Credit Horizons

Nobuhiro Kiyotaki, John Moore and Shengxing Zhang
Princeton, Edinburgh and LSE

21 September 2020
CFM Academic Seminar, LSE

Questions

To finance long-term investment, entrepreneurs raise external funds against collateral or future revenues

If the latter, why might entrepreneurs borrow largely against their near-term revenues (horizon)?

How do credit horizons interact with firm dynamics?

Can lower real interest rates stifle investment and growth?

Approach

An engineer/entrepreneur invests to construct, jointly, *plant* (in a building) and *tools* (human capital) for future production

Output depends upon plant quality (productivity), which evolves over time

Future plant quality depends on both current quality and engineers' maintenance

Engineer cannot precommit her future human capital. To finance investment, engineer sells plant to a saver

At each date, plant owner (saver) needs to pay a fixed cost (rent or user cost of building) to operate plant

Plant owner hires engineers for maintenance in a competitive market

Engineer's wage today equals the present value of her marginal impact on entire future output. Engineer cannot precommit to work for less than this wage (non-exclusivity constraint)

Over time, the fraction of the quality of plant attributable to engineers' cumulative maintenance rises
$\longrightarrow$ Owner's fraction of gross return from *initial* plant falls
$\longrightarrow$ Investing engineer's borrowing capacity is governed by near-term revenues

A persistent fall in real interest rates $\longrightarrow$ Present value of fixed costs may rise more than that of pledgeable revenue $\longrightarrow$ Engineer's borrowing capacity may fall $\longrightarrow$ Investment and growth can be stifled

Model

Small open economy with an exogenous world real interest rate R

No aggregate uncertainty

For the moment, we consider steady state equilibrium (later, we examine effects of an unanticipated persistent drop in R)

Homogeneous perishable consumption/investment good at each date $t = 0, 1, 2, ...$(numeraire)

Continuum of agents, each maximizes utility of consumption

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln c_t \right], \ 0 < \beta < 1$$

Each agent sometimes has an investment opportunity (engineer) and sometimes not (saver)

$$\begin{aligned} \text{Prob}\,(\text{engineer at t} \mid \text{engineer at t-1}) &= \pi^E \\ \text{Prob}\,(\text{engineer at t} \mid \text{saver at t-1}) &= \pi^S \end{aligned}$$

At each date t, an engineer, say E, can jointly produce plant and tools from goods: within the period, per unit of plant,

$$x \text{ goods} \rightarrow \left\{ \begin{array}{c} \text{plant of quality 1} \\ \text{E-tool} \end{array} \right.$$

Plant and tools are ready to use from date $t+1$

Engineer raises funds by selling the plant to savers
Crucially, she cannot commit her future human capital

Each tool is specific to the engineer ("E-tool") in that only she knows how to use it – unless she sells it to another engineer and teaches him

At each date, the owner of plant of quality z can hire any number $h \geq 0$ of tools (hiring each tool along with the engineer who knows how to use it) at a competitive rental price w ("wage") to produce goods and maintain plant quality: within the period, per unit of plant,

$$\left.\begin{array}{c} \text{plant of quality } z \\ h \text{ tools} \\ f \text{ goods} \end{array}\right\} \to \left\{\begin{array}{c} y = az \quad \text{goods} \\ \lambda \text{ plant of quality } z' = z^{\theta} h^{\eta} \\ \lambda h \text{ tools} \end{array}\right.$$

where $\lambda < 1$ reflects depreciation in use, f is a fixed cost per unit of plant, and $\theta, \eta > 0$ with $\theta + \eta \leq 1$

The plant owner always has the option to stop, so his value of a unit of plant of quality z at the end of the period is given by

$$V(z) = \frac{1}{R} \max \left\{ 0, \max_{h \geq 0} \left[az - wh - f + \lambda V \left(z^{\theta} h^{\eta} \right) \right] \right\}$$

The plant owner must devise a long-term plan:

– stop after a finite number of periods T, or

– continue forever ($T = \infty$)?

For each $T = 0, 1, 2, ...$, define recursively owner's value of a unit of plant of current quality z stopping in T periods:

$$
\begin{aligned}
S^0(z) &= 0 \\
S^1(z) &= \frac{1}{R}\,(az - f) \\
S^2(z) &= \frac{1}{R}\max_{h \geq 0}\left[az - wh - f + \frac{\lambda}{R}(az^{\theta}h^{\eta} - f)\right] \\
&\vdots \\
S^T(z) &= \frac{1}{R}\max_{h \geq 0}\left[az - wh - f + \lambda S^{T-1}\left(z^{\theta}h^{\eta}\right)\right]
\end{aligned}
$$

For all value of z, $\quad V(z) = \sup_{T \geq 0} S^T(z)$

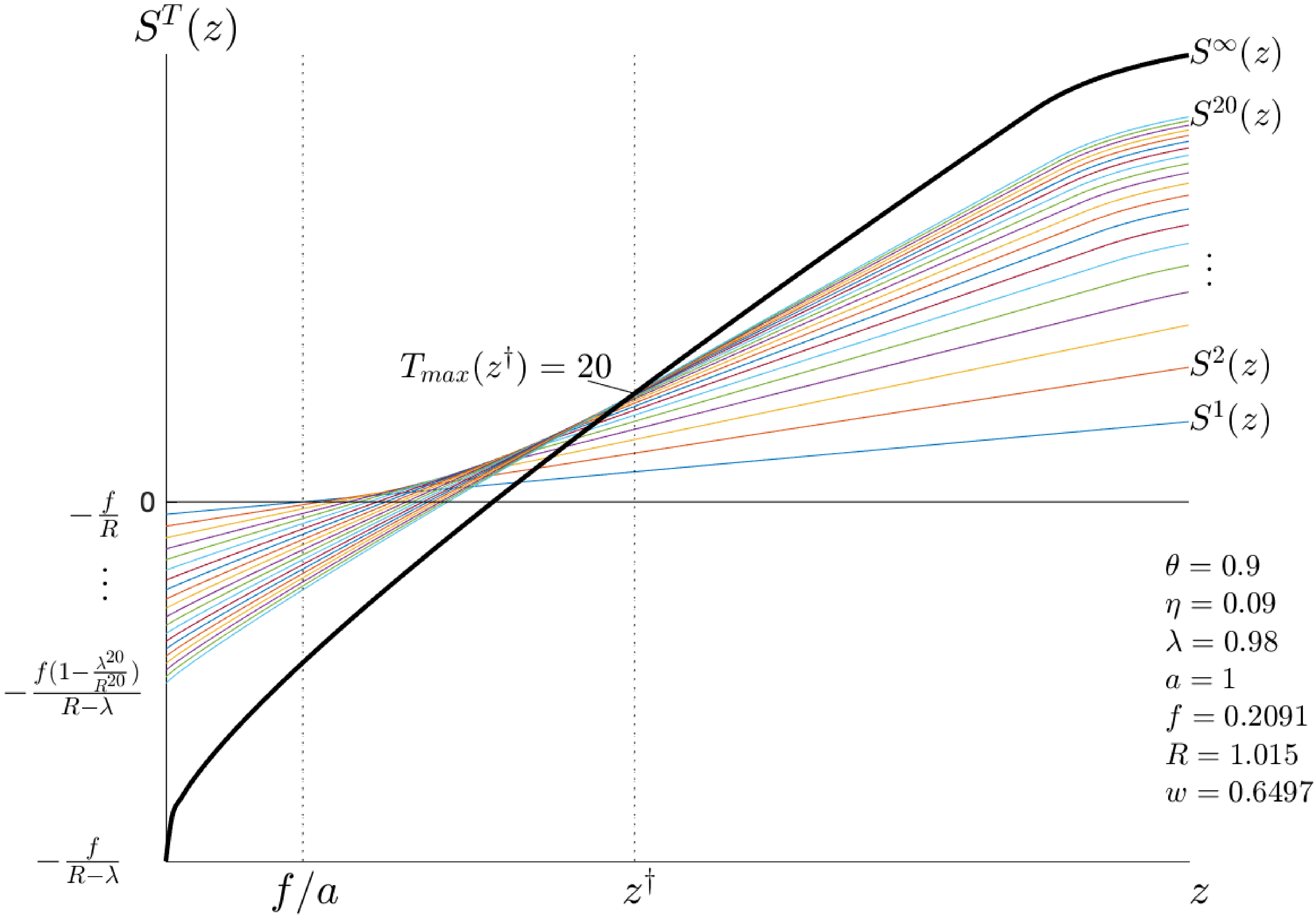
It turns out there is a clear dichotomy between stopping after a finite number of periods and continuing forever:

Lemma:

If the current plant quality z is below some cutoff value, $z^\dagger$, it is optimal for the plant owner to stop after, say, $T_{\mathsf{max}}(z) < \infty$ periods

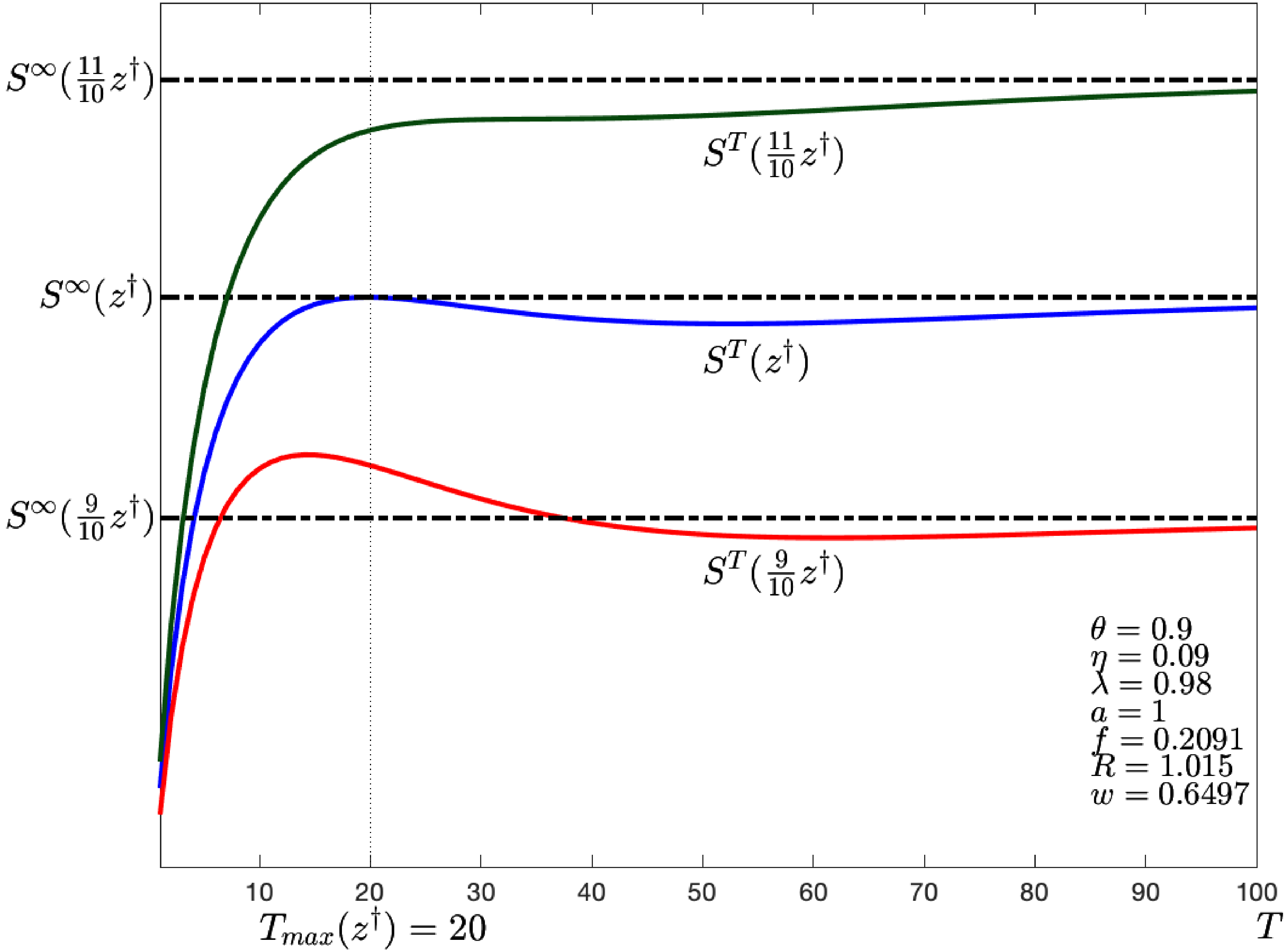
If z is above $z^\dagger$, it is optimal to continue forever

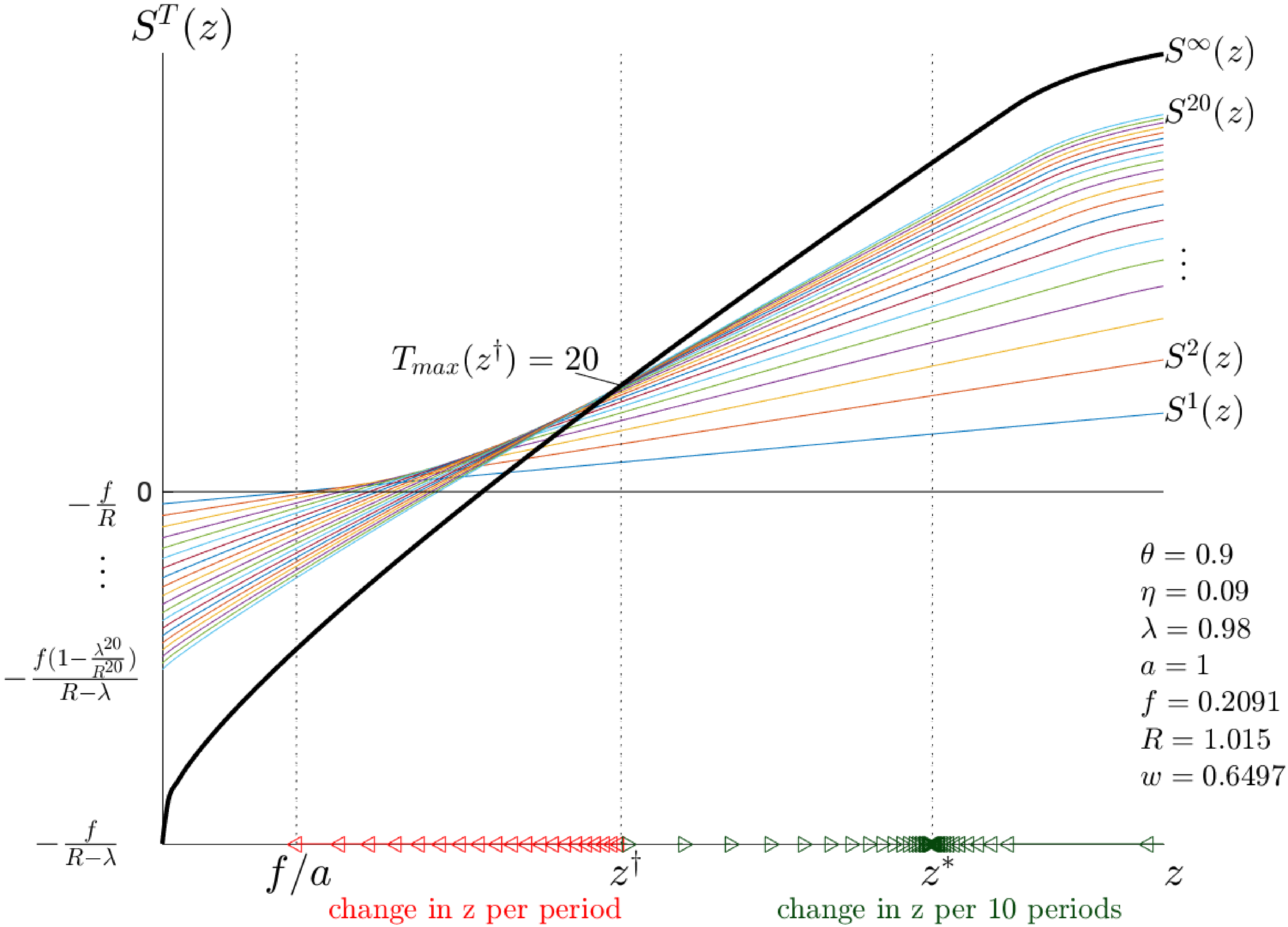
The cutoff value $z^\dagger$ increases with the fixed cost f and with the wage rate w



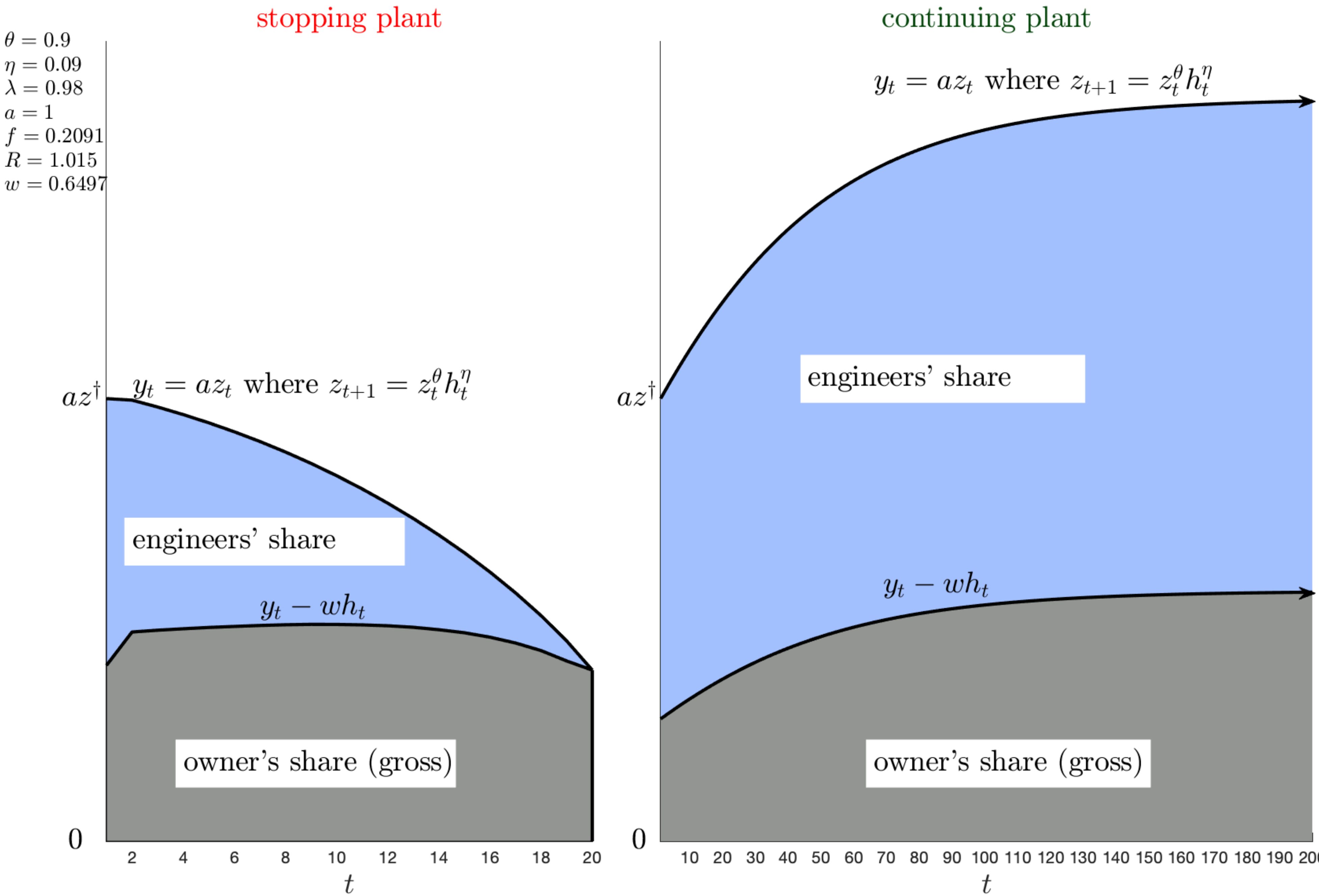
where $S^\infty(z) \equiv \lim_{T\to\infty} S^T(z)$

Figure 2: $S^T(z)$ as a function of horizon T





$S^T(z)$
$S^\infty(z)$
$S^{20}(z)$
$S^2(z)$
$S^1(z)$
$T_{max}(z^\dagger) = 20$
$-\frac{f}{R}$
0
$-\frac{f(1-\frac{\lambda^{20}}{R^{20}})}{R-\lambda}$
$-\frac{f}{R-\lambda}$
f/a
$z^\dagger$
z^*
z
change in z per period
change in z per 10 periods
$\theta = 0.9$
$\eta = 0.09$
$\lambda = 0.98$
$a = 1$
$f = 0.2091$
$R = 1.015$
$w = 0.6497$



Division of Cash Flows

At each date t, whether current z_t lies above or below cutoff $z^\dagger$, an optimal sequence $\{h_t, z_{t+1}, h_{t+1}, z_{t+2}, h_{t+2}, ...\}$ equates discounted sum of marginal product to wage:

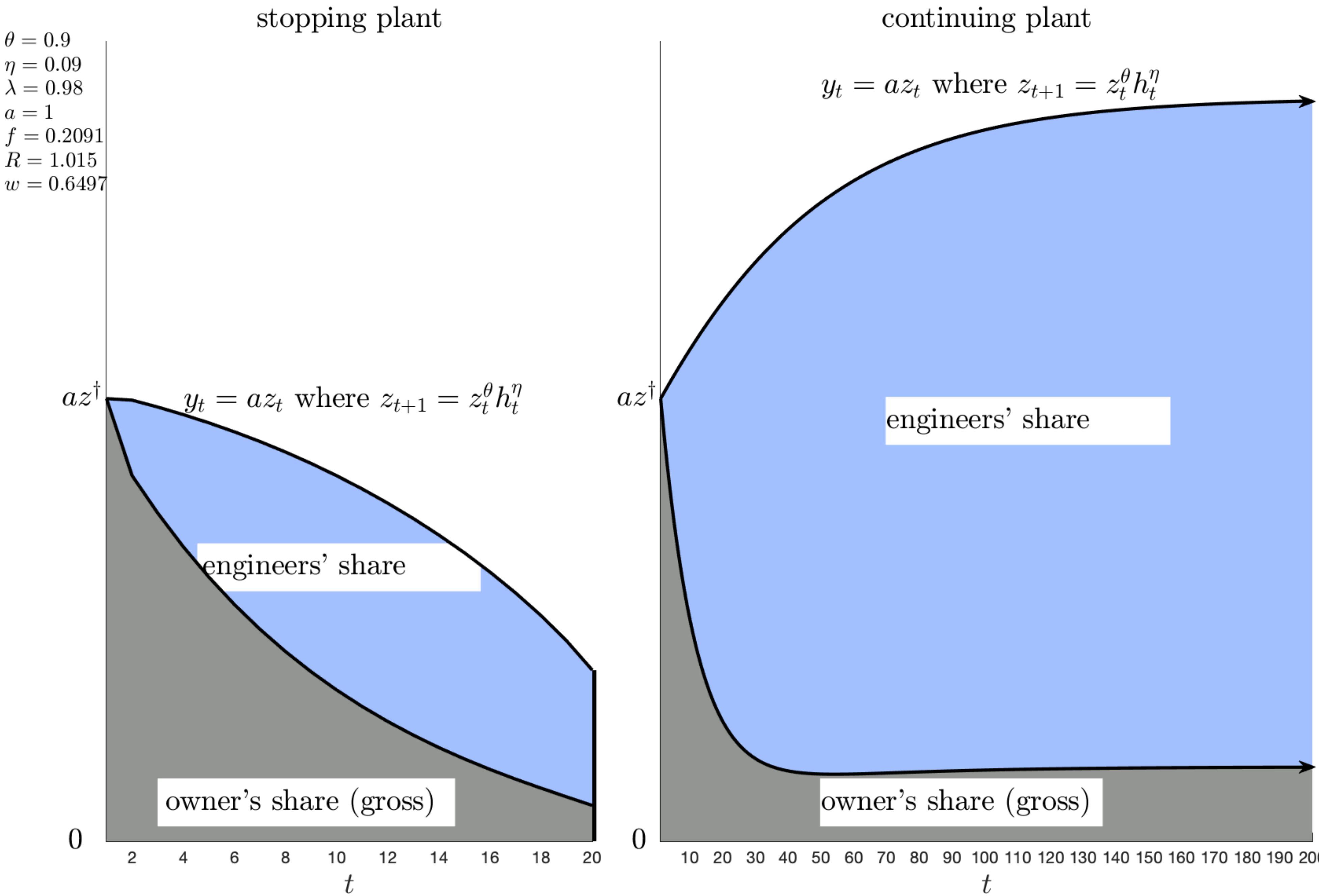
$$\begin{aligned} w \;=\; & \frac{\lambda}{R} a\eta \frac{z_{\mathrm{t+1}}}{h_{\mathrm{t}}} \;+\; \left(\frac{\lambda}{R}\right)^2 a\eta \frac{z_{\mathrm{t+1}}}{h_t}\theta\frac{z_{\mathrm{t+2}}}{z_{\mathrm{t+1}}} \\ & + \left(\frac{\lambda}{R}\right)^3 a\eta \frac{z_{\mathrm{t+1}}}{h_{\mathrm{t}}}\theta\frac{z_{\mathrm{t+2}}}{z_{\mathrm{t+1}}}\theta\frac{z_{\mathrm{t+3}}}{z_{\mathrm{t+2}}} \\ & + ... + \left(\frac{\lambda}{R}\right)^{T-t} a\eta \frac{z_{\mathrm{t+1}}}{h_t}\theta\frac{z_{\mathrm{t+2}}}{z_{\mathrm{t+1}}}\theta\frac{z_{\mathrm{t+3}}}{z_{\mathrm{t+2}}} \times ... \times \;\theta\frac{z_{\mathrm{T}}}{z_{\mathrm{T\text{-}1}}} \end{aligned}$$

Multiplying through by h_t, and simplifying

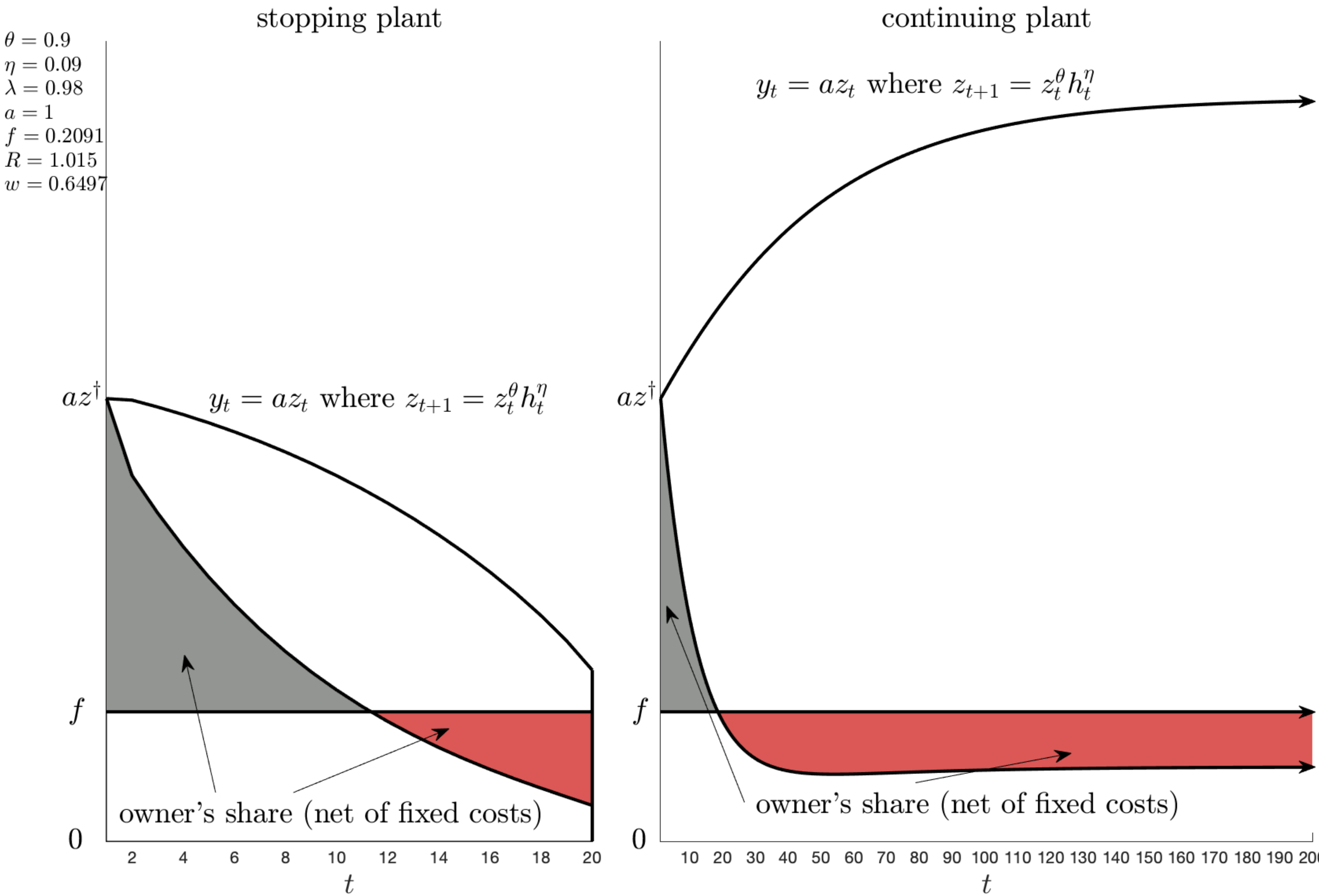
$$wh_{\mathrm{t}} = \frac{\lambda}{R}\eta y_{\mathrm{t+1}} + \frac{\lambda^2}{R^2}\eta\theta y_{\mathrm{t+2}} + \frac{\lambda^3}{R^3}\eta\theta^2 y_{\mathrm{t+3}} + ... + \frac{\lambda^{\mathrm{T\text{-}t}}}{R^{\mathrm{T\text{-}t}}}\eta\theta^{\mathrm{T\text{-}t\text{-}1}} y_{\mathrm{T}}$$

An engineer raises funds by selling new plant (which has quality 1) at price

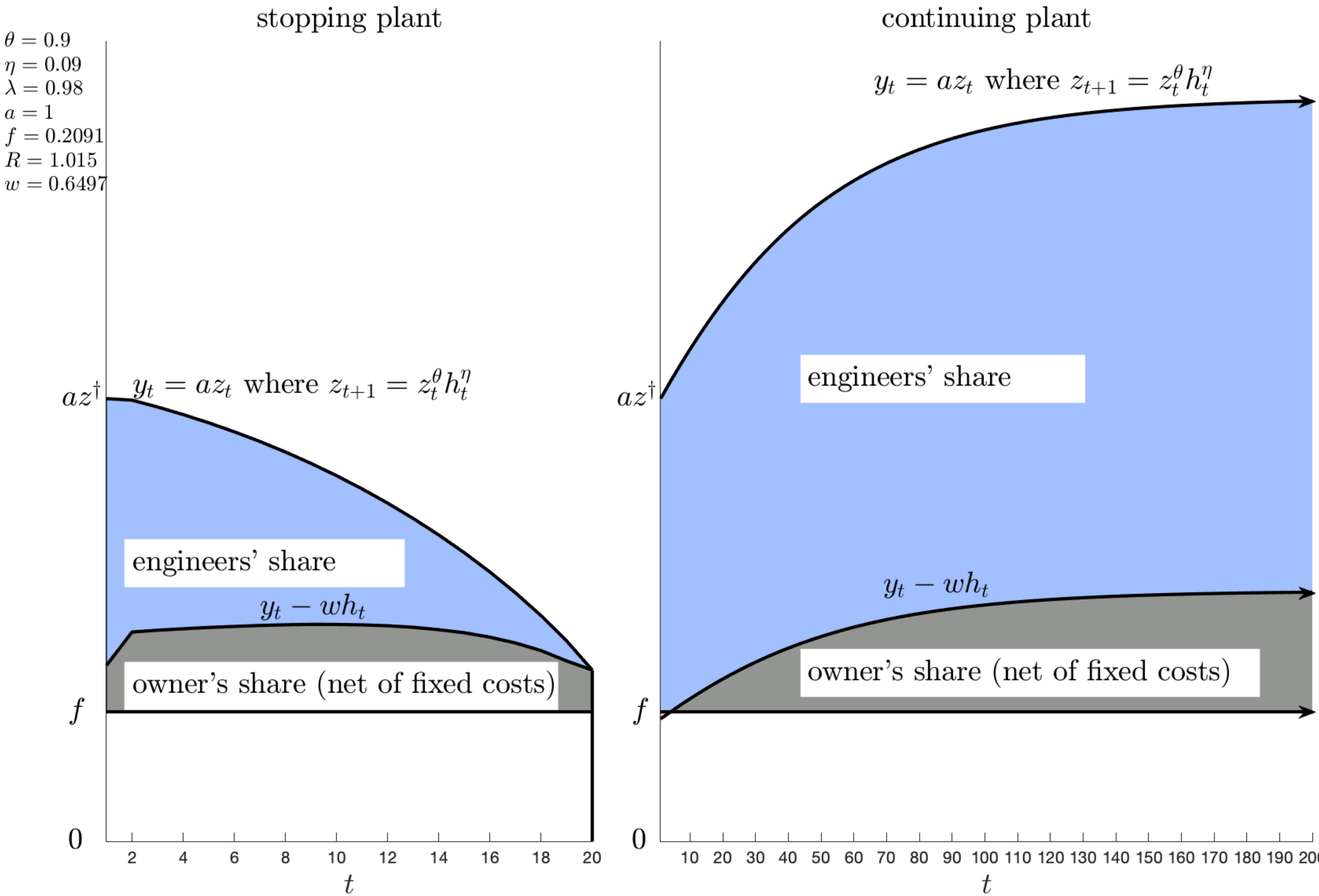
$$
\begin{aligned}
b = V(1) \; = \; & \frac{1}{R}(a - f) \;\; + \;\; \frac{\lambda}{R^2}[y_2(1 - \eta) - f] \\
+ \; & \frac{\lambda^2}{R^3}[y_3(1 - \eta - \eta\theta) - f] \\
& \ldots \\
+ \; & \frac{\lambda^{\text{T-2}}}{R^{\text{T-1}}}[y_{\text{T-1}}(1 - \eta - \eta\theta - \ldots - \eta\theta^{\text{T-3}}) - f] \\
+ \; & \frac{\lambda^{\text{T-1}}}{R^{\text{T}}}[y_{\text{T}}(1 - \eta - \eta\theta - \ldots - \eta\theta^{\text{T-2}}) - f]
\end{aligned}
$$



Underlying Division of Returns



Owner's Underlying Share of Returns (net of fixed costs)



Division of Cash Flows (net of fixed costs)

The budget constraint of an agent at date t who has h_t tools and d_t financial assets (maturing one-period discount bonds plus returns to plant ownership) is

$$c_t + (x - b)i_t + \frac{d_{t+1}}{R} = wh_t + d_t,$$

where h_t is positive iff the agent was an engineer yesterday; and investment i_t is positive iff the agent is an engineer today, in which case her tools tomorrow will be

$$h_{t+1} = \lambda h_t + i_t$$

The budget constraint can be written as

$$c_t + (x - b)h_{t+1} + \frac{d_{t+1}}{R} = [w + \lambda(x - b)]h_t + d_t \equiv n_t,$$

where n_t is net worth

When the rate of return on investment with maximal borrowing, R^E, exceeds the interest rate

$$R^E = \frac{w + \lambda(x - b)}{x - b} > R,$$

the engineer's consumption and investment are

$$\begin{aligned} c_t &= (1-\beta)n_t \\ (x-b)h_{t+1} &= \beta n_t \end{aligned}$$

A saver's consumption and asset holdings are

$$\begin{aligned} c_t &= (1-\beta)n_t \\ \frac{d_{t+1}}{R} &= \beta n_t \end{aligned}$$

A steady state equilibrium of our small open economy is characterized by the wage w and new-plant price b, together with the quantity choices of savers/plant owners (c, d, h, z, y), engineers (c, h, i), and foreigners (who have net asset holdings D^*), such that the markets for goods, tools, plant, and bonds all clear

Aggregating across engineers and savers, we obtain tool supply H, asset demand D, consumption C, and respective net worths $(N^E$ and $N^S)$:

$$
\begin{aligned}
(x-b) H_{t+1} &= \beta N_t^E \\
\frac{D_{t+1}}{R} &= \beta N_t^S \\
C_t &= (1-\beta)\left(N_t^E + N_t^S\right) \\
N_t^E &= \pi^E \left[w + \lambda(x-b)\right] H_t + \pi^S D_t \\
N_t^S &= (1-\pi^E)\left[w + \lambda(x-b)\right] H_t + (1-\pi^S) D_t
\end{aligned}
$$

The economy exhibits endogeneous growth: along a steady state path, growth rate G satisfies

$$
G = \pi^E R^E \beta + \pi^S R \beta \frac{(1-\pi^E) R^E \beta}{G - (1-\pi^S) R \beta}
$$

Proposition 1: There exists a critial value f^{critical} of the fixed cost such that

P-Region (Pure equilibrium with no stopping; low fixed cost): $f < f^{\text{critical}}$

(i) No plant owner stops: $z^{\dagger} < 1$

(ii) Aggregate ratio of tools-to-plant stays one-to-one (because equal initial supply, equal depreciation, no stopping): $h_t = 1$

(iii) All plant is maintained at initial quality: $z_t = z^* = 1$

M-Region (Mixed equilibrium; high fixed cost): $f > f^{\text{critical}}$

(i) Plant owners are initially indifferent between stopping after some finite time and continuing forever: $z^{\dagger} = 1$

(ii) Aggregate ratio of tools-to-plant is larger than one-to-one for continuing plant: for all t, $h_t > 1$

(iii) With decreasing returns to scale, $\theta + \eta < 1$, quality of continuing plant increases over time, converging to some $z^* \in (1, \infty)$

(iv) With constant returns to scale, $\theta + \eta = 1$, continuing plant quality grows at some constant rate $g > 1$

(v) Stopping plant decreases in quality over time; stop occurs just before z_t falls below f/a

Proposition 2P (P-Region):

(i) For an open set of parameters (in particular with R and λ not too far from 1), a pure equilibrium with no stopping exists such that

an unexpected permanent drop in the interest rate R leads to a lower steady state growth rate G

(ii) We show numerically that, immediately following the drop in R, all agents (engineers and savers) can be strictly worse off

Intuition

In P-Region, there is no stopping, and for all t, $h_t = 1$, $z_t = 1$, and $y_t = a$ so that the engineer's borrowing capacity is

$$b = \frac{a - w - f}{R - \lambda}$$

The wage is

$$w = \frac{\lambda}{R}\eta a + \frac{\lambda^2}{R^2}\eta\theta a + \frac{\lambda^3}{R^3}\eta\theta^2 a + \ldots = \frac{a\lambda\eta}{R - \lambda\theta}$$

which rises significantly with the fall in R – because the engineer's marginal product has a long horizon

Thus, e.g. with constant returns to scale, $\theta + \eta = 1$,

$$b \quad = \quad \frac{a}{R - \lambda\theta} \quad - \quad \frac{f}{R - \lambda}$$

engineer's borrowing capacity	present value of plant owner's share of gross revenues	present value of fixed costs

Because $\theta < 1$, the fall in R increases the present value of fixed costs proportionately more than the present value of the plant owner's share of gross revenues

Net, the fall in R can *decrease* the engineer's borrowing capacity

This effect can be strong enough – overcoming rise in net worth – to stifle investment and growth:

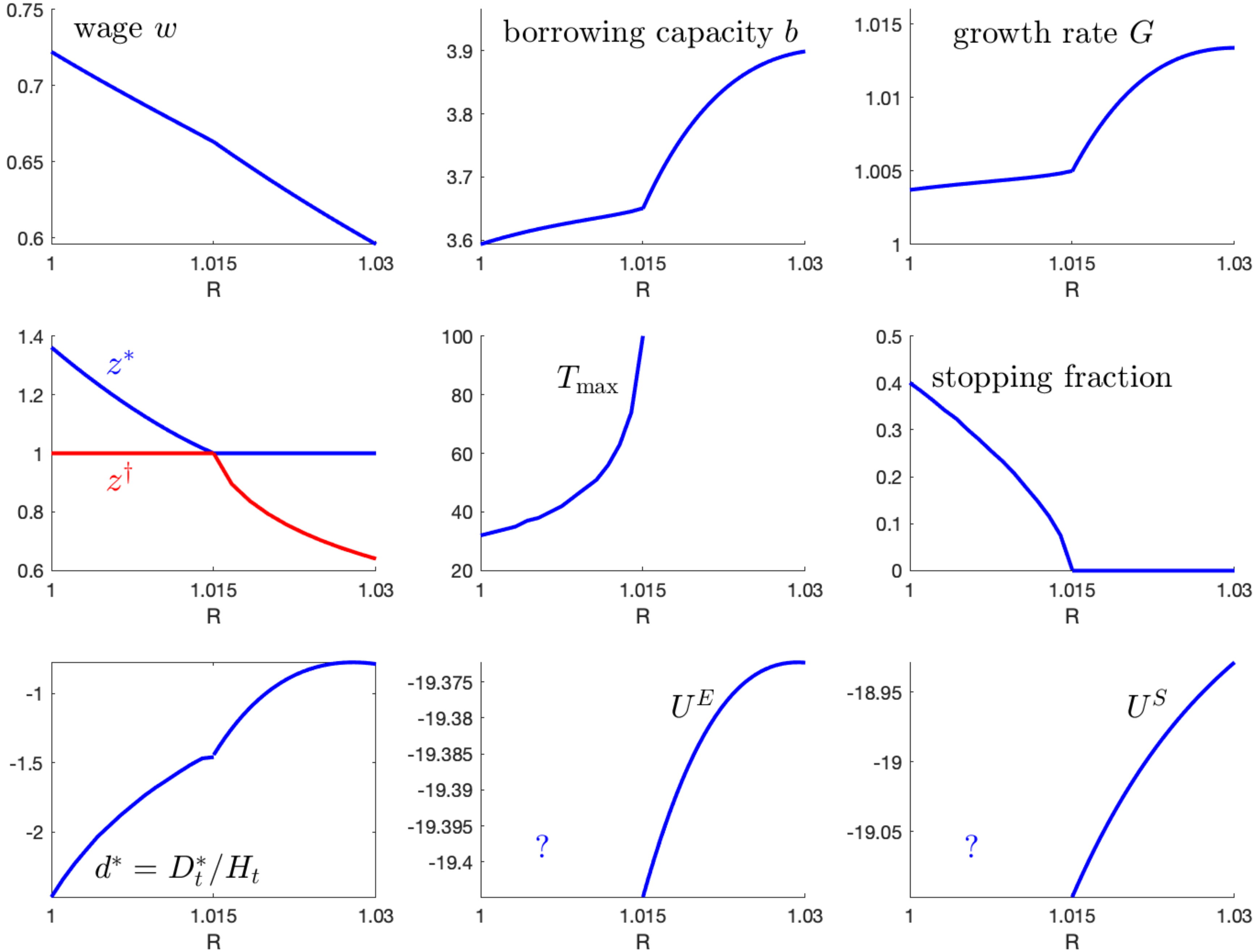
$$\text{gross investment } (H_{t+1}) \downarrow \; =$$

$$\text{saving rate } (\beta) \times \frac{\text{net worth of engineers } (N_t^E) \uparrow}{\text{investment cost } (x) - \text{borrowing capacity } (b) \downarrow}$$

Proposition 2M (M-Region):

In a mixed equilibrium, we demonstrate numerically that an unexpected permanent drop in the interest rate R can lead to a lower steady state growth rate G

e.g. $\theta = 0.9, \eta = 0.09, \lambda = 0.98, a = 1, f = 0.2091,$
$x = 6.127, \beta = 0.92, \pi^E = 0.7, \pi^S = 0.1$:



$R < 1.015$: M-Region

$R > 1.015$: P-Region

Extensions:

Heterogeneity across plants: in initial z and/or idiosyncratic shocks to subsequent $z', z'', ...$

Heterogeneity across engineers: in investment cost x

Choice of technique by engineers

Housing market: loan-to-value loan-to-income constraints

Land model

Bargaining model