

# Residence Time of Atmospheric $CO_2$

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## Abstract

In discussing carbon management and global warming, there is a need for the participants to make quantitative judgments. The concept (and significance) of a special parameter  $\tau_L$ , the “effective residence time” of atmospheric  $CO_2$ , does not seem to be widely recognized or appreciated. This brief note advocates more attention be paid to this parameter.

## 1 The Residence Time Concept

Suppose the trustees of a small four year college has decided—for good and sufficient reasons—to set the total number of enrolled students at 4000. How many freshmen should be admitted each year? Everybody knows the answer to this simple question! Since the “residence time” of each student is four years, the freshmen class should be approximately 1000 in the steady state (flunkouts and transfers can change the *effective* residence time slightly but not its order of magnitude).

Let us now suppose that the world has decided—for good and sufficient reasons—to limit the amount of atmospheric  $CO_2$  below the value  $C_*$  (in unit of GtC, gigatons of Carbon equivalent). How much anthropogenic  $CO_2$  annual emission,  $E_*$  (in GtC per year), would be allowed? An answer, by analogy with the above example, is given by:

$$E_* = \frac{C_* - 600}{\tau_L} \quad (1)$$

where  $\tau_L$  is an *effective residence time*<sup>1</sup> (in unit of years) of  $CO_2$  in the atmosphere. Note that eq.(1) has been made consistent with the preindustrial historical data which said that  $C_*$  was approximately constant at 600 GtC when  $E_*$  was essentially zero for many centuries.

## 1.1 What is the value of $\tau_L$ ?

The short answer is:  $\tau_L \approx 400$  years, plus or minus 20%.

This value is arrived at by studying all the available published IPCC global warming calculations for “stabilized” scenarios.<sup>2</sup> Once simply plots the computed  $E_*$  against the specified  $C_*$  in the “steady state” regime. The value of  $\tau_L$  is estimated from the reciprocal of the slope of the correlation straight line.

There exists no observation data to validate this value. In fact, it is not possible to experimentally measure the value of  $\tau_L$  (and the claim of its constancy) unless reliable data taken over many centuries (with *constant* emission rate) are available.

$\tau_L \approx 400$  years is the *consensus* value of all the published IPCC models.

## 2 The Response of $C$ to $E(t)$

Given  $E(t)$ , some assumed time function of the anthropogenic  $CO_2$  annual emission rate, what is the response of  $C$ , the amount of atmospheric  $CO_2$  of the whole world?

This problem is dealt with in the IPCC community using highly sophisticated models and supercomputers. A very simple “model” is proposed here:<sup>3</sup>

$$\frac{dC}{dt} = \beta \left( E(t) - \frac{C - 600}{\tau_L} \right). \quad (2)$$

At the beginning of the 21st century, the initial condition is  $C(0) = 750$  GtC. The value of  $E(0)$  is approximately 6 GtC/year.

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<sup>1</sup>The subscript  $L$  of  $\tau_L$  emphasizes that this is the *long* effective time scale.

<sup>2</sup>[http://www.ipcc.ch/pub/IPCCCTP.III\(E\).pdf](http://www.ipcc.ch/pub/IPCCCTP.III(E).pdf)

<sup>3</sup>This is in fact the model adopted by William Nordhaus, *Managing the Global Commons: The Economics of Climate Change*, p. 23, MIT Press, Cambridge, MA 1994. Nordhaus used  $\beta = 0.64$  and  $\tau_L = 120 \times 0.64 \approx 77$  years (instead of 400 years; so Nordhaus’ model is *not* consistent with all the IPCC simulations). See also Nordhaus and Boyer, *Warming the World; Economic Models of Global Warming*, Internet Version, October 25, 1999, particularly Chapter 3. (<http://www.econ.yale.edu/~nordhaus/homepage/dicemodels.htm>)

This ODE contains only three empirical constants,  $\beta$ ,  $\tau_L$  and 600, the latter two are already chosen as mentioned previously. A recommended value for  $\beta$  is 0.5 which is suggested by the well known observation in the past few decades that the measured annual increase of  $C$  is approximately half the actual documented value of  $E(t)$ .<sup>4</sup>

Try eq.(2) out on your home computer and compare your own numerical results with the sophisticated IPCC supercomputer calculations.<sup>5</sup> Go to [http://www.ipcc.ch/pub/IPCCCTP.III\(E\).pdf](http://www.ipcc.ch/pub/IPCCCTP.III(E).pdf) and see how well eq.(2) *reproduces* the IPCC simulation data from the graphs on its page 15.

Strictly speaking, eq.(2) is not a model. It is a rough (order of magnitude) *emulator* of **all** the IPCC models

### 3 The Response of $\Delta T$ to $C$

Let  $\Delta T$  denote the increase of the average global temperature (in degrees Fahrenheit from the preindustrial level) due to  $C$ . The following is an empirical relation:

$$\Delta T = \alpha(C - 600) \tag{3}$$

where  $\alpha$  is usually called the *climate sensitivity* in the global warming community. Empirical observational data says  $\alpha$  is approximately  $0.01^\circ F/GtC$  (in other words,  $6^\circ F$  increase for  $C = 1200$  GtC, *i.e.* the “doubling” scenario), with a relatively large error bar ( $\pm 50\%$ ).

It is useful to look up the difference of average temperatures between Chicago and Kansas City ( $5.8^\circ F$ ) to put things in perspective.

### 4 So What?

The purpose of this note is to provide simple quantitative tools for everybody to use in global warming discourse.

The question posed in the *Introduction*—if  $C_*$  is 1200 GtC (doubling), what is the allowed  $E_*$ ?—can now be answered. Surprised? It

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<sup>4</sup>The claim here is that both parameters can “adequately” be represented by constants. An alternative choice is to empirically determine  $\beta(C, t)$  (and perhaps also  $\tau_L(C, t)$ ) to fit the published IPCC results. See P. Schultz and J. Kasting, Optimal Reductions in  $CO_2$  emissions, *Energy Policy*, **25**, 5, pp. 491-500, 1997.

<sup>5</sup>Numerical (or analytical) integration of eq.(2) may be beyond the capability of some people in the general public, but surely eq.(1) cannot possibly intimidate anybody.

is often said that Kyoto is only the first step in the right direction. How big is the second step? In the global warming literature, advocates often say “... this policy option will decrease  $CO_2$  emission by one billion tons per year ...” and assume that a decrease of  $E(t)$  by one billion tons per year is *obviously* a very good deed. How much cooler is the world afterwards? How much more remains to be done? Suppose all anthropogenic  $CO_2$  emissions are suddenly stopped forever. How long does it take for  $C$  to decay back to 600 GtC?

You can now *roughly* figure these numbers out for yourself.

## 4.1 Parting Remarks

How reliable are these answers? It is worth repeating that none of the long term predictions of the IPCC models can ever be “validated” (until centuries later) because of the lack of observational data. It is interesting that all IPCC models are consistent in their modeling of the **natural** carbon “graduation” processes—all gave approximately the same  $\tau_L$ . The order of magnitude of  $\tau_L$  is *probably* correct, but it is *not* validated. Many investigators used “pulse-response” computer results from highly sophisticated computer models (or observational “bomb” data) to “calibrate” their simplified models for use in policy and economic studies. If indeed  $\tau_L$  is roughly 400 years, such calibration data must span *at least* 800 years in order for the calibration to be mathematically meaningful! Unless one chooses to pooh-poo *all* the IPCC stabilization calculations in the past decade, their “steady state” results are ideal for the *estimation* of  $\tau_L$ .

There ought to be some debates on the reasonableness of this predicted “theoretical” value of  $\tau_L$ . Once a credible *order of magnitude* of  $\tau_L$  is agreed upon (is it 77 or 400 years?), the simple formula for  $E_*$  presented here should receive its fair share of attention in global warming policy discussions. How can an intelligent public debate be conducted if the public does not know the order of magnitude of  $\tau_L$ ?

The low IPCC values of the  $E_*$ ’s for selected values of  $C_*$  are well known to all insiders in the global warming community. For some unknown reasons, the actual numbers do not appear in the IPCC *Summary for Policymakers*, nor in most (nearly all) of the articles written for the general public. If indeed our policymakers are betting the future of our world on the consensus IPCC value of  $\tau_L$  being wrong (too big by a large factor), the general public ought to be told.