)\,w_t X_t^y, && (22)
\end{aligned}$$

where

$$A_t^h = \gamma\left(Q_t H_{t-1}^h - B_{t-1}^h\right)$$

is the net worth of young home owners. In steady state, the borrowing constraint pins down the debt taken on by the young home owners at

$$B_t^h = E_t Q_{t+1} H_t^h. \tag{23}$$

Aggregate demand for consumption and housing by old households are

$$\begin{aligned}
C_t^o &= \phi\nu_t^o \overline{W}_t^o && (24)\\
H_t^o &= \frac{1-\phi}{r_t^h}\nu_t^o \overline{W}_t^o, && (25)
\end{aligned}$$

where $\overline{W}_t^o$ is aggregate total wealth of old households which equals to the sum of non-human and human wealth

$$\begin{aligned}
\overline{W}_t^o &= W_t^{ho} X_t^o + Q_t\left(H_{t-1}^o + H_{t-1}^r\right) - B_{t-1}^o + (1-\gamma)\left(Q_t H_{t-1}^h - B_{t-1}^h\right),\\
W_t^{ho} &= (1-\tau_t)\,w_t + \frac{\sigma g^o}{R_t}(1-\tau_{t+1})\,w_{t+1} + \frac{\sigma g^o}{R_t}\frac{\sigma g^o}{R_{t+1}}(1-\tau_{t+2})\,w_{t+2} + \ldots && (26)
\end{aligned}$$

[10] We assume that the wage tax finances home ownership subsidy, the property tax component of the maintenance cost finances government purchase of goods, and the government balances the budget in every period. We make this assumption to prevent government fiscal policy from affecting the liquidity of different households.

where B_t^o is aggregate net borrowing of the old households at the beginning of date t. The bond market clearing condition at date t-1 implies that the sum of net debt of old and young households and the foreign sector must add up to zero as:

$$B_t^o + B_t^h + B_t^* = 0. \tag{27}$$

B_t^* is the net debt of the foreign sector (or the net foreign asset position of the home country) which evolves with net exports as follows:

$$\frac{B_t^*}{R_t} = B_{t-1}^* + NX_t.$$

Combining the goods market and net foreign asset accumulation, we have

$$\frac{B_t^*}{R_t} = B_{t-1}^* + Y_t - C_t - \mu_t Q_t \overline{H} \tag{28}$$

We assume foreigners do not own home housing. Housing market equilibrium is given by

$$\overline{H} = H_t^h + H_t^r + H_t^o. \tag{29}$$

The endogenous state variables are $\left(B_{t-1}^*, H_{t-1}^h, B_{t-1}^h\right)$, and $N_t^y, N_t^o, X_t^y, X_t^o$, A_t and R_t follows an exogenous process. Then fifteen endogenous variables Q_t, r_t^h, r_t^r, ξ_t, R_t^u, ν_t^o, C_t^y, C_t^o, H_t^h, H_t^r, H_t^o, $\overline{W}_t^o$, B_t^h, B_t^o, and B_t^* are determined by the fifteen equilibrium conditions $(5, 8, 13, 16, 18, 20 - 29)$ as a function of the state variables.

4.6 Stationary Representation

In the following we focus on the case of constant population growth and constant age-related labor productivity growth, i.e., G_{Nt}, g_t^y and g_t^o are all constant at G_N, g^y and g^o. In contrast, aggregate productivity growth rate G_{At} and the real interest rate R_t may have a once-for-all permanent change unexpectedly.

Then, the population shares of young and old are constant and given by:

$$n^y = \frac{G_N - \sigma}{G_N - \sigma + 1 - \gamma}, \text{ and } n^o = \frac{1-\gamma}{G_N - \sigma + 1 - \gamma}.$$

The average age-related labor per capita is constant and given by

$$\frac{X_t}{N_t} = \frac{X_t^y}{N_t^y}\frac{N_t^y}{N_t} + \frac{X_t^o}{N_t^o}\frac{N_t^o}{N_t} = x^y n^y + x^o n^o \equiv \overline{x},$$

We detrend the following variables by dividing by $A_t N_t$, because they have the same trend with $A_t N_t$:

$$Y_t, C_t^h, C_t^r, C_t^o, B_t^h, B_t^o, B_t^*, Q_t, r_t^h, r_t^r, r_t^o.$$

Because w_t has the same trend with A_t, we detrend w_t by dividing by A_t. We detrend the following variables by dividing by N_t, because they have the same trend with N_t

$$X_t^y, X_t^o, N_t^y, N_t^o.$$

We do not need to detrend the following variables as they do not have a trend

$$H_t^h, H_t^r, H_t^o, \Delta_t^h, \Delta_t^r, \Delta_t^o, \xi_t, \nu_t^o.$$

5 Calibration

We calibrate the annual frequency model to U.S. data. All data series were obtained either from FRED of the Federal Reserve Bank of St. Louis or from the Lincoln Institute for Real Estate. All data definitions and sources are described in the Appendix A.

Tables 1 and 2 below show the baseline parameter values and the data fit that the baseline calibration delivers.

Table 1: Baseline parameter values

G^N	1.0110	Population growth rate
G^A	1.0150	GDP per capita growth rate
g^y	1.0442	Endowment growth during youth
g^o	0.9497	Endowment growth when old
β	0.9562	Discount factor
γ	0.9676	Probability of remaining young
σ	0.9640	Probability of survival when old
μ	0.0278	Housing maintenance cost factor
υ	0.0135	Home ownership subsidy
$\overline{\chi}$	2.9504	Upper bound (housing preference)
ϕ	0.9380	Expenditure share non-durables
ω	0.5000	Speed of debt repayment
R	1.0458	Real mortgage rate

Table 2: Model vs Data

Moment	Data	Model
Housing to GDP ratio	1.680	1.683
Net foreign assets to GDP ratio	-0.350	-0.353
Pop aged 20-54 as % of al aged over 20	0.600	0.592
Housing rent to price ratio	0.047	0.047
Home ownership rate	0.656	0.691
Mortgages to GDP ratio	0.683	0.666
(Home-owner imputed rent)/(rental price)	0.710	0.710
Income at age 55/income at age 25	1.400	1.348
Income at age 65/income at age 25	1.200	1.225

Notes: 'Data' refers to the US data moments we target in the calibration procedure. For data moment definitions and sources, see Appendix A. 'Model' refers to the model implications for these moments as computed using the deterministic steady state.

In Table 1, there are a number of parameters that can be directly calibrated from the data. Starting with the growth rates, we set the population growth rate $G^N - 1$ to 1.1% based on the average annual growth rate of the Civilian Non-Institutional Population in the 1980-2019 period. Over the same period, aggregate real GDP grew at an annual rate of 2.6% implying an annual growth rate of GDP per capita ($G^A - 1$) of approximately 1.5%. We measure the real interest rate available to house buyers ($R - 1$) using the average 30-year mortgage rate minus the current inflation rate between 1980 and 2019. This gives a net real interest rate of 4.58% per annum.

Since our model has a life cycle element to it, it is important to calibrate the average path of earnings over the life cycle. Ours is a simple framework with stochastic switching between 'youth' characterized by rising endowments and 'old age' characterized by falling endowments. The average life cycle profile of earnings is characterized by the endowment growth in youth (g^y), the endowment growth rate in old age (g^o). We calibrate these parameters to match 2 moments from the profile of life cycle earnings in the US obtained from Gourinchas and Parker (2002) and to one additional moment from the age distribution of the population. Gourinchas and Parker (2002) estimate that earnings at age 50 and 65 are respectively 40% and 20% higher than at age 25.

We calibrate the discount factor of households (β) at 0.9562 which is exactly the inverse of the long term real interest rate in the baseline calibration. This implies a flat consumption profile in retirement. We set the home ownership subsidy (υ) to match the estimates of Poterba and Sinai (2008) who use the 2004 Survey of Consumer Finances and estimate that home ownership subsidy is 29% of the use cost of house for homeowners (before subsidy) in the US.

There remain 5 additional parameters: the annual tax on housing (μ), the expenditure share on housing (ϕ), the probability of death for old households (σ), the upper bound on the uniform distribution for the rental preference parameter ($\overline{\chi}$) and the probability of switching from 'youth' to 'old age' (γ). We pick these parameters in order to minimize the sum of squared deviations of 6 key data moments from the US housing market and their model counterparts.

(1) We target the ratio of the value of housing to the value of the aggregate endowment to ensure that the average ratio of housing and consumer durables to GDP which is equal to its average value of 1.68 over the 1980-2019 period. This moment is identified by the expenditure share on housing (ϕ).

(2) The average US housing rent-price ratio in the period 1980-2015 is equal to 4.66% according to data by Davis et. al. (2008). This helps identify the annual tax on housing (μ) which has a strong effect on the price-rent ratio of housing.

(3) The average ratio of household debt to GDP is equal to 68.3% in the 1980-2019 period. The probability of switching from 'youth' to 'old age' (γ) is the most important parameter which controls the model's implications for this moment because it determines the size of the borrower (young) population.

(4) The average home ownership rate in the US is equal to 65.6% in the 1980-2019 period. This is matched by the upper bound on the uniform distribution for the rental preference parameter ($\overline{\chi}$).

(5) The net foreign asset position is targeted at the (negative of the) net worth of the Rest of the World vis-a-vis the US. This has been trending down over the past 20 years so we use the average for the 2010-2019 period which is around 35% of GDP. This moment is identified by the probability of death for old households (σ) which determines the size of the population of old savers as well as the size of their savings.

(6) The size of the population aged 20 to 54 is approximately 60% of the total population aged over 20. We use the demographic parameters σ and γ to match this moment.

Whenever we conduct simulations with a stochastic version of the model where the long term level of growth or real interest rates can undergo probabilistic switches, we calibrate the probability of switches to 5% which makes such events a once-in-twenty-years occurrence.[11]

[11] The probability of an interest rate decrease is 1.5% per annum while the probability of an

Finally, we set ω - the adjustment speed with which households in negative equity must go back to positive housing equity - to 0.5. This implies that households eliminate half of the remaining negative equity in each period, taking 5-6 years to get back to zero net worth following an adverse shock.

We can see in Table 2 that despite having more moments than parameters, the model matches the data moments very well.

6 Housing Cycles and Welfare

Having built our OLG model with housing, we now use it to analyse the macroeconomic and welfare impact of changes in fundamentals. We focus on the role of long-term real interest rates and growth rates and emphasize the way a lower $R-G$ increases the volatility of housing prices and the redistribution between old and young, home owners and renters.

Figure 4 below implements a 0.5 percent permanent increase in the world real interest rate (the red line) and compares it to a 0.5 percent permanent reduction in the growth rate of the per capita endowment (the black line). The dashed lines are the levels of the new steady state.

increase is 3.5%. This generates an expected housing price equal to the one in stochastic steady state and therefore generates no capital gains for leveraged agents. This assumption is essential to achieve aggregation in the stochastic steady state.

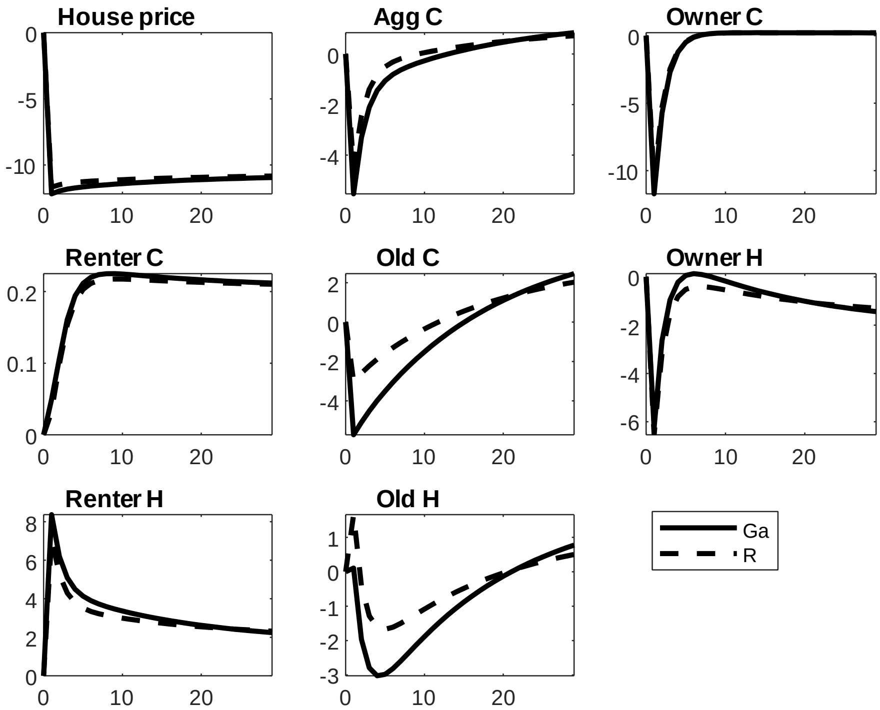

Figure 4: Comparing a 0.5% R^* increase and 0.5% G^A decrease

Note: The dashed line is the IRF to a 0.5% permanent increase in the world real interest rate in the baseline model. The solid line is the IRF to a permanent 0.5% fall in the endowment growth rate in the baseline model. All IRFs are expressed as a percentage change from the baseline steady state. Baseline parameter values are in Table 1.

The key message from the graph is that the real effects of the two shocks are rather similar after detrending. House prices fall substantially (around 10%), and this leads to a significant decline in the consumption of leveraged owners (down by around 10%). Older home owners experience a smaller fall in consumption (3-5%) since they hold housing without leverage. The consumption of renters is mostly unaffected since they are not exposed to movements in housing prices.

The interest/growth rate shock also leads to a substantial redistribution of housing usage. The housing used by those exposed to housing prices falls (the old and the young home-owners) while the housing consumption of renters goes up, reflecting the decline in the housing rental price.

Table 3 below shows the welfare impact of the two shocks on the three groups we are focusing on. The identity of the three groups is defined before the shock occurs. All welfare measures in the table are 'consumption equivalents': they show the permanent increase in consumer expenditure that would deliver the same welfare increase to the household as the interest and growth rate shocks we consider. The measures fully take the transition following the shock into account.

The first column shows the impact of higher interest rates. Young home owners with leverage lose significantly from the increase in real interest rates. The welfare loss equals 0.52% permanent reduction of consumption. They suffer from the leveraged loss of net worth associated with a lower housing price. Old home owners gain from the increase in interest income on their saving but lose from the fall in housing prices. Overall, the impact of capital losses dominates, leading to a small decline in welfare. Renters' welfare increases significantly (equivalent to a 0.97% permanent increase in consumption) due to the decline in rents following the interest rate increase. This allows renters to consume more housing at a time of their life when they are credit constrained and would like to increase their expenditure beyond current income. The renters also gain from a lower housing price because they can buy a house cheaper when they get old in future.

The second column of Table 3 shows the welfare impact of a decline in the per capita endowment growth rate. Households lose much more heavily from this shock compared to the interest rate increase. This is because the lower growth rate leads to a fall in lifetime resources rather than merely representing a change in the relative price of future consumption.

The ranking of the welfare impacts is the same as for the real interest rate increase. Young home owners lose the most (equivalent of 5.47% permanent decrease in consumption) due to leverage. Old home owners' welfare falls as the housing price falls because they are net sellers of housing. But the welfare decline is smaller compared to young home-owners because they have a shorter expected life-time ahead of them (so human wealth is less important) and they are not leveraged (so financial wealth is less sensitive to housing prices). In comparison to home-owners, renters lose less reflecting the reduction of current rents as well as the fact that they are future house buyers and lower prices benefit them. Still young renters loose significantly because their permanent income falls substantially.

Table 3: Welfare impact of shocks

Shock	R	G^A
Old	-0.10	-5.35
Young owners	-0.52	-5.47
Young renters	0.97	-3.95

Note: The table computes the percentage change in steady state expenditure for different groups (Old, Young owners, Young renters) which are equivalent to each shock in terms of their welfare impact in the baseline model. The R column examines a permanent 0.5% increase in the world real interest rate while the G^A column examines a permanent 0.5% reduction in the endowment growth rate. Baseline parameter values are in Table 1.

Just as we argued in section 2, the impact of changes in long-term interest and growth rates depends crucially on $R - G$. In Figure 5 below we compare the impact of a 0.5 percent increase in real interest rates under the baseline calibration and under an alternative calibration where $R - G$ is lower by 1 percent because the long term real interest rate is 3.58% instead of 4.58% as in the baseline calibration. We can see that the fall in the housing price is greater when the initial $R - G$ is lower. The larger movement of the housing price means that leveraged owners are more adversely affected while renters increase their consumption of housing and non-durables to a greater extent.

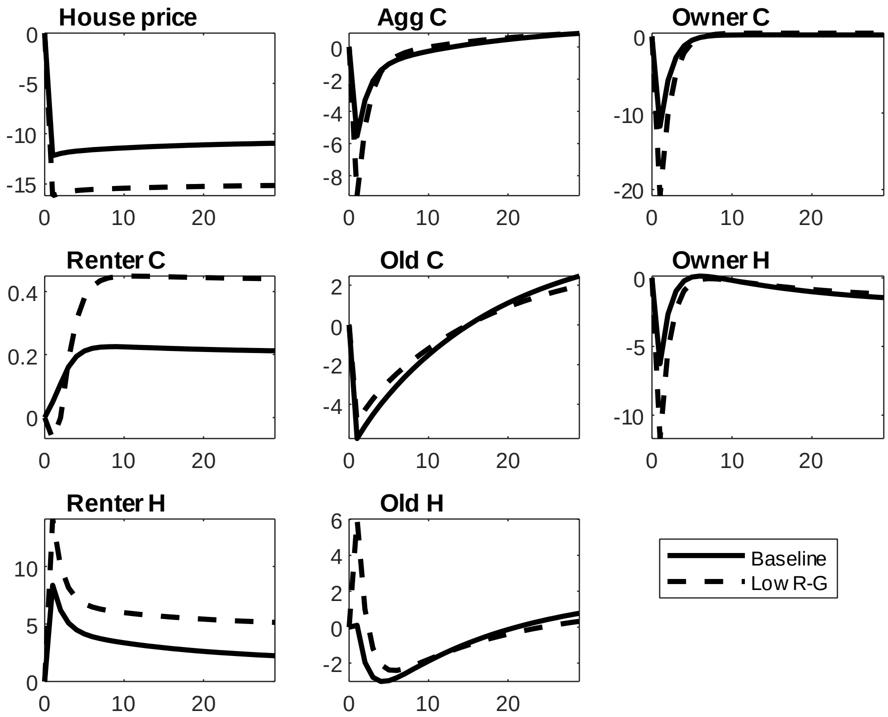

Figure 5: The impact of a 0.5% R increase under

the baseline and under a smaller value for $R-G$

Note: The solid line is the IRF to a 0.5% permanent increase in the world real interest rate under the baseline calibration. The dashed line is the IRF to the same permanent 0.5% increase in the world interest rate in the model in which the world real interest rate is 1pp lower than the baseline value of 4.58% while the endowment growth rate (and all other parameter values) are at their baseline values (see Table 1). All IRFs are expressed as a percentage change from the baseline steady state.

Table 4 below displays the welfare effect of a 0.5 percent interest rate increase as well as a 0.5 percent growth rate decrease at the lower value of $R-G$. The table shows that the level of $R-G$ also matters for the welfare effect of the shock. Compared to Table 3 above, there are a number of key differences. Old home-owners now experience a larger decline in welfare compared to young ones under both shocks because, as net sellers of housing, they lose more from the fall of housing value. In contrast, young home-owners remain net buyers of housing (due to their growing endowment) and this helps to temper the welfare loss due to the net worth decline. Still young home-owners are also more adversely affected compared to the baseline case because the larger fall in the housing price hurts their net worth to a greater extent.

Table 4: Welfare impact of shocks with a smaller $R-G$

Shock	R	G^A
Old	-1.41	-7.35
Young owners	-0.84	-6.38
Young renters	1.70	-3.93

Note: The table computes the percentage change in steady state expenditure for different groups (Old, Young owners, Young renters) which are equivalent to each shock in terms of their welfare impact in the model in which the world real interest rate is 1pp lower than its 4.58% baseline value while the endowment growth rate (and all other parameters) are at their baseline values (see Table 1). The R column examines a permanent 0.5% increase in the world real interest rate while the G^A column examines a permanent 0.5% reduction in the endowment growth rate.

The results in this section are largely unchanged even when the arrival of shocks to the interest rate and/or growth rate is anticipated. In Appendix D we solve for the stochastic steady state of our economy under the assumption that households know that permanent shocks to the world real interest rate or the per capita endowment growth rate could hit with a certain probability. The differences between stochastic and deterministic steady states are discussed in more detailed in Appendix D. Intuitively, the anticipation of shocks leads to some precautionary behaviour. The home-ownership rate declines and the remaining young home-owners choose slightly smaller housing. The old save more and reduce the share of housing in their portfolios. However, the qualitative nature of the way the economy responds to R and G^A

shocks does not change very much. The welfare impact is also similar. Hence, we continue with the analysis of policy measures in the next section using the simpler deterministic version of the model.

7 Managing and mismanaging the Housing Cycle

We saw in the previous section that the long lived nature of housing as an asset makes it highly vulnerable to shifts in expected growth and real interest rates, especially when $R-G$ is low. In turn, the fluctuations in housing values create large-scale redistribution between borrowers and lenders, with young mortgagors particularly vulnerable to housing price fluctuations. In this Section, we use our model to evaluate a number of housing policies that are either widely used, or that have been proposed in order to deal with the housing cycle.

7.1 State contingent debt

We start by analyzing the role of the key missing market in our economy - the market for state contingent mortgage debt. We do this by comparing the reaction to a 0.5 percent permanent real interest rate increase in an economy with simple and with state contingent mortgage debt (respectively, the black and the red line). This is shown in Figure 6 below.

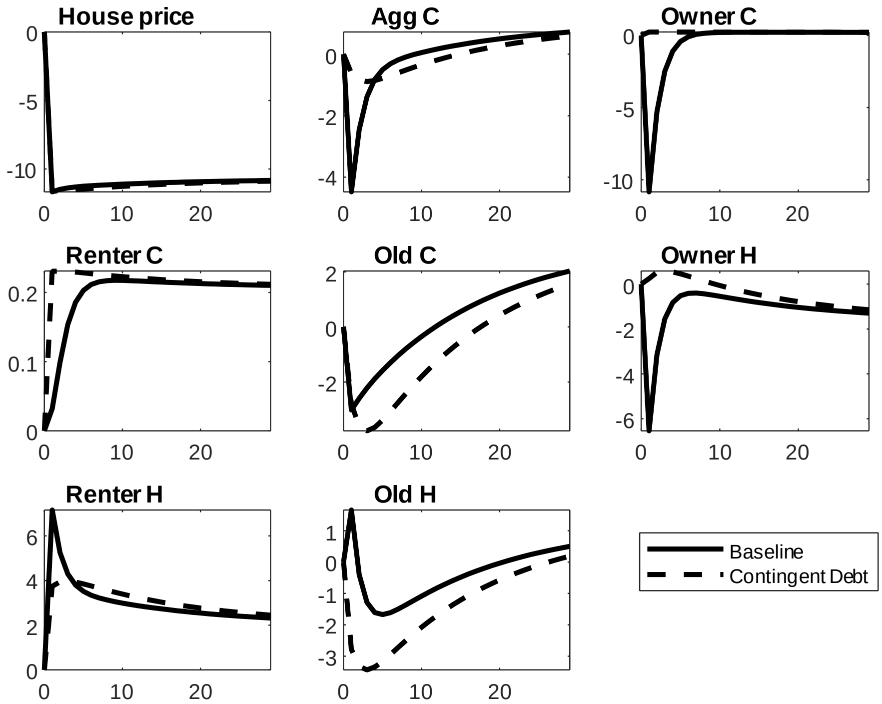

Figure 6: 0.5% permanent R increase under simple and contingent debt

Note: The solid line is the IRF to a 0.5% permanent increase in the world real interest rate in the baseline model with uncontingent debt. The dashed line is the IRF to the same permanent 0.5% increase in the world real interest rate in the model in which debt repayments are contingent upon the housing price realization. All IRFs are expressed as a percentage change from the baseline steady state. All parameter values are at their baseline values (see Table 1).

We can see straight away that state contingent debt mostly shields the consumption of borrower households from the effects of the shock. Their housing usage actually increases under state contingent debt due to the decline in the user cost of housing following the shock. The consumption and housing usage of the old decline by more under state contingent debt because they absorb all the losses from lower housing prices and they are net sellers of houses in future.[12]

Despite the fact that fully state contingent contracts do not exist, there are practical alternatives that offer net worth protection to young credit constrained people.[13] Notice in Figure 6 that the evolution of renters' consumption and housing usage under simple debt contracts is extremely similar to that of leveraged home owners under state contingent debt markets. Tenants are shielded from housing price fluctuations and their net worth and consumption do not move in response to shocks. In this sense, the rental market mimics the effects of state contingent debt on net worth.

The main reason why the rental market is not a full substitute for state contingent debt or a housing equity market is because it cannot deal with different households' preference for owning versus renting. In our model, some households have a strong preference for owning and the rental market would not be useful for them. Nevertheless, our model suggests that, with a healthy rental market, at least some of the credit constrained young households choose to rent and become shielded from the consumption volatility induced by housing price fluctuations.

7.2 The distortionary effect of home ownership subsidies

Despite the stabilizing risk-sharing properties of the rental market, US public policy encourages home ownership via the tax system in three main ways. First, the interest on debt secured by the household's main residence is deductible for income tax. Second, the imputed rental income from owned houses is untaxed. Third, owner occupied dwellings are exempt from capital gains tax. Residential property held for rental purposes does not enjoy any of these tax advantages although depreciation and property taxes can be deducted from landlords' tax bills. Sinai and Gyourko (2004) and Poterba and Sinai (2008) compute the size of the subsidies that are handed out to home owners. Using the 2004 Survey of Consumer Finances, they

[12] If young people can adjust labor supply more than old people, then young people would absorb some fraction of the loses from lower housing prices.

[13] Equity contracts could implement a considerable amount of risk sharing but, despite efforts to introduce them, these are not yet widespread in the housing market.

find that for the average household age and income, implicit subsidies through the tax system reduce the user cost of owner-occupied housing by around 29%.

In line with this evidence, we set the home ownership subsidy at 1.35% of housing value in the baseline calibration, implying a 29% decline in the user cost of housing for owner occupied relative to landlord-owned rental properties. The effect of this policy on the model's steady state is to increase the home-ownership rate and boost the price of housing leading to higher leverage among mortgagors.

In addition to changing the model's steady state, the home ownership subsidy also alters the way it reacts to shocks. Figure 7 below compares the way the economy without a home ownership subsidy and with the baseline 1.35% home ownership subsidy react to a permanent 0.5 percent increase in the world real interest rate R.

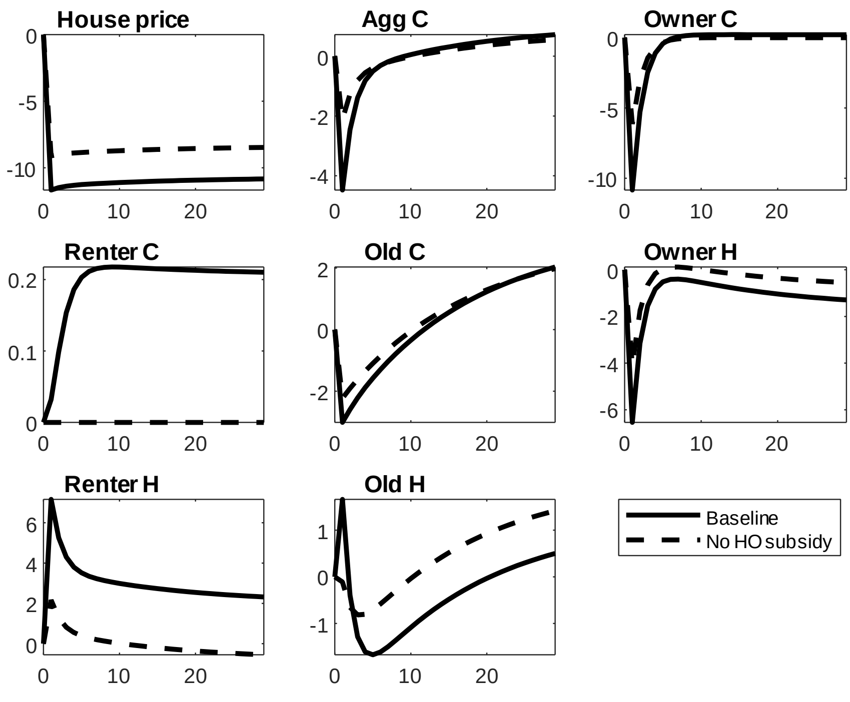

Figure 7: 0.5% permanent R increase with and without

a home ownership subsidy

Note: The solid line is the IRF to a 0.5% permanent increase in the world real interest rate in the baseline model with a home-ownership subsidy which is consistent with the estimates in Poterba and Sinai (2008). The dashed line is the IRF to the same permanent 0.5% increase in the world real interest rate without a home ownership subsidy. All IRFs are expressed as a percentage change from the baseline steady state. All other parameter values are at their baseline values (see Table 1).

We can see straight away that the economy in which home ownership is subsidized experiences larger housing price fluctuations and more consumption volatility. This happens for three main reasons. First, home ownership subsidies reduce the effective cost of owning a home and this has a similar effect on housing prices to a reduction in $R - G$. The higher price-to-rent ratio makes the housing price more responsive to shocks to R or G. Second, the higher home ownership rate means that there are more leveraged households who experience a hit to their net worth when housing prices fall. Third, the subsidy allows young borrowers to become even more leveraged. All these factors make aggregate consumption more volatile.

7.3 Policy options

So far we have seen that the combination of volatile housing prices, non-contingent debt and high leverage for young home-owners leads to large-scale redistribution and consumption volatility following shocks to long term real interest rates and income growth rates. This is compounded by policies that encourage home ownership and boost the price-to-rent ratio via subsidies to owner occupation.

How to reduce the vulnerability of the economy with non-contingent debt and home ownership subsidy? In this section we consider two policy options. One is the reduction of home ownership subsidies, the other is the imposition of constraints on borrowers' Loan-to-value (LTV) ratios. Here we evaluate these policies taking into account the transition from the equilibrium of the baseline economy (with home ownership subsidies and no leverage restrictions) to an equilibrium with reduced subsidies or reduced household leverage.

Figure 8 below compares the impact of reducing the home ownership subsidy from 1.35% to 0.8% with that of an LTV cap. The latter policy is implemented in the model via a borrowing tax which is imposed on young borrowers but its proceeds are then distributed lump-sum back to the young home owners.[14] The tax is 1.3% and the size of both policy interventions is chosen so as to limit the fall in the housing price to 10%.

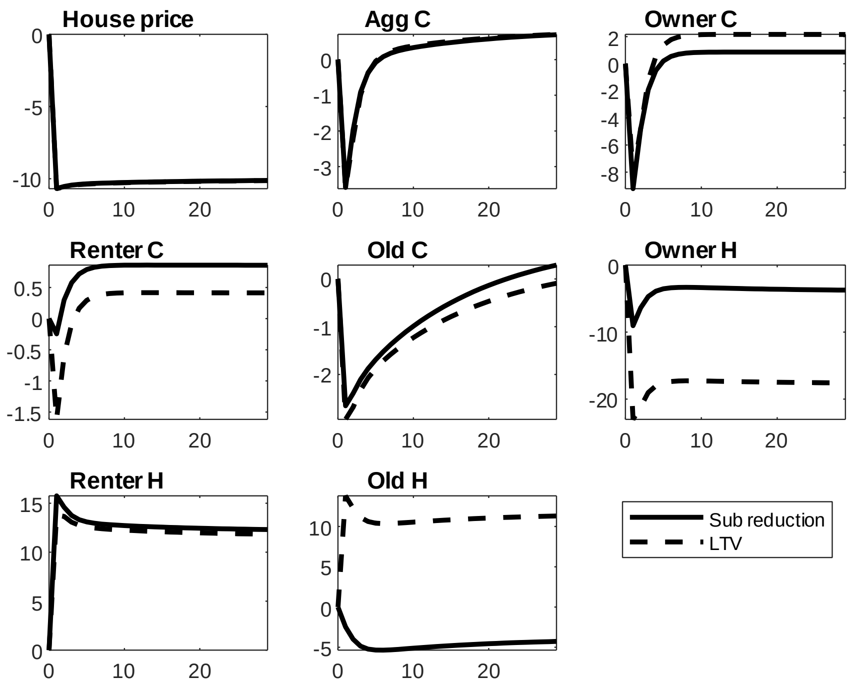

Figure 8: Comparing the impact of a home-ownership subsidy reduction and a LTV cap imposition

Note: The solid line is the IRF to a permanent reduction in the home ownership subsidy in the baseline model. The dashed line is the IRF to a permanent LTV cap for young borrowers. For comparability, both policy interventions have been chosen so as to generate a decline of

[14]The tax (which is rebated lump-sum back to the individual household) produces the same effect as an LTV constraint because it keeps the resources available to the household unchanged while inducing it to choose a lower level of leverage.

housing prices of apprxomately 10%. All IRFs are expressed as a percentage change from the baseline steady state. All other parameter values are at their baseline values (see Table 1).

In many respects the two policies have similar effects. Housing prices decline, non-durable consumption falls for all households. The home ownership rate falls and housing usage rises for the renters who absorb a lot of the housing vacated by owners. Consumption also falls for renters despite the fact that their net worth is insulated from housing price fluctuations. This is due to the fact that the removal of the subsidy to home-owners reduces rents and leads renters to substitute from non-durables and into housing.

The impact of the two policy options differs strongly in one respect. LTV caps curtail severely the housing usage of young owners while boosting the housing usage of the old. In contrast, the removal of the subsidy is more 'democratic' in the sense that it crowds out all owners (young and old).

Table 5 below compares the welfare impact of removing home ownership subsidies (the first column) and imposing an LTV cap (the second column). Several things stand out from the table. First, only renters benefit from either of these two policies. The renters gain from a lower housing price because they enjoy lower rents and can buy housing cheaper in future.[15]

Secondly, home owners suffer differently from these two policies. Old home-owners lose more from the removal of home-ownership subsidies, equivalent of 2.74% reduction of permanent consumption. Young leveraged home-owners are hurt more by an LTV cap, equivalent of 1.45% reduction in permanent consumption. That these two policies create such prominent losers explains why it is so difficult to implement them in practice.

[15] The reason why renters marginally prefer the removal of home-ownership subsidies lies in the interplay of two opposing forces. Renters will become old home-owners in future and would therefore gain from a policy that preserves home ownership subsidies. However, they are currently tax payers and would like a policy that reduces the taxes they have to pay. Which of these effects dominates depends on the probability of becoming old and on renters' discount factor. For our baseline calibration, renters marginally prefer the policy that reduces their taxes today at the cost of lower housing subsidies in future.

Table 5: Welfare impact of policies

Shock	Sub	LTV
Old	-2.74	-2.08
Young owners	-0.35	-1.45
Young renters	1.62	1.34

Note: The table computes the percentage change in steady state expenditure for different groups (Old, Young owners, Young renters) which are equivalent to each shock in terms of their welfare impact in the baseline model (see Table 1 for parameter values). The 'Sub' column examines a permanent reduction in the home ownership subsidy while the 'LTV' column examines the imposition of an LTV cap on young borrowers. For comparability, the size of the two policy interventions has been chosen so as to generate approximately a 10% decline in the housing price.

8 Conclusions

Housing is the ultimate durable good and its value derives mostly from the value of the land (or location) on which the structure is built. Land is also difficult to steal and its value does not depreciate if it is transferred to a different owner. These two features of housing make it extremely good collateral, affording housing buyers very substantial leverage.

However, the long lived nature of housing also make its price highly sensitive to changes in long run real interest rates and growth rates. Hence, housing is a risky asset and its riskiness grows as the difference between long term real interest rates and growth rates diminishes as it has done over the past 20 years. Leveraged young home-owners are particularly vulnerable to housing price fluctuations because their net worth is highly exposed to changes in home prices.

We build a tractable OLG model of housing credit cycles that captures the above narrative and use it to examine the impact of housing market policies. The model produces very large redistributions following unanticipated shocks to long-term interest rates and growth rates. Old savers and renters gain from higher interest rates: the former because they enjoy a higher rate of return on savings and the latter because they gain from lower housing prices that lead to cheaper rents. Leveraged home owners suffer from a prolonged period of negative equity that they have to clear by enduring lower consumption of both housing and non-durables. Lower growth rates hurt all households because they reduce the value of human wealth. Young home owners are especially hurt from a fall in the growth rate due to their leveraged loss on non-human (housing) wealth. The impact from such shocks is magnified in a highly non-linear fashion when the real interest rate is close to the economy's growth rate (as is currently the case).

The redistribution between different groups would be moderated by state contingent debt although in practice such debt does not exist so far. Outside equity for housing is also not used and equity-like products such as 'Shared Appreciation Mortgages' have not proved popular. However, we argue that the rental market has many features that make its aggregate impact similar to that of state-contingent debt or housing equity. In particular, it limits the tendency of negative shocks to redistribute from leveraged and credit-constrained young households towards unconstrained older households. The rental market is therefore a practical way to insulate vulnerable households from fluctuations in housing prices.

Unfortunately, fiscal policy in many countries penalizes the rental market through a policy of home ownership subsidies. We show that home ownership subsidies amplify the impact of shocks on housing prices and increase the number of leveraged young households who are most affected by fluctuations in home values. We examine the welfare effect of removing home ownership subsidies and compare it to the welfare effect of imposing borrowing limits on young home owners. On impact, both policies hurt home owners and benefit renters through their negative impact on housing prices. Young owners are hurt the most by borrowing limits while old households are hurt the most by the reductions in home ownership subsidies. That both policies create very substantial losers explains why the tax treatment of owner-occupied housing is hard to reform and the use of loan-to-value or loan-to-income caps remains politically controversial.

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Appendices

A Data Appendix

(1) GDP (1980 - 2019) - average real GDP growth of 2.6% (calibrate $G^A G^N$) - Data source: NIPA.

(2) Civilian non-institutional population (1980 - 2019) - average growth of 1.1% (calibrate G^N) - Data source: Current Population Survey (Bureau of Labour Statistics).

(3) US housing rent-price ratio (1980-2015) - average of 4.66% (calibrate μ) - Data source: Davis, M., Lehnert, A., and Martin, R., (2008), "The Rent-Price Ratio for the Aggregate Stock of Owner-Occupied Housing," Review of Income and Wealth, vol. 54 (2), p. 279-284; data located at Land and Property Values in the U.S., Lincoln Institute of Land Policy.

(4) Value of housing + durables to GDP (1980 - 2019) - average of 1.68 - Data source: US Flow of Funds (Balance Sheets of Households and Non-profit Organizations - Real estate at market value divided by annual GDP).

(5) Real mortgage interest rate (1980 - 2019) - average of 4.58% (30 year mortgage rate minus the current rate of inflation) - Data source: Primary Mortgage Market Survey (Freddie Mac).

(6) Life cycle income process (1): ratio of income aged 50 to income aged 25: 1.4 (Gourinchas and Parker (2002).

(7) Life cycle income process (2): ratio of income aged 65 to income aged 25: 1.2 (Gourinchas and Parker (2002).

(8) Demographic mix - Population aged 20 to 54/(Population aged over 20) = 0.6 (2019 value) - Data source: Current Population Survey (Bureau of Labour Statistics).

(9) Home ownership rate for the US (1980 - 2019) - 65.6%.

(10) Mortgage debt to GDP (1980-2019) - average since 1980 = 68.3% - Data source: US Flow of Funds (Balance Sheets of Households and Non-profit Organizations).

(11) Net foreign liabilities to GDP (2010-2019) - average net foreign liabilities of approximately 35% of annual GDP Data source: US Flow of Funds (Balance Sheets of Foreign Sector: Net worth divided by annual GDP).

B Population and Aggregate Income Dynamics

Let N_t^y and N_t^o be population of young and old households. Superscript y denotes young and o old. The population of young and old evolve as follows:

$$\begin{aligned} N_t^y &= \gamma N_{t-1}^y + (G_N - \gamma) N_{t-1}^y = G_N N_{t-1}^y \\ N_t^o &= \sigma N_{t-1}^o + (1 - \gamma) N_{t-1}^y. \end{aligned}$$

The number of young households grows at rate G_N and consists of γN_{t-1}^y households who continue to be young and $(G_N - \gamma) N_{t-1}^y$ newborns.

Because the population growth rate is constant, the ratio of the number of young and old households is constant at

$$N_t^y : N_t^o = G_N - \sigma : 1 - \gamma.$$

We normalize the efficiency unit labor of newborns to be unity as $x_t = 1$. Let g^y and g^o be the age-related labor productivity growth rate (plus one) of young and old agents at date t:

$$\begin{aligned} \frac{x_t}{x_{t-1}} &= g^y > 1, \text{ when young at date } t-1 \\ \frac{x_t}{x_{t-1}} &= g^o < 1, \text{ when old at } t-1 \text{ and survives at } t. \end{aligned}$$

Let X_t^y and X_t^o be the aggregate age-related (efficiency unit) labor of young and old agents. These are determined as follows:

$$\begin{aligned} X_t^y &= \gamma g^y X_{t-1}^y + 1 \cdot (G_N - \gamma) N_{t-1}^y \\ X_t^o &= \sigma g^o X_{t-1}^o + (1-\gamma) g^y X_{t-1}^y. \end{aligned}$$

Age-related labour productivity per capita $x_t^y = \frac{X_t^y}{N_t^y}$ and $x_t^o = \frac{X_t^o}{N_t^o}$ are given by:

$$\begin{aligned} G_{Nt} x_t^y &= \gamma g^y x_{t-1}^y + G_{Nt} - \gamma \\ G_{Nt} N_{t-1}^o x_t^o &= \sigma g^o N_{t-1}^o x_{t-1}^o + (1-\gamma) g^y N_{t-1}^y x_{t-1}^y. \end{aligned}$$

Because the age-related labor productivity growth rates and population growth rate are constant, the age-related labor of young and old grow at the same rate with population grow rate, $\frac{X_t^y}{X_{t-1}^y} = \frac{X_t^o}{X_{t-1}^o} = G_N$, and the average age-related labor productivity of young is constant as

$$x^y = \frac{X_t^y}{N_t^y} = \frac{G_N - \gamma}{G_N - \gamma g^y} > 1.$$

The average age-related labor productivity of old $x^o = \frac{X_t^o}{N_t^o}$ is also constant and satisfy

$$(G_N - \sigma g^o) N_{t-1}^o x^o = (1-\gamma) g^y N_{t-1}^y x^y,$$

or

$$x^o = \frac{G_N - \sigma}{G_N - \sigma g^o} g^y x^y < g^y x^y.$$

Aggregate efficiency unit labor is

$$X_t = X_t^y + X_t^o,$$

and aggregate income is

$$Y_t = A_t X_t$$

where A_t is the aggregate labor productivity which is equal to the productivity of the new born and grows at

$$\frac{A_t}{A_{t-1}} = G_{At}.$$

C Perfect Foresight Equilibrium: The Consumption-Savings Problem of the Old

After solving the static choice between housing usage and non-durable consumption, the problem of the old households can be reduced to the problem of choosing current expenditure (e_t) versus savings (a_{t+1}).

$$V_t^o = \max_{e_t, a_{t+1}} \left[\left(u_t^o\right)^{\frac{\eta-1}{\eta}} + \beta\sigma \left(E_t V_{t+1}^o\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{30}$$

where

$$u_t^o = \frac{e_t}{\left(r_t^h\right)^{1-\phi}}.$$

The maximization is subject to the budget constraint:

$$e_t + \frac{\sigma a_{t+1}}{R_t} = \left(1 - \tau_t\right) w_t x_t + a_t. \tag{31}$$

The envelope theorem implies:

$$\frac{\partial V_t^o}{\partial a_t} = \frac{1}{\left(r_t^h\right)^{1-\phi}} \left(u_t^o\right)^{\frac{-1}{\eta}} V_t^{\frac{1}{\eta}}.$$

The first order condition with respect to savings (a_{t+1}) is given by:

$$\begin{aligned} 0 &= -\frac{\sigma}{R_t}\frac{1}{\left(r_t^h\right)^{1-\phi}}\left(u_t^o\right)^{\frac{-1}{\eta}}\left(V_t^o\right)^{\frac{1}{\eta}}+\beta\sigma\left(E_tV_{t+1}^o\right)^{\frac{-1}{\eta}}\left(V_t^o\right)^{\frac{1}{\eta}}\frac{\partial V_{t+1}^o}{\partial a_{t+1}} \qquad (32) \\ &= -\frac{1}{\left(r_t^h\right)^{1-\phi}}\left(u_t^o\right)^{\frac{-1}{\eta}}+\beta R_t\left(E_tV_{t+1}^o\right)^{\frac{-1}{\eta}}\frac{1}{\left(r_{t+1}^h\right)^{1-\phi}}\left(u_{t+1}^o\right)^{\frac{-1}{\eta}}\left(E_tV_{t+1}^o\right)^{\frac{1}{\eta}} \\ &= -\left(u_{t+1}^o\right)^{\frac{1}{\eta}}+\beta R_t\left(\frac{r_t^h}{r_{t+1}^h}\right)^{1-\phi}\left(u_t^o\right)^{\frac{1}{\eta}}. \end{aligned}$$

This implies equation (15) in the main text.

We now verify the guess that expenditure is proportional to total (human and non-human) wealth

$$e_t = \nu_t^o W_t^o,$$

where

$$\begin{aligned} W_t^o &= a_t + W_t^{ho}x_t \\ W_t^{ho} &= \left(1-\tau_t\right)w_t + \frac{\sigma}{R_t}g^oW_{t+1}^{ho}. \end{aligned}$$

Under our guess the budget constraint becomes

$$\nu_t^oW_t^o + \frac{\sigma}{R_t}W_{t+1}^o = W_t^o, \text{ or}$$

$$\frac{W_{t+1}^o}{W_t^o} = \frac{R_t}{\sigma}\left(1-\nu_t^o\right). \qquad (33)$$

Since

$$u_t^o = \frac{e_t}{\left(r_t^h\right)^{1-\phi}},$$

we know that the guess above implies that:

$$u_t^o = \frac{\nu_t^o W_t^o}{\left(r_t^h\right)^{1-\phi}}.$$

Substituting into equation (15), we have:

$$\frac{\nu_{t+1}^o W_{t+1}^o}{\left(r_{t+1}^h\right)^{1-\phi}} = \left(\beta R_t^u\right)^\eta \frac{\nu_t^o W_t^o}{\left(r_t^h\right)^{1-\phi}}. \tag{34}$$

Putting together this and the budget constraint and rearranging we get implies equation (18):

$$\frac{1}{\nu_t^o} = 1 + \sigma\beta^\eta \left(R_t^u\right)^{\eta-1} \frac{1}{\nu_{t+1}^o}. \tag{35}$$

This confirms the initial guess.

Finally, we verify the guess that the value function is proportional to current utility

$$V_t^o = \Delta_t^o u_t^o.$$

Then because

$$V_t^o = \left[\left(u_t^o\right)^{\frac{\eta-1}{\eta}} + \beta\sigma\left(E_t V_{t+1}^o\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

we have

$$\begin{aligned}
\left(\Delta_t^o u_t^o\right)^{\frac{\eta-1}{\eta}} &= \left(u_t^o\right)^{\frac{\eta-1}{\eta}} + \beta\sigma\left(\Delta_{t+1}^o u_{t+1}^o\right)^{\frac{\eta-1}{\eta}}, \text{ or} \\
\left(\Delta_t^o\right)^{\frac{\eta-1}{\eta}} &= 1 + \beta\sigma\left(\Delta_{t+1}^o \frac{u_{t+1}^o}{u_t^o}\right)^{\frac{\eta-1}{\eta}}, \text{ or} \\
\left(\Delta_t^o\right)^{\frac{\eta-1}{\eta}} &= 1 + \beta\sigma\left(\Delta_{t+1}^o \left(\beta R_t^u\right)^\eta\right)^{\frac{\eta-1}{\eta}}, \text{ or} \\
\left(\Delta_t^o\right)^{\frac{\eta-1}{\eta}} &= 1 + \sigma\beta^\eta \left(R_t^u\right)^{\eta-1}\left(\Delta_{t+1}^o\right)^{\frac{\eta-1}{\eta}}.
\end{aligned}$$

This confirms the initial guess about the value function. Comparing the above equation with (18) we can see that:

$$\frac{1}{\nu_t^o} = (\Delta_t^o)^{\frac{\eta-1}{\eta}}$$

This also confirms the result in the main text that

$$\Delta_t^o = (\nu_t^o)^{\frac{\eta}{1-\eta}} . \tag{36}$$

D Online Appendix: A stochastic version of the model

D.1 Main results

The analysis in the main body of the paper was conducted under the assumption that the shock occurred in a fully unanticipated 'one-time' shock fashion. Here we consider how our analysis changes if households know that a shock may occur with a certain probability.

We introduce uncertainty in a simple way: we assume that at some point in future a one time shock will occur with a known probability. Once the shock occurs it is permanent and no shocks will ever hit from that point onwards. We assume that the shock can take two values - a positive shock (lower world real interest rate or a higher growth rate which occurs with a 1.5% probability) and a negative one (higher real interest rate or a lower growth rate which occurs with a 3.5% probability). Thus the arrival rate of a permanent shift is 5% per annum. Our analysis focuses on the stochastic steady state - the point to which the economy would converge if no shocks have occurred for a long time but households are aware that shocks will occur with some probability in the future.

In this section, we focus on explaining the main ways in which the model changes as a result of allowing for uncertainty and on accounting for the quantitative importance of uncertainty for the model. The full model equations are derived in section D.2 and the solution method is described in section D.4.

The existence of uncertainty has an effect on all households but especially on leveraged home owners. A leveraged housing purchase is especially risky when fundamentals may change, triggering a big fall in housing values. Despite linear period utility, households are risk averse in terms of their value functions: they value wealth more in states when consumption is low and is expected to grow in future. In addition, they value wealth in states of the world when the user cost of housing is cheap.

This is precisely the time when a negative shock hits.

Table 6 below compares the values of several key variables in the stochastic and deterministic equilibria of our economy. The table contains several noteworthy and surprising results.

Table 6: Comparing the Deterministic and Stochastic Steady States

	h^h	h^o	c^h	c^o	qh^o-B^o	$\frac{qh^o}{qh^o\text{-}B^o}$	ψ	$E(R^r)$	q	HO
Det. SS	1.37	0.77	66.8	37.7	79.5	1.23	0.938	4.58	97.6	69.1
Stoch. SS	1.35	0.78	67.0	37.5	81.9	1.20	0.939	4.66	98.0	68.7
% Diff	-1.44	1.27	0.21	0.31	3.04	-0.03	0.001	0.08	0.43	-0.40

Note: Lower case housing usage h^h, h^o and consumption c^h, c^o variables are in per capita terms. Upper case variables: (B^o) is the aggregate debt owned by the old. $qh^o - B^o$ is total assets of the old. The '% Diff ' row is expressed in per cent with the exception of the share of expenditure devoted to non-durables for the young home-owners (ψ), the Home Ownership Rate (HO), the share of housing in the portfolio of the old ($\frac{qh^o}{qh^o - B^o}$) and the expected rate of return to owning rental housing ($E(R^r)$): Those three variables are a simple difference between the Deterministic and Stochastic steady states.

First, we can see that, in the stochastic steady state, there is 'precautionary housing under-consumption'. Young home owners allocate a little less of their expenditure to housing than the share of housing services in utility. This can be seen in the column showing the value of ψ (the share of expenditure allocated to non-durables) which increases very marginally in the stochastic steady state from its deterministic steady state value of ϕ. This is because home ownership carries the risk that a big fall in housing prices will bring about a substantial decline in net worth and a prolonged period of depressed consumption. This risk premium in housing manifests itself in a somewhat lower share of housing in total expenditure although this effect is quantitatively very small. As a result, the per capita consumption of housing services by leveraged home owners declines (h^h).

Second, the stochastic steady state features a lower home ownership rate than in the deterministic steady state. The risks inherent in leveraged home ownership lead to a fall in the value of being a home owner. This leads to a shift in the marginal young household which is indifferent between owning and renting. This effect, which we term 'precautionary renting' brings about a 0.4 pp decline in the home ownership rate.

Somewhat surprisingly, housing prices actually increase by a small amount (0.4 %). This is despite the fact that the stochastic steady state features a risk premium on housing as evidenced by the expected return on holding rental housing of 4.66% which is 8bps higher than the world risk free real interest rate.

Going a little deeper into the balance sheet of the old (who are the households who price assets in the economy) we see that their total wealth ($q_t h_t^o - B_t^o$) is around 3% higher in the stochastic steady state compared to the deterministic one. However, the share of housing in the portfolio of the old ($q_t h_t^o / (q_t h_t^o - B_t^o)$) declines to 1.2 in the stochastic steady state compared to 1.23 in the deterministic one. In the end, the old respond to the greater uncertainty in an intuitive way: they increase overall saving but decrease the fraction of wealth devoted to the risky asset. The impact of higher saving dominates and pushes up the price of housing slightly.

The final point we want to emphasize in this section is that the anticipation of shocks to housing prices does not change the behavior of the economy very much. Figure 9 below shows how the IRF from an increase in R^* changes due to the precautionary behavior discussed above. The figure shows the IRF from two different starting points - the deterministic steady state (red solid line) and the stochastic steady state (black line). The IRFs are very similar both qualitatively and quantitatively.

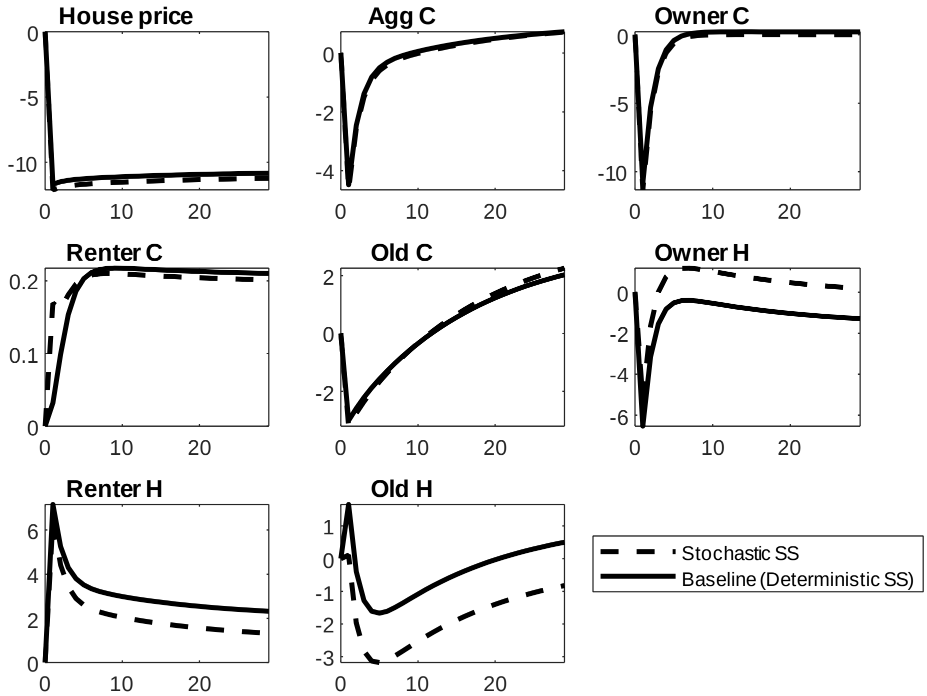

Figure 9: Comparing the impact of a 0.5% increase in the world real interest rate: stochastic vs deterministic steady state

Note: The solid line is the IRF to a 0.5% permanent increase in the world real interest rate starting from the deterministic steady state. The dashed line is the IRF to the same permanent 0.5% increase in the world real interest rate starting from the stochastic steady state in which agents anticipate that the world real interest rate may permanently increase or decrease by 0.5% with a 5% probability. All IRFs are expressed as a percentage change from the respective baseline steady states (determinstic or stochastic). All parameter values are at their baseline values (see Table 1).

Table 7 below also shows the welfare impact of the shocks to the long term real interest rate and growth rate starting from the stochastic steady state. We see that the welfare losses from the shock are similar to those suffered under an unexpected shock (Table 3 in the main text). All groups lose from a fall in the long-term endowment growth rate and young owners lose significantly from an increase in the long-term real interest rate.

Table 7: Welfare impact of shocks

Shock	R	G^A
Old	-0.11	-5.36
Young owners	-0.52	-5.47
Young renters	1.66	-3.12

Note: The table computes the percentage change in steady state expenditure for different groups (Old, Young owners, Young renters) which are equivalent to each shock in terms of their welfare impact in the baseline model. The R column examines a permanent 0.5% increase in the world real interest rate starting from the stochastic steady state while the G^A column examines a permanent 0.5% reduction in the endowment growth rate starting from the stochastic steady state. Baseline parameter values are in Table 1.

D.2 Individual Household Problem

In this section we outline the equations that define the stochastic equilibrium of the economy.

D.2.1 Set-up

The preference of young and old households are given by

$$V_t^y = \left\{ (u_t^y)^{1-\frac{1}{\eta}} + \beta[\gamma E_t V_{t+1}^y + (1-\gamma) E_t V_{t+1}^o]^{1-\frac{1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}, \tag{37}$$

$$V_t^o = \left[(u_t^o)^{\frac{\eta-1}{\eta}} + \beta\sigma \left(E_t V_{t+1}^o \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{38}$$

The period utility is

$$u_t^i = \left(\frac{c_t}{\phi}\right)^{\phi} \left(\frac{h_t}{1-\phi}\right)^{1-\phi}$$

for young and old home owners, $i = h$ and o, and

$$u_t^r = \left(\frac{c_t}{\phi}\right)^{\phi} \left(\frac{\chi h_t}{1-\phi}\right)^{1-\phi}$$

for a young renter of type χ where χ is distributed uniformly on $[0, 2]$.

The budget constraint of an old home owner is

$$c_t + (1 + \mu_t - \upsilon_t) Q_t h_t + \left[(1 + \mu_t) Q_t - r_t^r\right] s_t - \frac{b_t}{R_t} = (1 - \tau_t) w_t x_t + a_t, \tag{39}$$

where

$$a_t = \frac{1}{\sigma} \left[Q_t (h_{t-1} + s_{t-1}) - b_{t-1}\right].$$

is the household's net financial wealth (or net worth).

The budget constraint of a young renter is

$$c_t + r_t^r h_t - \frac{b_t}{R_t} = (1 - \tau_t) w_t x_t - b_{t-1},$$

and the borrowing constraint is

$$b_t \leq 0.$$

The budget constraint of a young home owner is

$$c_t + (1 + \mu_t - \upsilon_t) Q_t h_t - \frac{b_t}{R_t} = (1 - \tau_t) w_t x_t + a_t.$$

where yet again $a_t = Q_t h_{t-1} - b_{t-1}$ is net worth. The household also faces a borrowing constraint:

$$E_t \left(a_{t+1}\right) \geq \omega a_t,$$

where $\omega \in [0,1)$ is a parameter that governs the speed with which the household must adjust its balance sheet following a stochastic regime shift. It is a simple and tractable way to capture long term debt.

We can rewrite the budget constraint as follows:

$$c_t + r_t^h h_t + \frac{E_t a_{t+1}}{R_t} = (1-\tau_t)\, w_t x_t + a_t,$$

where

$$r_t^h = (1+\mu_t - \upsilon_t)Q_t - \frac{E_t Q_{t+1}}{R_t}. \tag{40}$$

is the required downpayment on a housing purchase when the borrowing constraint is binding.

D.2.2 Consumption and saving choice of the old household

Defining λ_t as the Lagrangian multiplier of the date-t budget constraint, the first order condition for consumption c_t is

$$\begin{aligned}\lambda_t &= \phi \frac{u_t^o}{c_t}(u_t^o)^{-\frac{1}{\eta}}(V_t^o)^{\frac{1}{\eta}} \\ &= \left(\frac{\phi}{1-\phi}\frac{h_t}{c_t}\right)^{1-\phi}(\Delta_t^o)^{\frac{1}{\eta}},\end{aligned}$$

where

$$\Delta_t^i = V_t^i / u_t^i$$

is the ratio of value to utility of type i household for $i = o, h, r$. The first order condition for the home ownership h_t is

$$(1+\mu_t - \upsilon_t)Q_t\lambda_t = (1-\phi)\frac{u_t^o}{h_t}(u_t^o)^{-\frac{1}{\eta}}(V_t^o)^{\frac{1}{\eta}} + \beta\sigma\frac{(V_t^o)^{\frac{1}{\eta}}}{(E_t V_{t+1}^o)^{\frac{1}{\eta}}}E_t\left(\frac{\partial V_{t+1}^o}{\partial a_{t+1}}\frac{Q_{t+1}}{\sigma}\right). \tag{41}$$

From (38) we have

$$\beta\sigma\left(\frac{V_t^o}{E_t V_{t+1}^o}\right)^{\frac{1}{\eta}} = (\beta\sigma)^{\frac{\eta}{\eta-1}}\left(1-(\Delta_t^o)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}}.$$

Substituting into (41) we get:

$$(1+\mu_t-\upsilon_t)Q_t\lambda_t = \left(\frac{1-\phi}{\phi}\frac{c_t}{h_t}\right)^{\phi}(\Delta_t^o)^{\frac{1}{\eta}}+(\beta\sigma)^{\frac{\eta}{\eta-1}}\left(1-(\Delta_t^o)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}}E_t\left(\lambda_{t+1}\frac{Q_{t+1}}{\sigma}\right),$$

Putting together the first order conditions of c_t and h_t, we get

$$\begin{aligned}\frac{1-\phi}{\phi}\frac{c_t}{h_t} &= (1+\mu_t-\upsilon_t)Q_t-(\beta\sigma)^{\frac{\eta}{\eta-1}}\left(1-(\Delta_t^o)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}}E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{Q_{t+1}}{\sigma}\right)\\ &= (1+\mu_t-\upsilon_t)Q_t-E_t\left(\Lambda_{t+1}\frac{Q_{t+1}}{\sigma}\right)\equiv r_t^o, \qquad (42)\end{aligned}$$

where r_t^o is the imputed rent for an old home-owner and

$$\begin{aligned}\Lambda_{t+1} &= (\beta\sigma)^{\frac{\eta}{\eta-1}}\left(1-(\Delta_t^o)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}}\frac{\lambda_{t+1}}{\lambda_t}\\ &= (\beta\sigma)^{\frac{\eta}{\eta-1}}\left(1-(\Delta_t^o)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}}\left(\frac{\Delta_{t+1}^o}{\Delta_t^o}\right)^{\frac{1}{\eta}}\left(\frac{r_t^o}{r_{t+1}^o}\right)^{1-\phi} \qquad (43)\end{aligned}$$

is the stochastic discount factor of the old household.

From the first order condition for debt b_t

$$\frac{1}{R_t}\lambda_t = \beta\sigma\frac{(V_t^o)^{\frac{1}{\eta}}}{(E_t V_{t+1}^o)^{\frac{1}{\eta}}}E_t\left(\frac{1}{\sigma}\lambda_{t+1}\right),$$

we get

$$1 = \frac{R_t}{\sigma}E_t\left(\Lambda_{t+1}\right). \qquad (44)$$

Also from the first order condition for the purchase of house for rent s_t

$$[(1+\mu_t)Q_t - r_t^r]\,\lambda_t = \beta\sigma \frac{(V_t^o)^{\frac{1}{\eta}}}{(E_t V_{t+1}^o)^{\frac{1}{\eta}}} E_t\left(\lambda_{t+1}\frac{Q_{t+1}}{\sigma}\right),$$

we get

$$r_t^r = (1+\mu_t)Q_t - E_t\left(\Lambda_{t+1}\frac{Q_{t+1}}{\sigma}\right). \tag{45}$$

Next we derive the old household's marginal propensity to consume out of total (human and non-human) wealth. We start from the date-t budget constraint; we lead it one period forward and multiply it by the household's stochastic discount factor:

$$c_t + (1+\mu_t - \upsilon_t)Q_t h_t + [(1+\mu_t)Q_t - r_t^r]\,s_t - \frac{b_t}{R_t} = (1-\tau_t)\,w_t x_t + a_t,$$

$$\begin{aligned} & E_t\Lambda_{t+1}\left(c_{t+1} + (1+\mu_{t+1} - \upsilon_{t+1})Q_{t+1}h_{t+1} + \left[(1+\mu_{t+1})Q_{t+1} - r_{t+1}^r\right]s_{t+1} - \frac{b_{t+1}}{R_{t+1}}\right) \\ = \;& E_t\Lambda_{t+1}\left((1-\tau_{t+1})\,w_{t+1}x_{t+1} + \frac{1}{\sigma}\left(Q_{t+1}\left(h_t + s_t\right) - b_t\right)\right), \end{aligned}$$

This allows us to solve for $\frac{b_t}{R_t}$

$$\begin{aligned} \frac{b_t}{R_t} = \;& E_t\Lambda_{t+1}\left((1-\tau_{t+1})\,w_{t+1}x_{t+1} + \frac{1}{\sigma}Q_{t+1}\left(h_t + s_t\right)\right) - \\ & E_t\Lambda_{t+1}\left(c_{t+1} + (1+\mu_{t+1} - \upsilon_{t+1})Q_{t+1}h_{t+1} + \left[(1+\mu_{t+1})Q_{t+1} - r_{t+1}^r\right]s_{t+1} - \frac{b_{t+1}}{R_{t+1}}\right). \end{aligned}$$

Substitute the above expression into the date-t budget constraint:

$$\begin{aligned} & c_t + (1+\mu_t - \upsilon_t)Q_t h_t + [(1+\mu_t)Q_t - r_t^r]\,s_t - E_t\Lambda_{t+1}\frac{1}{\sigma}Q_{t+1}\left(h_t + s_t\right) + \\ & E_t\Lambda_{t+1}\left(c_{t+1} + (1+\mu_{t+1} - \upsilon_{t+1})Q_{t+1}h_{t+1} + \left[(1+\mu_{t+1})Q_{t+1} - r_{t+1}^r\right]s_{t+1} - \frac{b_{t+1}}{R_{t+1}}\right) \\ = \;& (1-\tau_t)\,w_t x_t + E_t\Lambda_{t+1}\,(1-\tau_{t+1})\,w_{t+1}x_{t+1} + a_t \end{aligned}$$

Using the first order condition for s_t and the r_t^o definition, we get:

$$\begin{aligned} & c_t + r_t^o h_t + E_t \Lambda_{t+1} \left(c_{t+1} + (1+\mu_{t+1} - \upsilon_{t+1}) Q_{t+1} h_{t+1} + \left[(1+\mu_{t+1}) Q_{t+1} - r_{t+1}^r \right] s_{t+1} - \frac{b_{t+1}}{R_{t+1}} \right) \\ = \; & (1-\tau_t) w_t x_t + E_t \Lambda_{t+1} (1-\tau_{t+1}) w_{t+1} x_{t+1} + a_t \end{aligned}$$

Using $e_t = c_t + r_t^o h_t$, iterating the above condition forward and using the Transversality Condition

$$\lim_{s\to\infty} E_t \left[\Lambda_{t+1}\Lambda_{t+2}..\Lambda_{t+s} Q_{t+s} \right] = \lim_{s\to\infty} E_t \left[\Lambda_{t+1}\Lambda_{t+2}..\Lambda_{t+s} b_{t+s} \right] = 0,$$

we get

$$\frac{1}{\nu_t^o} e_t = a_t + W_t^{ho} x_t, \tag{46}$$

$$u_t^o = \frac{e_t}{(r_t^o)^{1-\phi}} \tag{47}$$

where the human capital of the old W_t^{ho} satisfies

$$W_t^{ho} = w_t(1-\tau_t) + g^o E_t \left(\Lambda_{t+1} W_{t+1}^{ho} \right), \tag{48}$$

and the expenditure-wealth ratio of the household ν_t^o satisfies

$$\frac{1}{\nu_t^o} = 1 + E_t \left(\Lambda_{t+1} \frac{e_{t+1}}{e_t} \frac{1}{\nu_{t+1}^o} \right). \tag{49}$$

From (38), the ratio of value to current utility $\Delta_t^o = V_t^o / u_t^o$ satisfies

$$(\Delta_t^o)^{\frac{\eta-1}{\eta}} = 1 + \beta\sigma \left[E_t \left(\frac{u_{t+1}^o}{u_t^o} \right) \Delta_{t+1}^o \right]^{\frac{\eta-1}{\eta}} \tag{50}$$

In the problem of the old household, eleven endogenous variables, c_t, h_t, r_t^o, Λ_{t+1}, b_t, s_t, e_t, u_t^o, W_t^{ho}, ν_t^o and Δ_t^o are determined by the sequence of eleven equations (39), (42), (43), (44), (45), (46), (47), (48), (49) and (50) where (42) has two equations.

D.2.3 Consumption and saving choice of a young renter

Because households expect their income to rise in future, we guess that they face binding borrowing constraints and later verify this guess. Under this guess $b_t = b_{t-1} = 0$, the young household's choice of consumption and utility is simple as

$$c_t = \phi \left(1 - \tau_t\right) w_t x_t \tag{51}$$

$$h_t = \frac{1-\phi}{r_t^r} \left(1 - \tau_t\right) w_t x_t \tag{52}$$

$$u_t^r = \left(\frac{\chi}{r_t^r}\right)^{1-\phi} \left(1 - \tau_t\right) w_t x_t. \tag{53}$$

Defining the value-to-utility ratio as $\Delta_t^r = V_t^r / u_t^r$, we get

$$\left(\Delta_t^r\right)^{\frac{\eta-1}{\eta}} = 1 + \beta \left\{ E_t \left[\gamma \frac{u_{t+1}^r}{u_t^r} \Delta_{t+1}^r + (1-\gamma) \frac{u_{t+1}^o}{u_t^r} \Delta_{t+1}^o \right] \right\}^{\frac{\eta-1}{\eta}}. \tag{54}$$

(51), (52), (53) and (54) determine c_t, h_t, u_t^r and Δ_t^r of the young renter.

D.2.4 Consumption and saving choice of a young home owner

Under uncertainty, the expenditure shares for leveraged agents will be different from the perfect foresight case. Assuming that the borrowing constraint is binding, we get

$$b_t = E_t Q_{t+1} h_t - \omega a_t = E_t Q_{t+1} h_t - \omega \left(Q_t h_{t-1} - b_{t-1}\right).$$

Let us suppose that

$$c_t = \psi_t \left[\left(1 - \tau_t\right) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t \right] \tag{55}$$

$$r_t^h h_t = \left(1 - \psi_t\right) \left[\left(1 - \tau_t\right) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t \right]. \tag{56}$$

Then period utility is given by:

$$u_t^h = \left(\frac{\psi_t}{\phi}\right)^{\phi}\left(\frac{1-\psi_t}{r_t^h(1-\phi)}\right)^{1-\phi}\left[(1-\tau_t)\,w_t x_t + \left(1-\frac{\omega}{R_t}\right)a_t\right]. \tag{57}$$

Defining λ_t as the Lagrangian multiplier of the date-t budget constraint, the first order condition for consumption c_t is

$$\begin{aligned}\lambda_t &= \phi\frac{u_t^h}{c_t}(u_t^h)^{-\frac{1}{\eta}}(V_t^h)^{\frac{1}{\eta}}\\ &= \left(\frac{\phi}{1-\phi}\frac{h_t}{c_t}\right)^{1-\phi}(\Delta_t^h)^{\frac{1}{\eta}},\end{aligned}$$

where $\Delta_t^o = V_t^o/u_t^o$. The first order condition for the home ownership h_t is

$$\begin{aligned}r_t^h\lambda_t &= \left(\frac{1-\phi}{\phi}\frac{c_t}{h_t}\right)^{\phi}\left(\frac{V_t^h}{u_t^h}\right)^{\frac{1}{\eta}} + \beta\left(\frac{V_t^h}{E_tV_{t+1}}\right)^{\frac{1}{\eta}}\left[\gamma E_t\left(\frac{\partial V_{t+1}^h}{\partial a_{t+1}}\frac{\partial a_{t+1}}{\partial h_t}\right) + (1-\gamma)E_t\left(\frac{\partial V_{t+1}^o}{\partial a_{t+1}}\frac{\partial a_{t+1}}{\partial h_t}\right)\right]\\ &= \left(\frac{1-\phi}{\phi}\frac{c_t}{h_t}\right)^{\phi}(\Delta_t^h)^{\frac{1}{\eta}}\\ &\quad+\beta^{\frac{\eta}{\eta-1}}\left(1-(\Delta_t^h)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}}E_t\left\{\left[\gamma\frac{\partial V_{t+1}^h}{\partial a_{t+1}} + (1-\gamma)\frac{(\Delta_{t+1}^o)^{\frac{1}{\eta}}}{(r_{t+1}^o)^{1-\phi}}\right](Q_{t+1}-E_tQ_{t+1})\right\}.\end{aligned}$$

Thus

$$\begin{aligned}r_t^h &= \frac{1-\phi}{\phi}\frac{\psi_t}{1-\psi_t} \\ &\quad+\frac{\beta^{\frac{\eta}{\eta-1}}\left(\frac{1-\phi}{\phi}\frac{\psi_t}{1-\psi_t}\right)^{1-\phi}}{\left(\left(\Delta_t^h\right)^{1-\frac{1}{\eta}}-1\right)^{\frac{1}{\eta-1}}}E_t\left\{\left[\gamma\frac{\partial V_{t+1}^h}{\partial a_{t+1}} + (1-\gamma)\frac{(\Delta_{t+1}^o)^{\frac{1}{\eta}}}{(r_{t+1}^o)^{1-\phi}}\right](Q_{t+1}-E_tQ_{t+1})\right\}.\end{aligned} \tag{58}$$

All that remains is to derive an expression for $\frac{\partial V_{t+1}^h}{\partial a_{t+1}}$

$$\begin{aligned}
\frac{\partial V_t^h}{\partial a_t} &= \frac{\partial e_t}{\partial a_t}\frac{\partial u_t}{\partial e_t}\frac{\partial V_t^h}{\partial u_t} + \beta E_t\left[\left(\frac{\partial V_t^h}{\partial V_{t+1}}\right)\left(\gamma\frac{\partial V_{t+1}^h}{\partial a_{t+1}} + (1-\gamma)\frac{\partial V_{t+1}^o}{\partial a_{t+1}}\frac{\partial a_{t+1}}{\partial a_t}\right)\right] \\
&= \left(1-\frac{\omega}{R_t}\right)\frac{\left(\Delta_t^h\right)^{\frac{1}{\eta}}}{\left(r_t^h\right)^{1-\phi}} \\
&\quad + \omega\beta^{\frac{\eta}{\eta-1}}\left(1-\left(\Delta_t^h\right)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}} E_t\left[\gamma\frac{\partial V_{t+1}^h}{\partial a_{t+1}} + (1-\gamma)\frac{\left(\Delta_{t+1}^o\right)^{\frac{1}{\eta}}}{\left(r_{t+1}^o\right)^{1-\phi}}\right]. \qquad (59)
\end{aligned}$$

When $\omega = 0$, the above expression is equal to the standard condition that the value of assets is equal to MU of current expenditure.

$$\frac{\partial V_t^h}{\partial a_t} = \frac{\left(\Delta_t^h\right)^{\frac{1}{\eta}}}{\left(r_t^h\right)^{1-\phi}}.$$

However, when the borrowing constraint stipulates a gradual repayment of debt or run-down of assets ($\omega > 0$), the value of assets is a recursive expression that depends on the entire future evolution of marginal utilities of expenditure. The ratio of value to utility satisfies

$$\left(\Delta_t^h\right)^{\frac{\eta-1}{\eta}} = 1 + \beta\left\{E_t\left[\gamma\frac{u_{t+1}^h}{u_t^h}\Delta_{t+1}^h + (1-\gamma)\frac{u_{t+1}^o}{u_t^h}\Delta_{t+1}^o\right]\right\}^{\frac{\eta-1}{\eta}}. \qquad (60)$$

Seven variables r_t^h, c_t, h_t, u_t^h, ψ_t, $\partial V_t^h/\partial a_t$ and Δ_t^h are determined the sequence of seven equations (40), (55), (56), (57), (58), (59) and (60) in the problem of the young home owner.

D.3 Aggregation and Market Clearing

In this section we aggregate the above individual equilibrium condition in a way that will make them suitable for a numerical solution. This includes also detrending the model. We detrend the following variables by dividing by $A_t N_t$, because they have the same trend with $A_t N_t$

$$Y_t, C_t^h, C_t^r, C_t^o, B_t^h, B_t^o, B_t^*, Q_t, r_t^h, r_t^r, r_t^o, A_t^o.$$

We detrend the following variables by dividing by A_t, because they have the same trend with A_t

$$w_t, W_t^{ho}.$$

We detrend the following variables by dividing by N_t, because they have the same trend with N_t

$$X_t^y, X_t^o, N_t^y, N_t^o.$$

We detrend $\frac{\partial V_t^h}{\partial a_t}$ by multiplying by $(A_t N_t)^{1-\phi}$. We do not need to detrend the following variables as they do not have trend

$$H_t^h, H_t^r, H_t^o, S_t, \Delta_t^h, \Delta_t^r, \Delta_t^o, \psi_t, \xi_t, \nu_t^o, \Lambda_{t+1}.$$

D.3.1 Market clearing conditions and aggregate state variable evolution

In our simple open economy, housing supply is constant in which housing requires the exogenous maintenance per unit. Aggregate output is sum of aggregate consumption C_t, housing maintenance $\mu_t Q_t$,and net export NX_t as

$$Y_t = C_t^h + C_t^r + C_t^o + \mu_t Q_t + NX_t.$$

The aggregate output is exogenously evolving as

$$Y_t = A_t \left(X_t^y + X_t^o \right), \tag{61}$$

and the wage rate equals to the labor productivity as

$$w_t = A_t, \text{ where} \tag{62}$$

$$\frac{A_t}{A_{t-1}} = G_t^A.$$

The bond market clearing condition at date t implies that the sum of net debt of old and young households and foreign sector must adds up to zero as:

$$B_t^o + B_t^h + B_t^* = 0, \tag{63}$$

Net debt of the foreign sector (or net foreign asset of home country) evolves with net exports as

$$\frac{B_t^*}{R_t} = \frac{B_{t-1}^*}{\left(G_t^N G_t^A\right)} + NX_t.$$

Normalizing total housing supply to unity, housing market equilibrium is given by

$$1 = H_t^y + S_t + H_t^o \tag{64}$$

The rental market equilibrium implies

$$H_t^r = S_t. \tag{65}$$

Combining the goods market and net foreign asset accumulation together, we have

$$\frac{B_t^*}{R_t} = \frac{B_{t-1}^*}{\left(G_t^N G_t^A\right)} + Y_t - (C_t^h + C_t^r + C_t^o) - \mu_t Q_t \tag{66}$$

The normalized government budget constraint implies

$$\upsilon_t Q_t \left(H_t^y + H_t^o\right) = \tau_t \left(X_t^y + X_t^o\right). \tag{67}$$

Young borrowers Let

$$r_t^h = Q_t\left(1+\mu_t-\upsilon_t\right)-\frac{E_t(G_{t+1}^N G_{t+1}^A Q_{t+1})}{R_t} \tag{68}$$

denote the downpayment for young borrowers. Aggregate demand for consumption and housing by young home owning households are

$$C_t^h = (1-\psi_t)\left\{\xi_t\left(1-\tau_t\right)w_t X_t^y + \left(1-\frac{\omega}{R_t}\right)\gamma\left(Q_t H_{t-1}^h - \frac{B_{t-1}^h}{G_t^N G_t^A}\right)\right\} \tag{69}$$

$$r_t^h H_t^h = \psi_t\left\{\xi_t\left(1-\tau_t\right)w_t X_t^y + \left(1-\frac{\omega}{R_t}\right)\gamma\left(Q_t H_{t-1}^h - \frac{B_{t-1}^h}{G_t^N G_t^A}\right)\right\}, \tag{70}$$

where ξ_t is the share of the young who are home owners and ψ_t is the expenditure share on housing of those young home owners.

The following two expressions pin down the housing expenditure share for young home owners.

$$\left(\frac{\phi}{1-\phi}\frac{1-\psi_t}{\psi_t}-1\right)\left(\frac{1-\phi}{\phi}\frac{\psi_t}{1-\psi_t}r_t^h\right)^{\phi} =$$

$$\frac{\beta^{\frac{\eta}{\eta-1}}}{\left(\left(\Delta_t^h\right)^{1-\frac{1}{\eta}}-1\right)^{\frac{1}{\eta-1}}} \qquad E_t\{\left[\gamma\left(\frac{\partial V_{t+1}^h}{\partial a_{t+1}}\right)+(1-\gamma)\left(\frac{\left(\Delta_{t+1}^o\right)^{\frac{1}{\eta}}}{\left(r_{t+1}^o\right)^{1-\phi}}\right)\right] *$$

$$\frac{1}{(G_{t+1}^N G_{t+1}^A)^{1-\phi}} \qquad [G_{t+1}^N G_{t+1}^A Q_{t+1} - E_t(G_{t+1}^N G_{t+1}^A Q_{t+1})]\}, \tag{71}$$

and

$$\frac{\partial V_t^h}{\partial a_t} = \left(1-\frac{\omega}{R_t}\right)\frac{\left(\Delta_t^h\right)^{\frac{1}{\eta}}}{\left(r_t^h\right)^{1-\phi}}+$$

$$\omega\beta^{\frac{\eta}{\eta-1}}\left(1-\left(\Delta_t^h\right)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}}\left[\frac{\gamma}{\left(G_{t+1}^N G_{t+1}^A\right)^{1-\phi}}\frac{\partial V_{t+1}^h}{\partial a_{t+1}}+(1-\gamma)\frac{\left(\Delta_{t+1}^o\right)^{\frac{1}{\eta}}}{\left(G_{t+1}^N G_{t+1}^A r_{t+1}^o\right)^{1-\phi}}\right]. \tag{72}$$

In the expression above I include a growth term for $\frac{\partial V^h_{t+1}}{\partial a_{t+1}}$. This growth term is $\frac{1}{\left(G^N_{t+1}G^A_{t+1}\right)^{1-\phi}}$; it is the same growth term as $\frac{(\Delta^o_t)^{\frac{1}{\eta}}}{\left(r^h_t\right)^{1-\phi}}$ and $\frac{\left(\Delta^y_t\right)^{\frac{1}{\eta}}}{\left(r^h_t\right)^{1-\phi}}$. We reason that $\frac{\partial V^h_t}{\partial a_t}$ would inherit the same growth term from the term it depends on.

The binding borrowing constraint determines the value of current borrowing:

$$B^h_t = E_t\left(G^N_{t+1}G^A_{t+1}Q_{t+1}\right)H^h_t - \omega\gamma\left(Q_tH^h_{t-1} - \frac{B^h_{t-1}}{G^N_tG^A_t}\right). \tag{73}$$

The value function of young borrowers is given by:

$$\left(\Delta^h_t\right)^{\frac{\eta-1}{\eta}} = 1 + \beta E_t\left\{\left(\gamma\Delta^h_{t+1}\frac{u^h_{t+1}}{u^h_t} + (1-\gamma)\,\Delta^o_{t+1}\frac{u^o_{t+1}}{u^h_t}\right)^{\frac{\eta-1}{\eta}}\right\}. \tag{74}$$

For easy aggregation in computing the welfare of individuals of different types (home owners, renters, the old) we need to show that the growth rate of utility is the same regardless of wealth level. Consider a household with income equal to x. Next we provide conditions under which the growth rate of utility is independent of x.

Young renters Aggregate demand for consumption and housing by young renters are

$$C^r_t = \phi\,(1-\xi_t)\,(1-\tau_t)\,w_tX^y_t \tag{75}$$

$$r^r_tH^r_t = (1-\phi)\,(1-\xi_t)\,(1-\tau_t)\,w_tX^y_t, \tag{76}$$

where ξ_t is the home ownership rate among young households.

$$(\Delta^r_t)^{\frac{\eta-1}{\eta}} = 1 + \beta E_t\left\{\left(\gamma\Delta^r_{t+1}\frac{u^r_{t+1}}{u^r_t} + (1-\gamma)\,\Delta^o_{t+1}\frac{u^o_{t+1}}{u^r_t}\right)^{\frac{\eta-1}{\eta}}\right\} \tag{77}$$

where

$$\frac{u^r_{t+1}}{u^r_t} = \frac{(1-\tau_{t+1})}{(1-\tau_t)}g^yG^A_{t+1}\left(\frac{r^r_t}{G^N_{t+1}G^A_{t+1}r^r_{t+1}}\right)^{1-\phi}$$

is the utility growth rate of the current young renters (conditional on remaining young) and

$$\frac{u^o_{t+1}}{u^r_t} = \frac{(1-\tau_{t+1})}{(1-\tau_t)} g^y G^A_{t+1} P^w_{t+1} \left(\frac{r^r_t}{\chi G^N_{t+1} G^A_{t+1} r^o_{t+1}} \right)^{1-\phi}$$

is the utility growth rate of the current young renters (conditional on becoming old).

Following from the assumed uniform distribution of rental preference parameter χ the home ownership rate ξ_t is given by comparing the value of becoming a home owner and the value of becoming a renter:

$$\xi_t = \frac{1}{\chi} \left(\frac{\Delta^h_t}{\Delta^r_t} \right)^{\frac{1}{1-\phi}} \left(\frac{r^r_t}{r^h_t} \right). \tag{78}$$

Old Households Aggregate demand for consumption and housing by old households are

$$C^o_t = \phi \nu^o_t \left(W^{ho}_t X^o_t + A^o_t \right), \tag{79}$$

$$H^o_t = \frac{1-\phi}{r^o_t} \nu^o_t \left(W^{ho}_t X^o_t + A^o_t \right), \tag{80}$$

$$A^o_t = Q_t \left(H^o_{t-1} + S_{t-1} \right) - \frac{B^o_{t-1}}{G^N_t G^A_t} + (1-\gamma) \left(Q_t H^h_{t-1} - \frac{B^h_{t-1}}{G^N_t G^A_t} \right). \tag{81}$$

where B^o_t is aggregate net borrowing of the old households at the beginning of date t. The first order conditions for s_t and b_t imply

$$r^r_t = Q_t \left(1 + \mu_t \right) - \frac{1}{\sigma} E_t (G^N_{t+1} G^A_{t+1} Q_{t+1} \Lambda_{t+1})$$

$$\frac{R_t}{\sigma} E_t \left(\Lambda_{t+1} \right) = 1 \tag{82}$$

$$r^o_t = (1 + \mu_t - \upsilon_t) Q_t - E_t \left(\Lambda_{t+1} \frac{Q_{t+1}}{\sigma} \right), \tag{83}$$

where

$$\Lambda_{t+1} = (\sigma\beta)^{\frac{\eta}{\eta-1}} \left(1 - (\Delta_t^o)^{\frac{1-\eta}{\eta}}\right)^{\frac{1}{1-\eta}} \left(\frac{\Delta_{t+1}^o}{\Delta_t^o}\right)^{\frac{1}{\eta}} \left(\frac{r_t^o}{G_{t+1}^N G_{t+1}^A r_{t+1}^o}\right)^{1-\phi} \tag{84}$$

is the stochastic discount factor. We also get

$$\frac{1}{\nu_t^o} = 1 + E_t \left(\Lambda_{t+1} G_{t+1}^o \frac{1}{\nu_{t+1}^o}\right) \tag{85}$$

and

$$W_t^{ho} = w_t(1-\tau_t) + g^o E_t \left(G_{t+1}^A \Lambda_{t+1} W_{t+1}^{ho}\right). \tag{86}$$

Finally we make the substitution into (38) to get an expression for Δ_t^o

$$(\Delta_t^o)^{\frac{\eta-1}{\eta}} = 1 + \beta\sigma E_t \left\{ (\Delta_{t+1}^o)^{\frac{\eta-1}{\eta}} \left(\left(\frac{r_t^o}{G_{t+1}^N G_{t+1}^A r_{t+1}^o} \right)^{1-\phi} G_{t+1}^o \right)^{\frac{\eta-1}{\eta}} \right\} \tag{87}$$

where

$$\begin{aligned} G_{t+1}^o &= \frac{e_{t+1}}{e_t} \\ &= \frac{\nu_{t+1}^o}{\nu_t^o} \frac{G_{t+1}^A g^o W_{t+1}^{ho} X_t^o + \frac{1}{\sigma}\left[Q_{t+1}\left(H_t^o + H_t^r\right) - \frac{B_t^o}{G_{t+1}^N G_{t+1}^A}\right]}{W_t^{ho} X_t^o + A_t^o} \end{aligned}$$

is the expenditure growth (conditional on survival) of the currently alive old.

The endogenous state variables are $\left(B_{t-1}^*, B_{t-1}^h, H_{t-1}^h, S_{t-1}\right)$, and $N_t^y, N_t^o, X_t^y, X_t^o$, A_t, G_t^A and R_t follows an exogenous process. Then twenty eight endogenous variables Y_t, w_t, Q_t, r_t^h, r_t^r, r_t^o, $\xi_t, \psi_t, C_t^h, C_t^r, C_t^o$, H_t^h, H_t^r, H_t^o, S_t, B_t^h, B_t^o, B_t^*, $\frac{\partial V_t^h}{\partial a_t}$, ν_t^o, Λ_{t+1}, $W_t^{ho}, \Delta_t^o, \Delta_t^h, \Delta_t^r$ and A_t^o by twenty eight independent equilibrium conditions ($61-87$) as a function of the state variables. Here one of the market clearing conditions ($63, 64, 65, 66$) is not independent due to Walras' Law.

When we introduce LTV constraint, we consider the lump sum transfer in the budget of young borrowers for the effect of the 'tax' on their borrowing. This is necessary because we are implementing an LTV cap through a tax on borrowing together with a lump sum rebate back to borrowing households.

D.4 Computational Method

Our stochastic model is solved under the assumption that the economy has not experienced any shocks in the past but that, in any time period, the world real interest rate (or the growth rate of the per capita endowment) could undergo a permanent change with a known probability. We are therefore looking for a stationary steady state together with the correct expectations of what happens to forward looking variables (e.g. house prices, rents, etc) when a shock does eventually hit. We solve for a stochastic steady state as follows

(1) Start with an initial guess that all variables are equal to their determinstic steady state values also when the shock to the world real interest rate/growth rate occurs. This initializes expectations conditional on a shock hitting.

(2) Solve the stochastic steady state using the initialized expectations of variables. We use Matlab's fsolve.m routine.

(3) With the stochastic steady state solved in (2) above as initial condition, do permanent positive and negative shocks to the world real interest rate (or the growth rate of per capita growth). We use Dynare's simul command to compute the perfect foresign dynamics following the unanticipated shock. Compare all variable values in the period when the shock hits to the previous guess. If they differ by more than a small tolerance amount, update the guess for the expectation conditional on a shock hitting..

(4) Solve for the new stochastic steady state and repeat until convergence.