Credit Horizons

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Questions

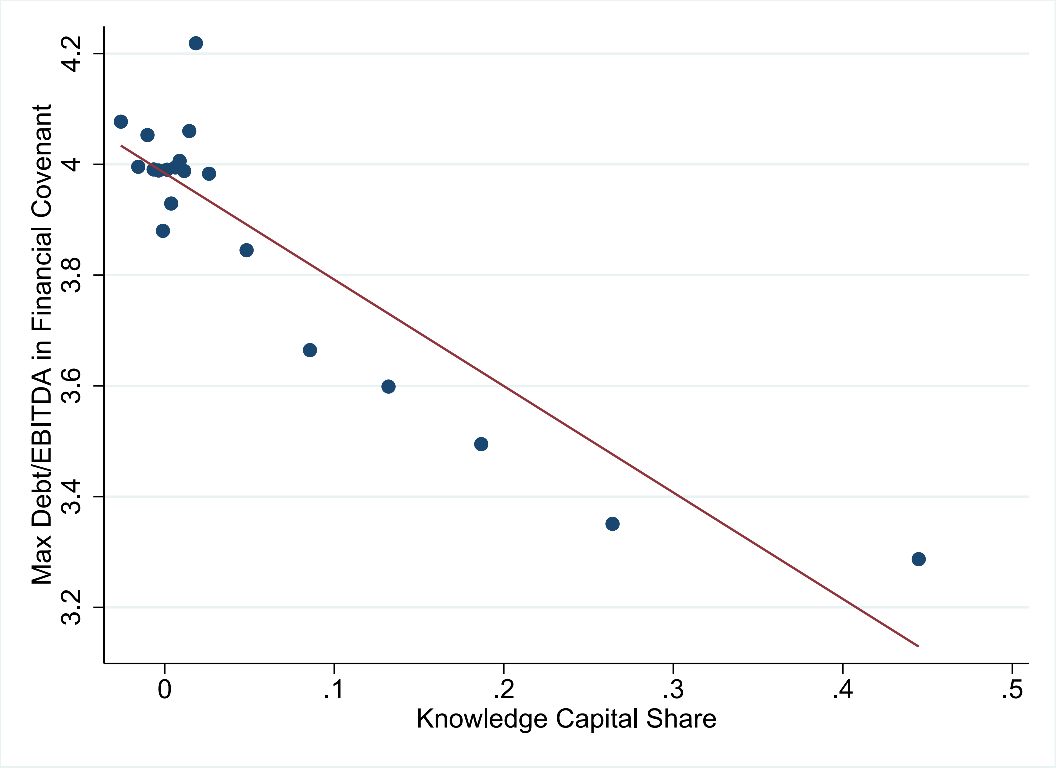
To finance investment, many businesses raise external funds against their collateral assets and future cash flow

Cash-flow-based borrowing: debt is limited mostly by 3 to 4.5 years worth of EBITDA, according to Lian-Ma (2021)

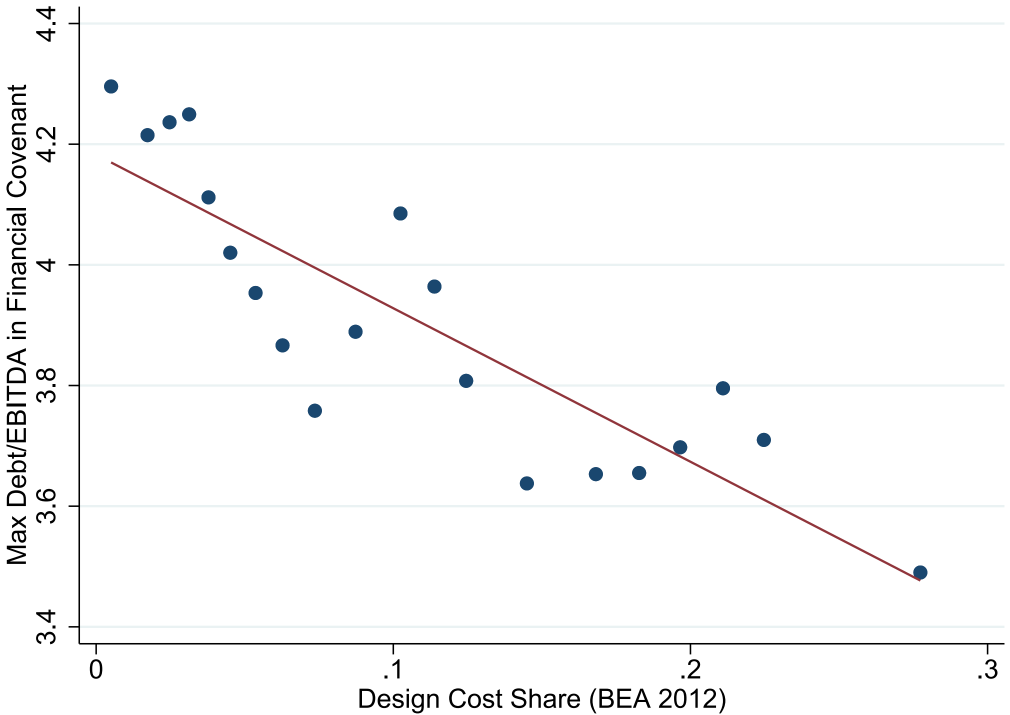
Stock analysts typically provide 5-year earning forecast

Why are the horizons of external financiers short, even when the durations of businesses are long?

How does a persistent fall in interest rate affect the economy? What if the change of interest rate is medium-term?



Max Debt/EBITDA in Financial Covenant
3.2
3.4
3.6
3.8
4
4.2
0
.1
.2
.3
.4
.5
Knowledge Capital Share



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Design Cost Share (BEA 2012)

Introductory Example

Engineer's investment technology:

$$\left.\begin{array}{c}\text{goods}\\ \text{a building}\end{array}\right\} \longrightarrow \left\{\begin{array}{c}\text{a plant}\\ \text{maintenance capacity}\end{array}\right.$$

To finance investment, engineer sells the plant ownership to saver

With the engineer's continual maintenance, the plant yields returns $y_{t+1},\ y_{t+2},\ y_{t+3},\dots$. Once missed the maintenance, the plant stops yielding returns forever

Engineer cannot commit to maintain the plant in future. Match between engineer and plant owner can change over time

The owner and engineer bilaterally match and bargain over "wage" every period. The stake for the owner is the franchise value. The engineer's stake is wage of this period

$$\underset{w_{t+1}}{Max}(V_{t+1} - w_{t+1})^{\theta}(w_{t+1})^{1-\theta}$$

Engineer receives a fraction $1-\theta$ of continuation value as wage *every period*

$$w_{t+1} = (1 - \theta)V_{t+1}$$

$$V_t = \frac{1}{R}(y_{t+1} - w_{t+1} + V_{t+1})$$

The plant owner retains θ fraction of the continuation value, and derives the value largely from near-future revenue

$$V_t = \frac{1}{R}y_{t+1} + \frac{\theta}{R^2}y_{t+2} + \frac{\theta^2}{R^3}y_{t+3} + \frac{\theta^3}{R^4}y_{t+4}....$$

The engineer sells plant at price V_t to finance investment: her borrowing capacity has a short horizon

Model

Small open economy with an exogenous real interest rate R

Homogeneous perishable consumption/investment good at each date $t = 0, 1, 2, ...$(numeraire)

Continuum of agents, utility $U = E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln c_t\right]$

Each agent sometimes has an investment opportunity (entrepreneur/engineer) and sometimes not (saver), Markov process

At each date, an engineer E can jointly produce plant and tools from goods and building: per unit of plant,

$$\left.\begin{array}{c} x \text{ goods} \\ 1 \text{ building} \end{array}\right\} \rightarrow \left\{\begin{array}{c} \text{plant of productivity } 1 \\ 1 \text{ E-tool} \end{array}\right.$$

Engineer raises funds by selling the plant to savers. Match between plant and engineer is not specific $\longrightarrow$ Plant owner hires engineers for maintenance in a competitive market at wage w. Engineer cannot precommit to work for less

At each date, the owner of plant of productivity z can hire any number h of tools (the engineer's expertise) to produce goods and maintain plant productivity: within the period, per unit of plant,

$$\left.\begin{array}{c}\text{productivity } z \text{ plant}\\ h \text{ tools}\end{array}\right\} \longrightarrow \left\{\begin{array}{c} y = az \quad \text{goods} \\ \lambda \text{ productivity } z' = z^{\theta} h^{\eta} \text{ plant} \\ \lambda h \text{ tools}\end{array}\right.$$

New buildings are supplied by foreigners

Alternative use of building by foreigners:

$$1 \text{ building} \longrightarrow \begin{cases} f \ \text{ goods} \\ \lambda \text{ building} \end{cases}$$

Building supply is perfectly elastic at a constant rent f

$\longrightarrow$ Price of buildings

$$q = \frac{f}{R - \lambda}$$

The plant owner always has the option to stop and liquidate his plant into generic building. So his value of a unit of plant of productivity z at the end of the period is given by

$$V(z) = Max\left\{q,\ \frac{1}{R}\max_h\left[az - wh + \lambda V\left(z^{\theta}h^{\eta}\right)\right]\right\}$$

The plant owner must devise a long-term plan:

stop after a finite number of periods T, or

continue forever ($T = \infty$)?

An engineer raises fund by selling a new plant at price $b = V(1)$

The budget constraint of an agent at date t who has h_t tools and d_t financial assets (maturing one-period discount bonds plus returns to plant ownership) is

$$c_t + (x + q - b)i_t + \frac{d_{t+1}}{R} = wh_t + d_t,$$

where h_t is positive iff the agent was engineer yesterday. Iff the agent is an engineer today, investment i_t is positive, and her tools tomorrow will be

$$h_{t+1} = \lambda h_t + i_t$$

The budget constraint can be written as

$$c_t + (x + q - b)h_{t+1} + \frac{d_{t+1}}{R} = [w + \lambda(x + q - b)]h_t + d_t \equiv n_t,$$

where n_t is net worth

When the rate of return on investment with maximal leverage, R^E, exceeds the interest rate

$$R^E = \frac{w + \lambda(x + q - b)}{x + q - b} > R,$$

the engineer's consumption and investment are

$$\begin{aligned} c_t &= (1 - \beta)n_t \\ (x + q - b)h_{t+1} &= \beta n_t \end{aligned}$$

A saver's are

$$\begin{aligned} c_t &= (1 - \beta)n_t \\ \frac{d_{t+1}}{R} &= \beta n_t \end{aligned}$$

A steady state equilibrium of our small open economy is characterized by

(i) wage rate w and new-plant price b,

(ii) quantity choices of savers/plant owners (c, d, h, z, y), engineers (c, h, i), and foreigners (who have net asset holdings D^*),

such that the markets for goods, tools, plant, and bonds all clear

Proposition. Pure Equilibrium with No Stopping: Low opportunity cost $f < f^{\text{critical}}$

(a) No plant owner stops

(b) Aggregate ratio of tools-to-plant stays one-to-one (because equal initial supply, equal depreciation, no stopping): $h_t = 1$

(c) All plant is maintained at initial productivity: $z_t = 1$

(d) All plan has output: $y_t = a$

(e) If $z' = z^{\theta} h^{1-\theta}$, the competitive equilibrium and the bilateral bargaining are equivalent

Optimal maintenance choice, $z_{t+1} = z_t^{\theta} h_t^{\eta}$ and $h_t = z_t = 1$

$$w = \eta \left(0 + \frac{\lambda}{R} a + \frac{\lambda^2 \theta}{R^2} a + \frac{\lambda^3 \theta^2}{R^3} a + ... \right)$$

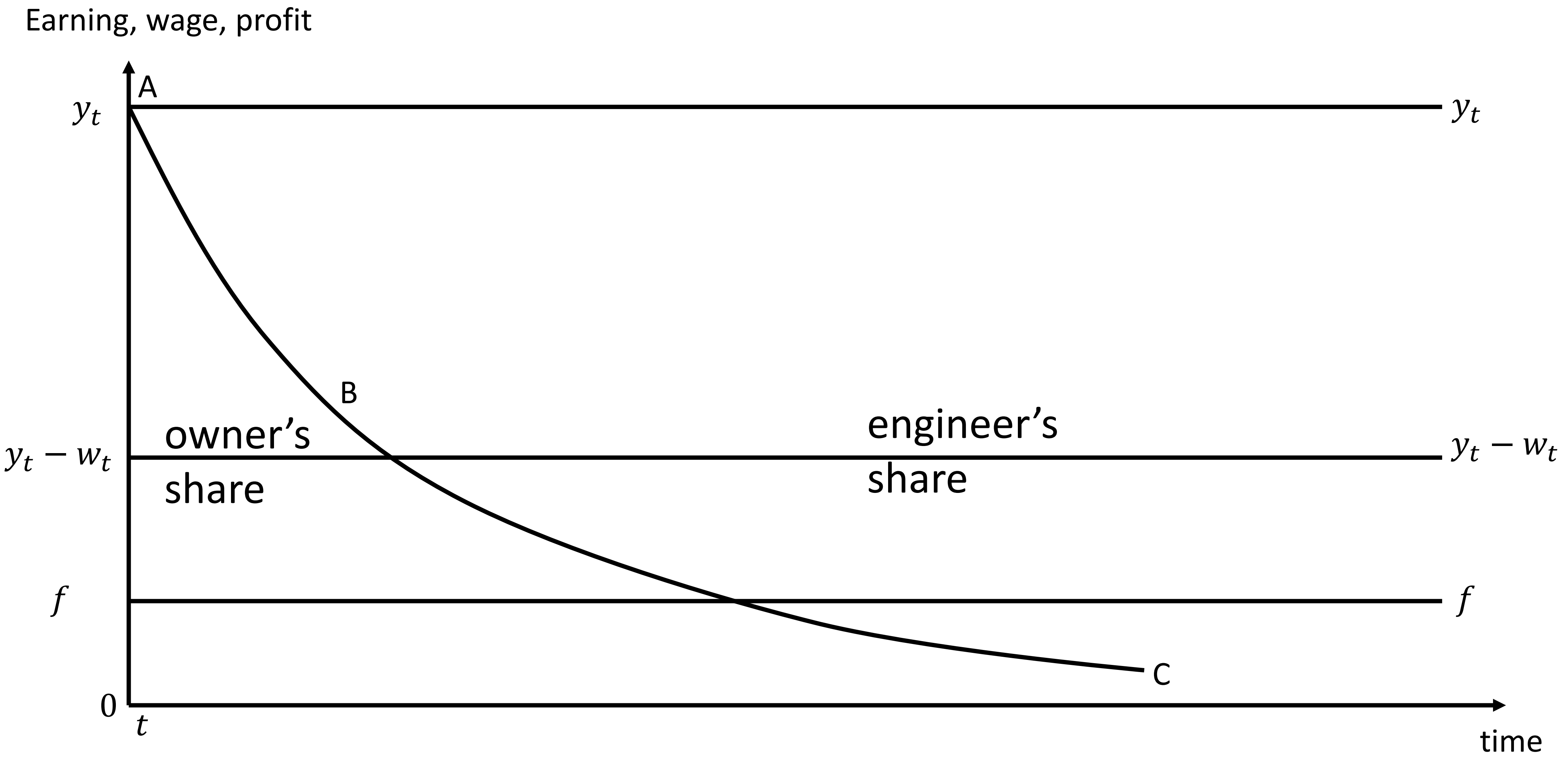
$$= \text{ PV of marginal product of } h$$

$$b = \frac{1}{R} a \; + \; \frac{\lambda}{R^2} a(1 - \eta) + \frac{\lambda^2}{R^3} a(1 - \eta - \eta\theta) + ..$$

Engineers' share of output rises with horizon as 0, η, $\eta(1+\theta)$, $\eta(1 + \theta + \theta^2)$, ...

Plant owner's share from present plant declines with horizon as 1, $1 - \eta$, $1 - \eta - \eta\theta$, $1 - \eta - \eta\theta - \eta\theta^2$, ...

Figure 1: Shares of Owner and Engineer



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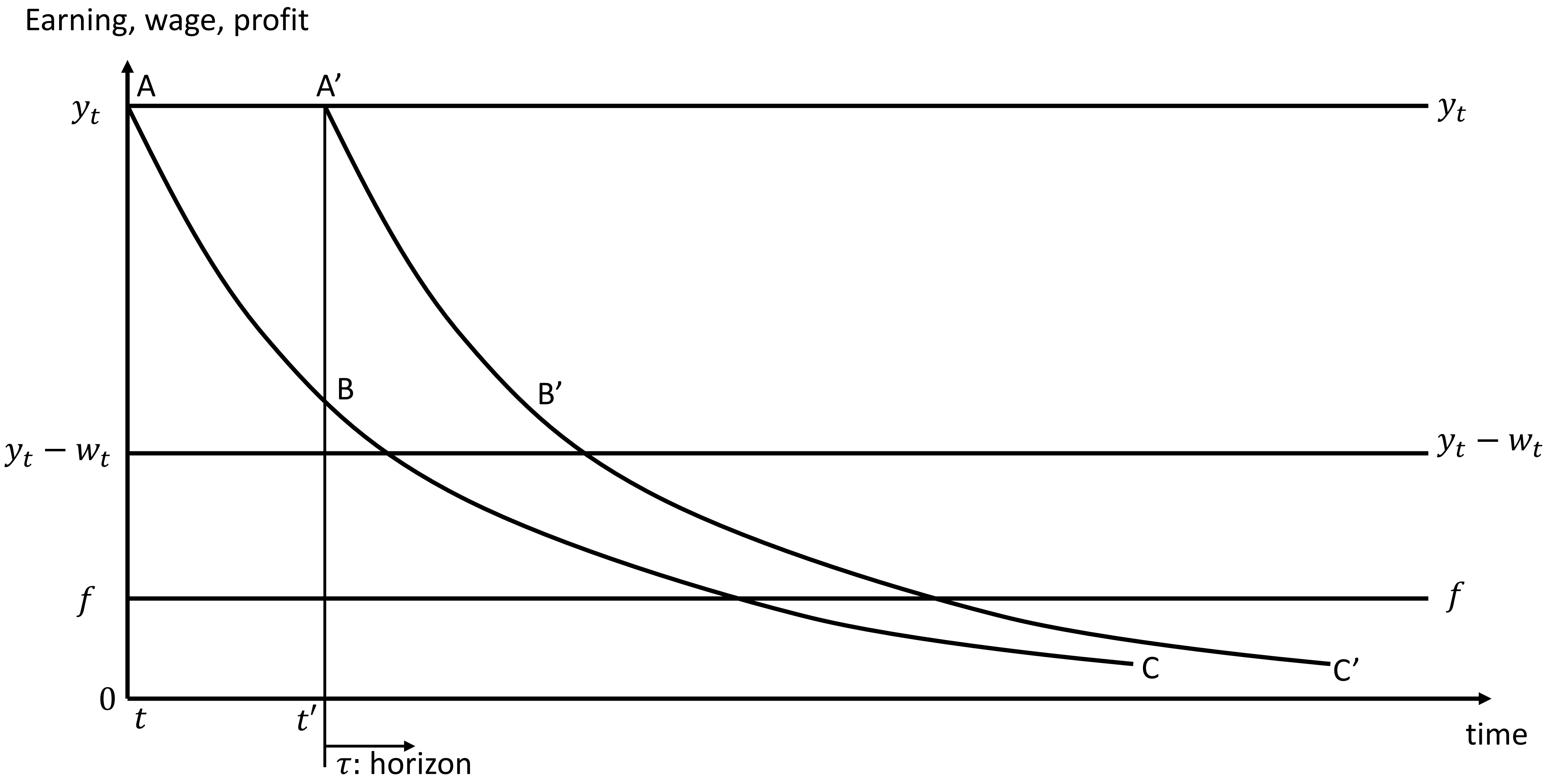

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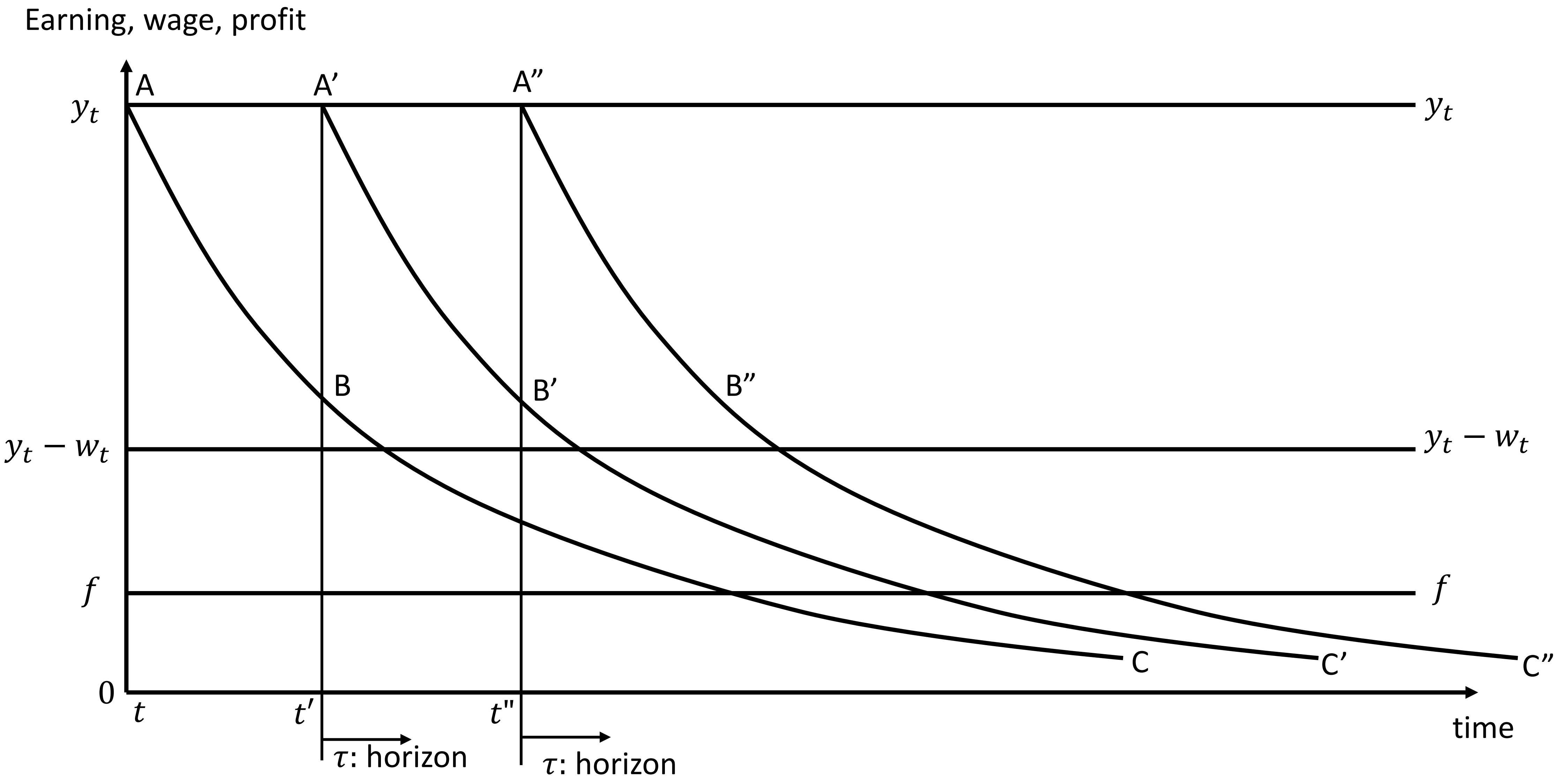


Effects of a Permanent Fall in Interest Rate

Engineer's investment is contrained iff

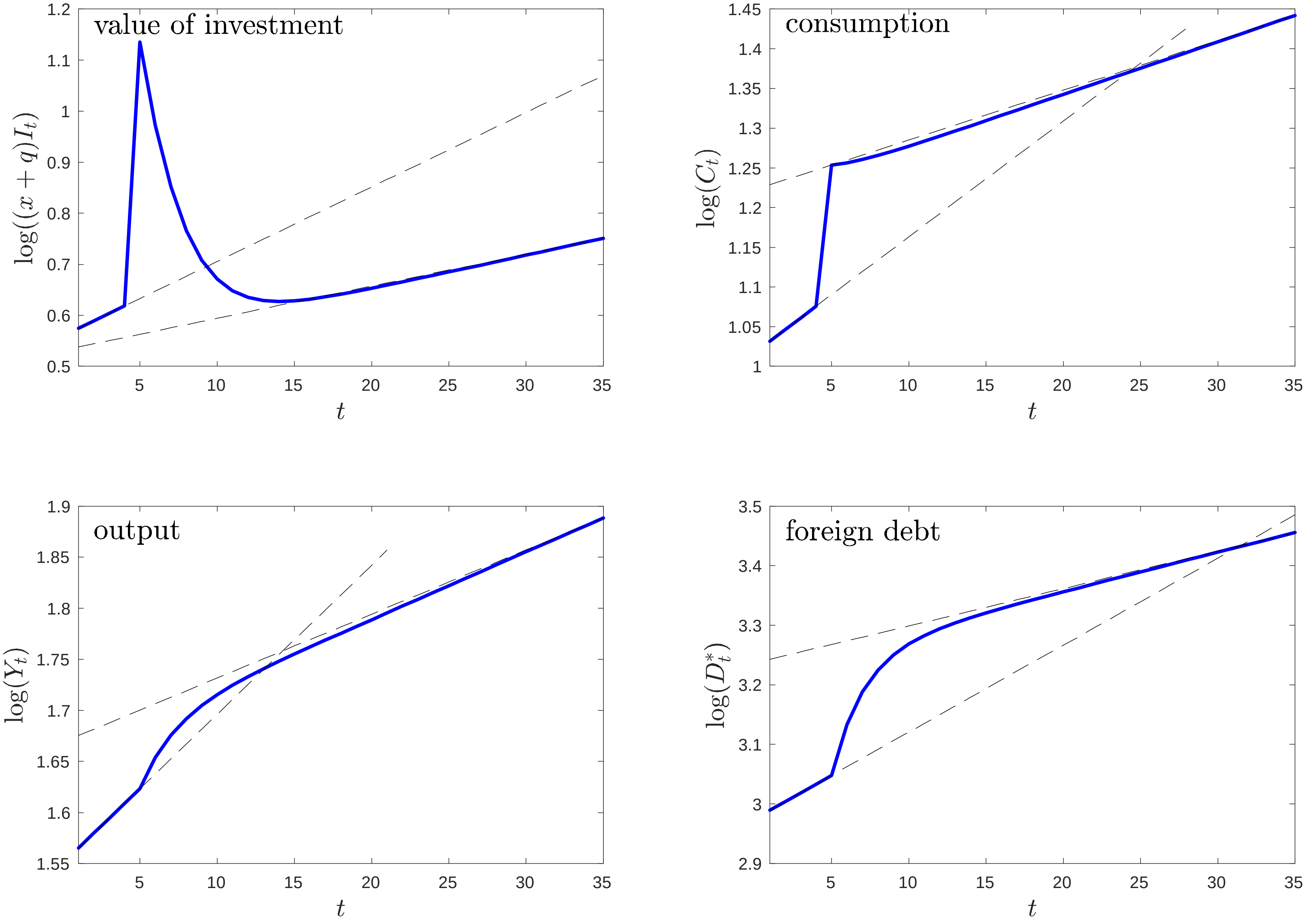
$$b < x + q < \frac{a}{R - \lambda}$$

The engineer's borrowing capacity has a shorter duration than building

With a permanent fall in interest rate, the borrowing capacity may fail to catch up with the investment cost. Can offset a rise in net worth – to stifle investment and growth:

$$\text{gross investment} \downarrow \; =$$

$$\beta \times \frac{\text{net worth of engineers} \uparrow}{\text{investment cost } \; (x+q) \uparrow\uparrow - \; \text{fund-raising capacity } \; (b) \uparrow}$$


value of investment
$\log((x+q)I_t)$
consumption
$\log(C_t)$
output
$\log(Y_t)$
foreign debt
$\log(D_t^*)$
t

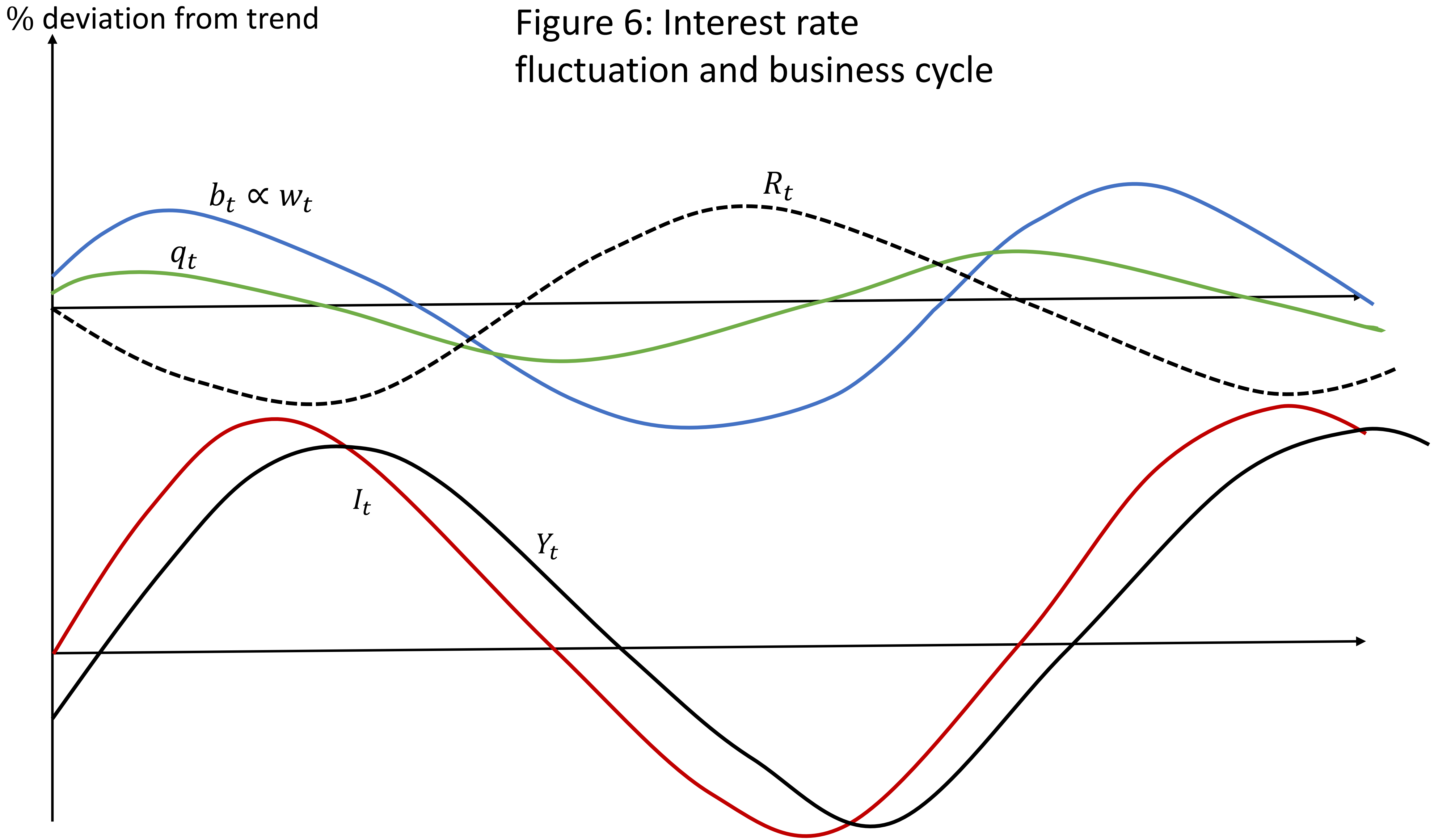
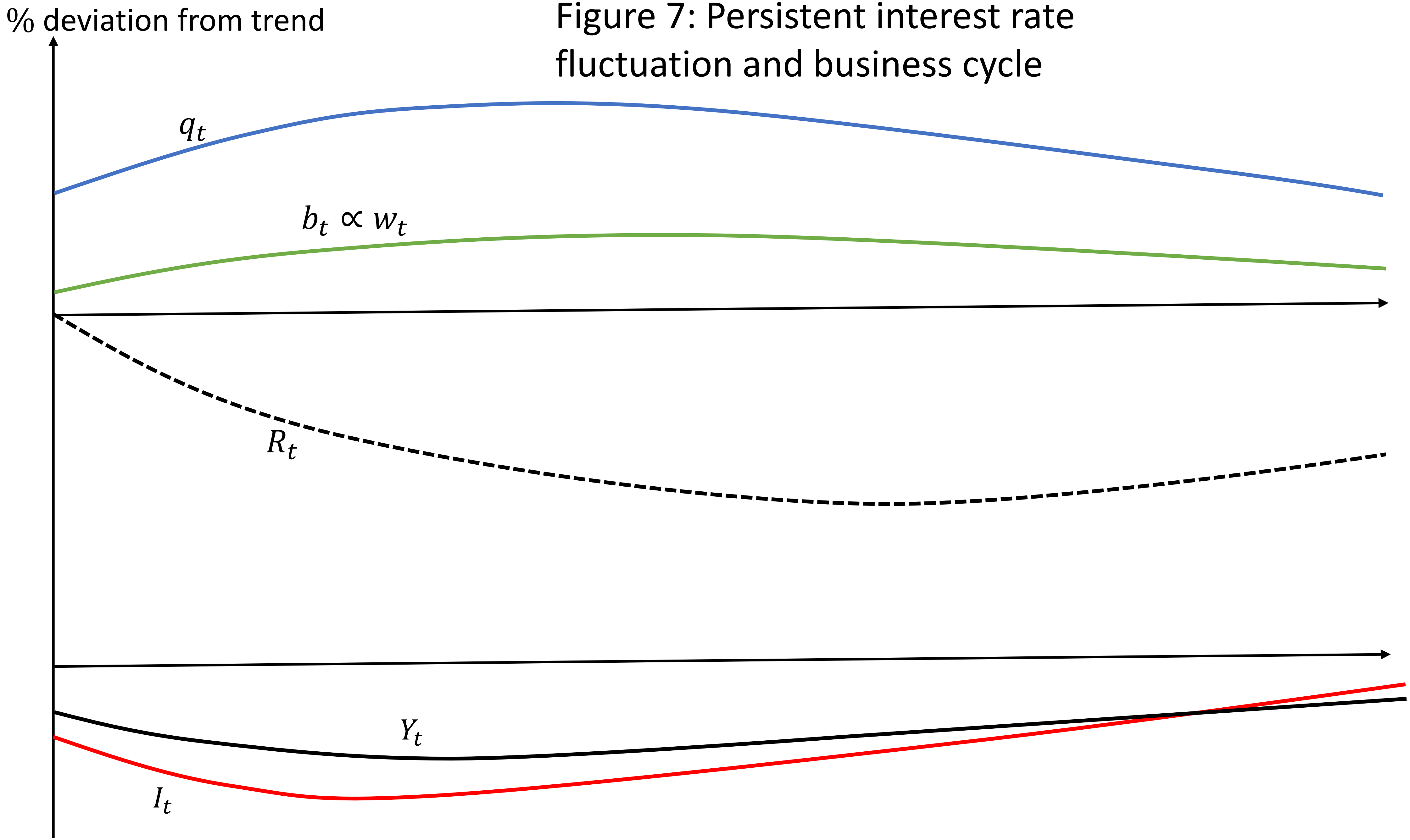


Figure 6: Interest rate fluctuation and business cycle

Figure 7: Persistent interest rate fluctuation and business cycle



Proposition. Mixed Equilibrium: High rent $f > f^{critical}$

(a) Plant owners are initially indifferent between stopping in some finite time and continuing forever

(b) Aggregate ratio of tools-to-plant is larger than one-to-one for continuing plant: $h_t > 1$

(c) The productivity of continuing plant increases over time

(d) The productivity of stopping plant decreases over time

Lemma: There is no equilibrium in which all plant shut down in finite time

Policy

Engineers do not maintain unless paid every period: Impossible to keep track of each engineer's trading history

If plant is easy to locate, then perhaps government could tax the plant owner's payroll at rate τ. Use the revenue to subsidize investment at rate s

$$\text{investment of engineers} = \frac{\text{engineers' net worth} \downarrow}{x + q - s - b \downdownarrows} \uparrow$$

$\longrightarrow \frac{\partial G}{\partial \tau} > 0$ in pure equilibrium with no stopping

Population-weighted average of the expected discounted utilities of engineers and savers rises with τ for small τ

Government acts as a social creditor to engineers