Credit Horizons

Nobuhiro Kiyotaki, John Moore and Shengxing Zhang
Princeton, Edinburgh and LSE

23 July 2020

Keynote Lecture

CESifo Area Conference on Macro, Money and International Finance
Munich

Question

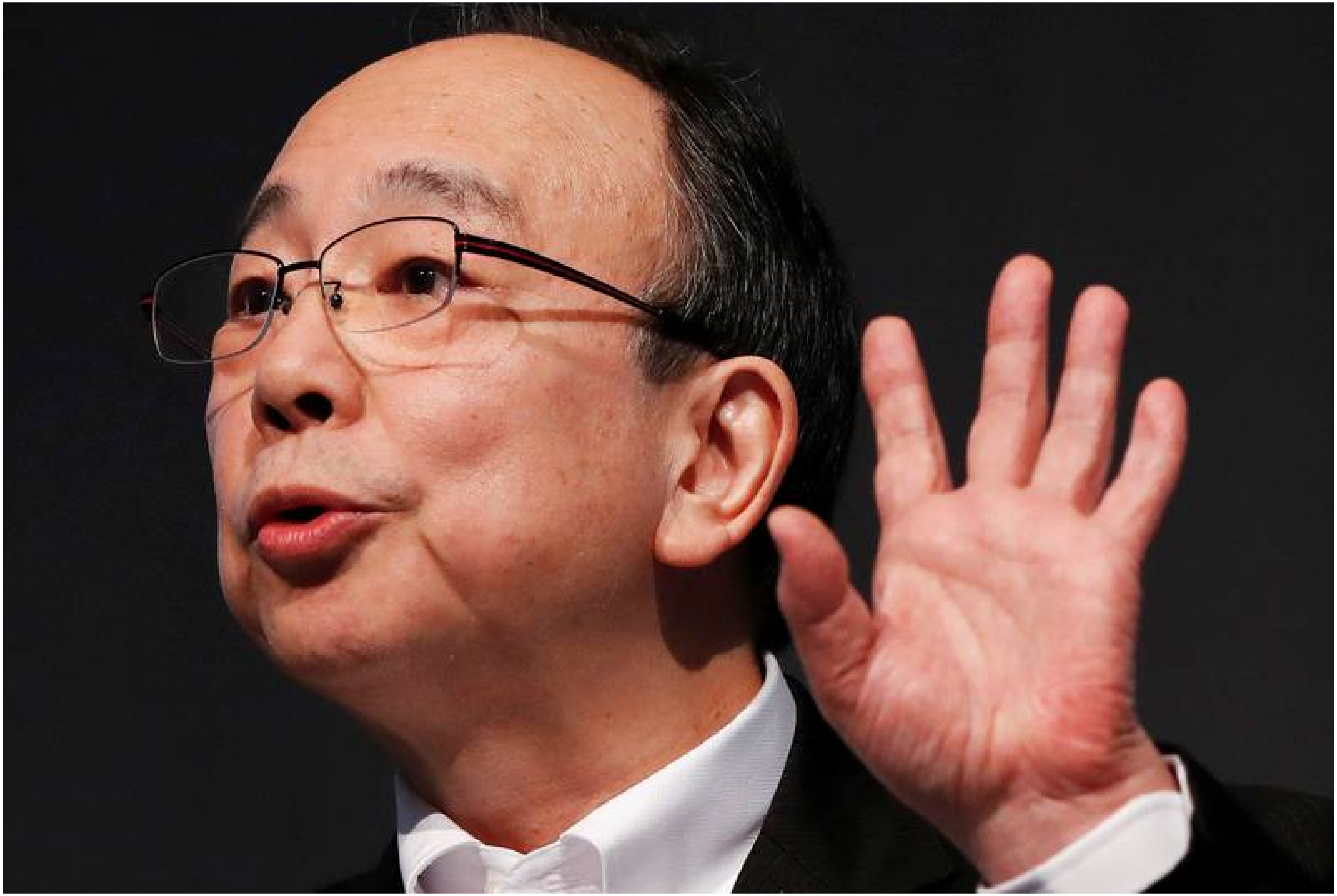
Why might entrepreneurs borrow largely against their near-term revenues? Are they unable to borrow much against the long-term horizon?

Today

We build a model of credit horizons

We explore firm dynamics
and the evolution of productivities

We examine the impact of low interest rates
on aggregate investment and growth
(The Amamiya Effect)



Mr. M. Amamiya
Deputy Governor, Bank of Japan

Model

Small open economy with an exogenous world real interest rate R

No aggregate uncertainty

For the moment, we consider steady state equilibrium (later, we examine effects of an unanticipated persistent drop in R)

Homogeneous perishable consumption/investment good at each date $t = 0, 1, 2, ...$(numeraire)

Continuum of agents, each maximizes utility of consumption

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln c_t \right], \ 0 < \beta < 1$$

Each agent sometimes has an investment opportunity (entrepreneur) and sometimes not (saver)

$$\text{Prob}\,(\text{entrepreneur at t} \mid \text{entrepreneur at t-1}) \;=\; \pi^E$$
$$\text{Prob}\,(\text{entrepreneur at t} \mid \text{saver at t-1}) \;=\; \pi^S$$

At each date t, an entrepreneur, say J, can jointly produce plant and tools from goods: within the period, per unit of plant,

$$x \text{ goods} \;\rightarrow\; \left\{ \begin{array}{c} \text{plant of quality 1} \\ \text{J-tool} \end{array} \right.$$

Plant and tools are ready to use from date $t+1$

Entrepreneur raises funds by selling the plant to savers
Crucially, she cannot commit her future human capital

Each tool is specific to the entrepreneur ("J-tool") in that only she knows how to use it – unless she sells it to another entrepreneur and teaches him

At each date, the owner of plant of quality z can hire any number $h \geq 0$ of tools (hiring each tool along with the entrepreneur who knows how to use it) at a competitive rental price w ("wage") to produce goods and maintain plant quality: within the period, per unit of plant,

$$\left.\begin{array}{c} \text{plant of quality } z \\ h \text{ tools} \\ f \text{ goods} \end{array}\right\} \rightarrow \left\{\begin{array}{c} y = az \quad \text{goods} \\ \lambda \text{ plant of quality } z' = z^{\theta} h^{\eta} \\ \lambda h \text{ tools} \end{array}\right.$$

where $\lambda < 1$ reflects depreciation in use, f is a fixed cost per unit of plant, and $\theta, \eta > 0$ with $\theta + \eta \leq 1$

The plant owner always has the option to stop, so his value of a unit of plant of quality z at the end of the period is given by

$$V(z) = \frac{1}{R} \max \left\{ 0, \max_{h \geq 0} \left[az - wh - f + \lambda V\left(z^{\theta} h^{\eta}\right) \right] \right\}$$

The plant owner must devise a long-term plan:

– stop after a finite number of periods T, or

– continue forever ($T = \infty$)?

For each $T = 0, 1, 2, \ldots$, define recursively owner's value of a unit of plant of current quality z stopping in T periods:

$$
\begin{aligned}
S^0(z) &= 0 \\
S^1(z) &= \frac{1}{R}(az - f) \\
S^2(z) &= \frac{1}{R}\max_{h \geq 0}\left[az - wh - f + \frac{\lambda}{R}(az^\theta h^\eta - f)\right] \\
&\vdots \\
S^T(z) &= \frac{1}{R}\max_{h \geq 0}\left[az - wh - f + \lambda S^{T-1}\left(z^\theta h^\eta\right)\right]
\end{aligned}
$$

For all value of z, $\quad V(z) = \sup_{T \geq 0} S^T(z)$

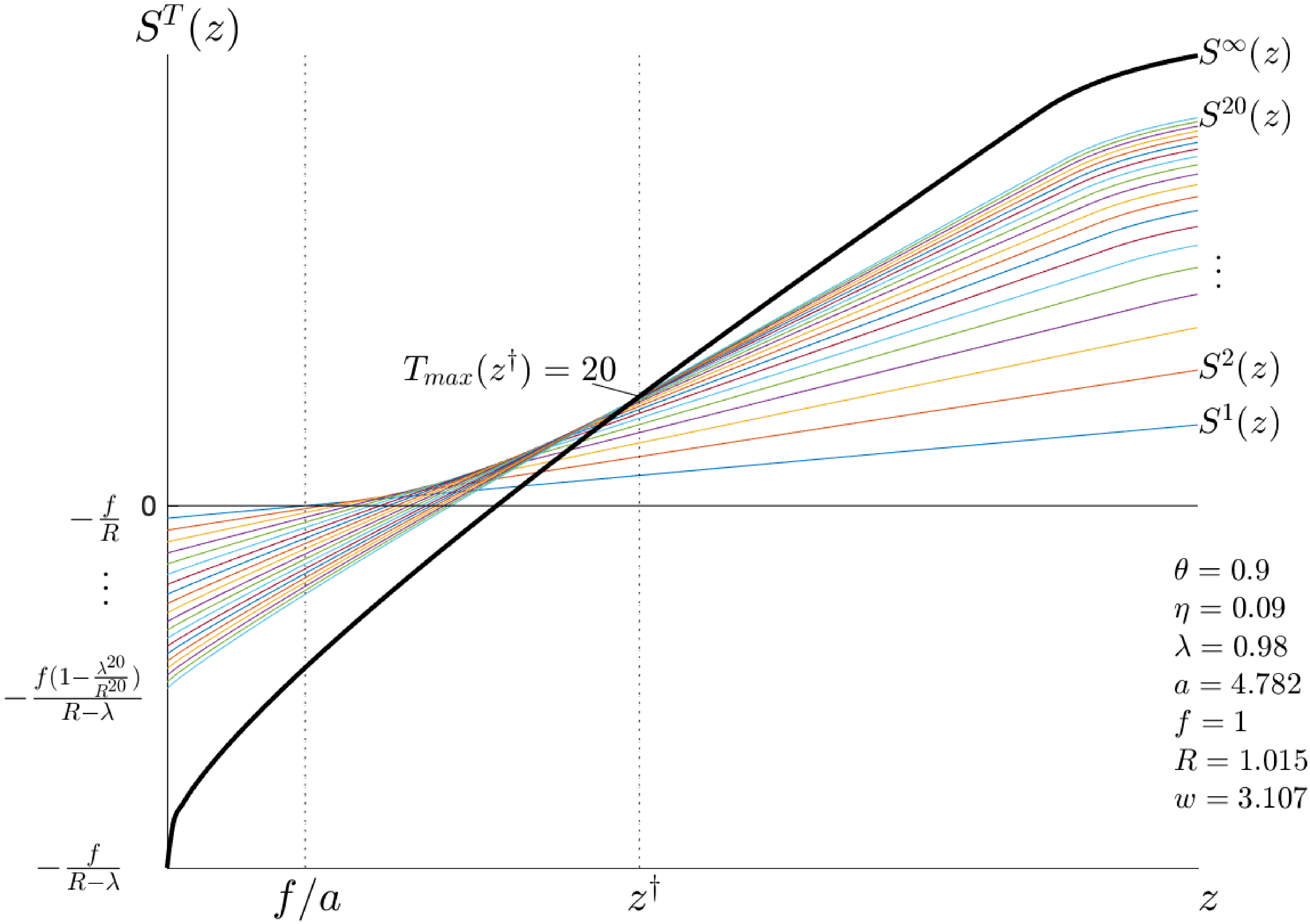
It turns out there is a clear dichotomy between stopping after a finite number of periods and continuing forever:

Lemma:

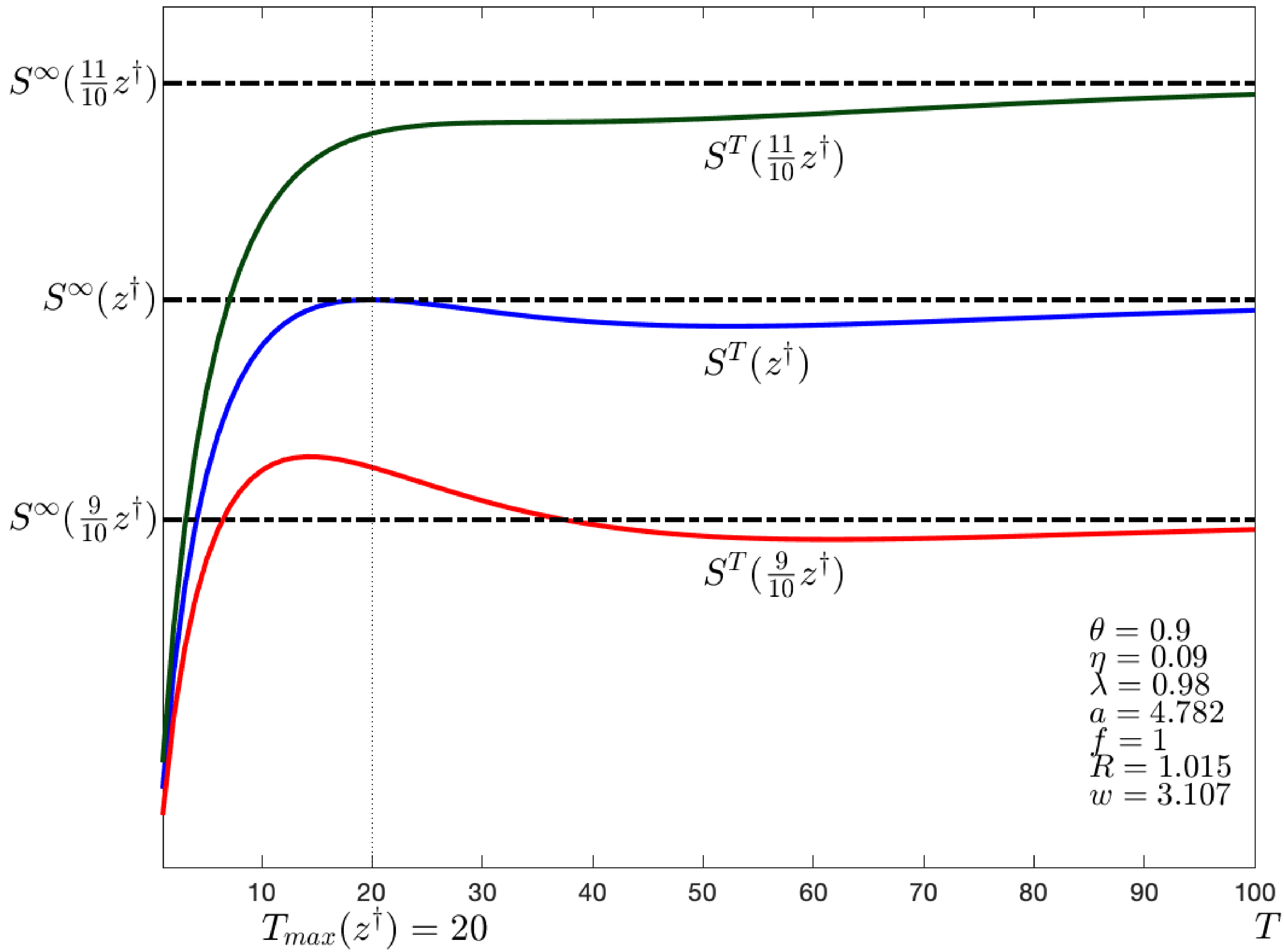
If the current plant quality z is below some cutoff value, $z^{\dagger}$, it is optimal for the plant owner to stop after, say, $T_{\text{max}}(z) < \infty$ periods

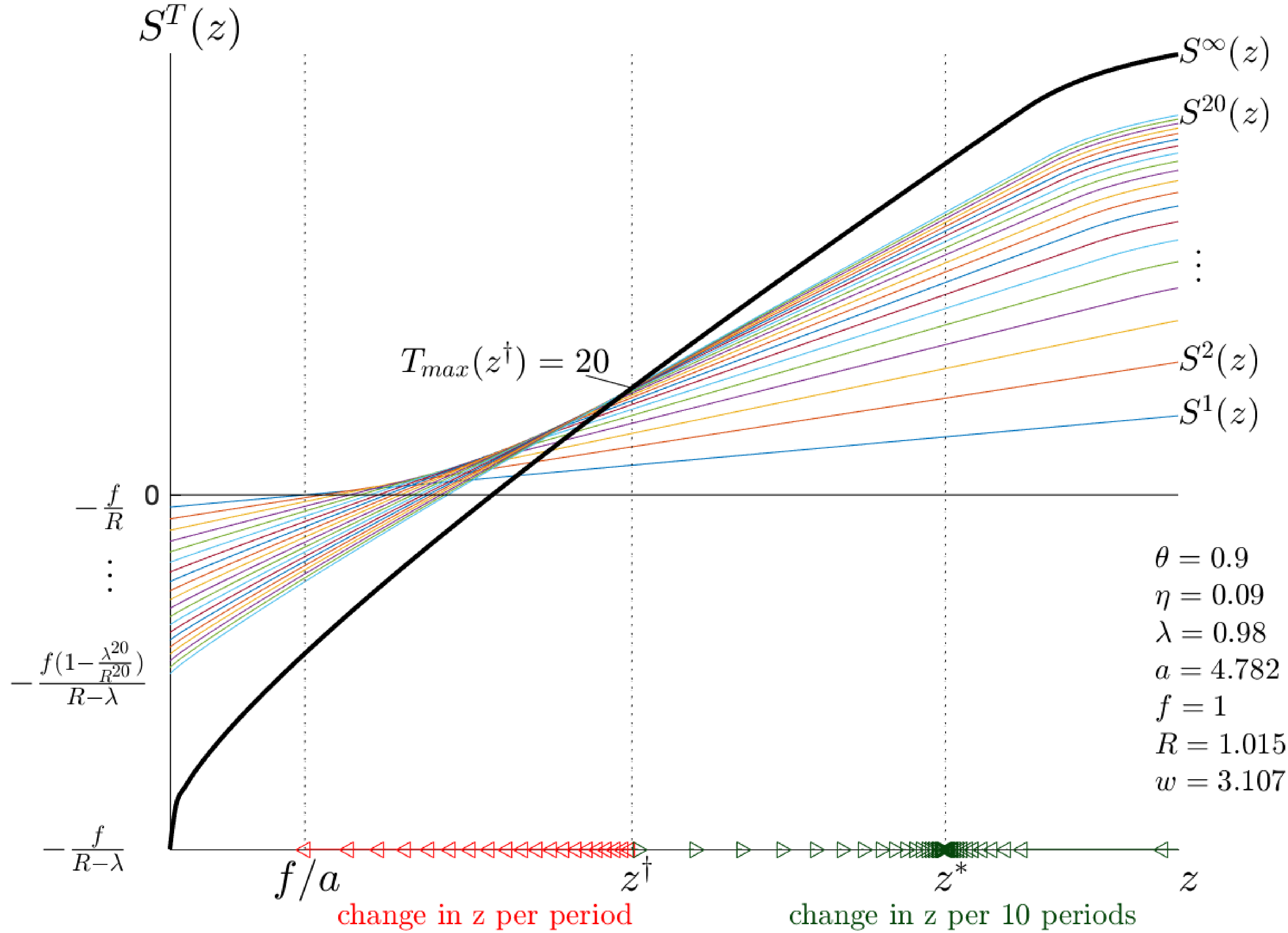
If z is above $z^{\dagger}$, it is optimal to continue forever

The cutoff value $z^{\dagger}$ increases with the fixed cost f and with the wage rate w

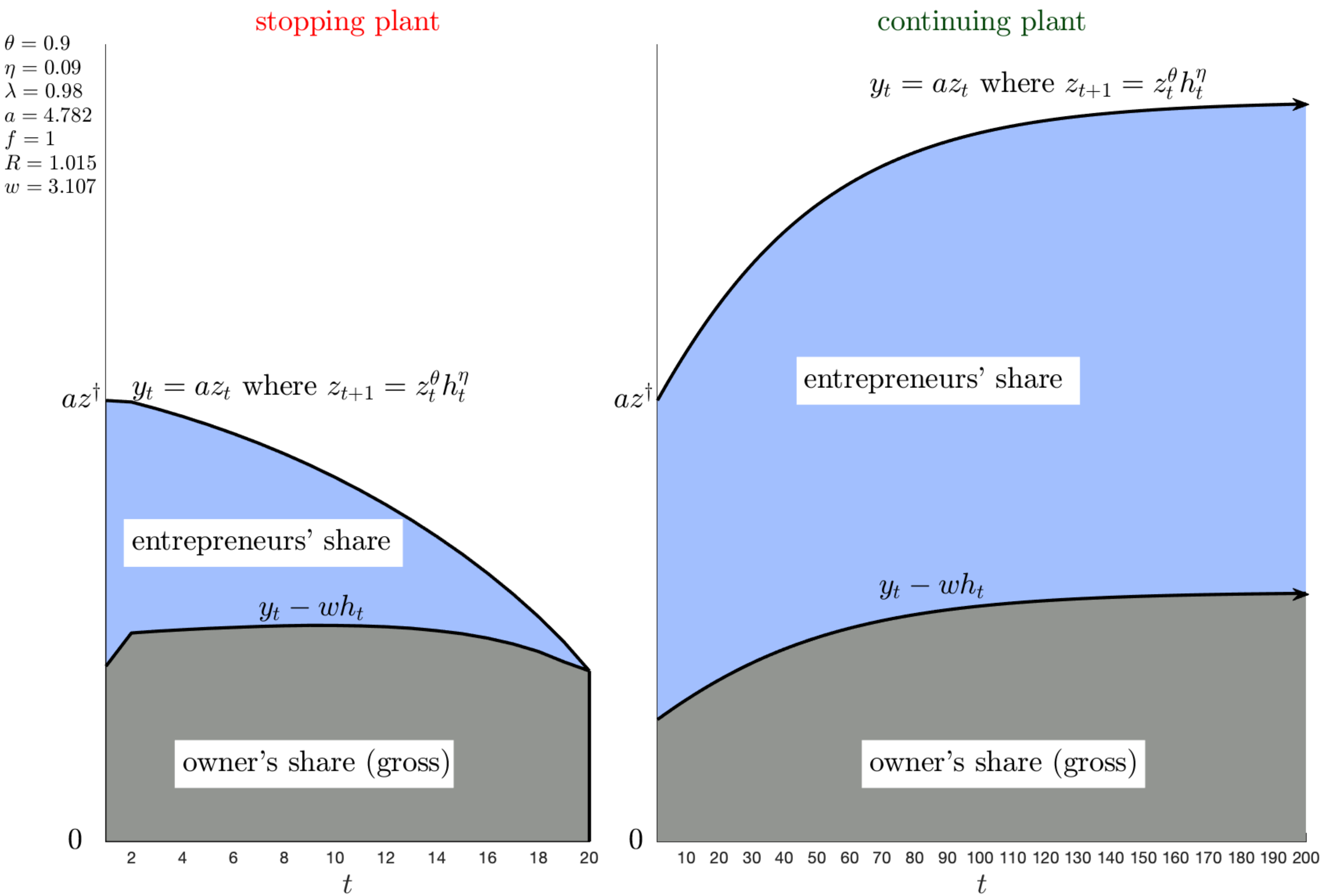


where $S^\infty(z) \equiv \lim_{T\to\infty} S^T(z)$


$S^\infty(\frac{11}{10}z^\dagger)$
$S^\infty(z^\dagger)$
$S^\infty(\frac{9}{10}z^\dagger)$
$S^T(\frac{11}{10}z^\dagger)$
$S^T(z^\dagger)$
$S^T(\frac{9}{10}z^\dagger)$
$\theta = 0.9$
$\eta = 0.09$
$\lambda = 0.98$
$a = 4.782$
$f = 1$
$R = 1.015$
$w = 3.107$
10
20
30
40
50
60
70
80
90
100
$T_{max}(z^\dagger) = 20$
T



$S^T(z)$
$S^\infty(z)$
$S^{20}(z)$
$S^2(z)$
$S^1(z)$
$T_{max}(z^\dagger) = 20$
$-\frac{f}{R}$
0
$-\frac{f(1-\frac{\lambda^{20}}{R^{20}})}{R-\lambda}$
$-\frac{f}{R-\lambda}$
f/a
$z^\dagger$
z^*
z
change in z per period
change in z per 10 periods
$\theta = 0.9$
$\eta = 0.09$
$\lambda = 0.98$
$a = 4.782$
$f = 1$
$R = 1.015$
$w = 3.107$



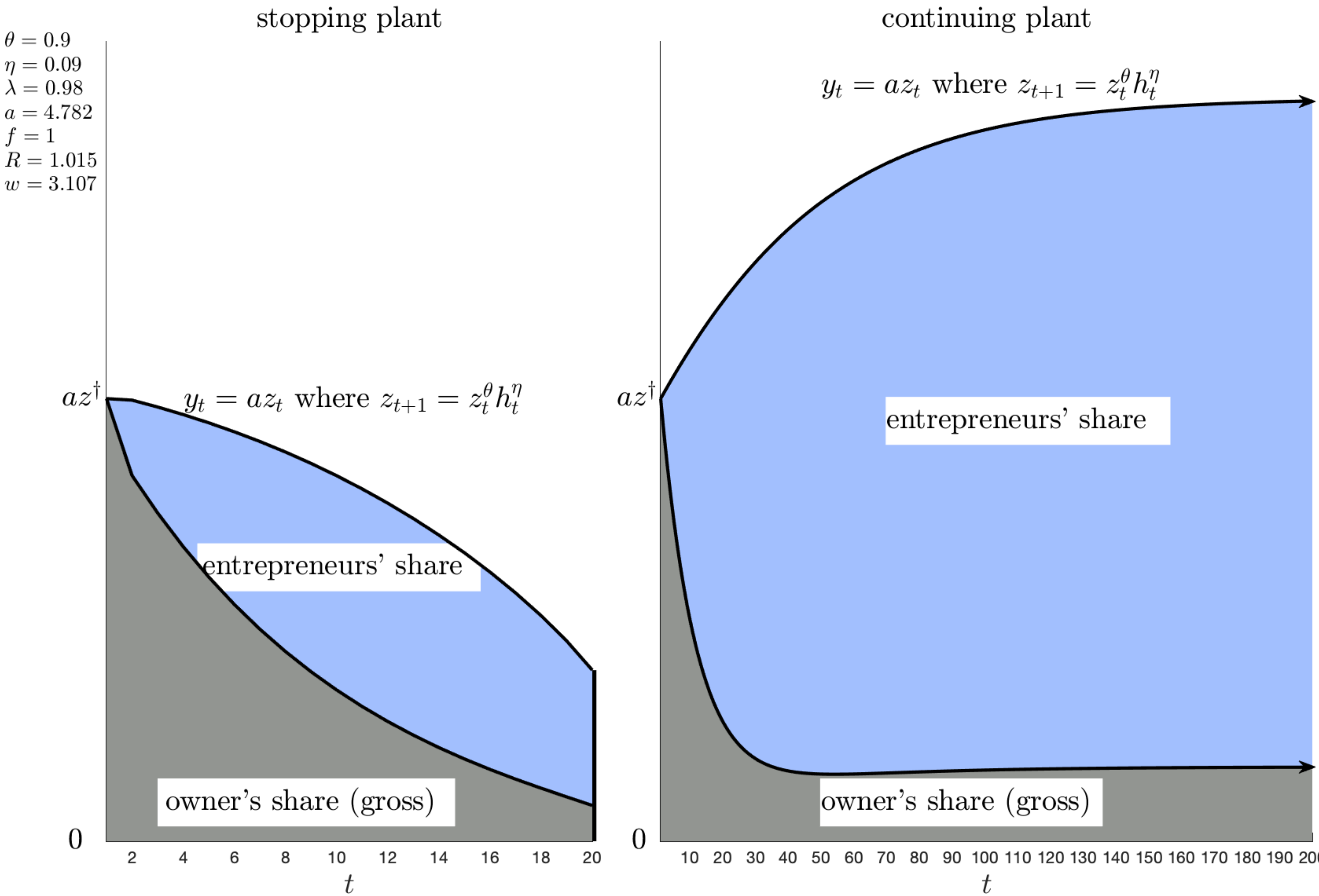
Division of Cash Flows

At each date t, whether current z_t lies above or below cutoff $z^\dagger$, an optimal sequence $\{h_t, z_{t+1}, h_{t+1}, z_{t+2}, h_{t+2}, ...\}$ equates discounted sum of marginal product to wage:

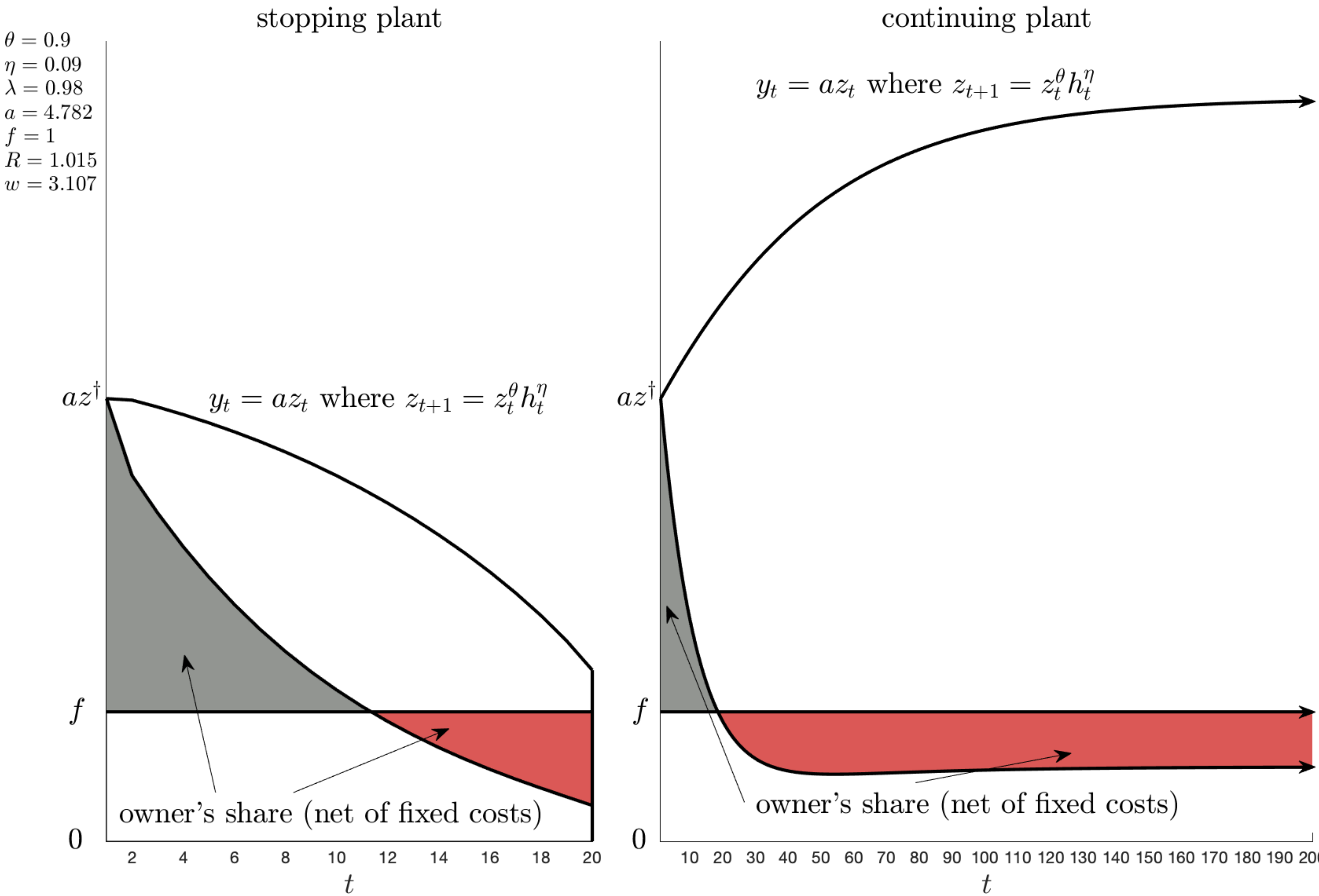
$$
\begin{aligned}
w &= \frac{\lambda}{R} a\eta \frac{z_{\mathrm{t+1}}}{h_{\mathrm{t}}} + \left(\frac{\lambda}{R}\right)^2 a\eta \frac{z_{\mathrm{t+1}}}{h_t} \theta \frac{z_{\mathrm{t+2}}}{z_{\mathrm{t+1}}} \\
&\quad + \left(\frac{\lambda}{R}\right)^3 a\eta \frac{z_{\mathrm{t+1}}}{h_{\mathrm{t}}} \theta \frac{z_{\mathrm{t+2}}}{z_{\mathrm{t+1}}} \theta \frac{z_{\mathrm{t+3}}}{z_{\mathrm{t+2}}} \\
&\quad + ... + \left(\frac{\lambda}{R}\right)^{T-t} a\eta \frac{z_{\mathrm{t+1}}}{h_t} \theta \frac{z_{\mathrm{t+2}}}{z_{\mathrm{t+1}}} \theta \frac{z_{\mathrm{t+3}}}{z_{\mathrm{t+2}}} \times ... \times \theta \frac{z_{\mathrm{T}}}{z_{\mathrm{T\text{-}1}}}
\end{aligned}
$$

Multiplying through by h_t, and simplifying

$$
wh_{\mathrm{t}} = \frac{\lambda}{R}\eta y_{\mathrm{t+1}} + \frac{\lambda^2}{R^2}\eta\theta y_{\mathrm{t+2}} + \frac{\lambda^3}{R^3}\eta\theta^2 y_{\mathrm{t+3}} + ... + \frac{\lambda^{\mathrm{T\text{-}t}}}{R^{\mathrm{T\text{-}t}}}\eta\theta^{\mathrm{T\text{-}t\text{-}1}} y_{\mathrm{T}}
$$



Underlying Division of Returns



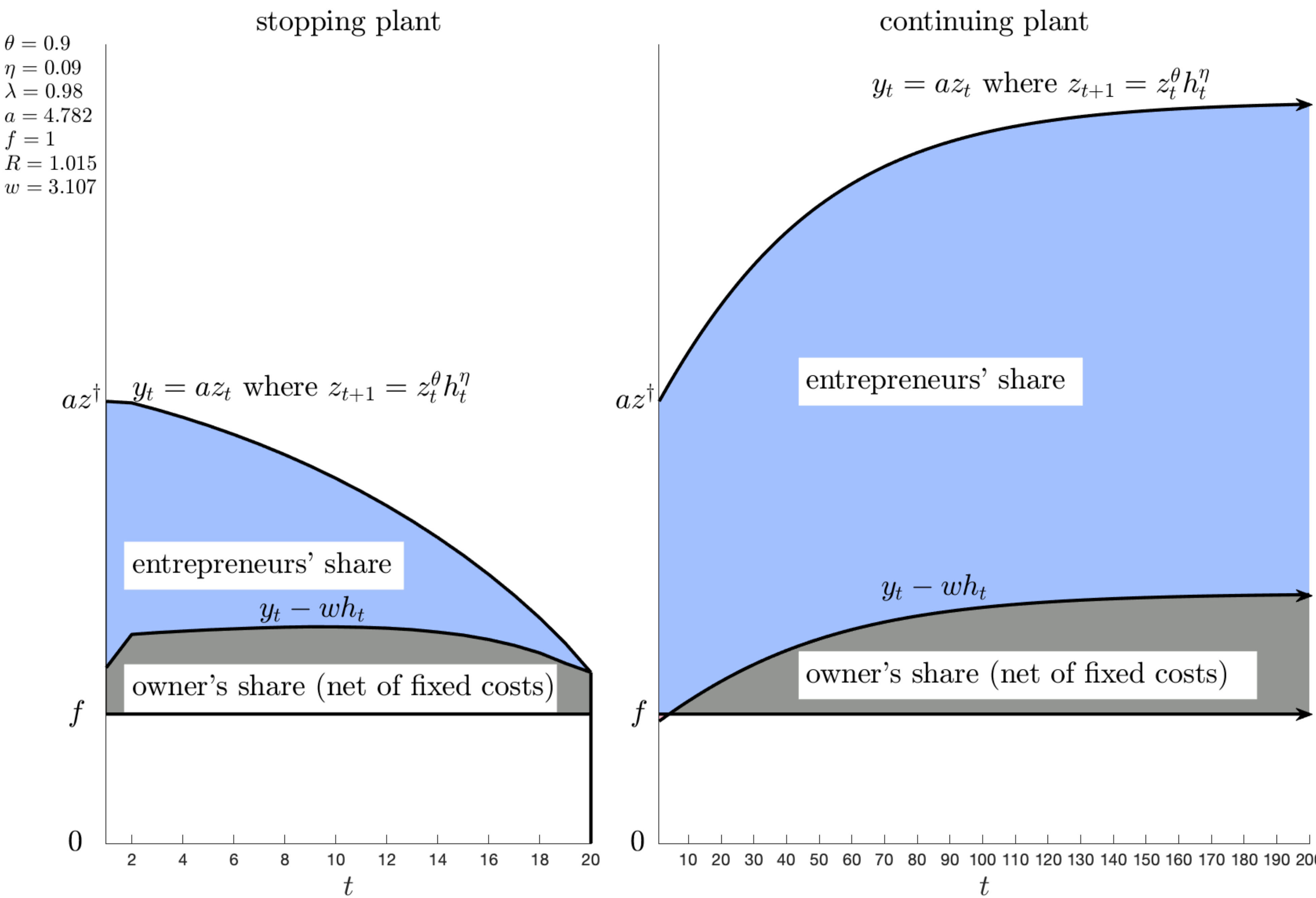
Owner's Underlying Share of Returns (net of fixed costs)

An entrepreneur raises funds by selling new plant (which has quality 1) at price

$$\begin{aligned} b = V(1) \ = \ & \frac{1}{R}(a-f) \ + \ \frac{\lambda}{R^2}[y_2(1-\eta)-f] \\ + \ & \frac{\lambda^2}{R^3}[y_3(1-\eta-\eta\theta)-f] \\ & \ldots \\ + \ & \frac{\lambda^{\text{T-2}}}{R^{\text{T-1}}}[y_{\text{T-1}}(1-\eta-\eta\theta-\ldots-\eta\theta^{\text{T-3}})-f] \\ + \ & \frac{\lambda^{\text{T-1}}}{R^{\text{T}}}[y_{\text{T}}(1-\eta-\eta\theta-\ldots-\eta\theta^{\text{T-2}})-f] \end{aligned}$$

b = borrowing capacity, per unit of investment

NB suggestive that borrowing capacity may fall as R falls



Division of Cash Flows (net of fixed costs)

The budget constraint of an agent at date t who has h_t tools and d_t financial assets (maturing one-period discount bonds plus returns to plant ownership) is

$$c_t + (x - b)i_t + \frac{d_{t+1}}{R} = wh_t + d_t,$$

where h_t is positive iff the agent was an entrepreneur yesterday; and investment i_t is positive iff the agent is an entrepreneur today, in which case her tools tomorrow will be

$$h_{t+1} = \lambda h_t + i_t$$

The budget constraint can be written as

$$c_t + (x - b)h_{t+1} + \frac{d_{t+1}}{R} = [w + \lambda(x - b)]h_t + d_t \equiv n_t,$$

where n_t is net worth

When the rate of return on investment with maximal borrowing, R^E, exceeds the interest rate

$$R^E = \frac{w + \lambda(x - b)}{x - b} > R,$$

the entrepreneur's consumption and investment are

$$\begin{aligned} c_t &= (1-\beta)n_t \\ (x-b)h_{t+1} &= \beta n_t \end{aligned}$$

A saver's consumption and asset holdings are

$$\begin{aligned} c_t &= (1-\beta)n_t \\ \frac{d_{t+1}}{R} &= \beta n_t \end{aligned}$$

A steady state equilibrium of our small open economy is characterized by the wage w and new-plant price b, together with the quantity choices of savers/plant owners (c, d, h, z, y), entrepreneurs (c, h, i), and foreigners (who have net asset holdings D^*), such that the markets for goods, tools, plant, and bonds all clear

Aggregating across entrepreneurs and savers, we obtain tool supply H, asset demand D, consumption C, and respective net worths (N^E and N^S):

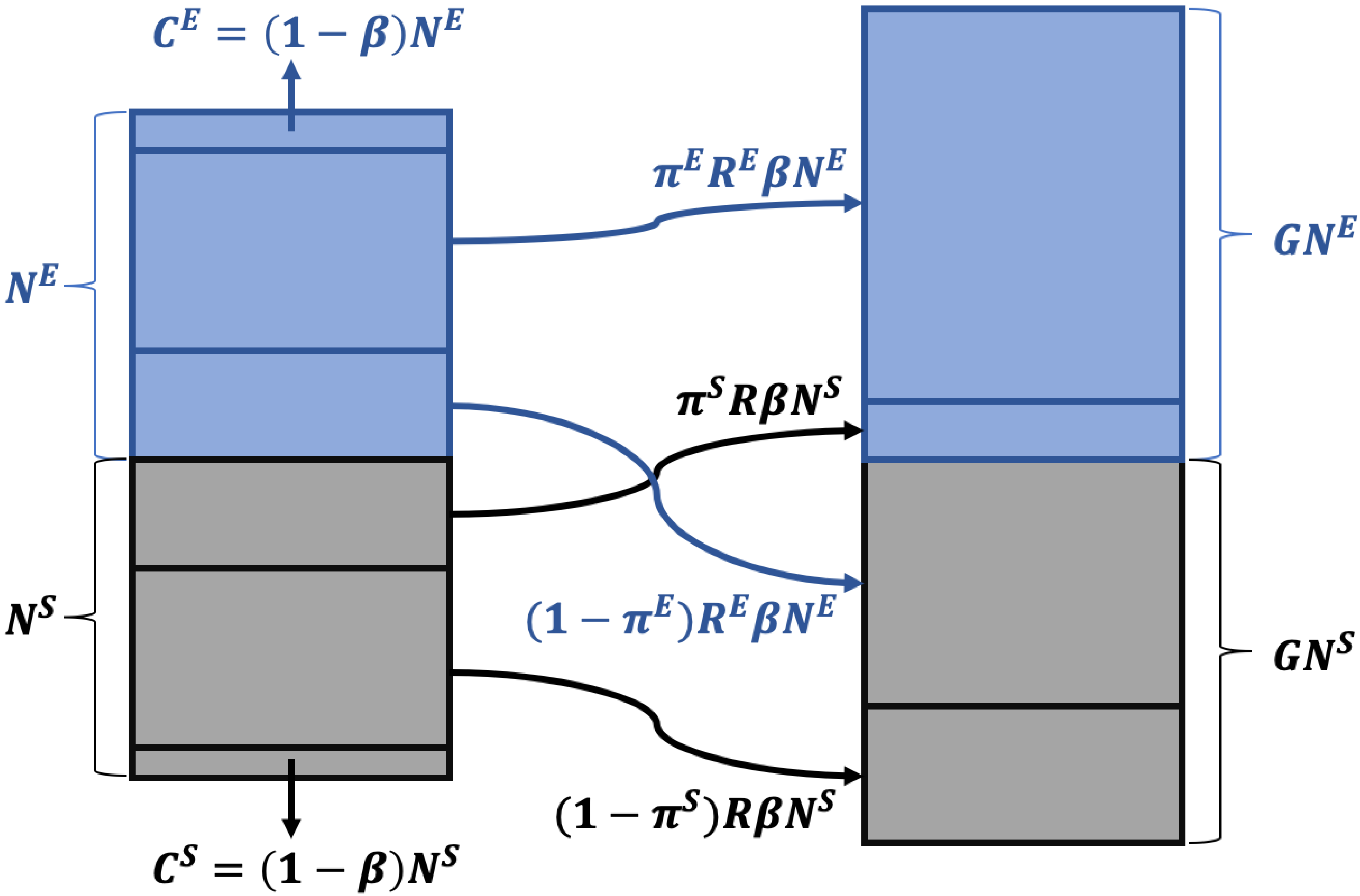
$$
\begin{aligned}
(x-b)H_{t+1} &= \beta N_t^E \\
\frac{D_{t+1}}{R} &= \beta N_t^S \\
C_t &= (1-\beta)\left(N_t^E + N_t^S\right) \\
N_t^E &= \pi^E\left[w + \lambda(x-b)\right]H_t + \pi^S D_t \\
N_t^S &= (1-\pi^E)\left[w + \lambda(x-b)\right]H_t + (1-\pi^S)D_t
\end{aligned}
$$

The economy exhibits endogeneous growth G: along a steady state path,

$$\frac{H_{t+1}}{H_t} = \frac{D_{t+1}}{D_t} = \frac{C_{t+1}}{C_t} = G$$

$$GN_t^E \;=\; N_{t+1}^E = \pi^E R^E \beta N_t^E + \pi^S R \beta N_t^S$$

$$GN_t^S \;=\; N_{t+1}^S = (1-\pi^E) R^E \beta N_t^E + (1-\pi^S) R \beta N_t^S$$



$C^E = (1-\beta)N^E$
N^E
$\pi^E R^E \beta N^E$
GN^E
$\pi^S R \beta N^S$
N^S
$(1-\pi^E) R^E \beta N^E$
GN^S
$(1-\pi^S) R \beta N^S$
$C^S = (1-\beta)N^S$

Proposition 1: There exists a critial value f^{critical} of the fixed cost such that

P-Region (Pure equilibrium with no stopping; low fixed cost): $f < f^{\text{critical}}$

(i) No plant owner stops: $z^{\dagger} < 1$

(ii) Aggregate ratio of tools-to-plant stays one-to-one (because equal initial supply, equal depreciation, no stopping): for all t, $h_t = 1$

$\rightarrow$ all plant is maintained at initial quality 1:
for all t, $z_t = 1$ $\quad (\because z_{t+1} = z_t^{\theta} h_t^{\eta})$
and $y_t = a$

M-Region (Mixed equilibrium; high fixed cost): $f > f^{\text{critical}}$

(i) Plant owners are initially indifferent between stopping after some finite time and continuing forever: $z^{\dagger} = 1$

(ii) Aggregate ratio of tools-to-plant is larger than one-to-one for continuing plant: for all t, $h_t > 1$

(iii) With decreasing returns to scale, $\theta + \eta < 1$, quality of continuing plant increases over time, converging to some $z^* \in (1, \infty)$

With constant returns to scale, $\theta + \eta = 1$, continuing plant quality grows at some constant rate $g > 1$

(iv) Stopping plant decreases in quality over time; stop occurs just before z_t falls below f/a

Proposition 2P (P-Region):

For an open set of parameters (in particular with R and λ not too far from 1), a pure equilibrium with no stopping exists such that

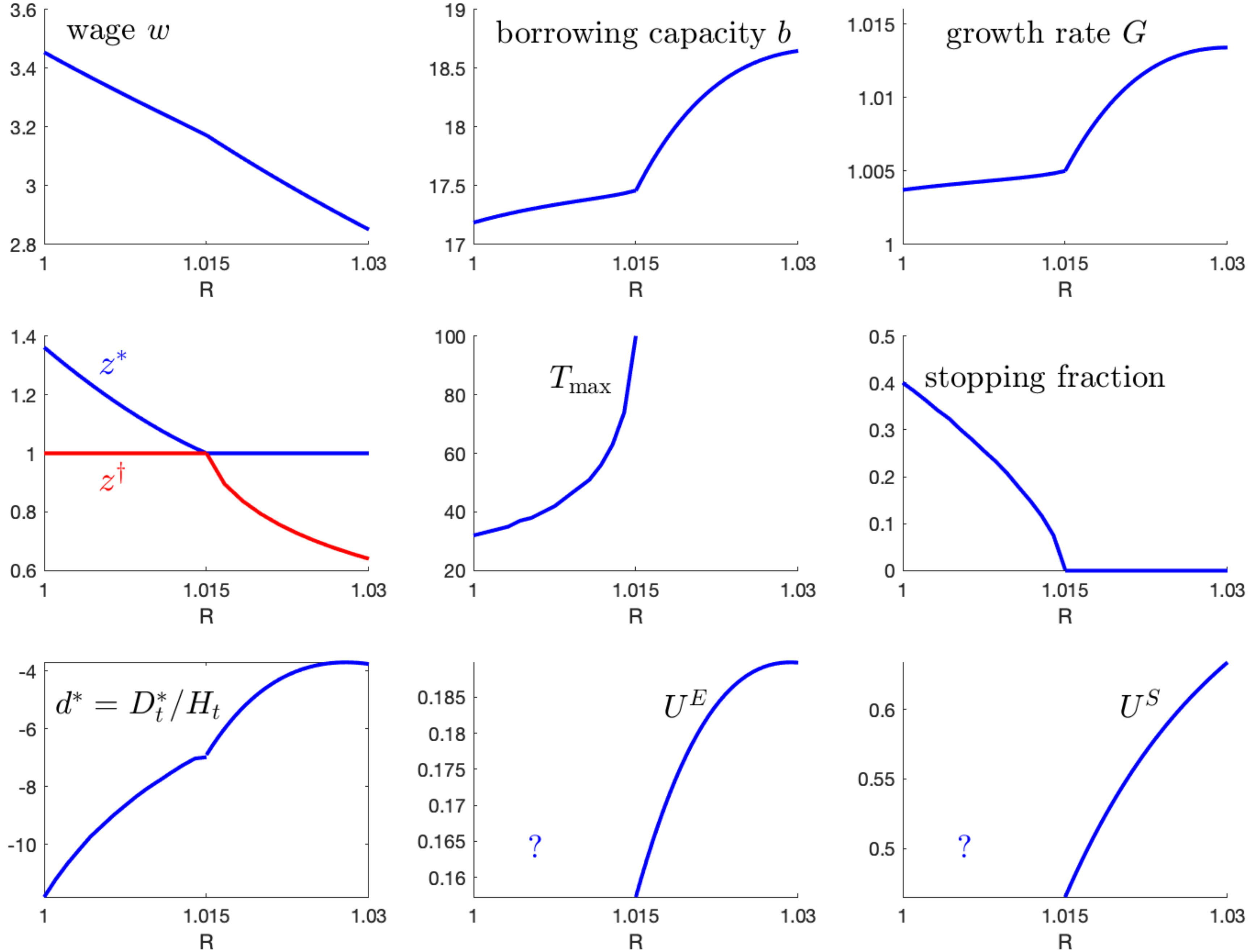
(i) an unexpected permanent drop in the interest rate R leads to a lower steady state growth rate G

(ii) immediately following the drop in R, all agents (entrepreneurs and savers) can be strictly worse off

Proposition 2M (M-Region):

In a mixed equilibrium, we demonstrate numerically that for an open set of parameters (in particular with R and λ not too far from 1), an unexpected permanent drop in the interest rate R can lead to a lower steady state growth rate G

e.g. $\theta = 0.9, \eta = 0.09, \lambda = 0.98, a = 4.782, f = 1,$
$x = 29.30, \beta = 0.92, \pi^E = 0.7, \pi^S = 0.1$:



$R < 1.015$: M-Region

$R > 1.015$: P-Region

Intuition (for P-Region)

In P-Region, there is no stopping ($T = \infty$) and, for all t, $h_t = 1$, $z_t = 1$, $y_t = a$, so the entrepreneur's borrowing capacity per unit of investment is simply

$$b = \frac{a - w - f}{R - \lambda}$$

and the wage (discounted sum of marginal product) is

$$w = \frac{\lambda}{R}\eta a + \frac{\lambda^2}{R^2}\eta\theta a + \frac{\lambda^3}{R^3}\eta\theta^2 a + \ldots = \frac{a\lambda\eta}{R - \lambda\theta}$$

which rises significantly with the fall in R – because the entrepreneur's marginal product has a long horizon

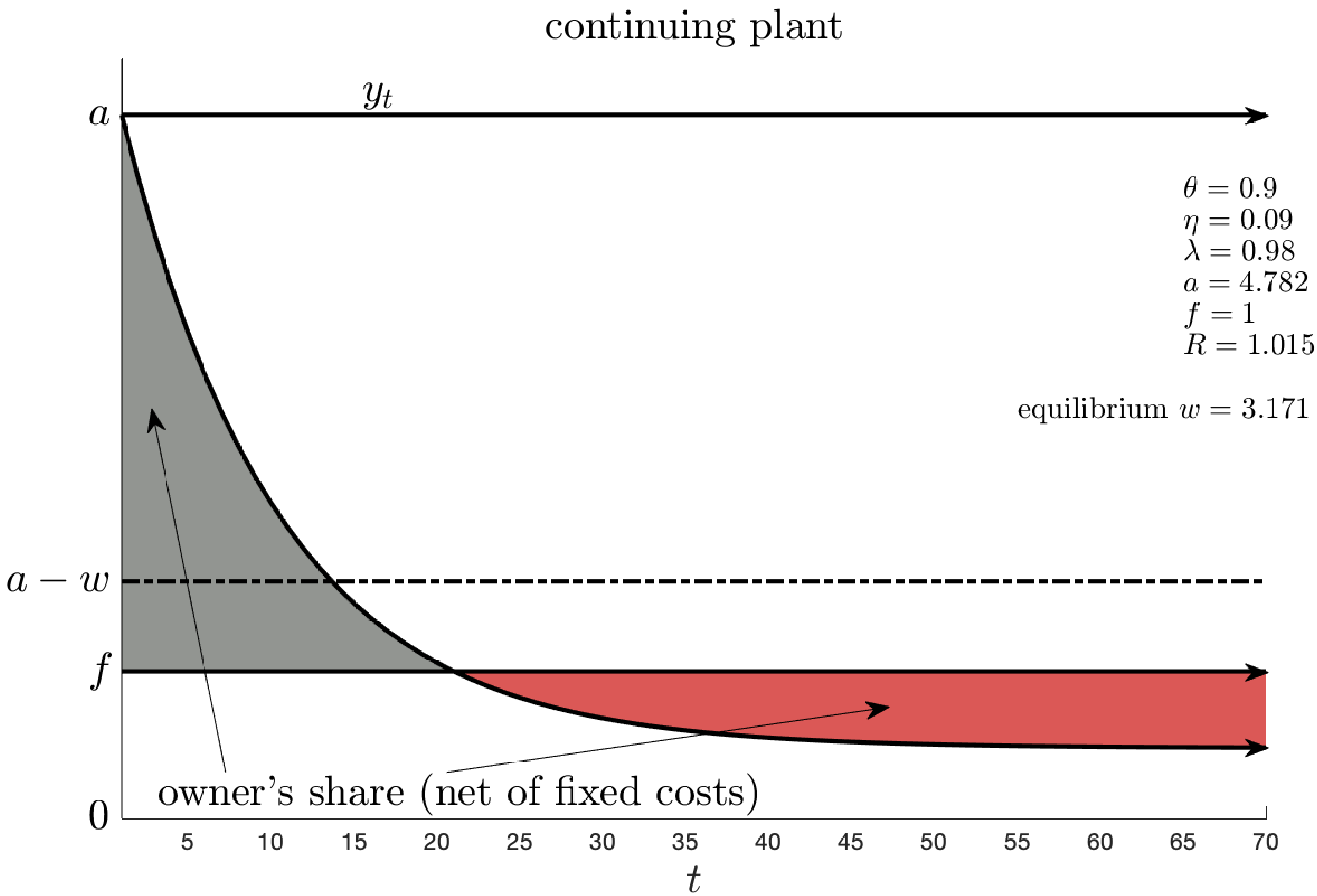
Thus, e.g. with constant returns to scale, $\theta + \eta = 1$,

$$b = \frac{a}{R - \lambda\theta} - \frac{f}{R - \lambda}$$

entrepreneur's borrowing capacity		present value of plant owner's share of gross revenues		present value of fixed costs

Because $\theta < 1$, the fall in R increases the present value of fixed costs proportionately more than the present value of the plant owner's share of gross revenues

Net, the fall in R can *decrease* the borrowing capacity



Owner's Underlying Share of Returns (net of fixed costs)

P-Region: no stopping and for all t, $h_t = 1$, $z_t = 1$, $y_t = a$

The fall in borrowing capacity b can be strong enough – overcoming any rise in net worth from, inter alia, the increase in wage – to stifle investment and growth:

$$\text{gross investment } (H_{t+1}) \downarrow \; = \; \text{saving rate } (\beta) \times \frac{\text{net worth of entrepreneurs } (N_t^E)\uparrow}{\text{investment cost } (x) - \text{borrowing capacity } (b)\downarrow}$$

AMAMIYA !

Extensions:

Heterogeneity across plants: in initial z and/or idiosyncratic shocks to subsequent $z', z'', \ldots$

Heterogeneity across entrepreneurs: in investment cost x

Choice of technique by entrepreneurs

Land model

Bargaining model