Incomplete: missing section 5 and references
appendix only contains proofs to results in section 2

INSIDE MONEY AND LIQUIDITY*

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1. Introduction

Inside money can be defined very broadly as any privately-issued long-term paper that is held by a number of agents in succession. Whenever paper circulates as a means of short-term saving (liquidity), it can properly be considered as money, or a medium of exchange, because agents hold it not for its maturity value but for its exchange value.[1] In this article, we construct a model of inside money and use the model to ask: When and why is the circulation of inside money essential to the smooth running of the economy? What are the symptoms of a shortage of liquidity? Does the economy generate enough inside money? In addressing these questions, we propose a theory of liquidity premia. We also build a rudimentary model of banks.

A traditional view of money is that it lubricates exchange among agents when there is no double coincidence of wants. The canonical example, due to Wicksell (1911), has three types of agent, and three physically distinct commodities. Type I wants a commodity supplied by type II, type II wants a commodity supplied by type III, and type III wants a commodity supplied by type I. Thus, the pattern of preferences is such that no pair of agents wants to consume each other's endowment. In the absence of a well-functioning market, money allows the agents to trade bilaterally: an agent accepts money not for its own sake, but because he can exchange it for what he wants. One of the three commodities could serve as money; or an "outside" object, such as fiat money, might be used.[2] Of course, there must be some trading friction, to justify why the agents cannot simply trade their commodities through a market. Search or matching frictions are often invoked, but unfortunately this class of model has proved rather difficult to work with and incorporate into a standard macroeconomic framework.[3]

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[1]Wicksell (1911) defined money as "an object which is taken in exchange, not for its own account, i.e. not to be consumed by the receiver or to be employed in technical production, but to be exchanged for something else within a longer or shorter period of time".

[2]See, for example: Hellwig (1976), Jones (1976), Diamond (1983), Iwai (19**), Kiyotaki and Wright (1989), Oh (1989), Trejos and Wright (1995).

[3]For alternative non-competitive approaches to modelling the lubrication role of money, see Shubik (19**).

Our approach to modelling inside money places limited commitment centre stage, rather than trading frictions. We assume a perfectly competitive environment where agents freely meet and trade in a marketplace, but they cannot necessarily pledge all of their future endowment or output. For moral hazard reasons, there may be an upper bound on the total value of the paper they can issue at any given date.

We also place emphasis on limits to the saleability, or negotiability, of paper. In the market, any agent D is free to issue his paper to any other agent C, promising to make a future delivery (up to D's commitment limit). But between the date of issue and the date of delivery, C may not be able to sell D's paper on to a new creditor C′ at a fair price because C′ may be less able than C to enforce D's promise. In other words, although at the time of the loan, ex ante, the terms of trade between the debtor D and his original creditor C are governed by the market, ex post there may be "lock-in" (we model the reasons for this in the text). At one extreme, where new creditors C′ cannot force D to deliver anything, the paper held by C is nonsaleable. In this case, D effectively makes only a bilateral commitment, to deliver to the agent who buys his paper at the date of issue. At the other extreme, where any new creditor C′ can enforce D's promise, the paper is perfectly saleable. In this case, D makes a multilateral commitment, to deliver to any bearer of his paper, and the paper can circulate as inside money. In our analysis, we consider both extremes -- nonsaleable paper and saleable paper.[4]

Our model is built around a lack of double coincidence of wants, not in physical goods, but in dated goods. Consider an intertemporal version of Wicksell's triangle, with three dates, 1, 2 and 3, and a single, non-storable good at each date. There are three types of agent, I, II and III, with an equal (and large) number of each type. An agent of type I wants to consume goods at date 1 but is endowed with date 3 goods. An agent of type II wants to consume goods at date 2 but is endowed with date 1 goods. An agent of type III wants to consume goods at date 3 but is endowed with date 2 goods.

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[4]This terminology is slightly misleading, since, ex ante, all paper can be freely sold in the marketplace. The question is whether or not "second-hand" paper is saleable: Can the original creditor trade it ex post?

Now, to overcome the lack of double coincidence of wants, it is enough to suppose, first, that each type I agent can make a bilateral commitment to deliver date 3 goods to someone who buys his paper at date 1, and, second, that each type III agent can make a similar commitment to deliver date 2 goods. In the date 1 market everyone trades their endowment for what they want, and the first-best is achieved. After date 1, there is no need for markets to reopen, because there are no further gains from trade. In particular, the type III agents who purchase type I agents' paper at date 1 hold on to it until it matures at date 3. It does not matter whether or not this paper is saleable, since there is no need for it to be sold.

We should say something about the nature of the date 1 market. For present purposes, any decentralised market trading mechanism will suffice. For example, we might suppose that coalitions of three agents, one of each type, get together and strike a three-way deal: agent I gives agent III paper promising to deliver goods at date 3; agent II gives goods to agent I; and agent III gives agent II paper promising to deliver date 2 goods. Alternatively, rather than assuming three-party transactions, we might suppose that there are consecutive two-party transactions at date 1: initially the type II agents use their endowment to buy paper from the type III agents; then the type III agents use these goods to buy paper from the type I agents. But, in the absence of any trading frictions, there is no need to examine the trading mechanism in this much detail. The virtue of a perfectly competitive framework is that we do not have to go inside the black box of the market.

However, there is an important caveat. We want to avoid the fiction of a Walrasian auctioneer, or any centralised mechanism involving a planner. The reason is that, in an intertemporal setting, we would need to decide on the degree to which the auctioneer or planner can make commitments. It is usual to assume that the auctioneer/planner is entirely trustworthy. But this would inadvertently add more "commitment power" (liquidity) to the economy[5] -- and thus confuse the purpose our enquiry, which is to understand

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[5]For an example, see footnote 7 below.

the workings of an economy with limited commitment. To assume less than full commitment on the part of the auctioneer/planner would mean that our conclusions hinged on the particular assumptions that we made. These problems do not arise if we confine ourselves to decentralised trading mechanisms with no auctioneer.

To return to the three-period example, the success of the date 1 market in resolving the lack of double coincidence of wants depends on the fact that both type I and III agents are able to commit fully. Suppose instead that there is less than full commitment. To illustrate what might happen, let us make the stark and asymmetric assumption that although each type I agent can make a full commitment to deliver at date 3, type III agents default on any promise to deliver at date 2.

Now if type I agents can only make bilateral commitments, the economy collapses to autarky, because type III agents have nothing to offer at date 1, and type I agents cannot sell their paper to type II agents who don't want date 3 goods.[6]

The situation is rescued, though, if type I agents can make multilateral commitments. See Figure 1. Markets open at dates 1 and 2. Initially, at date 1, type II agents use their endowment of goods to buy type I agents' paper promising to deliver (to the bearer) date 3 goods. Then, at date 2, type II agents sell this paper to type III agents, in exchange for goods. Finally, at date 3, the paper is redeemed: type III agents collect goods from type I agents. Everyone ends up with what they want, and the first-best is achieved. Notice that the type III agents' inability to make any future commitment is not a problem, because at date 2 they pay for their purchases of paper using their endowment of goods, in a spot transaction.

Here, type I agents' paper acts as a medium of intertemporal exchange. The type II agents do not buy the paper at date 1 because they intend to keep it until maturity at date 3 -- they don't want to consume date 3 goods.

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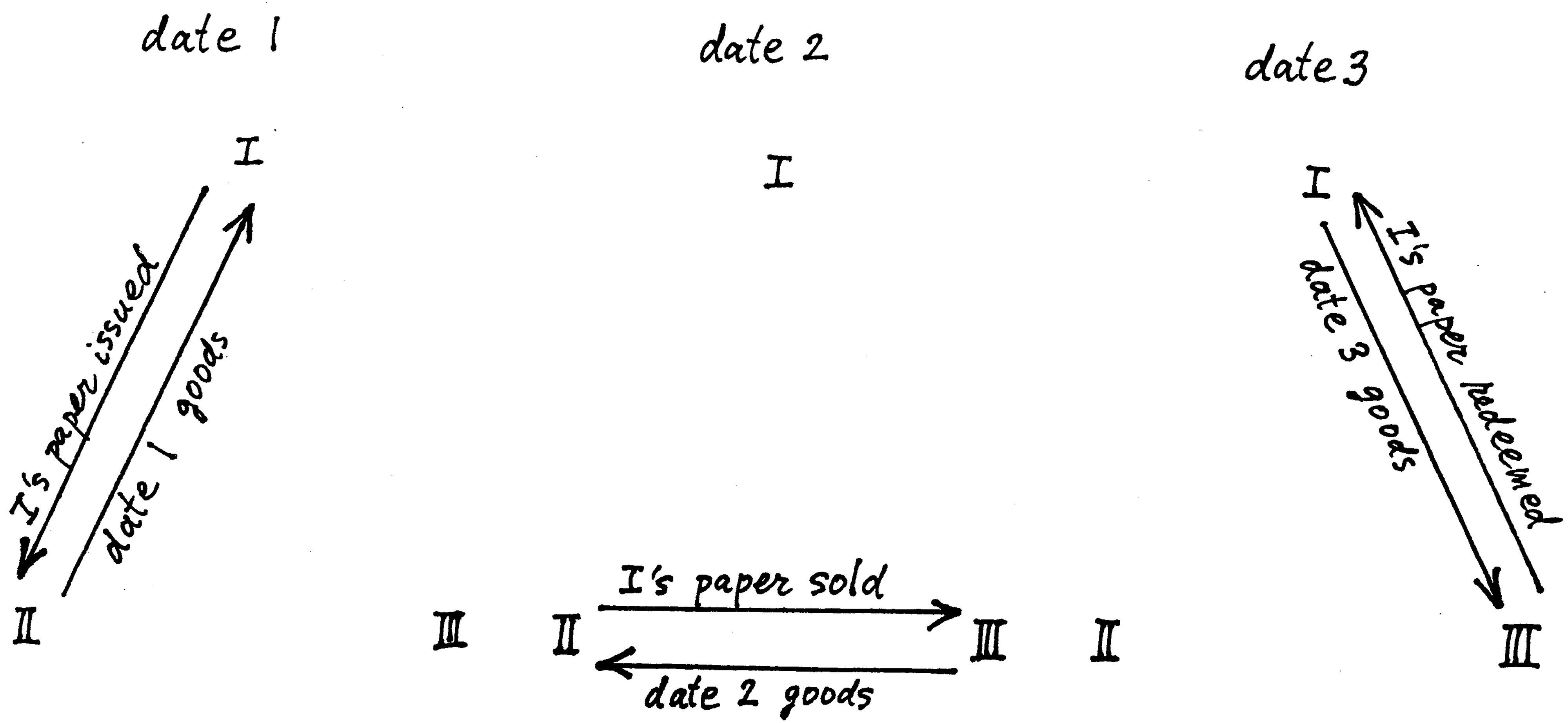
[6] We assume that everyone gets a small benefit from consuming their endowment, so there are no gifts.

Figure 1


date 1
I
I's paper issued
date 1 goods
II
date 2
I
III
II
I's paper sold
date 2 goods
III
II
date 3
I
I's paper redeemed
date 3 goods
III

Rather, they buy it because they expect to sell it at date 2. The paper is therefore inside money, held twice in succession as a means of short-term saving: by type II agents between dates 1 and 2, and by type III agents between dates 2 and 3.

This three-period example illustrates a very general idea. The power of one agent to make a multilateral commitment can substitute for another agent's lack of commitment. We believe that this is the key to explaining why the circulation of inside money is necessary to the smooth running of an economy.[7]

Although clean, the example does not really provide a satisfactory vehicle for exploring the issues further -- not least, because of the ad hoc assumption that type I agents can make multilateral commitments whereas type III agents cannot even make bilateral commitments. Since the focus of our analysis is on limited commitment and saleability, we need to put whatever assumptions we make about commitment and saleability on a sounder footing.

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[7]As we saw in the example, if the type I agents are unable to make multilateral commitments, so their paper cannot circulate as inside money, then the economy collapses to autarky. This is not true, though, if there is an auctioneer who can supply liquidity. In fact the first-best can be achieved. Markets open at dates 1 and 2. At date 1, there are three-way deals involving a type I agent, a type II agent and the auctioneer: agent I sells paper to the auctioneer, promising to deliver date 3 goods; agent II gives goods to agent I; and the auctioneer promises to deliver date 2 goods to agent II. At date 2, in exchange for a promise by the auctioneer to deliver date 3 goods, the type III agents give her goods, which she delivers to the type II agents as promised. Finally, at date 3 the auctioneer redeems the type I agents' paper, and she delivers the goods to the type III agents as promised. This sequence of trades relies on the ability of the auctioneer to supply liquidity (the means of short-term saving): she supplies liquidity to type II agents at date 1, and to type III agents at date 2. That is, the auctioneer's power to commit compensates for the fact that type III agents cannot make any commitments and type I agents can only make bilateral commitments (here, to the auctioneer). Since we are interested to know how much liquidity is generated by the agents themselves, not by a fictitious auctioneer, we restrict attention to decentralised trading mechanisms with no auctioneer.

Incidentally, in the standard Arrow-Debreu framework agents are assumed to commit fully and the auctioneer has no need to take a position in the market (e.g. to supply paper). It is only when there is limited commitment that the auctioneer can usefully contribute additional commitment power to the economy.

Our model, presented in Section 2, links commitment to production. We model an infinite-horizon economy, without uncertainty, where agents live for ever. (The infinite horizon is analytically convenient because we can look at steady states. Nothing hinges on "infinity".[8]) Production projects are of finite length, and in equilibrium agents take turns to invest. An agent starting a new project borrows by issuing paper against the project's future output. We assume not only that the agent is essential to the project, but also that he cannot guarantee to his creditor that he will not withdraw his human capital before the project is completed. The threat to default places an upper bound, θ say, on the fraction of the project's output that he can credibly mortgage at the time of investment.[9] He cannot issue paper at any other time since he has no other collateral to offer: he cannot borrow against future projects.

Our formal analysis, in Section 2, starts with the case of no multilateral commitment, because it provides a useful "non-monetary" benchmark. Arguably, the best way to appreciate the role of inside money is first to see how the economy copes with a restriction on the circulation of paper. We find that if there is enough bilateral commitment -- if θ is high enough -- the economy can achieve first-best, despite the absence of multilateral commitment. Even for lower values of θ, the economy does not reduce to autarky, given that everyone can partially commit ($\theta > 0$). But because too little paper is supplied to the economy, the price is too high. That is, the implied interest rate on the paper is lower than the agents' rate of time preference -- a symptom of a liquidity shortage. The economy fails to transfer enough resources from savers to investors relative to first-best: agents completing projects consume too much, and agents starting projects invest too little. Overall, the economy operates "too slowly", and output is too low. Moreover, agents wanting to save short-term may resort to using an inferior liquid technology such as storage, even though the return

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[8]This is not a model where money derives value because it lasts forever; nor is it a model where money passes between overlapping generations.

[9]The specific model we use is a simple variant of Hart and Moore (1994).

is even less than the return on the illiquid paper.[10] We should stress that, as the model is deterministic, ours is theory of liquidity premia not based on risk. For liquidity premia to arise in equilibrium, it is necessary that the economy has a shortage of liquidity.

In Sections 3 and 4, we introduce multilateral commitment and inside money into the model. In all cases, we find that money speeds the economy up: investment and output rise, and resources are directed away from inefficient storage.

Section 3 introduces a banking sector. A bank invests in a commitment technology that enables it to issue its own saleable paper against nonsaleable paper that it has purchased. Two conclusions emerge from our analysis. First, because banks are intrinsically unproductive, they can make a return on their investment only if there is a liquidity premium. If there were no liquidity shortage, there would be no banks. Second, when there is a liquidity shortage, the banking sector typically supplies the wrong amount of inside money into the economy, in the sense that discounted aggregate output is not maximised. In particular, we find that if the liquidity shortage is severe enough that agents resort to using storage, then the banking sector is too small.

Another way for inside money to be introduced into the model of Section 2 is to add to the economy some durable factor of production, land, which, unlike nonsaleable paper, can always be traded and so can serve as collateral for liquidity. In Section 4 we explore how the economy performs when paper secured against land circulates as inside money.

Finally, in Section 5 we briefly discuss the literature, and comment on what our analysis might suggest about the historic and future roles of money and banks.

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[10] This may shed light on the empirical puzzle that risk-free rates are so low (the flip side of the equity premium puzzle).

2. A model without inside money

Consider a deterministic, discrete-time economy with one homogeneous good, corn, that can be stored perfectly: one unit stored in period t yields one unit in period t+1. There is a continuum of identical, infinitely-lived agents who consume corn: at the start of period t, the utility of an agent is

$$(1) \qquad \sum_{s=0}^{\infty} \beta^s \log c_{t+s},$$

where c_{t+s} denotes consumption in period t+s, and the discount factor β lies strictly between 0 and 1. For reasons that will be clear shortly, it helps to normalize the population size to equal 3.

Each agent has a decreasing-returns-to-scale technology by which corn can be invested to produce corn two periods later. Specifically, per unit of population, x_t corn invested in period t yields a harvest of y_{t+2} corn in period t+2, where

$$(2) \qquad y_{t+2} = \left(\frac{x_t}{a(1-\lambda)} \right)^{1-\lambda}$$

and $0 < \lambda < 1$. For future reference, define the cost function

$$(3) \qquad x = a(1-\lambda)y^{1/(1-\lambda)} \equiv G(y).$$

We assume that the agent is fully occupied throughout production, investing in period t, growing in period t+1, and harvesting in period t+2. He is unable to operate overlapping investment projects. So if he invests in period t then he cannot invest again until period t+3.

In the absence of any constraints on contracting, first-best allocations are straightforward to characterize. A useful benchmark is the symmetric, steady-state allocation: in each period exactly a third of the agents invest, at a constant level x^*, to produce output y^* where $x^* = G(y^*)$; everyone consumes a constant amount $c^* = \frac{1}{3}(y^* - x^*)$; and marginal cost equals the discounted marginal return, using the marginal rate of substitution between consumption at the time of harvest and consumption at the time of investment as the discount factor,

$$(4) \qquad G'(y^*) \;=\; \beta^2.$$

Note that storage is not used in this first-best allocation, since the return on storage, 1, is strictly less than the rate of time preference, $1/\beta$. (Throughout the paper, we adopt the convention that the "return" on an asset is the gross return, and that the "rate of time preference" refers to the gross rate of time preference.)

In our economy, however, there are constraints on contracting. We make two critical assumptions.

First, we assume that it is only at the time of starting a new investment project that an agent can commit to share the project's output with anyone else; and, moreover, he cannot credibly commit to share out more than a fraction θ. This limits his ability to raise investment funds: for a project yielding y_{t+2} in period t+2, he can mortgage only a portion θy_{t+2}. In other words, he can issue only θy_{t+2} two-period paper in period t, where one unit of "two-period paper" refers to a credible promise to deliver one unit of corn in two periods time.

To justify this first assumption, we suppose that the agent's human capital is not only essential to the project but also inalienable, so that he cannot write a contract in period t that commits him to work through to period t+2. However, at the start of the project a creditor can be given a project key, by way of security. Without the key, no-one can gain access to the project. Since the debtor has project-specific human capital, and the creditor has the key, they are both needed to extract output at date t+2. After bargaining, they end up dividing the output in the ratio $1-\theta:\theta$.[11]

Second, we assume that anyone buying two-period paper in period t cannot sell it in period t+1. In terms of the discussion in the Introduction, since the paper is nonsaleable it cannot circulate as inside money.

To justify this second assumption, we suppose that the agent and his creditor are together able to strip the project, leaving just a shell that delivers no output, but which cannot be distinguished from an intact project by outsiders. This requires the cooperation of both parties, given that theft needs the agent's specific human capital plus access to the project (and the creditor has the project key). Aware of the possibility of theft between periods t and t+1, no third party is willing to buy second-hand paper in period t+1 since there is no guarantee of a return in period t+2. By contrast, the original creditor can be confident of getting a return, since the agent cannot unilaterally steal without the key. If (and only if) the creditor retains the key for the duration of the project, he can be confident of receiving a fraction θ of the return.[12,13]

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[11] Another justification is to suppose that the agent can steal a fraction $1-\theta$ of the output in period t+2, regardless of what contracts have been written in period t. (For details of these two kinds of models, see Hart and Moore, 1994 and 1998 respectively.)

[12] A related argument rules out multiple creditors for the same project. Suppose the agent borrows from several creditors and each is given a (duplicate) key as security. Then an individual creditor is vulnerable to the agent and one or more of the other creditors colluding to steal from the project. To combat this, creditors might be given different keys, where all the keys are jointly required to access the project. But if each creditor were given this kind of veto power, there would likely be considerable inefficiency in bargaining over the output from the project. Hence we suppose that there can be at most one creditor per project.

To avoid problems of divisibility -- an individual lender may not wish to be the sole creditor of a large project -- we suppose that a project is made up of a large number of small constituent projects, each with its own key. For future purposes, it helps to suppose that all constituent projects in the economy produce the same amount of output.

[13] Another, more standard explanation for why the creditor might not be able to

We must emphasize that we include these detailed "microfoundations" only to bolster our two critical assumptions: First, in period t, an agent investing x_t in a project that yields y_{t+2} in period t+2, can sell only θy_{t+2} two-period paper -- the agent can make only limited bilateral commitment, and no other borrowing is possible. Second, the buyer of this paper cannot sell it in period t+1, but has to hold it until it matures in period t+2 -- the agent cannot make multilateral commitments. We think both assumptions are interesting enough to warrant a study of their implications for the aggregate behaviour of the economy. The detailed justification for the assumptions is not essential to our argument.

Let q_t denote the price of newly-issued two-period paper in period t, i.e. the price of a nonsaleable claim to period t+2 corn in terms of period t corn. In what follows, let n_t denote an agent's holding of newly-issued two-period paper at the end of period t. And let z_t denote the amount of corn the agent stores between dates t and t+1. Both n_t and z_t must be nonnegative.

The nonsaleability of paper means that we must be somewhat careful in writing down agents' flow of funds constraints. Consider the flow of funds constraint of an agent investing in period t:

$$(5) \qquad x_t + c_t + z_t + q_t n_t = q_t \theta y_{t+2} + z_{t-1} + n_{t-2}.$$

The left-hand side (LHS) of (5) comprises expenditures: investment, plus consumption, plus storage, plus purchases of two-period paper newly issued by

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sell paper at date t+1 is that there is adverse selection in the second-hand market. Suppose the output from the investment project is stochastic, and the original creditor privately learns something about the distribution of output after the project has started in period t. If the "lemons" problem is severe, the resale market will break down completely (Akerlof, 1970). Although we have not modelled this, and so cannot be sure to what extent the introduction of stochastic returns and asymmetric information would impinge on the rest of the analysis, we are nevertheless confident that such a model could be constructed.

other agents. The right-hand side (RHS) comprises receipts. The first term is borrowing, i.e. the revenue from the sale of two-period paper, priced at q_t, issued against the mortgageable portion of his period t+2 output, θy_{t+2}. The second and third terms are the return from storage in period t-1 and the return from two-period paper purchased in period t-2.

In the next period t+1, the period of growing, the agent's flow of funds constraint is

$$c_{t+1} + z_{t+1} + q_{t+1}n_{t+1} = z_t + n_{t-1}. \tag{6}$$

Expenditures on the LHS of (6) are consumption, plus storage, plus purchases of newly-issued two-period paper. Receipts on the RHS are the return from storage in period t, plus the return from two-period paper purchased in period t-1. In period t+2, at the time of harvest, the agent's flow of funds constraint is

$$c_{t+2} + z_{t+2} + q_{t+2}n_{t+2} = (1-\theta)y_{t+2} + z_{t+1} + n_t. \tag{7}$$

Expenditures on the LHS of (7) are again consumption, plus storage, plus purchases of newly-issued two-period paper. Receipts on the RHS are the portion of the harvest that was not mortgaged in period t, plus the return from storage in period t+1, plus the return from two-period paper purchased in period t.

Competitive equilibrium is defined as a sequence of paper price, investment, output, consumption, storage and paper holdings, $\{q_t, x_t, y_{t+2}, c_t, z_t, n_t\}$, such that in each period t: first, each agent chooses $\{x_t, y_{t+2}, c_t, z_t, n_t\}$ to maximize utility (1) subject to the production function (2) and the flow of funds constraints (5), (6) and (7); and, second, the markets for corn and paper clear.

We focus on a symmetric steady state equilibrium in which exactly a third of the population invests in each period, each agent's choices of quantities are in a fixed 3-period pattern, and the price is constant. Let q denote the steady-state price of newly-issued two-period paper. Let x and y denote steady-state investment and production. Let c denote the consumption by agents when they are investing; and let n and z denote their end-of-period holding of newly-issued paper and storage. Let c′, n′ and z′ denote the corresponding quantities chosen when they are growing. And let c″, n″ and z″ denote the quantities chosen when they are harvesting. The flow of funds constraints (5), (6) and (7) reduce to

$$x + c + z + qn = q\theta y + z'' + n'; \tag{8}$$

$$c' + z' + qn' = z + n''; \tag{9}$$

$$c'' + z'' + qn'' = (1-\theta)y + z' + n. \tag{10}$$

In a symmetric steady-state equilibrium, the market-clearing condition for corn is

$$y = c + c' + c'' + x. \tag{11}$$

That is, output equals aggregate consumption plus investment; the return from storage one period earlier cancels out with new storage. And the market-clearing condition for newly-issued two-period paper is

$$\theta y = n + n' + n''. \tag{12}$$

That is, paper supply -- the mortgageable part of output in two periods time -- equals paper demand. By Walras' Law, one of (11) or (12) implies the other, given that the flow of funds constraints are satisfied.[14]

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[14]The equivalence of (11) and (12) is true provided $q \neq 1$. However, we will

Our goal is to characterise this equilibrium for different values of the parameters. In particular, we want to see how the equilibrium is affected by θ, the limit on bilateral commitment. To help understand the intuition, our discussion is somewhat informal, but it contains formal propositions that are proved in the Appendix.

Consider the flow of funds constraints (8), (9) and (10) corresponding to an agent's 3-period cycle of investing, growing and harvesting. Unless θ is very high, at the time of investment the production cost x will exceed borrowing $q\theta y$, and so the agent needs to save unmortgaged output from previous harvests, $(1-\theta)y$, to finance the downpayment for investment, $x - q\theta y$. Also, consumption at the time of investing and growing needs to financed by saving. Note that since the marginal product of investment and the marginal utility of consumption are both infinite at zero, investment and consumption will always be strictly positive in equilibrium.

There are two means of saving: storage and two-period paper. Storage is flexible in the sense that it can be used for one or more periods, but the rate of return is poor. Two-period paper, which is supplied by investing agents, will earn a higher return than storage but is less flexible since it cannot be sold prior to maturity and so cannot be used for saving from one period to the next. In particular, if an agent buys two-period paper at harvest time, he cannot use it to finance investment in the next period. Storage, on the other hand, is short-term saving and permits the agent to transfer harvest income directly to investment in the next period.

Can the symmetric steady-state equilibrium be first best efficient? As we noted earlier, since the return on storage is strictly less than the rate of time preference, storage is inefficient; so the only way to attain first-best is for agents to save using two-period paper, even though it is nonsaleable. Also, the first-best requires agents to invest every three periods, and not to idle between investment projects: to skip a production opportunity would lose surplus, given that the technology has strictly

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show that $q < 1$ in equilibrium.

decreasing returns to scale ($\lambda > 0$ in (2)). With this in mind, consider an agent who is investing in periods 1, 4, 7, ..., growing in periods 2, 5, 8, ..., and harvesting in periods 3, 6, 9, ... See Figure 2. The arcs above the time line denote production. Now there is a clever way for the agent to save income from harvest periods using the nonsaleable two-period paper: he holds the paper twice in succession: see the arcs below the time line that denote saving. For example, (levered) production started in period 1 yields the agent a return $(1-\theta)y$ in period 3, some of which the agent uses to purchase n'' paper; the rest is used for consumption c''. This n'' paper matures in period 5: some of the return is used for consumption c', and the rest is used to purchase n' paper, which in turn matures in time for investment in period 7. This sequence -- investment, followed by two successive purchases of paper, followed by more investment, and so on -- serves to link the budgets of odd-numbered periods. A similar sequence links the even-numbered periods. But notice that, in the absence of storage, there is no cross-linkage. It is akin to two parallel turnpikes that have no cross roads which join them. This arrangement emerges as the economy's ingenious response to the constraint imposed by the nonsaleability of paper, and the need to create double coincidences of wants.

However, the arrangement places great demands on the paper market. Along the odd-numbered periods, ignoring consumption, agents are saving twice as often as they are investing. Likewise along even-numbered periods. To get an idea of how much paper is needed to sustain the first-best, take an extreme case where consumption is negligible: $\beta \approx 1$ and $\lambda \approx 0$.[15] Almost all of the harvest that is not mortgaged, $(1-\theta)y$, is saved twice before being reinvested: so $n'' \approx (1-\theta)y/q$ and $n' \approx (1-\theta)y/q^2$, which makes a total demand for two-period paper of at least $(1-\theta)y[(1/q) + (1/q)^2]$. The supply is the mortgaged portion of harvest, θy. Now in a first-best equilibrium the implied one-period return on paper, $1/\sqrt{q}$, must equal the rate of time preference, $1/\beta$, which we are assuming is close to 1; so $q \approx 1$. Hence the minimum value of θ, θ^* say, needed to achieve the first-best is approximately the solution to $2(1-\theta)y = \theta y$. That is, $\theta^* \approx 2/3$.

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[15]We assume here that the technology has almost constant returns ($\lambda \approx 0$) so that the consumption of "profit" can be ignored.

Figure 2

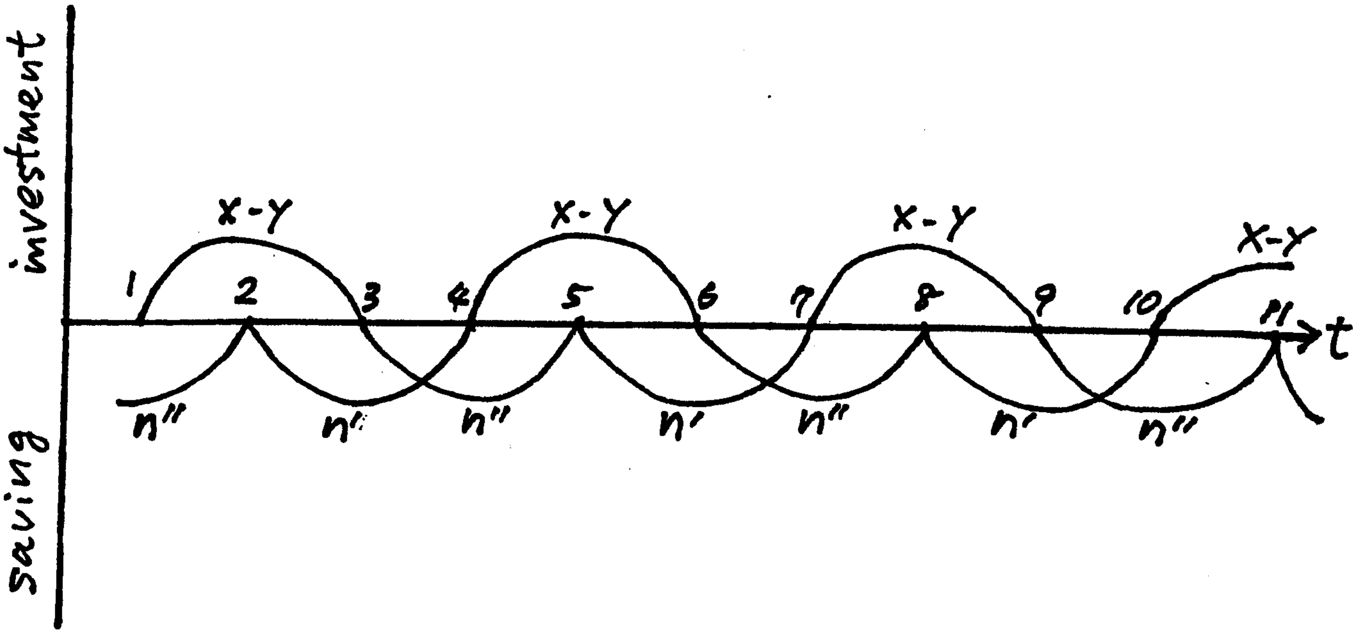
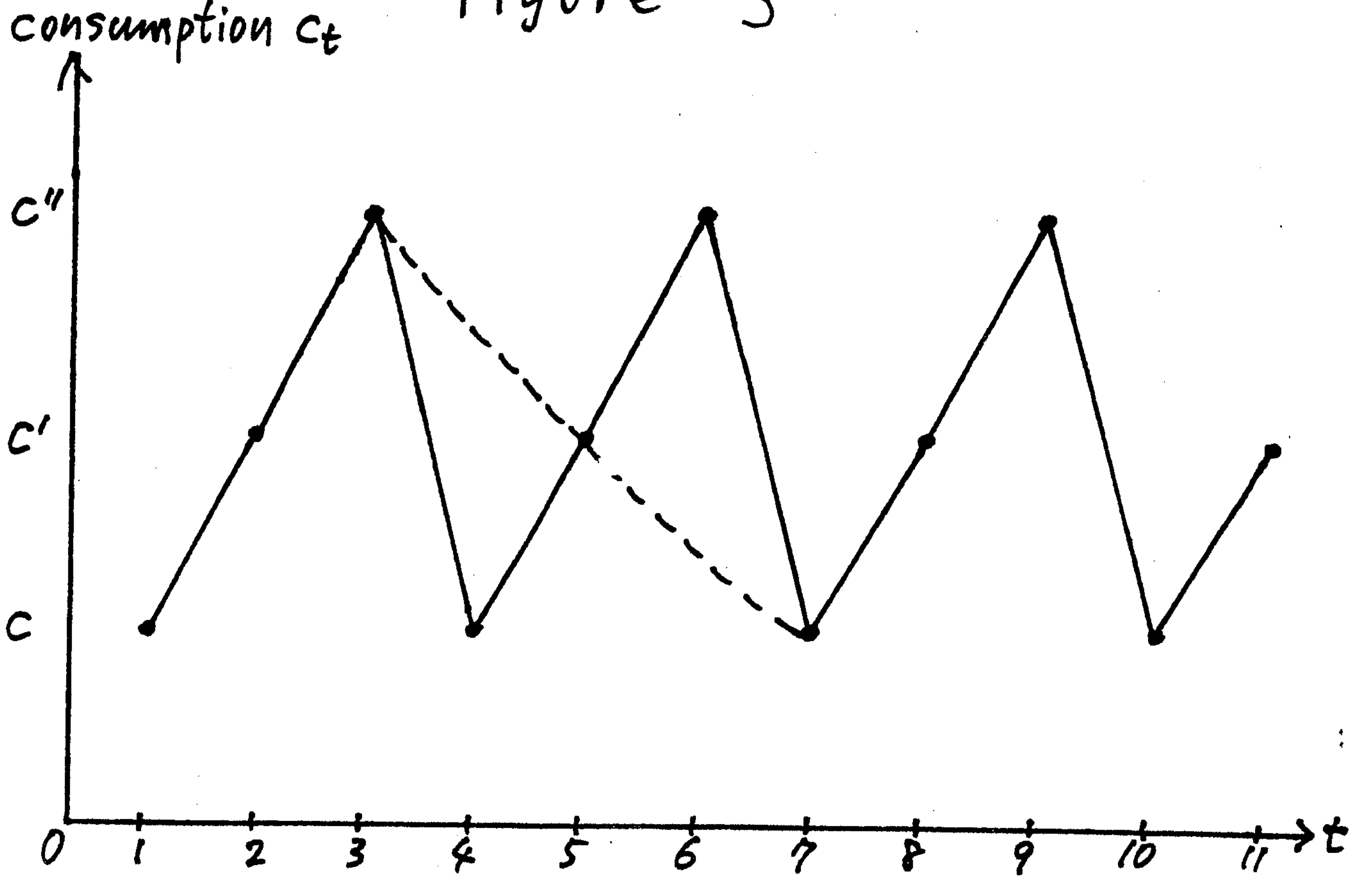


Figure 3



To ascertain if the need for a such high degree of bilateral commitment (the need for θ to be higher than approximately 2/3) can be attributed to the absence of multilateral commitment, we might ask, again for $\beta \approx 1$ and $\lambda \approx 0$: What is the minimum level of θ needed to sustain the first-best if paper were saleable rather than nonsaleable? The important difference is that an agent who buys saleable two-period paper with his unmortgaged portion of harvest, $(1-\theta)y$, can sell the paper in the next period -- in order to finance investment then, rather than in four periods time. Saleable paper is thus a means of short-term saving. And since two different agents successively hold two-period paper, each agent saving only short-term, the paper is used twice: it is inside money. If we let p denote the steady-state price of period t+1 corn in terms of period t corn, then the price of newly-issed saleable two-period paper is p^2. Neglecting consumption, the demand for short-term saving by the harvesting agents is $(1-\theta)y/p$. To determine supply, note that each period the investing agents issue θy two-period paper; so the total supply of short-term saving equals the stock of two-period paper maturing next period, θy, plus next period's value of the two-period paper issued this period, $p\theta y$. Since $1/p$ must equal $1/\beta$ in first-best, $p \approx 1$. Hence, when the paper is saleable, the critical value of θ below which there would be a supply shortage is approximately the solution to $(1-\theta)y = 2\theta y$. That is, the minimum level of θ needed to achieve the first-best is approximately 1/3 -- half that for the case of nonsaleable paper!

Return to the main case of nonsaleable paper. For general λ and β, the value of the θ^* is given in Proposition 1.

<u>Proposition 1</u> The symmetric, steady-state equilibrium is first-best efficient if $\theta \geq \theta^*$, where

$$\theta^* = \frac{1}{3}\left(2 - \frac{\lambda\beta^2(1 + 2\beta^2)}{1 + \beta^2 + \beta^4}\right). \tag{13}$$

In this region, the implied one-period return on two-period paper, $1/\sqrt{q}$, equals the rate of time preference, $1/\beta$. That is, $q = \beta^2$.

When θ lies below the critical value θ^*, the demands on the paper market are such that the first-best cannot be sustained: the economy is short of means of saving, and the price of two-period paper, q, is above β^2. One immediate symptom is that consumption is no longer smoothed. Consider the standard first-order conditions that determine an agent's choice between consumption and saving. Both the marginal rate of substitution between consumption at the time of harvesting and consumption two periods later (the next growing period), and the marginal rate of substitution between consumption at the time of growing and consumption two periods later (the next investment period), are equal to the return on saving using two-period paper:

$$(14) \qquad \frac{(1/c'')}{\beta^2(1/c')} = \frac{(1/c')}{\beta^2(1/c)} = \frac{1}{q}.$$

When $q > \beta^2$, we see from (14) that the path of consumption every other period, starting from the time of harvest, is downward-sloping: $c'' > c' > c$. Consumption is highest at the time of harvest and lowest at the time of investment, because the return on paper is less than the rate of time preference. See the dotted line in Figure 3 joining the odd-numbered periods 3, 5 and 7. Repeating the same exercise for the even-numbered periods, and then connecting adjacent periods, we obtain the jagged solid line.

Another symptom of the shortage of means of saving is that borrowing constraints bind at the time of investment. To see this, consider the first-order condition that determines an agent's choice between consumption and investment:

$$(15) \qquad \frac{(1/c)}{\beta^2(1/c'')} = \frac{1-\theta}{G'(y) - \theta q}.$$

The LHS of (15) is the marginal rate of substitution between consumption at the time of investment and consumption at the time of harvest. The RHS is the marginal rate of return on investment with maximum leverage -- the fraction of harvest not mortgaged, $1 - \theta$, divided by $G'(y) - \theta q$, the portion of marginal cost not covered by borrowing (the marginal downpayment). Now multiply (15) and the two equations in (14) to obtain:

$$(16) \qquad \left(\frac{1 - \theta}{G'(y) - \theta q} \right) \frac{1}{q^2} = \frac{1}{\beta^6}.$$

(16) is a steady state condition relating consumption at the time of investment with consumption at the time of investment six periods later: the composite return from investment followed by holding paper for four periods must equal the six-period rate of time preference. This tells us that, when $q > \beta^2$, the return on levered investment exceeds the two-period rate of time preference, $1/\beta^2$, which in turn exceeds the rate of return on paper, $1/q$. Hence the borrowing constraint $n \geq 0$ is binding.

The fact that investing agents face a borrowing constraint suggests that investment and output might be lower than in the first-best. This indeed turns out to be true, and is a further symptom of the shortage of means of saving. From (16), we can solve for the marginal cost of investment:

$$(17) \qquad G'(y) = \theta q + (1 - \theta)\frac{\beta^6}{q^2}.$$

(17) says that, at the margin, the cost of investment is met by a combination of external finance, θq, and internal finance, $(1-\theta)\beta^6/q^2$, saved from harvest income.[16] Notice that when q equals β^2, (17) reduces to (4): output is at its first-best level, y^*. However, when $\theta < \theta^*$, q exceeds β^2. Now, on the one hand, a high q allows agents to raise more external finance for investment -- the price of their paper is high. But, on the other hand, a low return on

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[16] The factor β^6 is the fraction of harvest income that is not consumed.

saving, $1/q$, reduces the internal finance available for investment. Also, a low θ shifts the balance from external to internal finance, which pushes investment down. Overall, as is proved in Proposition 2 below, the negative effects dominate: for $\theta < \theta^*$, when $q > \beta^2$, investment and output are less than their first-best levels x^* and y^*.

A simple, though incomplete, intuition for the underinvestment is that, because the economy is short of means of saving, agents tend to overconsume at the time of harvesting (and also at the time of growing). Roughly speaking, consumption in periods of surplus acts as an imperfect substitute for saving in other agents' production.

To sum up so far, we have seen that, for θ below θ^*, as a consequence of the shortage of means of saving, consumption is not smooth, borrowing constraints bind at the time of investment, and investment and output are too low. The nonsaleability of paper causes the economy to "run more slowly", in the sense that it takes longer for agents to reinvest the returns from their earlier production.

One way for the economy to "speed up" would be to use storage as a means of short-term saving. It is clear that for θ not too far below θ^*, in the neighbourhood of the first-best, storage will not be used. From (14), the marginal rate of substitution between consumption at the time of harvest and consumption at the time of investment, $(1/c'')/\beta(1/c)$, equals β^3/q^2. Provided β^3/q^2 is greater than the return on storage, 1, agents will not use storage. In Proposition 2 we show that this will be true whenever θ lies above some critical value, $\hat{\theta}$ (which is strictly below θ^*).[17]

When θ lies strictly below this critical value $\hat{\theta}$, storage is used by agents at the time of harvest to save for the next period's investment, $z'' > 0$. (But none of the other agents uses storage: $z = z' = 0$.) The price of two-period paper is given by the solution to

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[17] In this region, $n = 0$, $z = z' = z'' = 0$, and the equilibrium values of the eight unknowns, q, x, y, c, c', c'', n' and n'', can be solved from (3), (8), (9), (10), (12), (14) (two equations) and (15).

(18) $$\frac{(1/c'')}{\beta(1/c)} = \frac{\beta^3}{q^2} = 1;$$

i.e. $q = \beta^{3/2}$.[18] When storage is used, at the time of harvest agents in effect mix two portfolio/investment strategies, one fast, the other slow:

fast strategy: store in order to invest in the next period;

slow strategy: buy two-period paper twice in succession in order to invest four periods later.

See Figure 4. The drawback to the fast strategy is that the return on storage is lower than the return on paper:

(19) $$1 \quad < \quad \frac{1}{\sqrt{q}} = \frac{1}{\beta^{3/4}} \quad < \quad \frac{1}{\beta}.$$

return on storage	implied one-period return on nonsaleable two-period paper	one-period rate of time preference

Despite this return dominance, the agent is willing to put corn into storage at the time of harvest because it affords him the flexibility to invest in the next period rather than having to wait four periods. On balance, the agent is indifferent between the two strategies, since (from (16) and (18)) they have the same six-perid return (of $1/\beta^6$):

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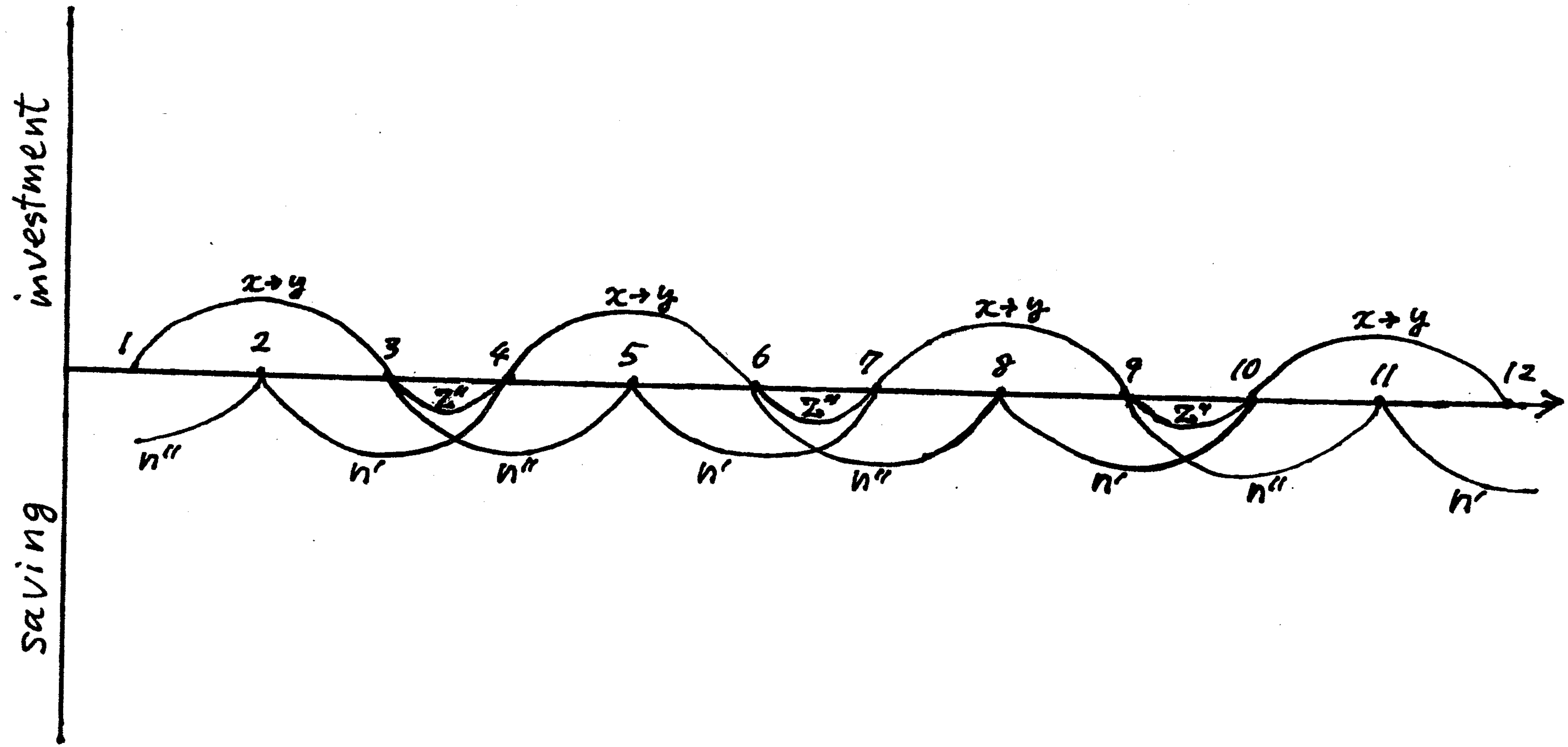
[18] In this region, $n = 0$, $z = z' = 0$, $q = \beta^{3/2}$, and the equilibrium values of the eight unknowns, x, y, z'', c, c', c'', n' and n'', can be solved from (3), (8), (9), (10), (12), (14) (two equations) and (15).

Figure 4


investment
saving
x+y
x+y
x+y
x+y
1
2
3
4
5
6
7
8
9
10
11
12
z"
z"
z"
n"
n'
n"
n'
n"
n'
n"
n'

$$(20) \qquad 1 \times \left(\frac{1-\theta}{G'(y) - \theta q} \right) \times 1 \times \left(\frac{1-\theta}{G'(y) - \theta q} \right) = \frac{1}{q^2} \times \left(\frac{1-\theta}{G'(y) - \theta q} \right).$$

$$\underbrace{\left(\begin{array}{c}\text{three-period return} \\ \text{on the fast strategy}\end{array}\right)^2}_{} \qquad \begin{array}{c}\text{six-period return} \\ \text{on the slow strategy}\end{array}$$

The shortfall in the return on storage relative to the return on nonsaleable two-period paper can be thought of as a liquidity premium, reflecting the preference agents have for liquid means of saving: holding nonsaleable two-period paper entails skipping the opportunity to finance the next period's investment from current saving.[19] This liquidity premium arises if and only if there is a shortage of liquidity. If $\theta \geq \theta^*$, there is no such premium.[20]

Incidentally, if θ falls to extremely low levels, so that very little two-period paper is supplied, then in equilibrium agents cease to use the slow strategy, and instead only hold paper to furnish consumption in periods of growing, i.e. $n'' > 0$ but $n' = 0$. We exclude this uninteresting case by assuming that the following weak condition holds:

$$(\text{C.1}) \qquad \theta \geq \underline{\theta} \equiv \beta^{1/2} \left(\frac{1 - \beta^3 + \lambda\beta^3}{1 + \beta^{1/2} + \beta + (1-\lambda)\beta^2(1 - \beta^{3/2})} \right).$$

[19] There is a parallel between our agents' use of storage and the conventional precautionary demand for money. However, the parallel is not exact, because precautionary money demand is essentially to do with individual uncertainty, whereas in our model agents' lives are deterministic.

[20] If the model is extended to allow for paper of longer maturity than two periods, then our assumption of nonsaleability might be relaxed to limited saleability. For example, if the time interval between periods is very short, we might suppose that paper has to be held for at least two periods (it takes that long to buy and sell). In this case, perfectly saleable paper would still command a liquidity premium over paper with limited saleability, provided the economy is short of liquidity.

Note that $\underline{\theta} \approx 0$ if $\beta \approx 1$ and $\lambda \approx 0$.

Proposition 2 underpins all the discussion since Proposition 1.

<u>Proposition 2</u> Suppose $\underline{\theta} \le \theta < \theta^*$, where $\underline{\theta}$ and θ^* are defined in (C.1) and (13). Then there exists a unique symmetric steady-state equilibrium in which:

(i) the price of two-period paper q strictly exceeds the two-period discount factor β^2, but is no greater than $\beta^{3/2}$;

(ii) consumption is highest at time of harvest and lowest at the time of investment: $c'' > c' > c$;

(iii) borrowing constraints bind at the time of investment: $n = 0$;

(iv) investment and output are lower than in first-best: $x < x^*$ and $y < y^*$.

Moreover, as θ falls, q, c''/c' and c'/c all rise, and x and y both fall.

There is a critical value of θ, which lies strictly between $\underline{\theta}$ and θ^*:

$$(21) \qquad \hat{\theta} = \frac{\beta^{1/2} + \beta - \lambda \dfrac{\beta^3(1 + \beta^{3/2} + \beta^2)}{1 + \beta^2 + \beta^4}}{1 + \beta^{1/2} + \beta + \lambda \dfrac{(1 + \beta^{3/2} + \beta^2)(\beta^{3/2} - \beta^3)}{1 + \beta^2 + \beta^4}}.$$

For $\hat{\theta} \le \theta < \theta^*$, no storage is used. For $\underline{\theta} \le \theta < \hat{\theta}$, agents store at the time of harvest, $z'' > 0$, but not at any other time. In this latter region, the price of two-period paper q is constant at $\beta^{3/2}$, so the return on storage is strictly less than the implied one-period return on paper: $1 < 1/\sqrt{q}$.

Before closing this section, we should remark on the possible circulation of fiat, or outside, money. Suppose the economy is endowed with a fixed stock of intrinsically useless, perfectly durable "seashells". Then for $\theta \geq \hat{\theta}$ seashells cannot have positive value in a symmetric steady state equilibrium. However, for $\theta < \hat{\theta}$ there can be an equilibrium in which seashells have positive value and substitute for storage as a means of liquid saving (i.e. saving from one period to the next, as opposed to saving using nonsaleable two-period paper). In such an equilibrium, the aggregate stock of liquid saving at the end of a period would be equal to the value of seashells plus the total quantity of corn in storage. Symmetric steady-state equilibrium allocations are the same whether or not seashells are used. But if the agents are in an equilibrium where seashells are not being used, then, as in Samuelson (1958), they could enjoy a one-off consumption gain if they switched to using seashells, since this would avoid the need to tie up corn in storage.

3. Banking and inside money

There are a number of ways in which inside money might be introduced into the economy. In this section, we consider the role of financial intermediaries, or banks. We suppose that, at a cost, banks are able to issue their own saleable paper against nonsaleable paper that they purchase. The banks' paper circulates as inside money.

For clarity, let us call the agents in Section 2 farmers. Add to the economy a continuum of new agents, bankers, who have the same preferences as farmers. We normalise the population of bankers to equal unity, so that we can think of a representative banker. In each period t, the representative banker has an available "banking capacity", k_t, which can be used to issue k_t saleable two-period paper against k_t nonsaleable two-period paper purchased from farmers. After two periods of use, banking capacity depreciates at a rate δ. But by investing $F(i_t)$ corn, the bank can (instantaneously) increase the available banking capacity in period t by i_t:

$$(22) \qquad k_t = (1-\delta)k_{t-2} + i_t,$$

where $F(0) = F'(0) = 0$ and $F''(i) > 0$ for $i > 0$.[21] Notice that banks are intrinsically unproductive in the sense that they use corn, but don't produce any.

To justify why bankers are able to issue saleable paper against nonsaleable paper, recall why two-period paper issued by a farmer investing in period t cannot be resold in period t+1. We supposed that the farmer and his original creditor can together steal from the project between periods t and t+1, leaving just a shell that delivers no output, but which cannot be distinguished from an intact project by outsiders. Given this possibility,

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[21]Because of the two-period nature of banking (capacity is used for two periods, to issue two-period saleable paper against two-period nonsaleable paper), there are in effect two distinct paths of banking capacity, one available for the odd-numbered periods, and one for the even.

no-one is willing to buy the paper at date t+1.

Theft requires the project key, which is held as security, together with the farmer's paper, by the original creditor. Now suppose a bank can invest in a strong box, just large enough to hold one project key and the accompanying paper. The box has two features. First, having been locked using the box key, it is specially designed so that it cannot be unlocked, even with the box key, until two periods later. Second, the box is "transparent" in the sense that, without opening it, everyone can verify what is inside, when it was last locked (and hence when it can next be unlocked), and whether the box key fits the lock. After each use, only a fraction $1-\delta$ of boxes can be re-used.[22]

The bank purchases two-period paper from an investing farmer in period t, and locks the paper and the project key in a box. The bank then sells its own two-period paper, secured by the box key. Unlike the farmer's paper, the bank's paper is saleable. The reason is that someone selling the bank's paper in period t+1 can not only show to potential buyers the farmer's paper and project key inside the box, and confirm that the box key fits the box's lock, but also prove that, thanks to the box's special design, no-one can have used the project key since the project started. In particular, the farmer cannot have gained access to the project to steal.

In rough terms, the idea is that whenever someone buys paper secured by a key, they don't know how the key has been used to date. In the case of a project key this matters a lot, because the key could have been used at any time for theft. But in the case of a box key it matters less, since boxes cannot be opened except at designated times. Box keys thus afford greater security than project keys.

Again, we should stress that these detailed microfoundations are not essential to our argument. Our main concern is with the aggregate effects of introducing banks and inside money into the economy, taking as given the "reduced form" notion of banking capacity k_t, together with the equation of

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[22]The function F(.) corresponds to the cost of building new boxes.

motion (22) and the cost function F(.).

In any period, there will be two different vintages of bankers' paper traded: newly-issued paper, and paper that was issued one period earlier. Since there is no uncertainty in the model, both vintages are perfect substitutes as means of short-term saving. Let p_t denote the price of period t+1 corn in terms of period t corn. Then a unit of bankers' paper, promising to deliver one unit of corn in two periods time, is priced at $p_t p_{t+1}$ when newly issued in period t, and is priced at p_{t+1} one period later. In aggregate, the bankers' supply of liquidity in period t -- i.e. the means of saving short-term to receive one unit of corn in period t+1 -- equals $p_{t+1}k_t$ (the value in period t+1 of the stock of paper the bankers issue in period t) plus k_{t-1} (the stock of paper they issued in period t-1 maturing in period t+1).

By contrast, farmers' two-period paper is traded only once, when it is newly issued. In equilibrium, the price of farmers' paper, q_t, can never be strictly greater than the price of newly-issued bankers' paper, $p_t p_{t+1}$, because the latter can always be used as an alternative means of saving for two periods. Equally, p_t can never be strictly greater than 1, since storage can always be used as an alternative means of short-term saving.

The representative banker chooses a sequence of consumption c_t^b, investment $F(i_t)$, and banking capacity k_t, to maximise discounted utility (1) subject to (22) and the flow of funds constraint

$$c_t^b + F(i_t) = (p_t p_{t+1} - q_t)k_t. \tag{23}$$

The RHS of (23) is the return in period t from issuing k_t bankers' paper at price $p_t p_{t+1}$ against k_t farmers' paper purchased at price q_t.[23]

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[23] Here we are implicitly assuming that a banker does not choose to save using paper. This is indeed the case, at least in steady state.

In a steady-state equilibrium (dropping time subscripts), $i = \delta k$,

$$(24) \qquad c^b = (p^2 - q)k - F(\delta k),$$

and banking capacity k is chosen to equate marginal cost to discounted return:

$$(25) \qquad F'(\delta k) = \frac{p^2 - q}{1 - \beta^2(1-\delta)}.$$

Notice from (25) that, given our assumptions about the cost function F(.), the banking sector is active if and only if the price of saleable two-period paper, p^2, strictly exceeds the price of nonsaleable two-period paper, q; i.e. if and only if there is a liquidity premium:

$$(26) \qquad \underbrace{\frac{1}{p}}_{\text{one-period return on saleable two-period paper}} < \underbrace{\frac{1}{\sqrt{q}}}_{\text{implied one-period return on nonsaleable two-period paper}}.$$

The rest of the economy is similar to before, except that now farmers can use bankers' paper (as well as storage) to save short-term. Clearly, storage is not used if the return on bankers' paper is superior:

$$(27) \qquad \text{either } p < 1 \text{ and } z = z' = z'' = 0, \quad \text{or } p = 1.$$

The farmers' flow of funds constraints (8), (9) and (10) now read

$$(28) \qquad x + c + pm + qn = q\theta y + m'' + n'$$

$$(29) \qquad c' + pm' + qn' = m + n''$$

$$(30) \qquad c'' + pm'' + qn'' = (1-\theta)y + m' + n,$$

where m, m′ and m″ denote their short-term saving at the time of investing, growing and harvesting. m, m′ and m″ must all be nonnegative.

The market-clearing conditions for corn and farmers' paper now include the bankers' activity:

$$(31) \qquad y = c + c' + c'' + c^{b} + x + F(\delta k);$$

$$(32) \qquad \theta y = n + n' + n'' + k.$$

In addition, we have the market-clearing condition for liquidity:

$$(33) \qquad pk + k + z + z' + z'' = m + m' + m''.$$

The LHS of (33) is the bankers' supply of liquidity, pk + k, plus storage.

The qualitative nature of the equilibrium depends on the region of the parameter space. In particular, if the banking sector is large -- if the costs F(.) are small -- then the banks soak up most of the farmers' paper, and farmers mainly save using the bankers' paper.[24] To facilitate comparisons with Propositions 1 and 2, we consider here a smaller banking sector, with higher costs. As in Section 2, our discussion will be informal; details are

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[24] Specifically, n′ drops to zero. That is, n = n′ = 0, and the farmers only buy just enough nonsaleable two-period paper at the time of harvesting (n″) to furnish consumption two periods later, at the next time of growing.

in the Appendix.

Recall from Propositions 1 and 2 that in the economy without bankers and inside money, there are three principal regions, indexed by the farmers' ability to commit, θ. Given that the banking sector is not too large, the same is true here. In the first region, θ is high enough that the first-best can be achieved. In the other two regions, where θ is lower, there is a shortage of paper. The second and third regions are distinguished by whether there is enough bankers' paper to meet liquidity demand, or whether storage has to be used too. Let us deal with each of these three regions in turn.

In the first region, where the first-best is attained, the critical lower bound on θ is still θ^*, defined in (13). For $\theta \geq \theta^*$, we know from Proposition 1 that the economy without bankers achieves the first-best, and that in equilibrium the return on farmers' paper equals the rate of time preference: $\sqrt{q} = \beta$. But this allocation is also the equilibrium of our economy, with the banking sector inactive ($k = 0$ and $c^b = 0$) because farmers are unwilling to pay more than β for liquidity, and without a liquidity premium there is no return to banking. For $\theta < \theta^*$, since the first-best can't be attained in the economy without bankers (see Proposition 2), it can't be attained in an economy with bankers either, since banks are intrinsically unproductive and so shouldn't operate in the first-best.

In the second and third regions ($\theta < \theta^*$), away from the first-best, there is a shortage of farmers' paper. As a result, the return is strictly less than the rate of time preference ($1/\sqrt{q} < 1/\beta$). This goes hand in hand with a liquidity premium ($1/p < 1/\sqrt{q}$). Hence banks are active ($k > 0$). And given that the bankers supply liquidity, the farmers must demand it. Thus, qualitatively, a typical farmer's behaviour is as depicted in Figure 4 -- except that he uses bankers' paper in addition to, or instead of, storage as a means of short-term saving. At the time of harvest, he mixes the fast strategy (save short-term to invest in the next period) and the slow strategy (save twice using other farmers' nonsaleable paper to invest four periods later). The indifference condition (20) is modified to:

$$(34) \qquad \frac{1}{p} \times \left(\frac{1 - \theta}{G'(y) - \theta q} \right) \times \frac{1}{p} \times \left(\frac{1 - \theta}{G'(y) - \theta q} \right) = \frac{1}{q^2} \times \left(\frac{1 - \theta}{G'(y) - \theta q} \right).$$

The farmer's first-order conditions (14) and (15) still apply.[25] Hence (16) holds, which, together with (34), implies

$$(35) \qquad p = \frac{q^2}{\beta^3}.$$

Equation (35) provides a useful relationship between the price of bankers' paper and the price of farmers' paper. In particular, away from the first-best where $p = \sqrt{q} = \beta$, (35) squares with our earlier assertions, that because of the shortage of farmers' paper the return is strictly less than the rate of time preference, and that there is a liquidity premium.[26]

In the second region, θ lies strictly between θ^* and some critical value $\hat{\theta}^b < \theta^*$. Here the bankers supply enough liquidity to meet the farmers' demand. p is strictly less than 1, and no storage is used. From (35), q is strictly less than $\beta\sqrt{\beta}$.

In the third region, θ lies strictly below $\hat{\theta}^b$. The liquidity shortage is sufficiently severe that the farmers resort to using storage, in addition to bankers' paper, as a means of short-term saving. p equals 1, and, from (35), q equals $\beta\sqrt{\beta}$.

The remainder of this section is devoted to asking: Does the banking sector supply "enough" liquidity in equilibrium? With this question in mind,

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[25] As in Section 2, the farmers do not save at the time of investing ($n = 0$ and $m = 0$, implying $z = 0$); and they do not save short-term at the time of growing ($m' = 0$, implying $z' = 0$). The equilibrium values of the thirteen unknowns, p, q, x, y, z'', k, c, c', c'', c^b, m'', n' and n'', can be solved from (3), (14) (two equations), (15), (24), (25), (27), (28), (29), (30), (32), (33) and (34). By Walras' Law, (32) and (33) imply (31).

[26] To see why, rewrite (35) as $p/\sqrt{q} = \left(\sqrt{q}/\beta\right)^3$. Hence $p > \sqrt{q}$ iff $\sqrt{q} > \beta$.

let us assume that $\theta < \theta^*$, so that there is a liquidity shortage, and the banks are active. We may be in either the second or the third region.

Consider the following policy experiment. Suppose that an outside agent, say the government, tries to increase banking capacity by subsidizing the bankers' investment at a rate $\tau > 0$. The subsidy is financed by a lump-sum tax on the bankers themselves (the government has to break even in each period). Our assumption is that the government has sufficient information about the bankers, and power over them, to implement these subsidies and lump-sum taxes. However, the government knows nothing about individual farmers (not even their whereabouts), and so cannot subsidize or tax them. A fortiori, the government cannot assist with enforcing debt repayment by farmers.[27]

The representative banker's flow of funds constraint (23) now reads

$$(36) \qquad c_t^b + (1-\tau)F(i_t) = (p_t p_{t+1} - q_t)k_t - \tau F(\bar{i}_t),$$

where the final term is the lump-sum tax, and $\bar{i}_t$ is the average investment of bankers (which individual bankers take as given). In steady-state, the first-order condition (25) is modified to

$$(37) \qquad (1-\tau)F'(\delta k) = \frac{p^2 - q}{1 - \beta^2(1-\delta)}.$$

Unfortunately it is not easy to analyse the welfare implications of τ, because the economy has heterogeneous agents: farmers at three different points in their production cycle, plus bankers. Moreover, a full-blooded welfare analysis would require consideration of transition dynamics in

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[27]Another interpretation of this policy experiment is that the government simply takes control over the banking sector, and at the end of each period pays out any net receipts to the bankers for them to consume.

response to policy changes, which would take us beyond the scope of this article. Instead, we simply consider one particular steady-state welfare measure, net output, defined as

$$(38) \qquad W = \underbrace{\left(\beta^2 y - G(y)\right)}_{\text{net output from farming}} + \underbrace{(\beta - 1)(z + z' + z'')}_{\text{net output from storage}} - \underbrace{[1 - \beta^2(1-\delta)]\frac{F(\delta k)}{\delta}}_{\text{net output from banking}}.$$

The first term, $\beta^2 y - G(y)$, is the output from farming net of cost, taking into account that output is obtained two periods later and so must be discounted by β^2. Similarly, the middle term is the discounted output from storage, $\beta(z + z' + z'')$, minus input, $z + z' + z''$. The final term is the user cost of banking capacity.[28] It should be borne in mind that W is only a partial measure of welfare, in the sense that it ignores distributional issues and consumption smoothing.

Notice that W is maximised at the first-best allocation, where $G'(y) = \beta^2$ and $z = z' = z'' = k = 0$. Only farming makes a positive contribution to W. In direct terms, storage and banking are both unproductive. However, they provide an indirect benefit because they lubricate the economy, thus giving a boost to farming.

Now the direct effect of τ is to raise the equilibrium level of banking capacity k. The question is: Does this increase W? The answer depends on whether the economy is in the second or third region.[29]

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[28] To see why, consider the case where a banker's investment cost is linear (constant F'). In this case, the user cost per unit of banking capacity is $[1 - \beta^2(1-\delta)]F'$ -- i.e. production cost minus discounted "resale" price. When $F(.)$ is convex, we value the cost of producing k units as $\left(\frac{F(\delta k)}{\delta k}\right)k$, by using the average cost of investment.

[29] Take τ to be small enough that the subsidy doesn't cause the economy to change region.

In the third region, prices are pegged ($p = 1$ and $q = \beta\sqrt{\beta}$), so there are only quantity effects. Moreover, farming output y is unaffected, since, from (16), y is a function of q alone. Therefore the increase in k comes about entirely through a reduction in storage. We show in the Appendix that, on balance, W rises. One interpretation of this result is that, by shifting resources from storage to banking, τ improves the efficiency with which liquidity is supplied to the economy. Loosely put, banking is the lesser of two evils. Also notice that, since the supply θy of farmers' paper is fixed, τ speeds the economy up in the sense that more of the paper is held by banks, and less by other farmers.[30]

In the second region, where there is no storage, an increase in k causes p and q to fall (maintaining the relationship given in (35)). The question is whether the indirect benefit of banking -- lubricating the economy to boost farming -- outweighs the direct cost. It turns out that, in terms of net output, the answer is no: W is reduced by a subsidy τ (see the Appendix). The intuition for this is that, without storage, there are only two activities in the economy, farming and banking, and it makes no sense to subsidize the unproductive activity if the objective is to maximize net output. However, this ignores the fact that τ reduces p and q -- i.e. raises the returns on saleable and nonsaleable paper nearer to the rate of time preference -- and hence improves consumption smoothing (for example, see (14)). Without further work, in this region we cannot say what the overall effect on welfare is of a subsidy τ.

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[30] There is an important note of caution to be added to the conclusion that the banking sector is "too small" if storage is used in equilibrium. As we briefly discussed at the end of Section 2, were fiat money -- seashells -- to circulate, then this would obviate the need for storage, and our welfare analysis would not hold.

However, if storage yielded a positive net return, storage would strictly dominate holding seashells as a means of short-term saving (assuming the government doesn't retire seashells). All our analysis would be essentially unchanged by having a positive net return on storage (but less than the net rate of time preference). We assumed a one-for-one storage technology simply to keep the algebra less cluttered.

4. Asset-backed money

Another way for inside money to be introduced into the model of Section 2 is to add to the economy some durable asset, land say, which, unlike nonsaleable paper, can always be traded and so can serve as collateral for liquidity. In this section we explore how the economy performs when paper secured against land circulates as inside money.

Return to the model of Section 2, where there are no bankers, and farmers are simply "agents". In that model, there are only two factors of production: corn and the human capital of the agents. By assumption, agents are unable to borrow against their human capital. And although they can issue paper secured against their corn output, there is a limit: they can only make a bilateral commitment to pay a fraction θ to the original purchaser of the paper. To allow for a greater degree of commitment, let us now include a third factor of production, land, in the model. Land is in fixed aggregate supply, 3L, and does not depreciate.

Specifically, let the structure of the model be the same as before, except that (2) is replaced by a Cobb-Douglas technology that uses ℓ_t land to convert x_t corn in period t into y_{t+2} corn in period t+2:

$$y_{t+2} = \left(\frac{\ell_t}{\alpha\lambda\mu} \right)^{\lambda\mu} \left(\frac{x_t}{\alpha(1-\lambda)} \right)^{1-\lambda}, \tag{39}$$

where $0 < \mu < 1$. (Notice that the sum of the exponents in (39) is strictly less than 1: the missing factor is the investing agent's human capital.) We assume that the land is fully utilized throughout production, in all three periods t, t+1 and t+2, and so only becomes available for further investment in period t+3.[31] Let v_t denote the price of land in terms of corn in period

[31] Note that in a symmetric allocation where land is used equally across agents, $\ell_t \equiv L$, and (39) reduces to (2) with $a = \left(\alpha^{1-\lambda+\lambda\mu}(\lambda\mu/L)^{\lambda\mu}\right)^{1/(1-\lambda)}$.

t.[32]

It seems reasonable to suppose that, unlike the corn output, land can be fully mortgaged. An agent using ℓ_t land to invest in period t can credibly issue $v_{t+3}\ell_t$ three-period paper -- where "three-period paper" refers to a promise, secured against land, to deliver one unit of corn in three periods time -- since in period t+3 the land will have collateral value $v_{t+3}\ell_t$.

It also seems reasonable to suppose that this three-period paper is saleable, unlike two-period paper secured against corn. The point is that land is a tangible asset that anyone can not only verify but also, if necessary, seize. In other words, an agent issuing three-period paper against land is able to make a multilateral commitment. Three-period paper can be used as inside money.

In any period, there will be three different vintages of three-period paper being traded: newly-issued paper, paper that was issued in the previous period, and paper that was issued two periods earlier. Since there is no uncertainty in the model, all three vintages are perfect substitutes as means of short-term saving. For convenience, we shall refer to them all simply as "saleable paper", irrespective of their vintage. Let p_t denote the price of period t+1 corn in terms of period t corn, using saleable paper. Hence three-period paper maturing in period t+3 is priced at $p_t p_{t+1} p_{t+2}$ when it is newly issued in period t; it is priced at $p_{t+1} p_{t+2}$ in period t+1; and it is priced at p_{t+2} in period t+2. By contrast, the two-period paper is marketed only once, when it is newly issued. This we shall refer to as simply

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[32] We assume land is purchased outright, rather than rented three periods at a time. This makes no difference to the steady-state equilibrium allocations.

The equilibrium would change, however, if land were rented period-by-period (although we believe the model would be qualitatively the same). But there are reasons to suppose that, at the time of investment, an agent might need to purchase control over the land he will be using for all three periods of production, insofar as he does not want to be held to ransom by a landlord before harvest. This would be especially true if he faced individual income uncertainty (see Hart and Moore (1998)) -- but this takes us much beyond the present analysis.

"nonsaleable paper".

We will see that saleable paper dominates storage as a means of saving, so there is no storage in equilibrium ($z_t \equiv 0$). Let m_t denote an agent's holding of saleable paper at the end of period t. As in Section 2, we let n_t denote his holding of newly-issued nonsaleable paper. Both m_t and n_t must be nonnegative. If the agent invests in period t then his flow of funds constraint is

$$(40) \qquad x_t + v_t\ell_t + c_t + p_tm_t + q_tn_t$$

$$= q_t\theta y_{t+2} + p_tp_{t+1}p_{t+2}v_{t+3}\ell_t + m_{t-1} + n_{t-2}.$$

Compared with (5), the LHS of (40) includes the expenditure on land, $v_t\ell_t$, and the expenditure on saleable paper, p_tm_t. The RHS includes the borrowing against land, $p_tp_{t+1}p_{t+2}v_{t+3}\ell_t$, and the return from saleable paper purchased in period t-1, m_{t-1}. (Note that any land, ℓ_{t-3}, that the agent purchased three periods earlier was fully mortgaged: the agent issued $v_t\ell_{t-3}$ three-period paper secured against it. The receipts from selling the land do not appear in (40), because they are exactly offset by repayment of the debt.) The agents' flow of funds constraints in period t+1 and t+2 are

$$(41) \qquad c_{t+1} + p_{t+1}m_{t+1} + q_{t+1}n_{t+1} = m_t + n_{t-1}$$

$$(42) \qquad c_{t+2} + p_{t+2}m_{t+2} + q_{t+2}n_{t+2} = (1-\theta)y_{t+2} + m_{t+1} + n_t$$

-- which are similar to (6) and (7).

Competitive equilibrium is defined as a sequence of paper prices, investment, output, consumption, and paper holdings, $\{p_t, q_t, x_t, y_{t+2}, c_t, m_t, n_t\}$, such that in each period t: first, each agent chooses $\{x_t, y_{t+2}, c_t, m_t, n_t\}$ to maximize utility (1) subject to the production function (39) and the flow of funds constraints (40), (41) and (42); and, second, the markets for corn, land, saleable paper and nonsaleable paper clear.

As in Section 2, we are concerned with characterising a symmetric steady-state equilibrium. Adopting the same notation convention as before, the flow of funds constraints (40), (41) and (42) reduce to

$$x + v\ell + c + pm + qn = q\theta y + p^3 v\ell + m'' + n'; \tag{43}$$

$$c' + pm' + qn' = m + n''; \tag{44}$$

$$c'' + pm'' + qn'' = (1-\theta)y + m' + n. \tag{45}$$

The market-clearing condition for corn is given by (11). Since land is used equally by agents who are investing, growing and harvesting, the land market clears when

$$\ell = L. \tag{46}$$

The supply of saleable paper is the stock of the three-period paper that matures next period, $v\ell$, plus next period's value of the three-period paper that matures in two periods time, $pv\ell$, plus next period's value of the three-period paper that is issued this period, $p^2 v\ell$. Hence the saleable paper market clears if

$$v\ell + pv\ell + p^2 v\ell = m + m' + m''. \tag{47}$$

Finally, the market-clearing condition for newly-issued nonsaleable paper is given by (12).[33]

The cost of production can be thought of as $x + w\ell$, where

$$(48) \qquad w = v - p^3 v$$

is the downpayment required to purchase a unit of land fully mortgaged. Minimizing cost subject to (39) we obtain the cost function

$$(49) \qquad G(y) \equiv \alpha(1-\lambda+\lambda\mu)\left(w^{\lambda\mu}y\right)^{\frac{1}{1-\lambda+\lambda\mu}}$$

and the input demand functions

$$(50) \qquad \ell = \alpha\lambda\mu\left(\frac{y}{w^{1-\lambda}}\right)^{\frac{1}{1-\lambda+\lambda\mu}}$$

$$(51) \qquad x = \alpha(1-\lambda)\left(w^{\lambda\mu}y\right)^{\frac{1}{1-\lambda+\lambda\mu}}.$$

As in Proposition 1, for high enough values of θ, the first-best can be achieved:

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[33] By Walras' Law, three of (11), (12), (46) and (47) imply the fourth.

Proposition 3 The symmetric, steady-state equilibrium is first-best efficient if $\theta \geq \theta^{**}$, where

$$\theta^{**} = \frac{1}{3}\left(2 - \frac{\lambda\beta^2}{1+\beta^2+\beta^4}\left(1 + 2\beta^2 + 3\mu\frac{\beta(1+\beta^2)}{1-\beta}\right)\right). \tag{52}$$

In this region, the one-period return on saleable paper, $1/p$, and the implied one-period return on nonsaleable paper, $1/\sqrt{q}$, both equal the rate of time preference, $1/\beta$. That is, $p = \beta$ and $q = \beta^2$.

Notice from (52) that the larger is μ, the smaller is θ^{**} (particularly given the factor $1/(1-\beta)$, which can be quite big). That is, the more important is land to the technology (39), so the price of land is higher and the stock of inside money is greater -- which means that less bilateral commitment is needed to sustain the first-best. In what follows, we should think of μ as small, otherwise the economy will be swamped with enough money to guarantee that the equilibrium is first-best efficient.

When θ is strictly less than θ^{**}, there is not enough paper supplied for the economy to achieve first-best, and both $p > \beta$ and $q > \beta^2$. The same symptoms occur as before. Consumption is jagged.[34] The investing agents face borrowing constraints: $m \geq 0$ and $n \geq 0$ both bind. Investment and output are lower than in the first-best. Since the investing agents sell saleable (three-period) and nonsaleable (two-period) paper, both papers must be used as means of saving by the other agents. The equilibrium is like in Figure 4, except that saleable paper is used for short-term saving rather than storage. In particular, at the time of harvest, agents mix the same fast and slow portfolio/investment strategies that we identified earlier. The three-period return on the fast strategy is now

[34] The first-order conditions (14) and (15) still hold.

(53) $$\frac{1}{p} \times \left(\frac{1 - \theta}{G'(y) - \theta q} \right) = \frac{1}{\beta^3},$$

given that saleable paper is used instead of storage. And the six-period return on the slow strategy is

(54) $$\frac{1}{q^2} \times \left(\frac{1 - \theta}{G'(y) - \theta q} \right) = \frac{1}{\beta^6}.$$

Hence, from (53) and (54), it follows that

(55) $$p = \frac{q^2}{\beta^3}.$$[35]

When $\theta < \theta^{**}$, so that $p > \beta$ and $q > \beta^2$, (55) implies that there is a liquidity premium:

(56) $$\underbrace{\frac{1}{p}}_{\substack{\text{one-period}\\ \text{return on}\\ \text{saleable}\\ \text{paper}}} < \underbrace{\frac{1}{\sqrt{q}}.}_{\substack{\text{implied}\\ \text{one-period}\\ \text{return on}\\ \text{nonsaleable}\\ \text{two-period}\\ \text{paper}}}$$

Nonsaleable paper has a higher return than saleable paper to compensate for its lack of flexibility. One measure of the liquidity premium is

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[35] The equilibrium values of the thirteen unknowns, p, q, v, w, x, y, ℓ, c, c', c", m", n' and n", can be solved from (11), (12), (14) (two equations), (15), (43), (44), (45), (46), (48), (50), (51) and (55). In this equilibrium, n = 0; and neither the investing nor the growing agents hold saleable paper: m = m' = 0.

$(p^2 - q)/q$, which is the additional cost of saleable two-period paper relative to nonsaleable two-period paper. Recall from Proposition 3 that for $\theta \geq \theta^{**}$, where the first-best is achieved, $p = \sqrt{q} = \beta$ and there is no liquidity premium. Thus the premium only arises when there is a shortage of paper ($\theta < \theta^{**}$). Moreover, the lower is θ, the higher are p and q, and the higher is the liquidity premium $(p^2 - q)/q$.[36]

The return on paper, $1/p$, is strictly greater than the return on storage, 1. The reason is that, as can be seen from (48), the value of land -- and hence the supply of inside money -- grows without limit as p approaches 1.[37] That is, the demand for liquidity will always be satisfied by saleable three-period paper secured against land, so storage is never needed.

As before, when θ falls to extremely low levels, agents cease to purchase nonsaleable paper twice in succession ($n' \geq 0$ binds). To rule out this possibility, we assume (C.1) and

$$\theta \geq \frac{1 - \beta^3 + \lambda(1-\mu)\beta^3}{(2 - \beta)(1 + \beta + \beta^2)}. \tag{C.2}$$

Note that the RHS of (C.2) is approximately zero when $\beta \approx 1$ and $\lambda \approx 0$.

Proposition 4 substantiates the claims we have made, and gives further details of the equilibrium.

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[36]From (55), $(p^2 - q)/q = (q^3 - \beta^6)/\beta^6$, which increases as q increases.

[37]The marginal product of land is always positive (and hence so is w), given the Cobb-Douglas production function (39).

Proposition 4 Suppose $\theta < \theta^{**}$, where θ^{**} is defined in (52); and assume conditions (C.1) and (C.2) both hold. Then there exists a unique symmetric steady state equilibrium in which:

(i) there is rate of return dominance: $\beta < \sqrt{q} < p < 1$;

(ii) consumption is jagged: $c'' > c' > c$;

(iii) borrowing constraints bind at the time of investment: $m = n = 0$;

(iv) investment and output are lower than in first-best: $x < x^*$ and $y < y^*$.

Moreover, as θ falls, p, q, c''/c', c'/c, and the liquidity premium $(p^2 - q)/q$ all rise, and x and y both fall.

To ascertain the effect of introducing inside money, we should compare the model with land to the model without land from Section 2. An easy way to make this comparison is to ask: What would happen in the land model if there were no inside money and, instead, three-period paper were not saleable, i.e. if there were no multilateral commitment? It turns out that the models would then coincide, in terms of corn and two-period paper allocations and prices. The reason is that, in the land model, nonsaleable three-period paper would not be traded in equilibrium: the price of such paper would be β^3, the three-period rate of time preference, and there would be no gains from trade. The same is true for land: in a symmetric allocation, all agents are using equal amounts of land, and there are no gains from trade.

Proposition 5 shows that the land model with inside money performs better than the model without inside money.

Proposition 5 In the land model, the symmetric steady-state equilibrium without inside money (i.e. when three-period paper is nonsaleable) has a higher price q, more jagged consumption, and lower investment x and output y, than the model with inside money (where three-period paper is saleable). And the minimum level of θ needed to achieve the first-best is higher: θ^* (from (13)) strictly exceeds θ^{**} (from (52)).

Notice that in moving from the case of saleable to nonsaleable three-period paper, we are not taking away any means of saving from the economy. Rather, we are making the paper less liquid; and this is what causes the economy to perform worse and slow down.

There as a nice insight that can be gleaned from the land model when three-period paper is nonsaleable. Given that the price of such paper is β^3, there is a triple rate of return dominance:

(57)	1	$<$	$\frac{1}{\sqrt{q}}$	$<$	$\frac{1}{\beta}$
	return on storage		implied one-period return on nonsaleable two-period paper		implied one-period return on nonsaleable three-period paper

The point to observe here is that, if we lengthen the maturity of nonsaleable paper, the implied one-period rate of return must rise in order to compensate for the reduction in flexibility.

Appendix

Proof of Proposition 1 and 2

Since consumption is always positive due to infinite marginal utility at zero consumption, the first order conditions for choice between consumption and purchasing two-period papers n, n′ and n″ are

$$\text{(A1)} \quad \frac{1}{q} \le \frac{(1/c)}{(\beta^2/c'')}, \quad \text{and} \quad \left(\frac{1}{q} - \frac{(1/c)}{(\beta^2/c'')} \right) n = 0,$$

$$\text{(A2)} \quad \frac{1}{q} \le \frac{(1/c')}{(\beta^2/c)}, \quad \text{and} \quad \left(\frac{1}{q} - \frac{(1/c')}{(\beta^2/c)} \right) n' = 0,$$

$$\text{(A3)} \quad \frac{1}{q} \le \frac{(1/c'')}{(\beta^2/c')}, \quad \text{and} \quad \left(\frac{1}{q} - \frac{(1/c'')}{(\beta^2/c')} \right) n'' = 0.$$

Similarly, the first order conditions for choice between consumption and storage z, z′ and z″ are

$$\text{(A4)} \quad 1 \le \frac{(1/c)}{(\beta/c')}, \quad \text{and} \quad \left(1 - \frac{(1/c)}{(\beta/c')} \right) z = 0,$$

$$\text{(A5)} \quad 1 \le \frac{(1/c')}{(\beta/c'')}, \quad \text{and} \quad \left(1 - \frac{(1/c')}{(\beta/c'')} \right) z' = 0,$$

$$\text{(A6)} \quad 1 \le \frac{(1/c'')}{(\beta/c)}, \quad \text{and} \quad \left(1 - \frac{(1/c'')}{(\beta/c)} \right) z'' = 0,$$

The first order condition for choice between investment and consumption is given by (15), where equality holds because investment is always positive due to infinite marginal product at zero investment. The market clearing condition for two-period paper is given by (12), and the goods market clearing condition (11) is not independent due to Walras's Law. The symmetric steady state equilibrium is characterized by (q,x,y,c,c′,c″,n,n′,n″, z,z′,z″) which satisfies (3),(8),(9),(10),(12),(15), (A1),(A2),(A3),(A4),(A5) and (A6).

First, we prove the following lemma.

<u>Lemma 1</u>: $\beta^2 \le q \le 1$.

(Proof) From multiplying three inequalities of (A1), (A2) and (A3) side by side, we learn $q \geq \beta^2$ in the steady state. Also, supply of two-period paper is positive, at least one of n, n′ and n″ is strictly positive. Then from (A4), (A5) and (A6), we get $q \leq 1$, because the return on two period paper cannot be dominated by return on storage for two period in succession. QED.

<u>Lemma 2</u>: There is no symmetric steady state with the following patterns of paper holdings under condition (C1).

(i) $n > 0$, and $n' = n'' = 0$,
(ii) $n' > 0$, and $n = n'' = 0$,
(iii) $n'' > 0$, and $n = n' = 0$,
(iv) $n > 0$, $n' > 0$, $n'' = 0$,
(v) $n > 0$, $n'' > 0$, $n' = 0$.

Thus, the equilibrium trading pattern must be the one considered in the text:

(vi) $n' > 0$ and $n'' > 0$.

(Proof) (i) From (12), we know $\theta y = n$. Then from (8), we know $z'' > 0$, because x and c are strictly positive. Then from (A1) and (A6), we get

$$\frac{c}{c''} = \frac{q}{\beta^2} = \beta, \text{ or } q = \beta^3, \text{ which violates Lemma 1.}$$

Using the very similar arguments, the trading patterns of (ii) and (iv) leads to the violation of Lemma 1.

(iii) Because $n'' > 0$, (A3) implies

(A7) $\frac{c''}{c'} = \frac{q}{\beta^2} > \beta$ by Lemma 1.

Then from (A5), we know $z' = 0$. Then we consider four alternative cases:

(iii-1) $z'' > 0$ and $z = 0$,
(iii-2) $z'' > 0$, $z > 0$,
(iii-3) $z'' = 0$, $z > 0$,
(iii-4) $z'' = 0$, $z = 0$.

(iii-1) Because $z'' > 0$, we know from (A6) that

$$\text{(A8)} \qquad \frac{c}{c''} = \beta.$$

Also (8),(9) and (10) can be simplified using (3) as:

$$\text{(A9)} \qquad \begin{aligned} (1-\lambda)G'(y)y + c &= q\theta y + z'' \\ c' &= n'' \\ c'' + qn'' + z'' &= (1-\theta)y. \end{aligned}$$

Then, using the first order conditions (15),(A7) and (A8), we get

$$\text{(A10)} \qquad c'' = (1-\beta)\left(1 - \theta + \frac{\lambda G'(y)}{1-\beta^3}\right) y.$$

Then from (A7),(A8),(A9) and (15), (12) becomes:

$$\theta\, y = \frac{\beta^2}{q}(1-\beta)\left(1 - \theta + \lambda\,\frac{\theta q + (1-\theta)\beta^3}{1 - \beta^3}\right) y, \text{ or}$$

$$\text{(A11)} \qquad q = \frac{1-\theta}{\theta}\,\frac{\beta^2(1-\beta^3+\lambda\beta^3)}{1+\beta+\beta^2-\lambda\beta^2}.$$

But, (A7), (A8) and (A2) with $n' = 0$ imply:

$$\frac{q}{\beta^2} > \frac{c'}{c} = \frac{\beta}{q}, \quad \text{or} \quad q > \beta^{3/2},$$

which contradicts (A11) and Assumption (C1).

By the similar arguments, we find that (iii-2),(iii-3),(iii-4) and (v) all inconsistent with symmetric steady state equilibrium under Assumption (C1). (The details are available from the authors upon request) Q.E.D. of Lemma 2.

From Lemma 2 with (A2) and (A3), we have (14) in the text. Then (A6) implies:

$$\text{(A12)} \qquad q \le \beta^{3/2}, \text{ and } z'' > 0 \text{ only if } q = \beta^{3/2}.$$

Then, (A4) and (A5) together with (14) imply $z = z' = 0$. Also from (A1) together with (14), we have

(A13) $\quad n > 0$, only if $q = \beta^2$.

Given Lemma 2, we first concentrate on the equilibrium without storage in order to see under what condition, the competitive equilibrium achieves the first best allocation. Then (8),(9) and (10) together with (3) become:

$$\begin{array}{lrcl}
\text{(A14)} & (1-\lambda)G'(y)y + c + qn & = & q\theta y + n' \\
\text{(A15)} & c' + qn' & = & n'' \\
\text{(A16)} & c'' + qn'' & = & (1-\theta)y + n
\end{array}$$

Then, we get

$$\text{(A17)} \quad (1-\theta)y + n = c'' + qc' + q^2[c + qn + (1-\lambda)yG'(y) - \theta qy].$$

From (14) and (15), we have (16). With (14),(16) and (A13), (A17) becomes:

$$\text{(A18)} \quad c'' = \left((1-\beta^2)(1-\theta) + q^2\frac{\lambda G'(y)}{1+\beta^2+\beta^4}\right) y + (1-\beta^2)n.$$

(A16) and (A18) implies

$$\text{(A19)} \quad qn'' = \left(\beta^2(1-\theta) - q^2\frac{\lambda G'(y)}{1+\beta^2+\beta^4}\right) y + \beta^2 n.$$

(A15),(A18) and (14) implies

$$\text{(A20)} \quad qn' = n'' - \frac{\beta^2}{q}\left[\left((1-\beta^2)(1-\theta) + q^2\frac{\lambda G'(y)}{1+\beta^2+\beta^4}\right) y + (1-\beta^2)n\right]$$

With (A19) and (A20), (12) becomes:

$$\text{(A21)} \quad q\theta y = \left\{ (1 + \frac{1}{q}) \left(\beta^2(1-\theta) - q^2\frac{\lambda G'(y)}{1+\beta^2+\beta^4}\right) - \frac{\beta^2}{q}\left((1-\beta^2)(1-\theta) + q^2\frac{\lambda G'(y)}{1+\beta^2+\beta^4}\right)\right\} y + n\left(q + \beta^2 + \frac{\beta^4}{q}\right).$$

From (A13),(A21) and (16), equilibrium q is a positive root of

$$\text{(A22)} \quad 0 = \theta q - \beta^2(1-\theta)\left(1 + \frac{\beta^2}{q}\right) + \frac{\lambda}{1+\beta^2+\beta^4}(1+\beta^2+q)\left[\theta q^2 + (1-\theta)\frac{\beta^6}{q}\right]$$

$$\equiv \phi(q;\ \theta,\lambda,\beta),$$

if the root is larger than β^2, and $q = \beta^2$ if the root of (A22) is smaller than or equal to β^2. Once equilibrium q is found, equilibrium y solves (17), which is equivalent to (16). From (17), y is the first best if and only if $q = \beta^2$. Also if $q = \beta^2$, the consumption is smooth by (14). Thus, the allocation is the first best, if and only if $q = \beta^2$. Therefore, if the root of (A22) is equal to or smaller than β^2, then the competitive equilibrium is the first best. We can easily check,

$$\frac{\partial \phi}{\partial q} > 0, \quad \frac{\partial \phi}{\partial \theta} > 0, \quad \frac{\partial \phi}{\partial \lambda} > 0,$$

and $\phi(q;\ \theta,\lambda,\beta) \to -\infty$ as $q \to 0$. Thus the root of $\phi(q;\ \theta,\lambda,\beta) = 0$ is smaller or equal to β^2, if and only if $\phi(\beta^2;\ \theta,\lambda,\beta) \geq 0$, or

$$\text{(A23)} \qquad \theta \geq \theta^* \equiv \frac{1}{3}\left(2 - \frac{\lambda\beta^2(1+2\beta^2)}{1+\beta^2+\beta^4}\right).$$

When $q = \beta^2$, (A12) implies $z'' = 0$. Also, under the first best, we can check

$$n = \frac{y}{3}\left(3\theta - 2 + \frac{\lambda\beta^2(1+2\beta^2)}{1+\beta^2+\beta^4}\right) = (\theta - \theta^*)y \geq 0.$$

and $n' > 0$ and $n'' > 0$. This completes the proof of Proposition 1.

In order to prove Proposition 2, we consider two cases of parameter θ separately: [Case 1: $\hat{\theta} < \theta < \theta^*$] and [Case 2: $\underline{\theta} < \theta \leq \hat{\theta}$].

[Case 1: $\hat{\theta} < \theta < \theta^*$]

For this parameter, (A22) implies:

$$\phi(\beta^2;\ \theta,\lambda,\beta) < 0 < \phi(\beta^{3/2};\ \theta,\lambda,\beta).$$

Thus there is a unique q^* in $(\beta^2, \beta^{3/2})$, which solves $\phi(q;\ \theta,\lambda,\beta) = 0$. In order to show this q^* is equilibrium q, we verify our assumptions $z'' = 0$ and $n' > 0$. From (A12), we learn $z'' = 0$, because $q < \beta^{3/2}$. Also, because $q > \beta^2$, we know $n = 0$ by (A13). Also, straightforward algebra shows that (A19) and (A20) with $n = 0$ implies $n' > 0$ at equilibrium q when $\theta \in (\hat{\theta}.\ \theta^*)$. Also (A22) implies

$$c < c' < c'',$$

since $q > \beta^2$.

In order to show that competitive investment is lower than the first best, we solve (17) for $1-\theta$ as a function of q and $G'(y)$ as:

$$1 - \theta = \frac{1 - G'(y)/q}{1 - \beta^6/q^3}$$

and substitute it into (A16) as:

$$\text{(A24)}\quad 0 = \tilde{\phi}(q;\ G'(y),\lambda,\beta) = q - q\,\frac{1 - G'(y)/q}{1 - \beta^6/q^3} + (1 + \beta^2 + q)\frac{\lambda q G'(y)}{1+\beta^2+\beta^4}.$$

Solving (A24) for $G'(y)$, we have

$$\text{(A25)}\quad G'(y) = \frac{\beta^2(1+\beta^2+\beta^4)}{\lambda(q-\beta^2)(1+\beta^2+q) + 1+\beta^2+\beta^4}$$

The right-hand side does not contain θ, and is equal to β^2 when $q = \beta^2$ and is a decreasing function of q. Since $G'(y)$ is an increasing function of y, we learn that equilibrium y is lower than the first best when $q > \beta^2$ in the equilibrium for $\theta \in (\hat{\theta},\ \theta^*)$.

[Case 2: $\bar{\theta} < \theta \leq \hat{\theta}$]

First we look into the economy with $\theta \in (\underline{\theta}, \hat{\theta})$ and then study the economy at margin with $\theta = \hat{\theta}$. For $\theta \in (\underline{\theta}, \hat{\theta})$, we first conjecture that $n' > 0$, $z'' > 0$, and construct an equilibrium under this conjecture, and then verify these conjecture in equilibrium. From (A12), for $z'' > 0$, we need

$$\text{(A26)} \qquad q = \beta^{3/2}.$$

Then (A13) implies $n = 0$. Now, flow-of-funds constraints (8),(9) and (10) are,

$$\text{(A27)} \quad (1-\lambda)G'(y)y + c = q\theta y + z'' + n',$$
$$\text{(A28)} \quad c' + qn' = n''$$
$$\text{(A29)} \quad c'' + z'' + qn'' = (1-\theta)y.$$

Combining these, we have

$$(1-\theta)y = c'' + c + (1-\lambda)G'(y)y - q\theta y - n' + q(c' + qn').$$

Then, from (14),(16),(A6) and (A26), we get

$$\text{(A30)} \quad c'' = (1-\beta)\left((1-\theta)y + n' + \frac{\lambda G'(y)}{1-\beta^3}y\right).$$

Then, from (A26) and (A29), we have

$$\text{(A31)} \quad n'' = \beta^{3/2}n' + \beta^{1/2}(1-\beta)\left((1-\theta)y + n' + \frac{\lambda G'(y)}{1-\beta^3}y\right).$$

Then, from (12),(17) and (A28), we have

$$\text{(A32)} \quad n' = \frac{1}{1+\beta^{1/2}}\left\{\theta - \beta^{1/2}\left((1-\beta)(1-\theta) + \lambda\frac{\theta\beta^{3/2} + (1-\theta)\beta^3}{1+\beta+\beta^2}\right)\right\}y.$$

Thus, $n' > 0$, if and only if $\theta > \underline{\theta}$. Also from (A29),(A31) and (A32), we can verify $z'' > 0$, if $\theta < \hat{\theta}$. This completes the construction of equilibrium in which $n'>0$, $z''>0$, and $z=z'=n=0$ for $\theta \in (\underline{\theta}, \hat{\theta})$.

For $\theta = \hat{\theta}$, from the continuity of equilibrium with respect to θ for $\theta \in (\underline{\theta}, \hat{\theta})$ and $\theta \in (\hat{\theta}, \theta^*)$, we can construct an equilibrium in which $q = \beta^{3/2}$, $n' > 0$, and $z'' = z = z' = n = 0$.

From (14) and (A26), we see

$$c < c' < c''.$$

Also from (17) and (A26), we have

(A33) $$G'(y) = \theta \beta^{3/2} + (1-\theta) \beta^3.$$

The right-hand side is an increasing function of θ. For the case of $\theta \in (\hat{\theta}, \theta^*)$, we already showed that y is lower than the first best level. And there was no discontinuity of equilibrium at $\theta = \hat{\theta}$. Therefore, (A33) implies that y is even lower than the economy with $\theta = \hat{\theta}$, which has already lower y than the first best. (Q.E.D. of Proposition 2.)