Ripplonic Lamb Shift

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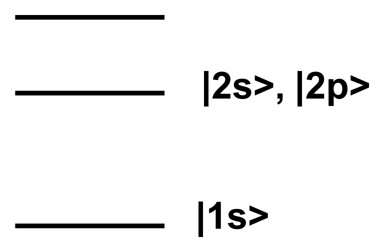
- The Lamb shift
- Direct and indirect two-ripplon processes
- Comparison with experiment

Disclaimer: unfinished work!



Hydrogen atom: $E_n = -R_0/n^2$

Quantum electrodynamics: **shift of the energy levels due to virtual emission/absorption of photons**

Non-relativistic picture (Bethe):

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + V(\mathbf{r})$$

$\mathbf{A}(\mathbf{r})$ is the radiation vector-potential

$$\mathbf{A}(\mathbf{r}) = \sum c(2\pi\hbar/\omega V)^{1/2}\mathbf{e}_{\mathbf{k}\alpha}a_{\mathbf{k}\alpha}e^{i\mathbf{kr}} + \mathrm{H.c.}$$

photon annihilation operator

Write the Hamiltonian as

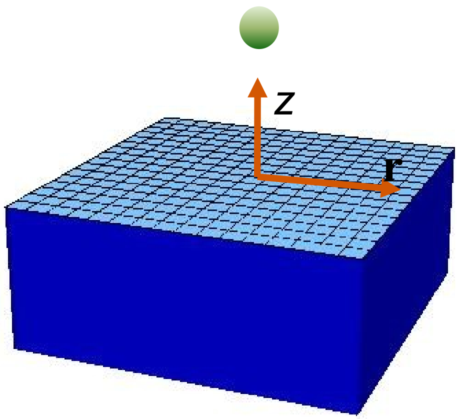
$$H = H_0 + H_1 + H_2, \quad H_1 = -\frac{e}{2mc}(\mathbf{p}\cdot\mathbf{A}(\mathbf{r}) + \mathbf{A}(\mathbf{r})\cdot\mathbf{p}), \quad H_2 = \frac{e^2}{2mc^2}\mathbf{A}^2(\mathbf{r})$$

photon emission/absorption with interstate electron transitioins

$$< n \,|\, H_2 \,|\, n > = \mathrm{const}$$

no frequency shift

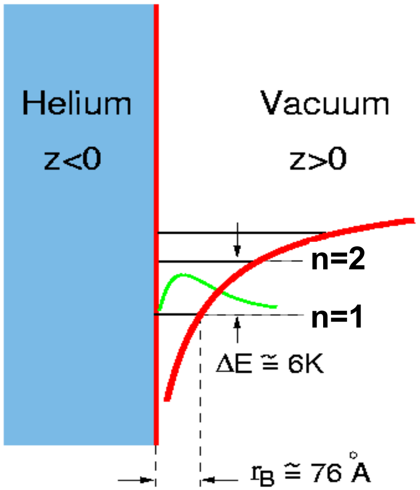
Idealized model: flat surface, infinite barrier



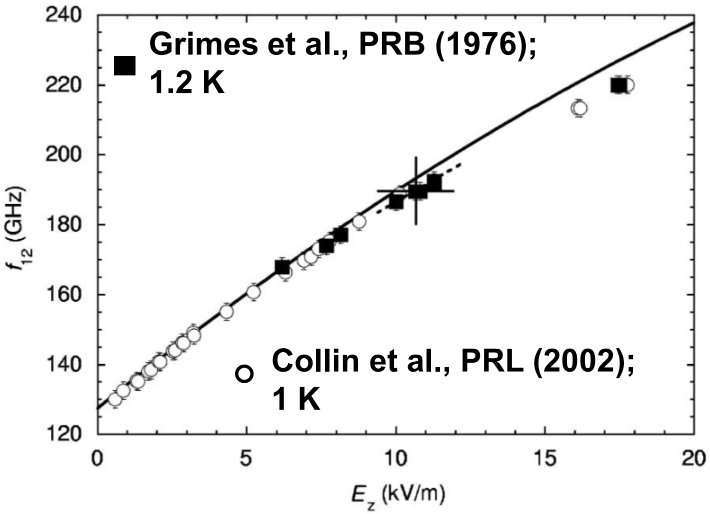
$$U(z) = -\Lambda/z \quad (z > 0)$$

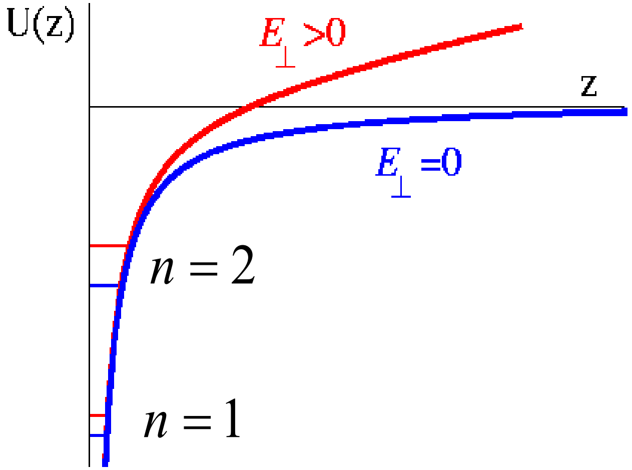
$$\Lambda = (\epsilon - 1)\, e^2/4(\epsilon + 1)$$

$$E_n = -R/n^2, \quad R = m\,\Lambda^2/2\hbar^2$$



Measurement: Stark-shift transition frequency by field $E_\perp$ to tune microwave radiation to 1- 2 resonance





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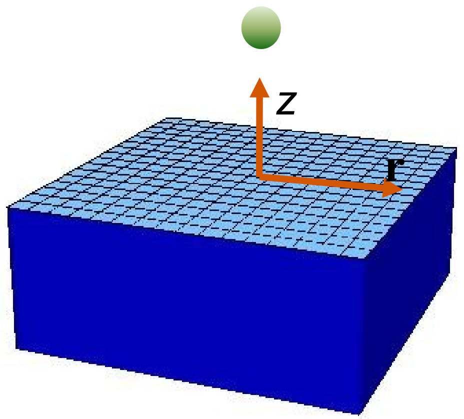

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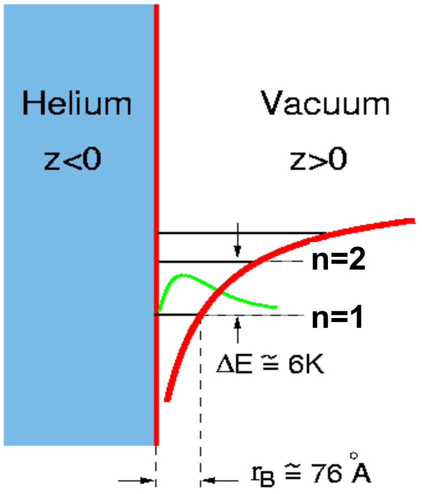

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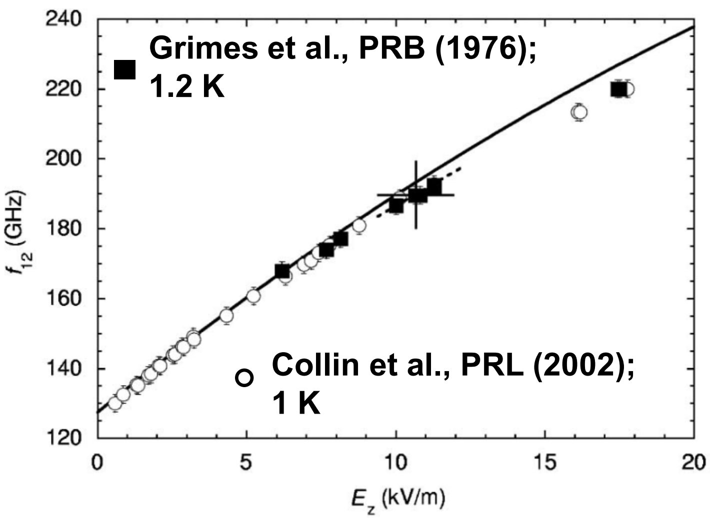


Complications at short distances, $|z| \leq$ 0.1 – 0.2 nm:

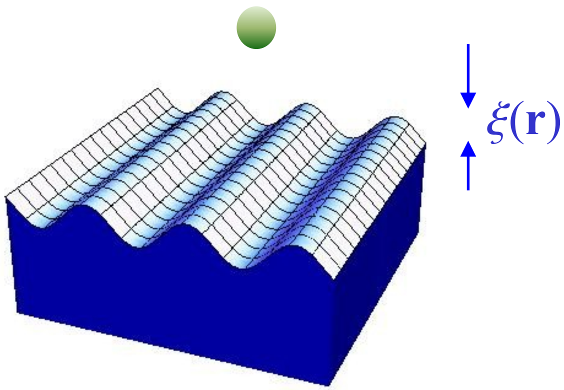
- **Finite barrier height, $U_0 \sim$ 1 eV**
- **Surface diffuseness**

Theory: modification of the 1D confining potential

Example: $U(z) = -\Lambda/(z+b)$ **[Grimes et al., 1976]**

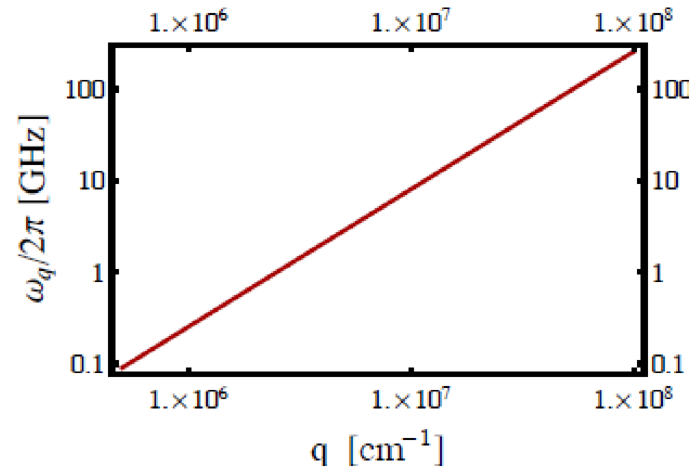
Cole (1970), Sanders & Weinreich (1976), Cheng et al. (1994), Nieto (2000), Patil (2001), Degani et al. (2005),…

Surface is not flat: capillary waves ⇔ ripplons. Ripplons are slow



$$\omega_q = \left(gq + \frac{\sigma}{\rho} q^3\right)^{1/2}$$

$$H_r = \sum \hbar\omega_q b^+_{\mathbf{q}} b_{\mathbf{q}}$$

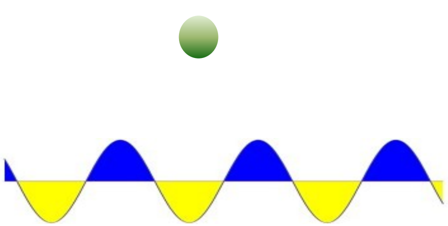


Surface displacement $\xi(\mathbf{r}) = \sum_{\mathbf{q}} \xi_{\mathbf{q}} \exp(i\mathbf{qr}),\qquad \xi_{\mathbf{q}} = S^{-1/2}(\hbar q/2\rho\omega_q)^{1/2}(b_{\mathbf{q}} + b^+_{-\mathbf{q}})$

Typically, $<\xi^2> \sim$ 0.1- 0.4 nm **for** $\mathrm{T} < 2\ \mathrm{K}$ **(Cole, 1970)**

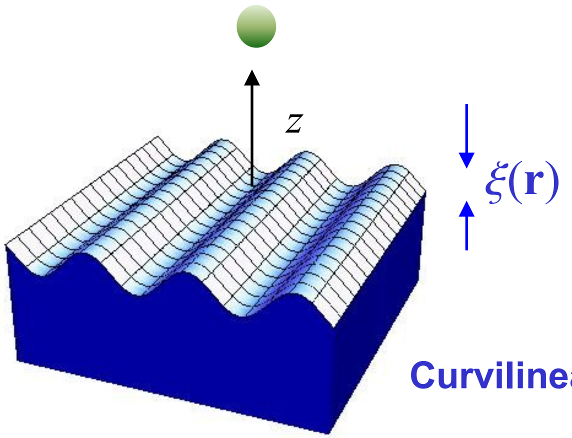
Effects on the electron dynamics

- **Broken translational symmetry: mixing of lateral and transverse motion ⇒ kinematic (inertial) coupling**



- **Change of polarization energy ⇒ polarization coupling**

Kinematic electron-ripplon coupling

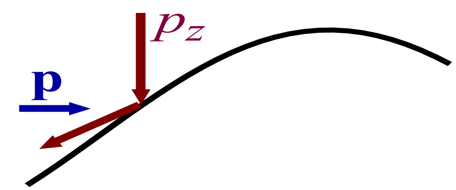


The shift transformation: $z \to z - \xi(\mathbf{r})$

(Shikin & Monarkha, 1974)

canonical transformation, $U = \exp[-i\hat{p}_z \xi(\mathbf{r})/\hbar]$

Curvilinear coordinates: $\mathbf{p} \Rightarrow \mathbf{p} - \nabla\xi\, p_z$



Electron kinetic energy: $$K \Rightarrow \frac{\mathbf{p}^2}{2m} + \frac{p_z^2}{2m} + K_{1r} + K_{2r}$$

One-ripplon kinematic coupling

$$K_{1r} = \frac{-1}{2m}(\mathbf{p}\cdot\nabla\xi + \nabla\xi\cdot\mathbf{p})p_z$$

small effect on 2D (lateral) motion, $<n|p_z|n>=0$

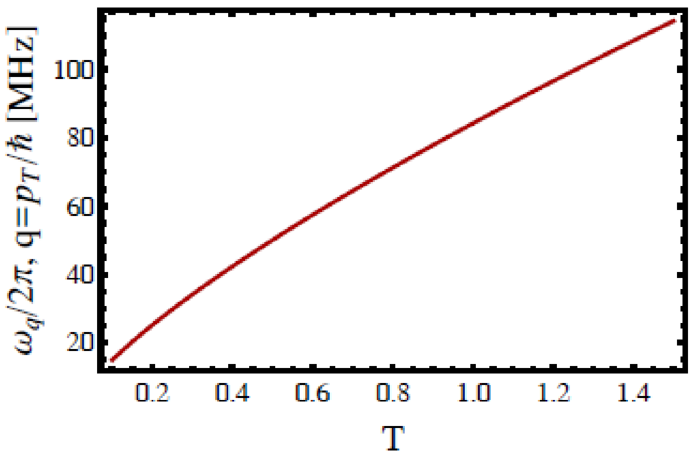
conventional wisdom

Two-ripplon kinematic coupling

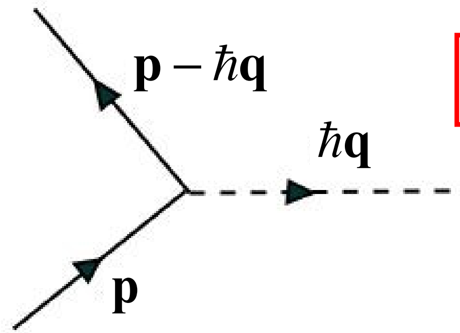
$$K_{2r} = \frac{p_z^2}{2m}(\nabla\xi)^2$$

electron energy relaxation via two-ripplon scattering

Ripplons are very slow

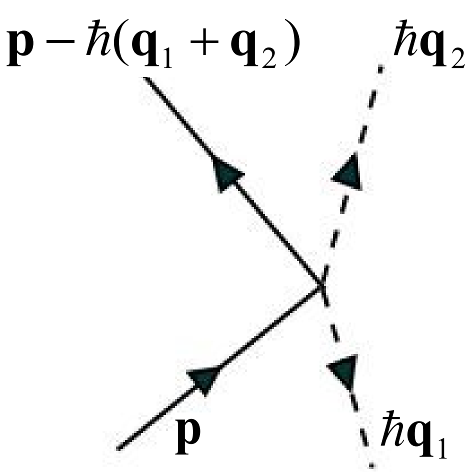


One-ripplon scattering is quasi-elastic



$$\hbar q \sim p_T \Rightarrow \hbar\omega_q << k_B T_e$$

Two-ripplon scattering: no constraint on ripplon energy from momentum conservation



$$\hbar\,|\,\mathbf{q}_1+\mathbf{q}_2\,|\le 2p_T, \quad \hbar q_{1,2} >> p_T \Rightarrow \hbar\omega_{q_1} \approx \hbar\omega_{q_2} \sim k_B T_e$$

energy relaxation; for quantized lateral states replace $p_T \Rightarrow \hbar/a$

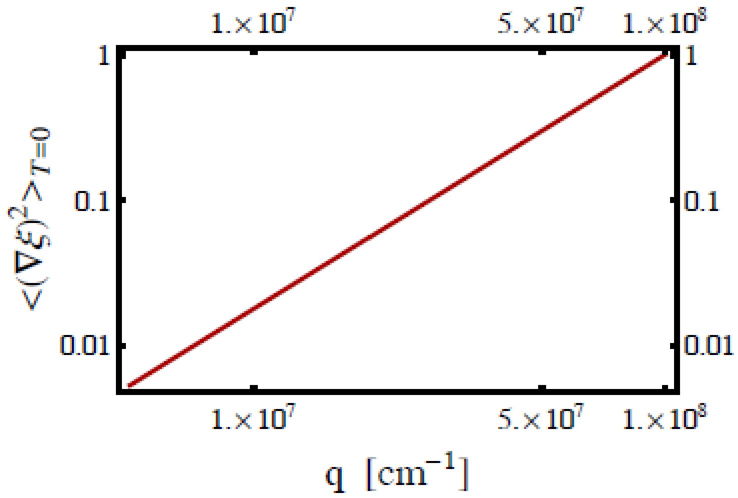
(MD, 1978; Monarkha, 1978)

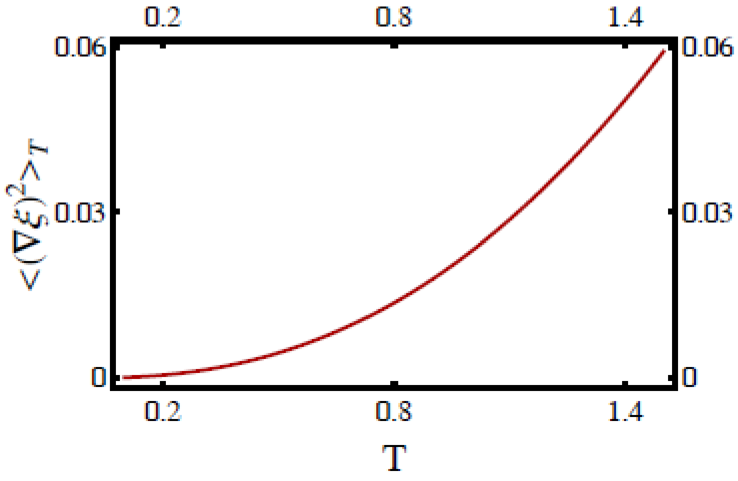
The gorilla in the room: two-ripplon kinematic coupling *is not weak!*

$$K_{2r} = \frac{p_z^2}{2m}(\nabla\xi)^2$$

$$<(\nabla\xi)^2>_{T=0} = \sum_{q<q_{\max}} q^2 <|\xi_{\mathbf{q}}^2|>_{T=0} \propto q_{\max}^{7/2}$$

large factor



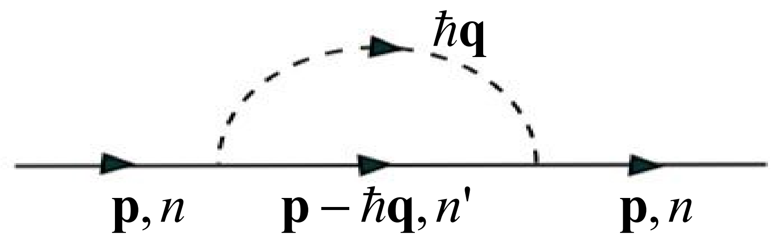


The thermal part is small (but important!)

The kinematic coupling due to the ripplon inertia is also ultraviolet-diverging, but is smaller

But… One-ripplon kinematic coupling *is not weak either!*

$$K_{1r} = \frac{-p_z}{2m}(\mathbf{p}\cdot\nabla\xi + \nabla\xi\cdot\mathbf{p}) = -ip_z\sum\left(\frac{\hbar q^2}{2m}\right)\xi_{\mathbf{q}}\mathrm{e}^{i\mathbf{qr}} + \ldots$$



Level shift:

$$\delta E^{(1k)}_{\mathbf{p},n} \sim \sum\left\langle |<\mathbf{p},n\,|\,K_{1r}\,|\,\mathbf{p}',n'>|^2\right\rangle/(E_{\mathbf{p},n} - E_{\mathbf{p}',n'} \pm \hbar\omega_q)$$

$\mathbf{q} = (\mathbf{p}-\mathbf{p}')/\hbar$

Large q: the large numerato[illegible] "overrides" the inter-subband energy difference

Virtual *inter-subband* transitions [illegible]mpensate the leading-order divergence of the two-ripplon coupling. The expressions for the s[illegible]ve similar structures:

$$K_{2r} = \frac{p_z^2}{2m}(\nabla\xi)^2$$

$$\delta E^{(2k)}_{\mathbf{p},n} \; [illegible] \; |\,K_{2r}\,|\,\mathbf{p},n>\rangle = <n\,|\,\frac{p_z^2}{2m}\,|\,n>\left\langle(\nabla\xi)^2\right\rangle$$

$$\delta E^{(1k)}_{\mathbf{p},n} \sim \left\langle(\nabla\xi)^2\right\rangle < n\,|\,\frac{p_z^2}{2m}\,|\,n>$$

Details….Transform the denominator using $E_{\mathbf{p},n} = E_n + p^2/2m,$

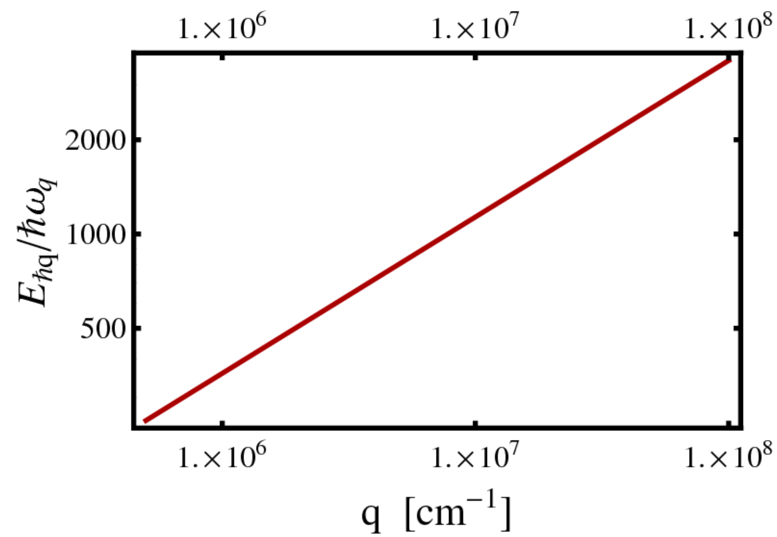
$$K_{1r} = \frac{-1}{2m}(\mathbf{p}\cdot\nabla\xi + \nabla\xi\cdot\mathbf{p})p_z \qquad \delta E^{(1k)}_{\mathbf{p},n} \sim \sum\left\langle |<\mathbf{p},n\,|\,K_{1r}\,|\,\mathbf{p}',n'>|^2\right\rangle / (E_{\mathbf{p},n} - E_{\mathbf{p}',n'} \pm \hbar\omega_q)$$

$$\frac{1}{E_{\mathbf{p},n} - E_{\mathbf{p}',n'} \pm \hbar\omega_q} = \frac{1}{E_{\mathbf{p}} - E_{\mathbf{p}'} \pm \hbar\omega_q}\left(1 - \frac{E_n - E_{n'}}{E_{\mathbf{p},n} - E_{\mathbf{p}',n'} \pm \hbar\omega_q}\right)$$

Use the sum rule $\sum_{n'} |<n\,|\,p_z\,|\,n'>|^2 = <n\,|p_z^2\,|\,n>$

Disregard $\hbar\omega_q$ and $p^2/2m$ **compared with** $\hbar^2 q^2/2m$

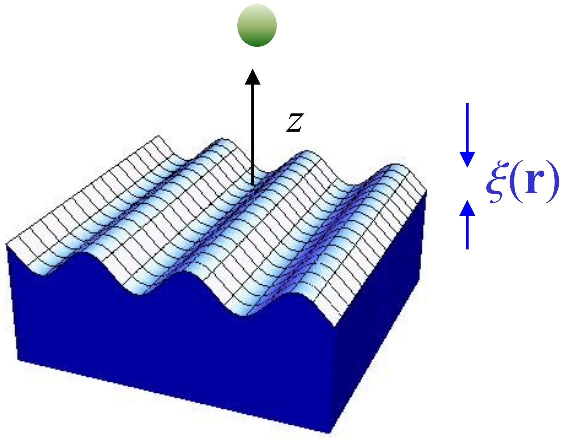
$(E_{\mathbf{p}'} \equiv E_{\mathbf{p}+\hbar\mathbf{q}} >> E_{\mathbf{p}}, \hbar\omega_q)$



exact compensation of the direct two-ripplon coupling,

$$\delta E^{(2k)}_{\mathbf{p},n} = \left\langle (\nabla\xi)^2 \right\rangle < \mathbf{p},n\,|\,p_z^2\,|\,\mathbf{p},n> / 2m$$

Uncompensated: $q \lesssim r_B^{-1}$

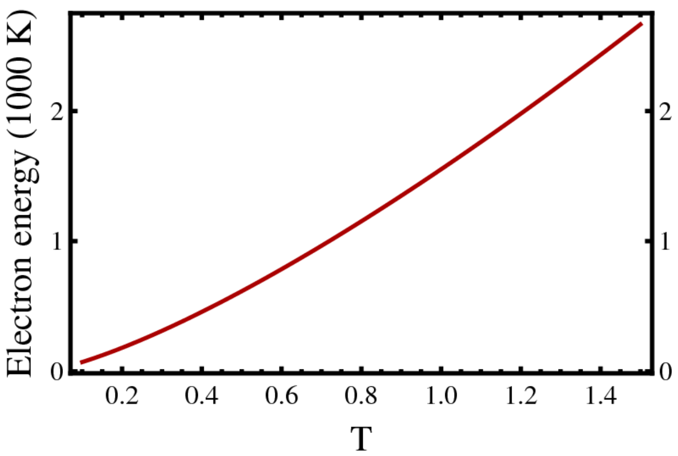


The direct two-ripplon polarization coupling, $U_{2r} \propto \xi^2$, also leads to the ultraviolet divergence, $\delta E_{\mathbf{p},n}^{(2p)} \propto q_{\max}^{3/2}$. It is compensated by interference of the single-ripplon kinematic and polarization coupling

Real tricks: calculating the wave functions of the excited states: a combination of the WKB, $z \sim r_B$, and asymptotic small-z expansion

Need highly excited states even to find the contribution from thermal ripplons

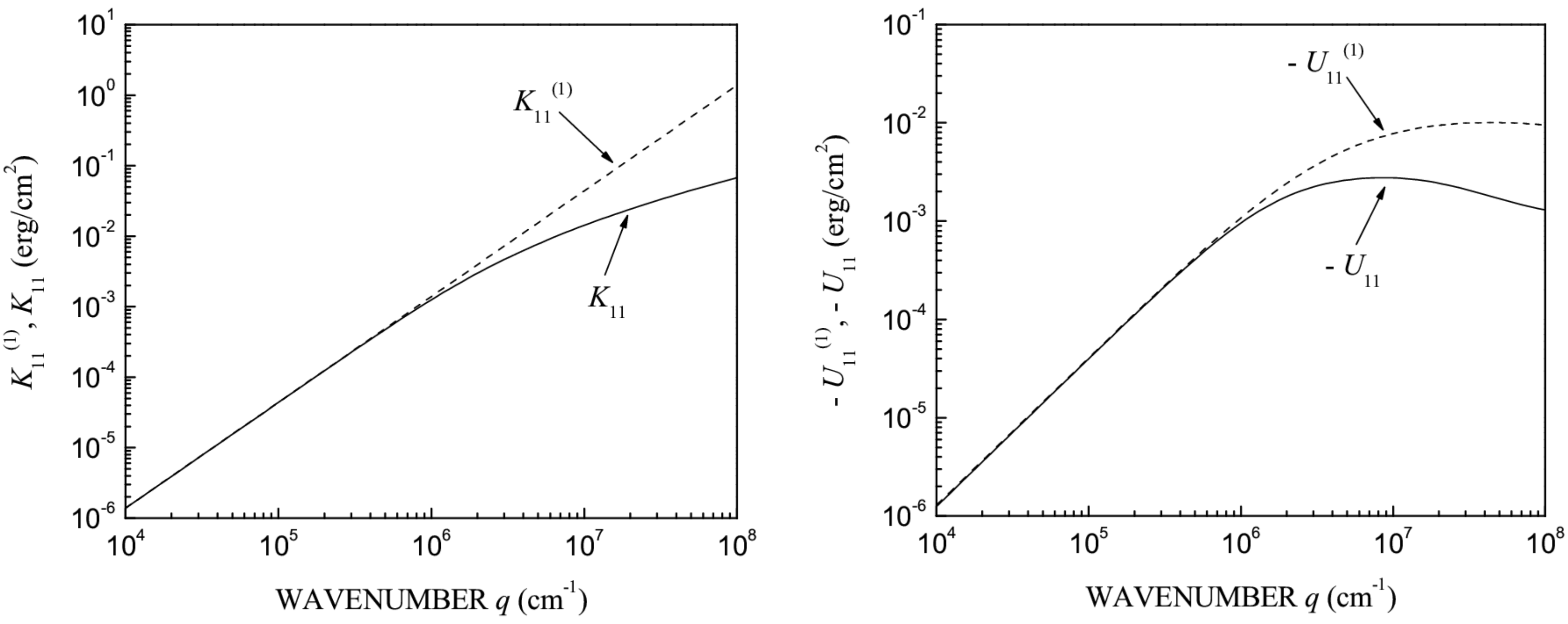
$$\hbar^2 q_{rT}^2/2m \ \text{ with } q_{rT} \ \text{ from condition } \hbar\omega_{q_{rT}} = \mathrm{k_B T}$$



***n*th level shift:**

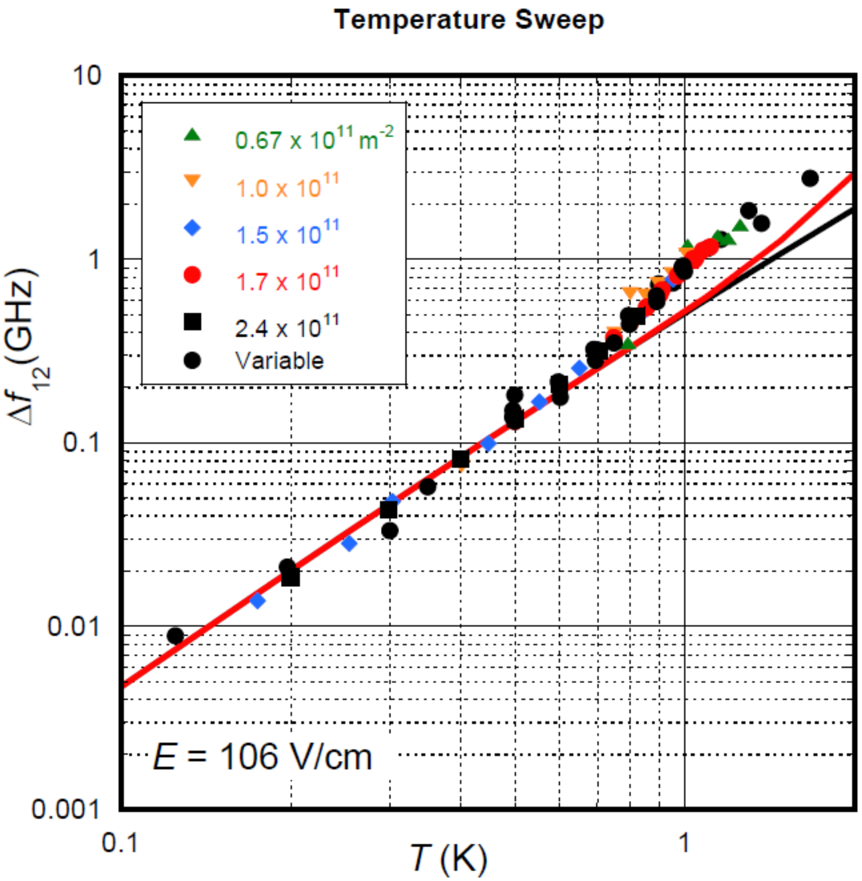
$$\delta E_n = \mathrm{v.p.}\sum_{\mathbf{q}} \frac{\hbar q}{2\rho\omega_q S}\left(2\bar{n}_{\mathrm{q}}+1\right)\left[K_{nn}(q)-U_{nn}(q)\right], \qquad \bar{n}_{\mathrm{q}} = \frac{1}{\exp(\hbar\omega_{\mathrm{q}}/k_B T)-1}$$

kinematic → $K_{nn}(q)$; polarizational → $U_{nn}(q)$



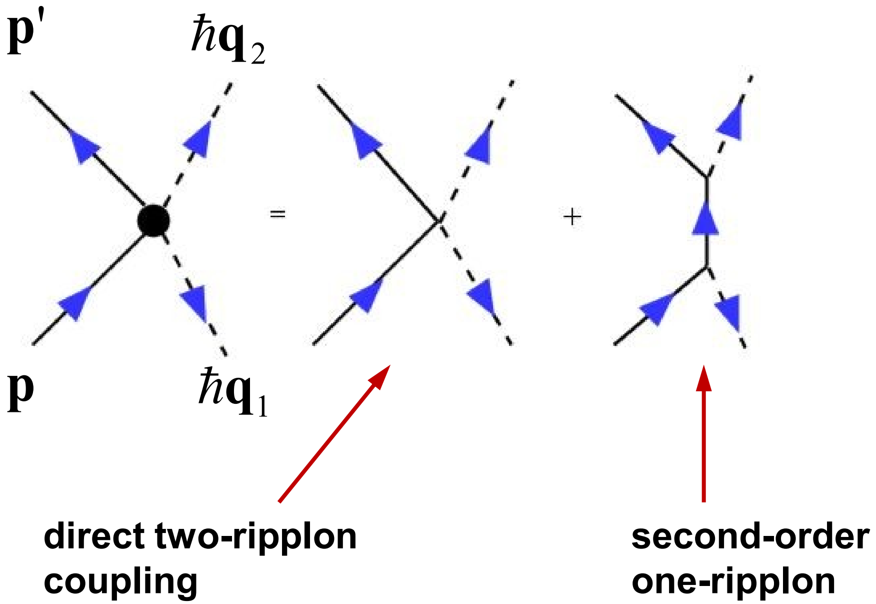
$$f_{12} = (E_2 - E_1)/h$$

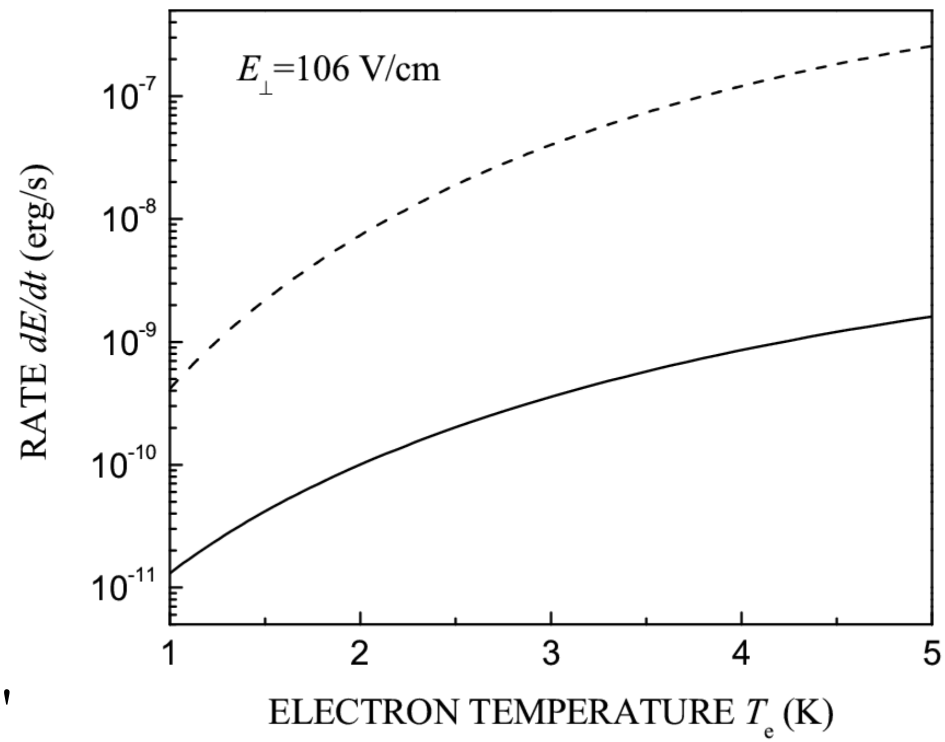
No adjustable parameters… almost



^{3}He displays a nonmonotonic behavior of f_{12} vs T

One-ripplon coupling renormalizes the two-ripplon matrix elements





Energy relaxation: $\mathbf{q}_1 \approx -\mathbf{q}_2, \quad \hbar q_{1,2} >> p, p'$

(MD 1978)

One-ripplon processes compensate the large-q terms in the two-ripplon coupling, and thus strongly reduce the energy relaxation rate due to two-ripplon processes

What else, simple and basic, have we overlooked?

Conclusions

- Two-ripplon processes lead to the Lamb-like shift of the electron energy levels, with a characteristic temperature dependence

- The ultraviolet catastrophe of the direct two-ripplon coupling is compensated by one-ripplon processes

- One-ripplon processes significantly reduce the energy relaxation rate due to two-ripplon emission