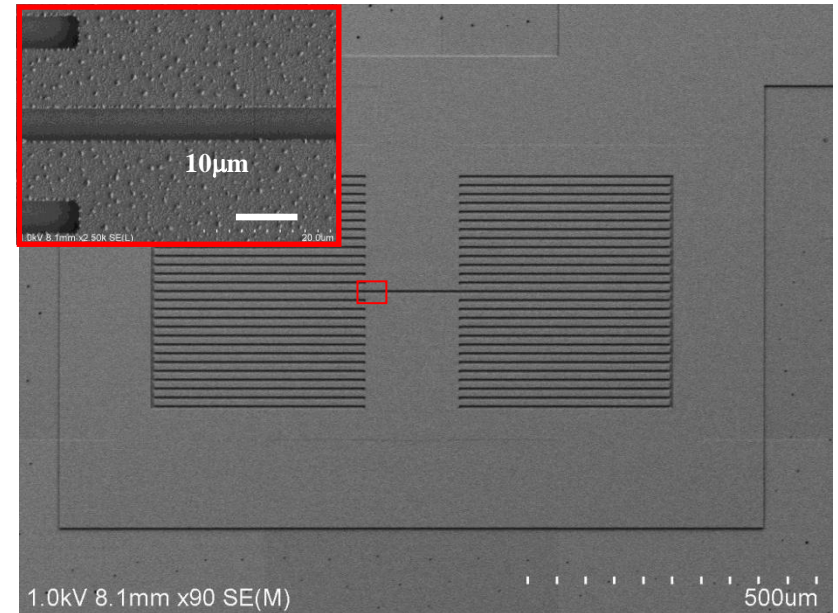


# Wigner Crystals Confined in Micrometer-wide Channels

Hiroki Ikegami  
*RIKEN, Japan*

Collaborators;  
Hikota Akimoto  
Kimitoshi Kono



# Outline

---

## 1. Introduction

## 2. Device

## 3. Nonlinear transports of a Wigner crystal

- Bragg-Cherenkov scattering of ripplons
- decoupling of a Wigner crystal from the dimple lattice

## 4. Melting of a Wigner crystal in quasi-1D geometry

- 1D-2D crossover

## 5. 1.6 $\mu\text{m}$ channel

## 6. Summary

# Wigner Crystal

## Electrons on helium surface

very low density

density:  $10^{11} \sim 10^{13} \text{ m}^{-2}$

electron spacing:  $0.1 \sim 1 \mu\text{m}$

Coulomb repulsion  $\gg$  kinetic energy

( $\sim 93 \text{ K}$  @  $1 \times 10^{13} \text{ m}^{-2}$ )



**Wigner Crystal coupled with Dimple Lattice**



## Unique transport properties of the Wigner crystal

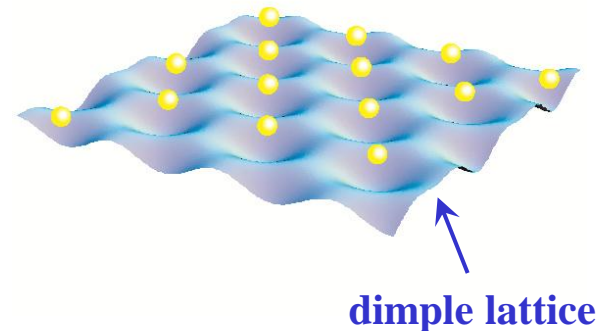
▪ Nonlinear transports of a Wigner crystal on superfluid  $^4\text{He}$

- Bragg-Cherenkov scattering of ripplons
- decoupling of a Wigner crystal from the dimple lattice

▪ Transports of a Wigner crystal on superfluid  $^3\text{He}$

- resistance caused by the quasiparticle scattering

K. Shirahama et al., PRL 1997, H. Ikegami and K. Kono PRL 2006



# Outline

---

## 1. Introduction

## 2. Device

## 3. Nonlinear transports of a Wigner crystal

- Bragg-Cherenkov scattering of ripplons
- decoupling of a Wigner crystal from the dimple lattice

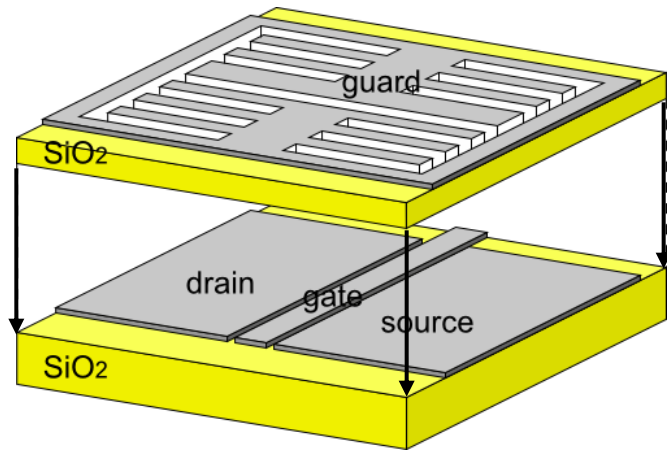
## 4. Melting of a Wigner crystal in quasi-1D geometry

- 1D-2D crossover

## 5. 1.6 $\mu\text{m}$ channel

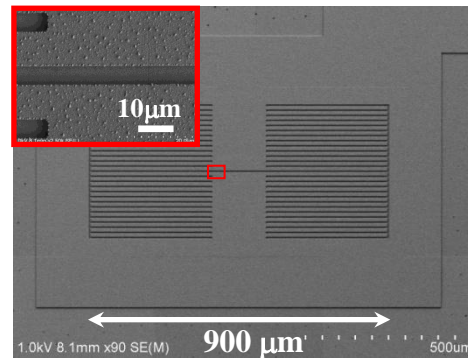
## 6. Summary

# Electrode

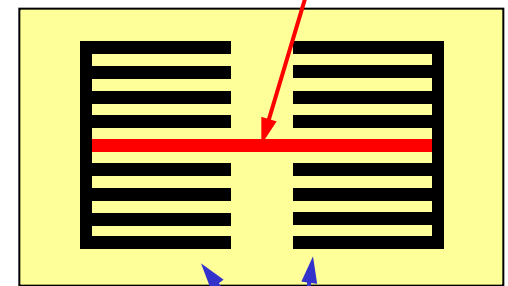


$W = 5, 8, 15 \mu\text{m}$

upper electrode (5  $\mu\text{m}$ )



center channel

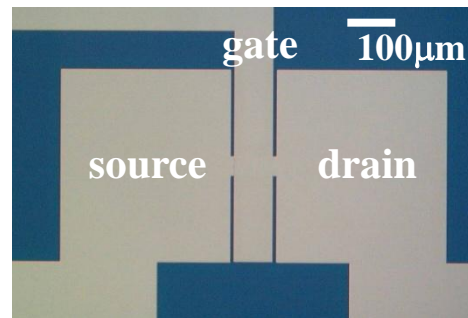


potential

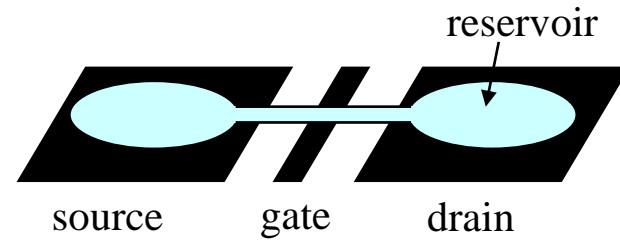
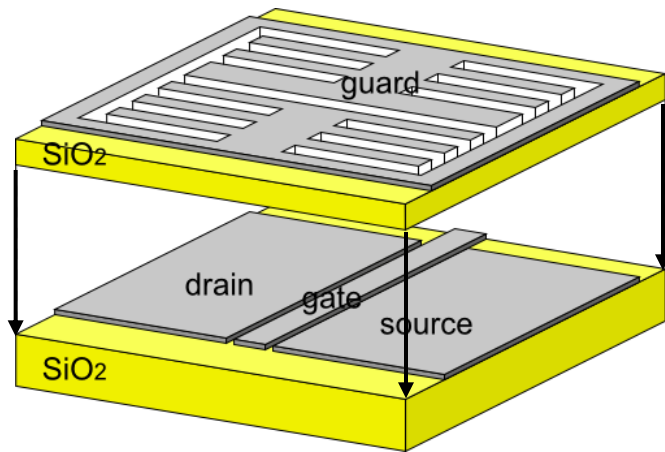


SiO<sub>2</sub>  
1.6  $\mu\text{m}$

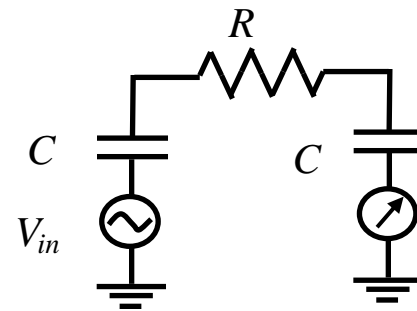
lower electrode



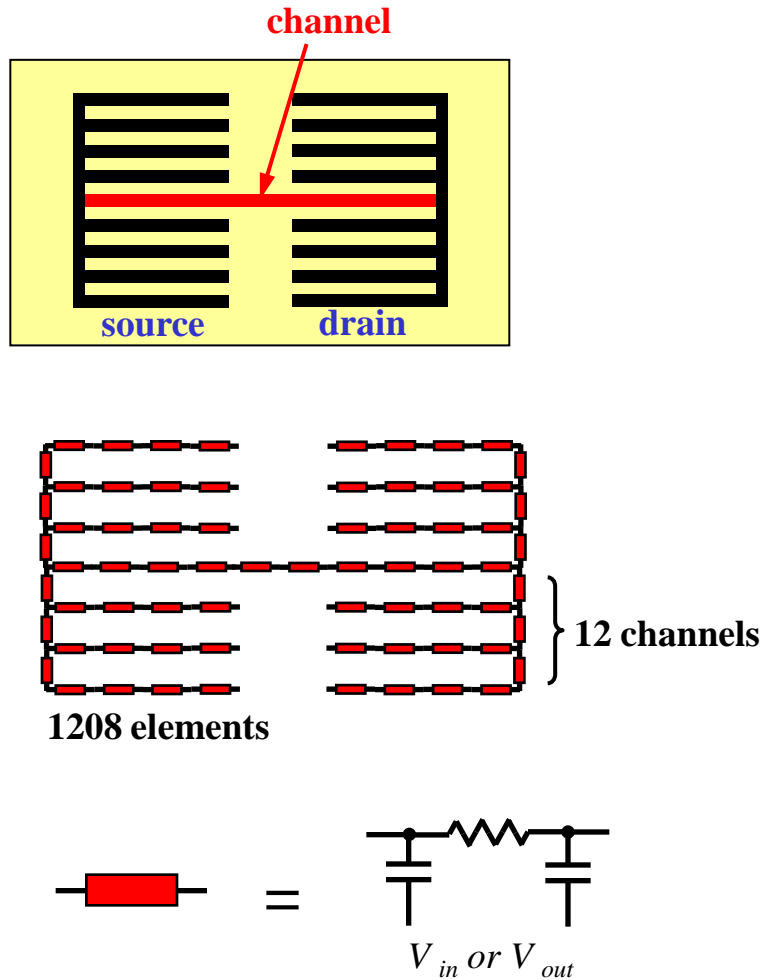
# Electrode



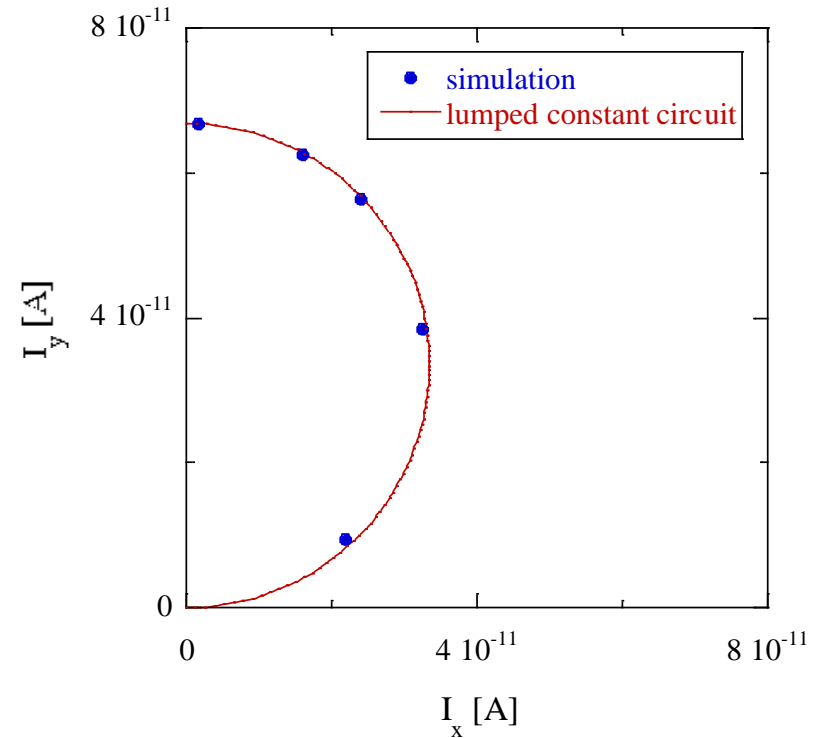
**equivalent circuit**



# Simulation of Electrode



## response of the circuit

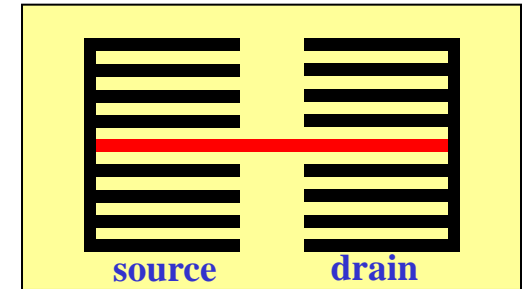
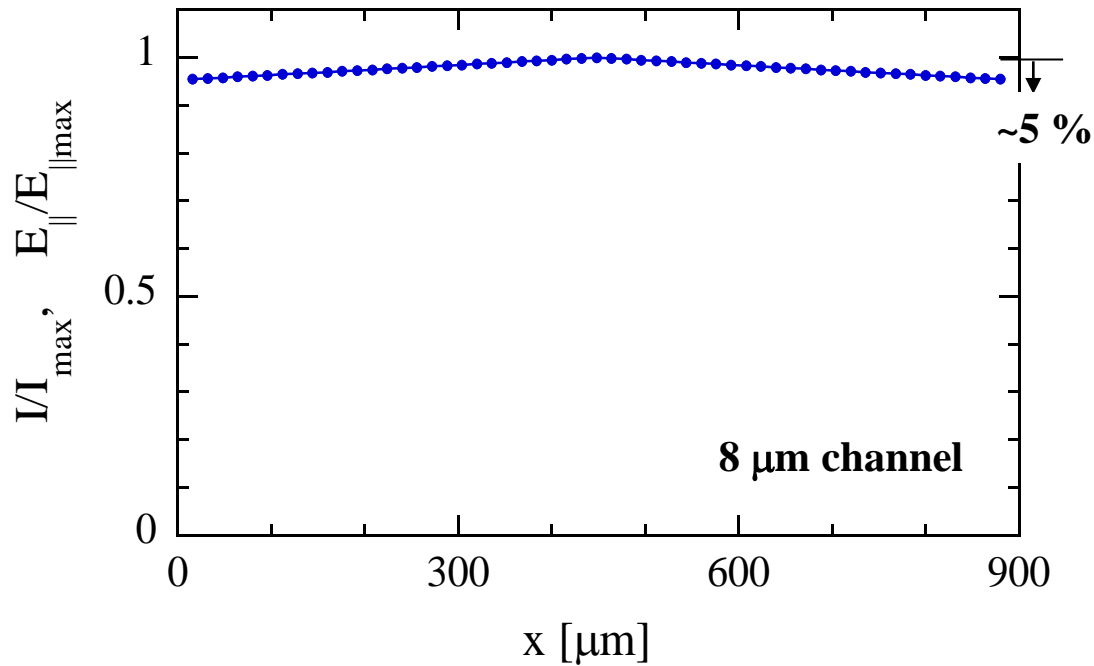


$\mu = 10, 30, 60, 100, 1000 \text{ m}^2/\text{Vs}$

**96% of the resistive component comes from the center-channel**

# Simulation of Electrode

distribution of  $I$  and  $E_{\parallel}$  along the channel



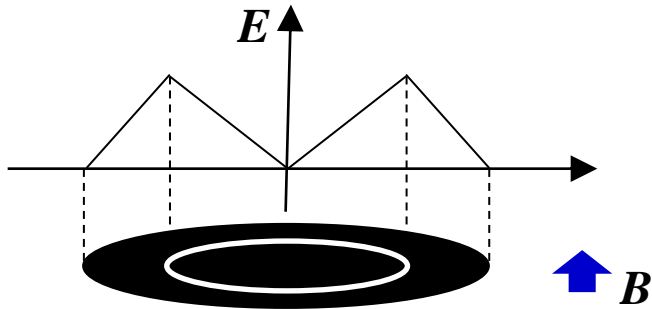
**$I$  and  $E_{\parallel}$  are homogeneous along the channel**



# Previous Experiments with Corbino electrode

---

## Current distribution in Corbino geometry



## Bragg-Cherenkov scattering

A. Kiristensen *et al*, PRL 77, 1350 ('96).

## Decoupling transition

K. Shirahama and K. Kono, PRL 74, 781 ('95).

In nonlinear regime,  
distribution of  $E$  smears the features.

# Outline

---

## 1. Introduction

## 2. Device

## 3. Nonlinear transports of a Wigner crystal

- Bragg-Cherenkov scattering of ripplons
- decoupling of a Wigner crystal from the dimple lattice

## 4. Melting of a Wigner crystal in quasi-1D geometry

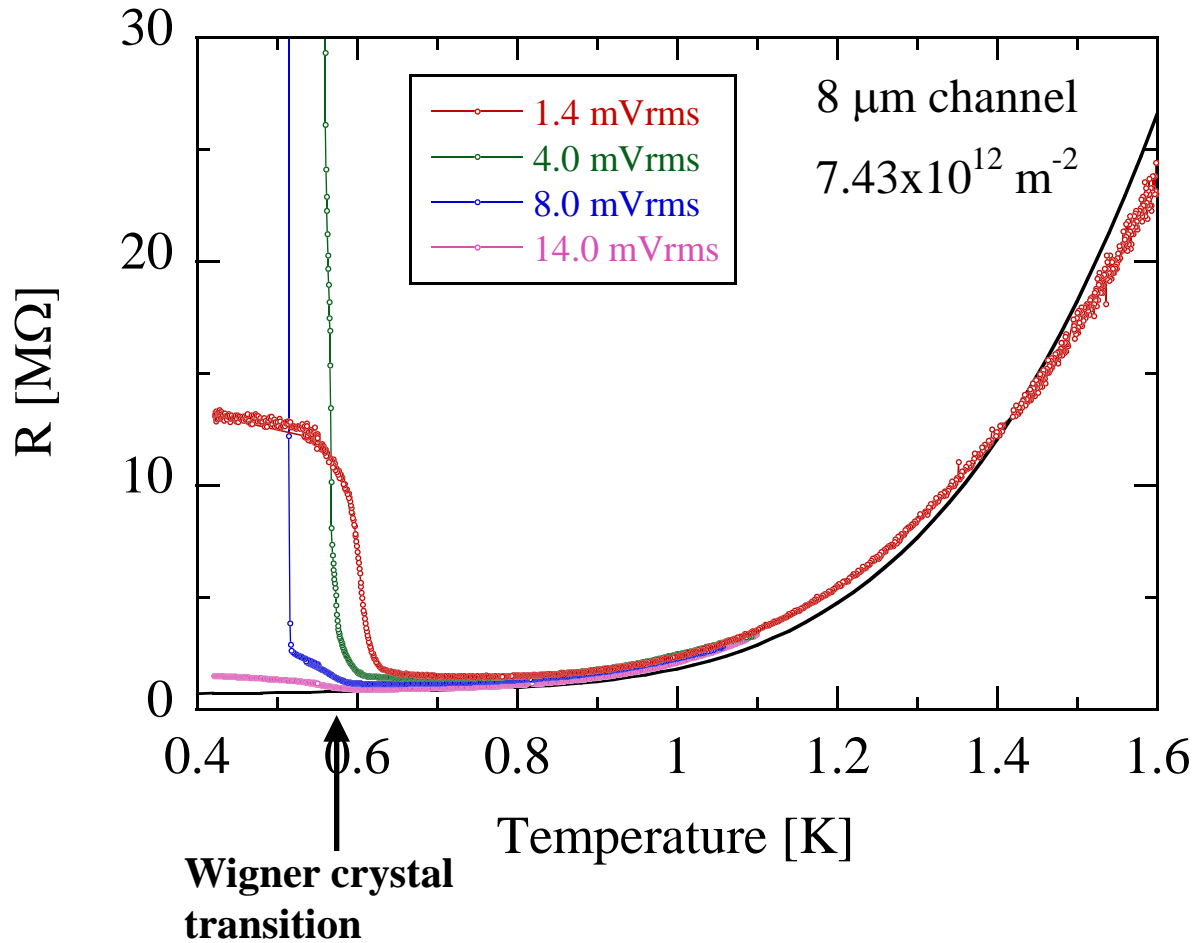
- 1D-2D crossover

## 5. 1.6 $\mu\text{m}$ channel

## 6. Summary

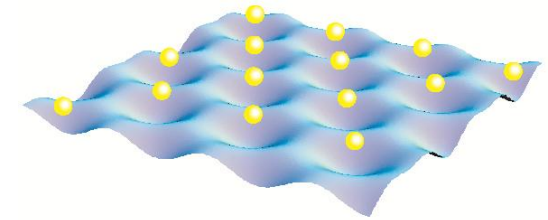
# Temperature Dependence of Resistance

8  $\mu\text{m}$  channel



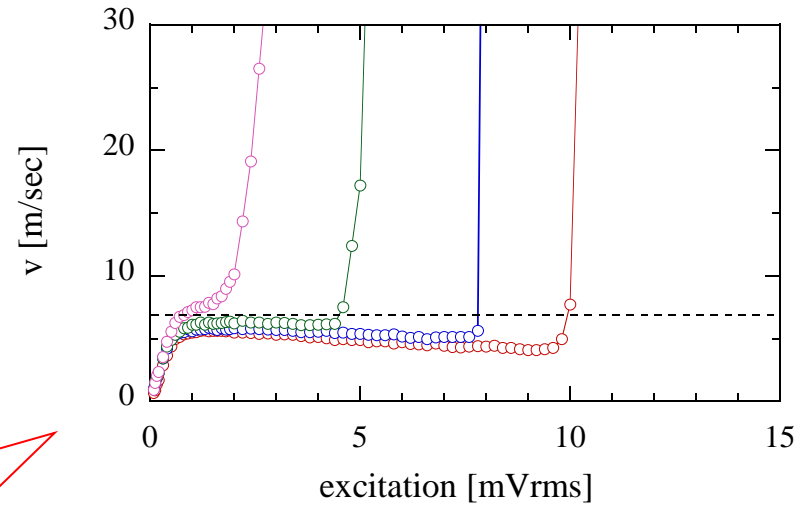
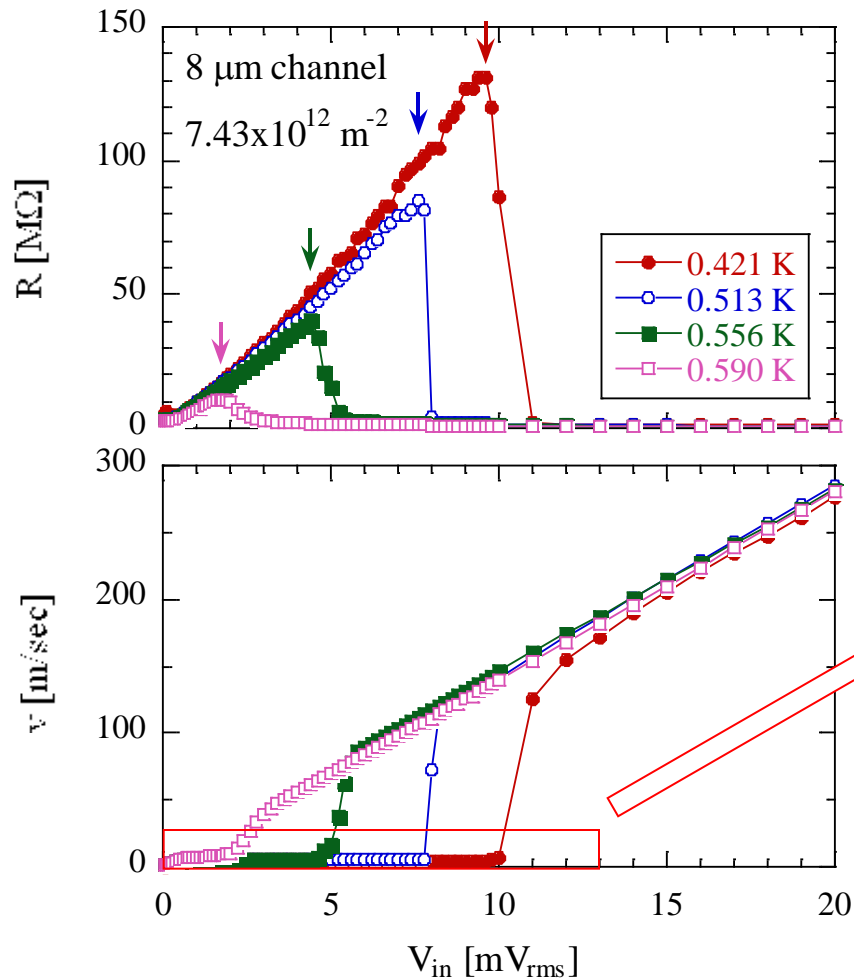
Solid line: Theory,  
M. Saitoh JPSJ **42**, 201 ('77).

Wigner crystal



**Strong nonlinear behavior in the Wigner crystal phase**

# Results: Nonlinear Behavior in Wigner Crystal Phase



H. Ikegami, et al., PRL 102, 046807 ('09)

- Electron velocity shows the saturation at low excitations
- Velocity jumps to a high value at large excitations

# Bragg-Cherenkov Scattering

Experiment: Kristensen *et al*, PRL **77**, 1350 ('96).

Theory: Dykman and Rubo, PRL **78**, 4813 ('97).

**Cherenkov emission of ripplon**

$$\mathbf{v} \cdot \mathbf{k} = \omega(\mathbf{k})$$

**Bragg condition**

$$\mathbf{k} = \mathbf{G}$$



**constructive interference**

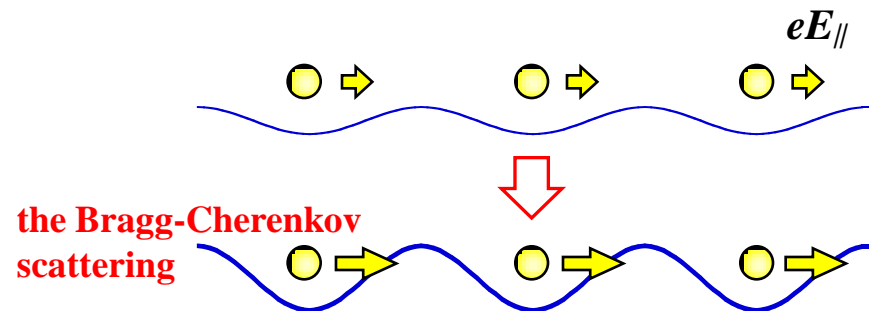
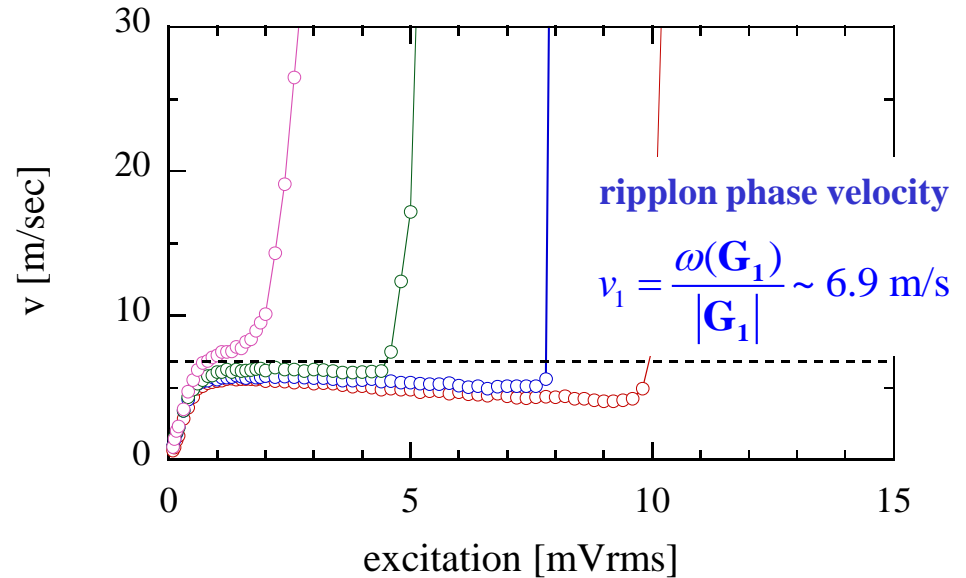
$$\mathbf{v} \cdot \mathbf{G} = \omega(\mathbf{G})$$

**v**: velocity of electron

**k**: wave number of ripplon

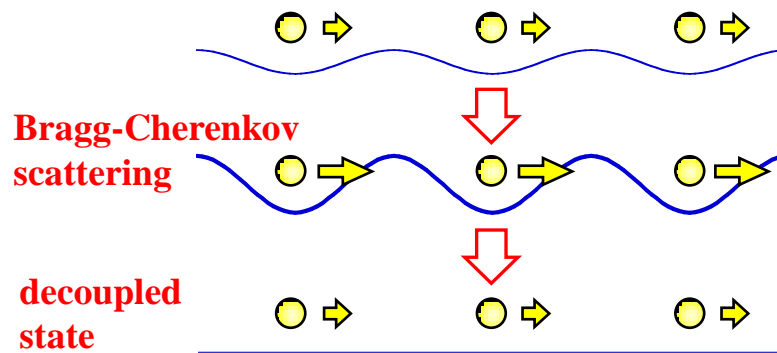
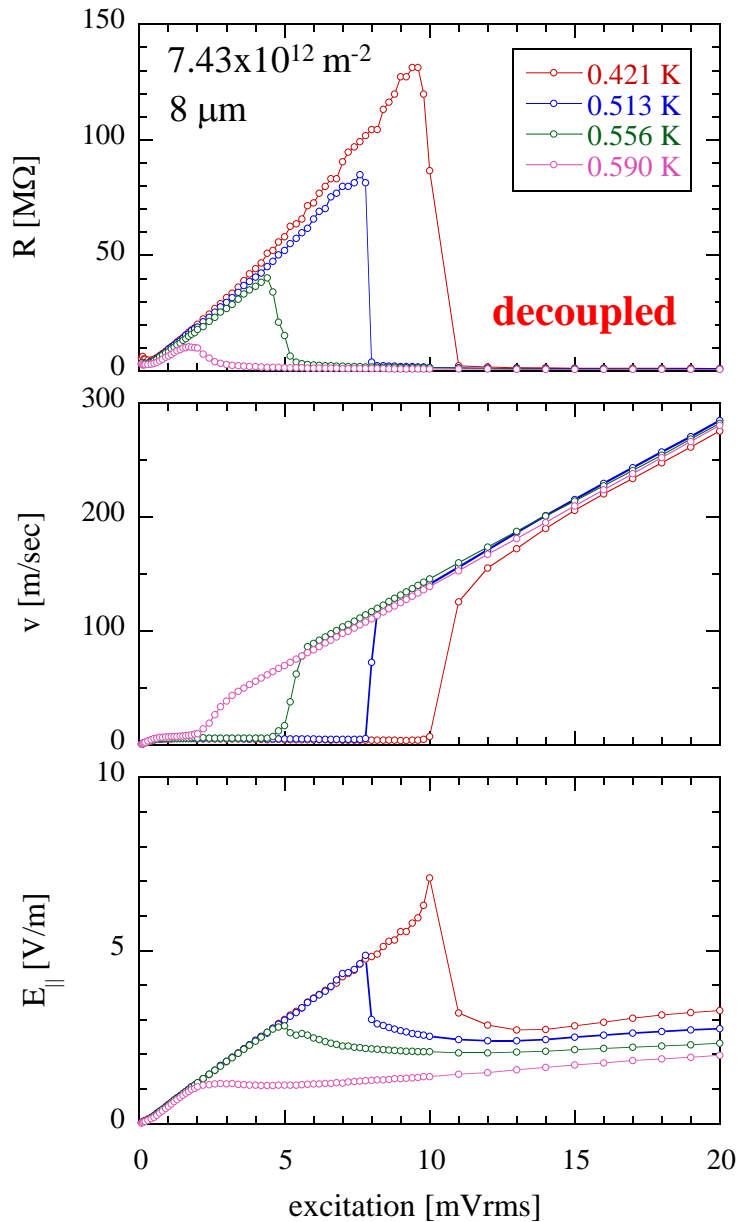
$\omega$ : energy of ripplon

**G**: reciprocal lattice vector  
of Wigner crystal



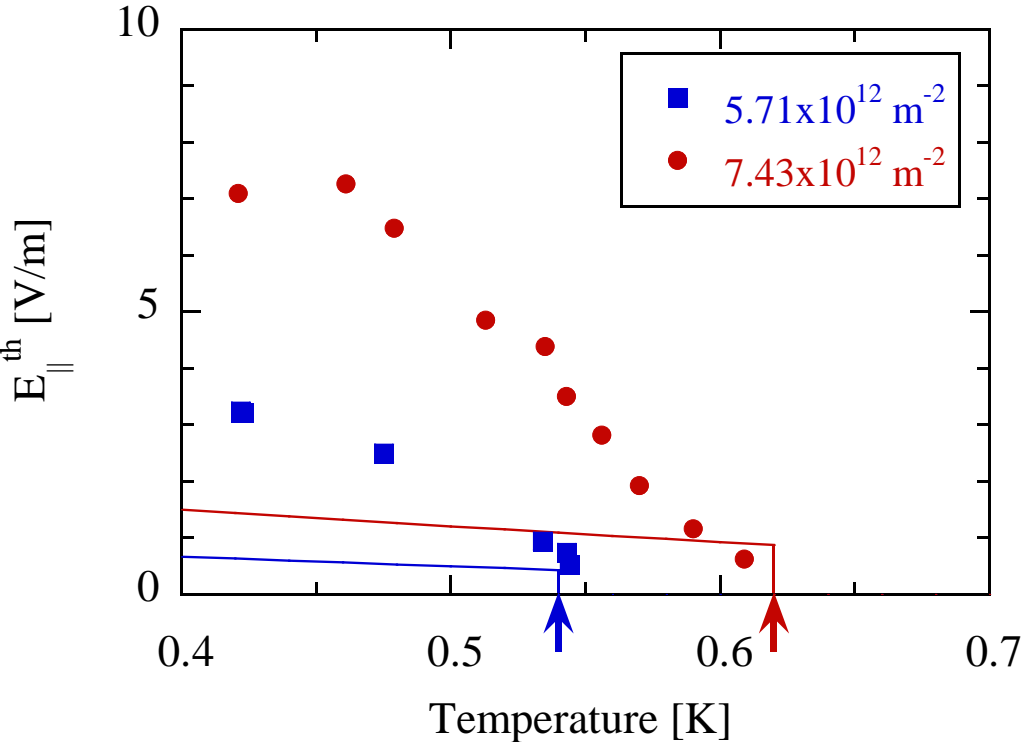
- **Velocity of electrons is limited by the phase-velocity of ripplon**
- **Dimple becomes deeper**

# Decoupling of a Wigner Crystal from Dimple Lattice



**Decoupling occurs from the Bragg-Cherenkov state**

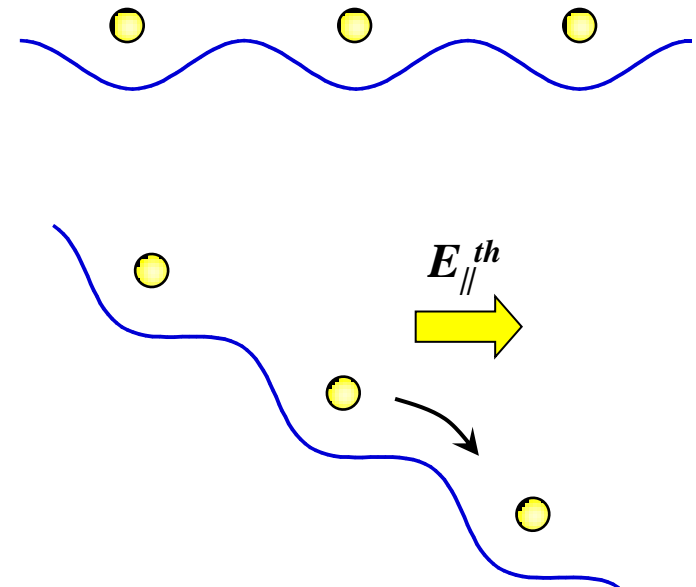
# Decoupling of a Wigner Crystal from Dimple Lattice



## Decoupling from rigid dimple lattice

K. Shirahama and K. Kono, PRL 1995

Decoupling occurs when the local minimum disappears.



**Decoupling from deepened dimple**

# Outline

---

## 1. Introduction

## 2. Device

## 3. Nonlinear transports of a Wigner crystal

- Bragg-Cherenkov scattering of ripplons
- decoupling of a Wigner crystal from the dimple lattice

## 4. Melting of a Wigner crystal in quasi-1D geometry

- 1D-2D crossover

## 5. 1.6 $\mu\text{m}$ channel

## 6. Summary



# Melting of a Wigner Crystal

In 2D system:

**Kosterlitz-Thouless- Halperin-Nelson-Young  
(KTHNY) mechanism**

Melting occurs at

plasma parameter  $\Gamma=U/K \sim 130$

$U$ : Coulomb interaction

$K=k_B T$ : kinetic energy

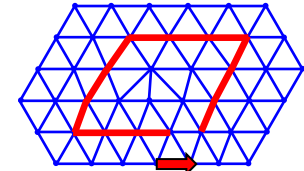
**unbinding of dislocation pairs**

In 1D system:

short range order at finite temperature

$$T > T_m^{2D}$$

**free dislocation**

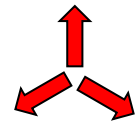


**Burgers vector**

$$T < T_m^{2D}$$



**dislocation-pair**



**three dislocations**



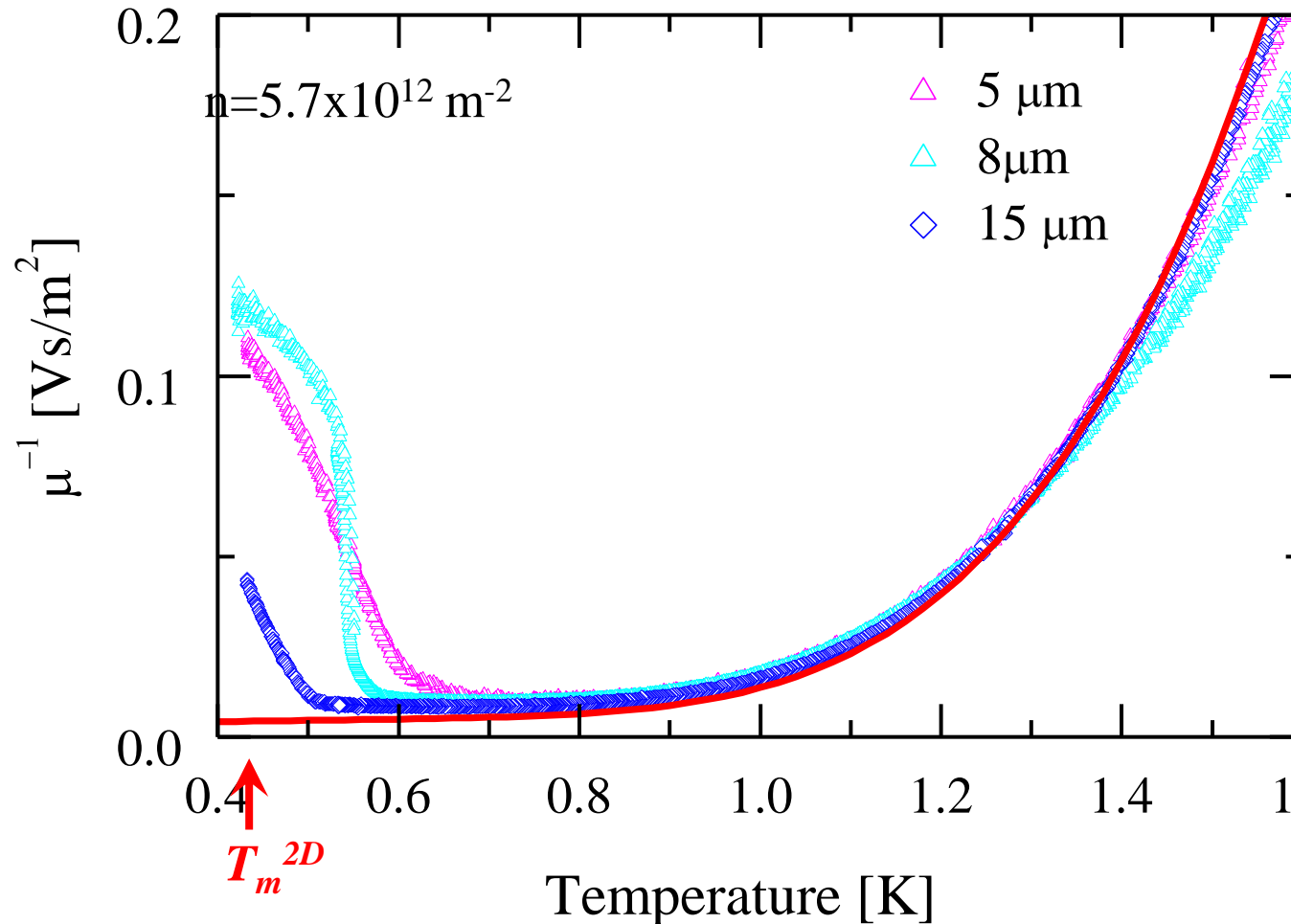
**Motivation**

**How the melting proceeds in the quasi-1D geometry?**



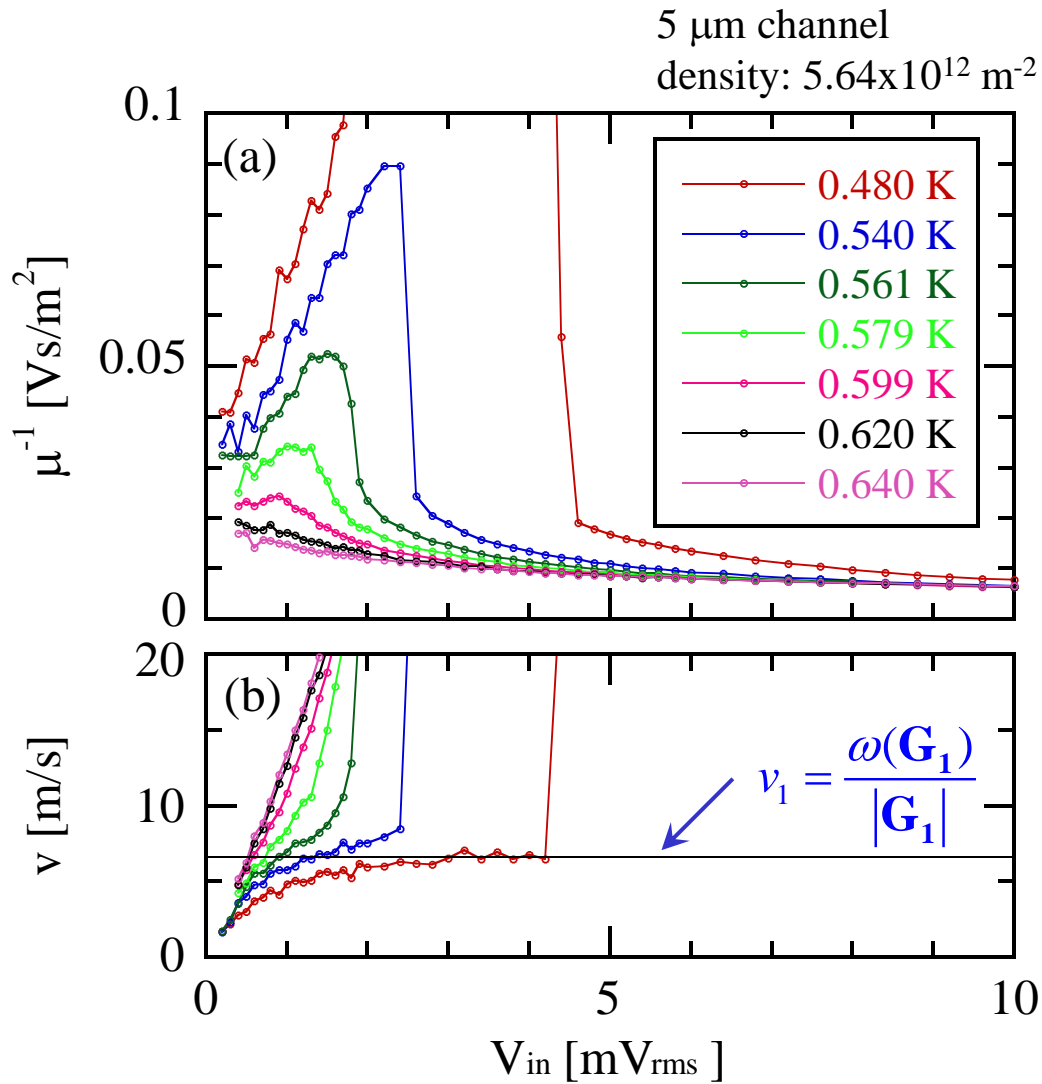
**Nonlinear transport measurements of electrons in quasi-1D channels**

# Temperature Dependence of Mobility



**For a narrower channel,  
 $\mu^{-1}$  starts to increase at a higher temperature.**

# Excitation Dependence of $\mu^{-1}$ and $v$



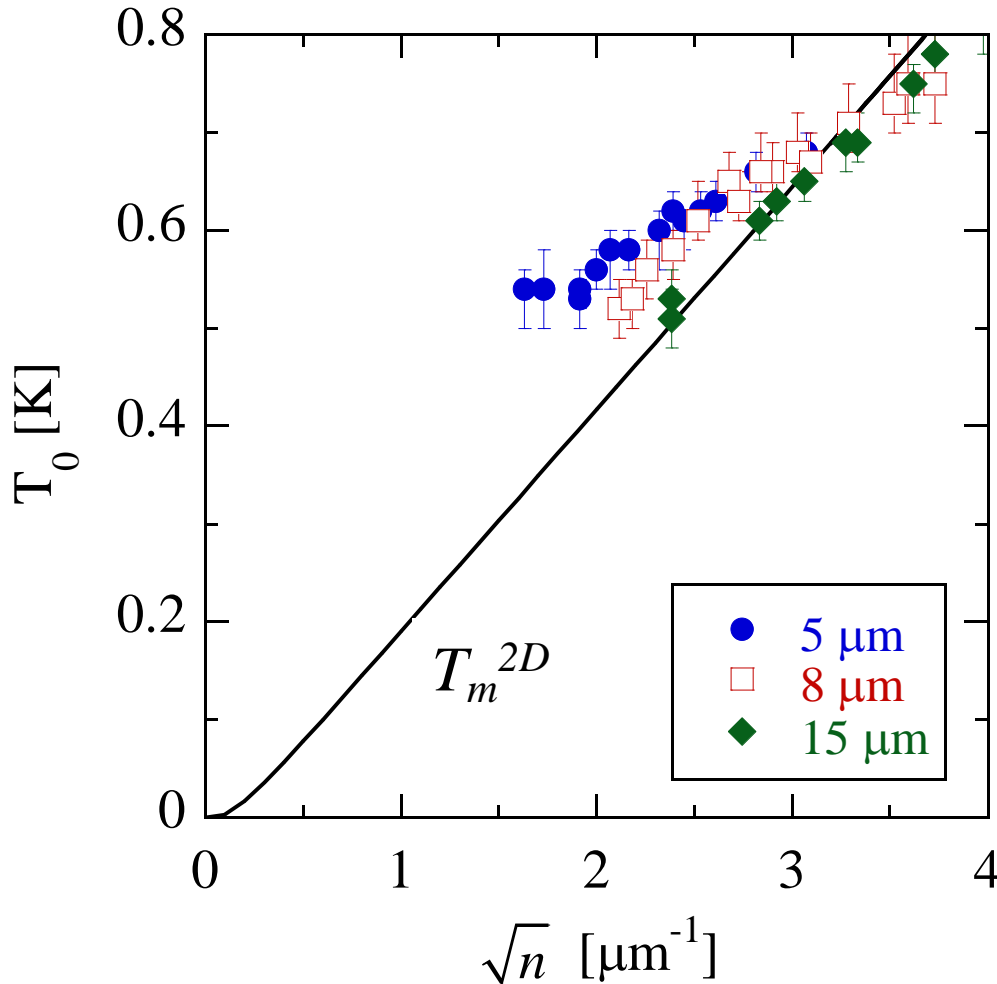
the BC scattering disappears:  
 $T_0 = 0.62 \pm 0.02 \text{ K}$

periodic structure



Bragg-Cherenkov scattering

# Density Dependence of $T_0$



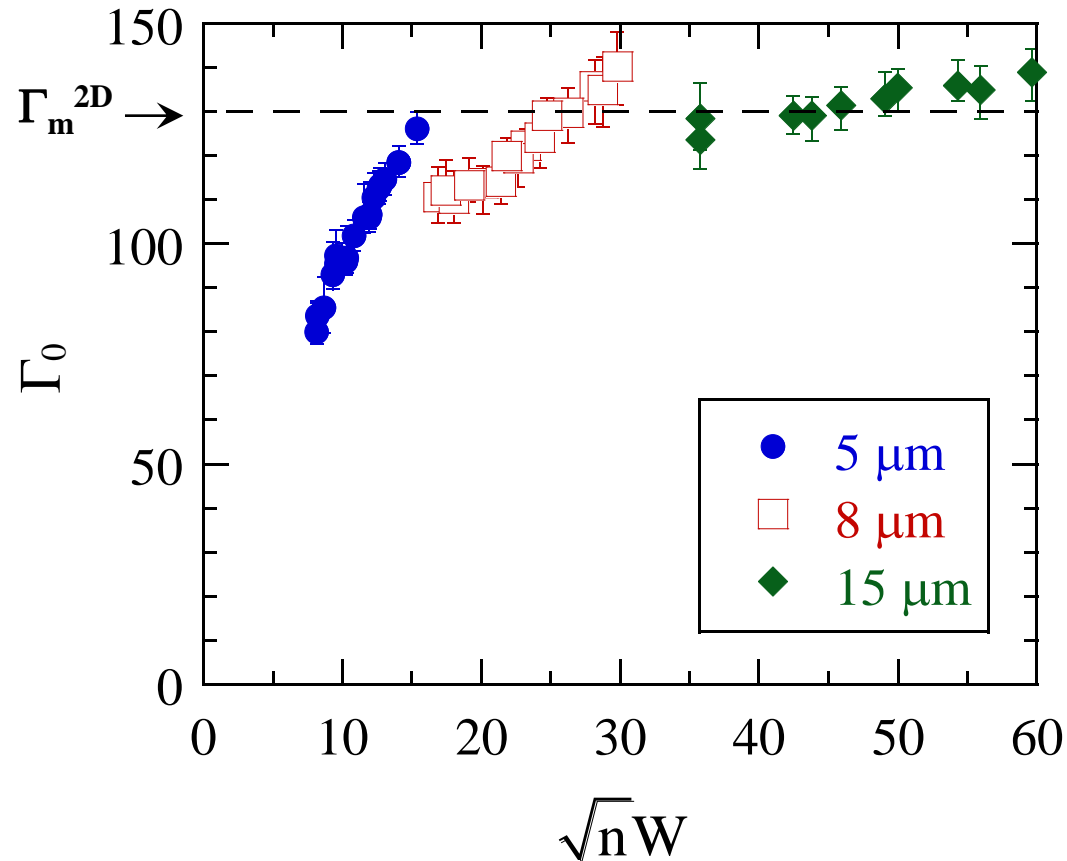
on bulk liquid helium

$$T_m^{2D}: \Gamma = U/K \sim 130$$

**Deviation from the 2D behavior is more significant for a narrower channel and at a lower density.**

# $\Gamma_0$ as a Function of the Number of Electron across Channel

plasma parameter at  $T_0$ :  $\Gamma_0 = U/k_B T_0$



**the plasma parameter at  $T_0$  is smaller  
for a fewer electrons across the channel**

# Melting of a Wigner Crystal in 2D

## Kosterlitz-Thouless-Halperin-Nelson-Young (KTHNY) mechanism

$$T < T_m^{2D}$$



dislocation-pair    three dislocations

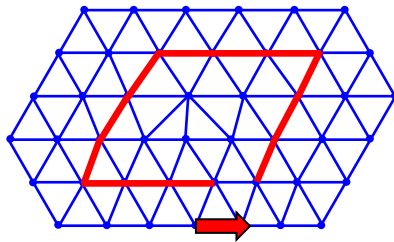
▪ positional-correlation function:

$\Psi \sim r^{-\eta}$  due to phonons

▪ quasi-long range order

$$T > T_m^{2D}$$

free dislocation



Burgers vector

▪ positional-correlation function:

$\Psi \sim \exp(-r/\xi_+)$  due to free dislocations

▪ short range order

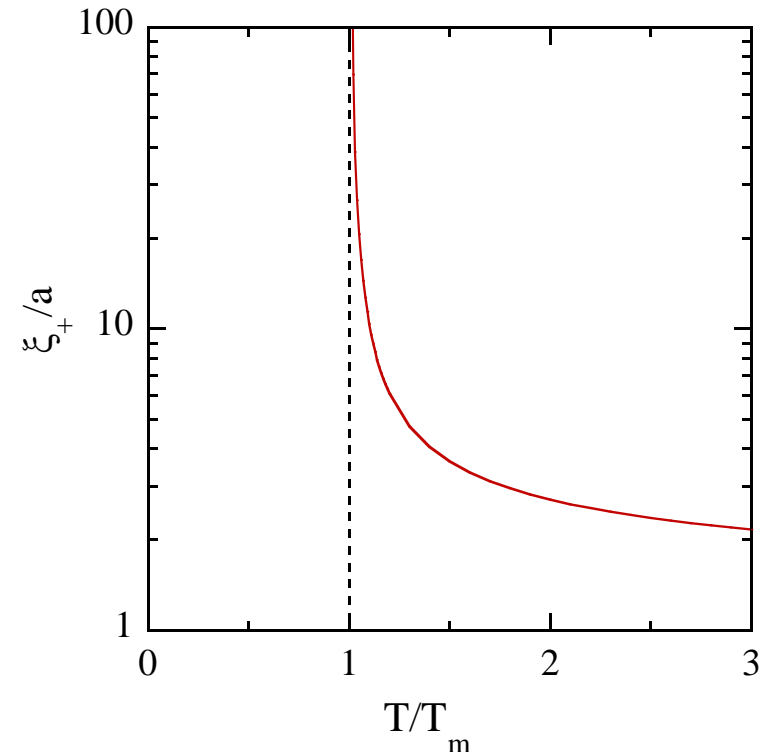
correlation length

(mean distance between dislocations)

$$\xi_+ = A a_0 \exp \left[ \frac{b}{(T/T_m^{2D} - 1)^\nu} \right]$$

$a_0$ : core size of a dislocation

$\nu = 0.36963$

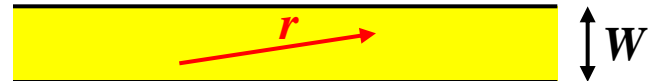


# Melting of a Wigner Crystal in quasi-1D Geometry

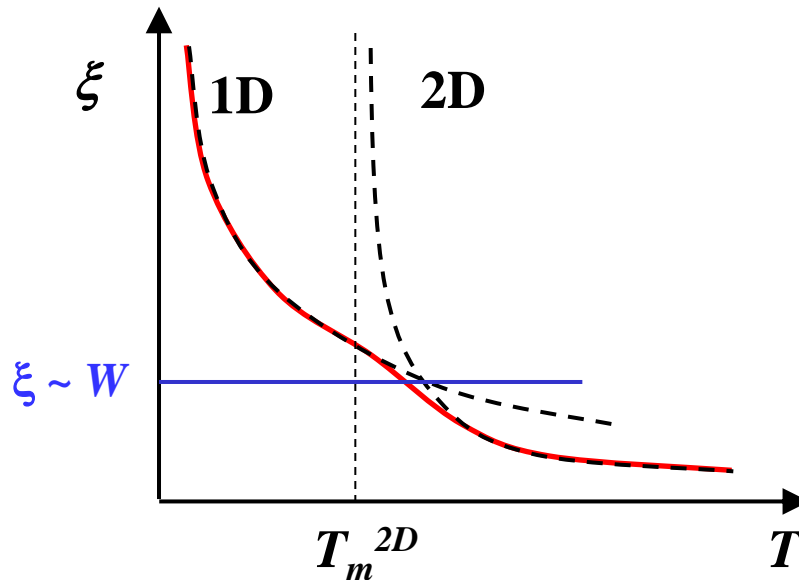
## Positional Correlation in quasi-1D geometry

$$r > W : \Psi \sim \exp(-r/\xi) \quad (1\text{D-like})$$

Short range order  
correlation length:  $\xi \sim T^{-1}$



$$r < W : 2\text{D-like}$$

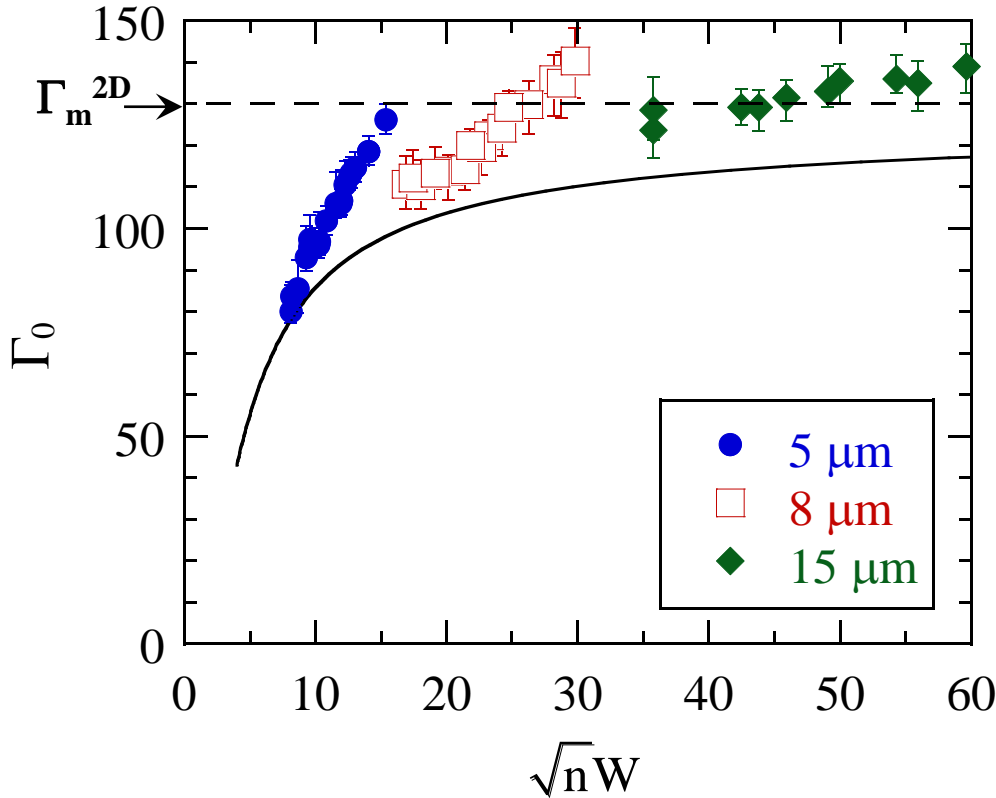


$\xi \sim W$ : 1D-2D crossover

# $\Gamma_0$ as a Function of the Number of Electron across Channel

plasma parameter at  $T_0$ :  $\Gamma_0 = U/k_B T_0$

solid line:  $\xi_+ = W$



$$\xi_+ = Aa_0 \exp \left[ \frac{b}{(T/T_m^{2D} - 1)^\nu} \right]$$

$$= Aa_0 \exp \left[ \frac{b}{(\Gamma_m^{2D}/\Gamma - 1)^\nu} \right]$$

$a_0$ : lattice constant ( )  
 $\nu=0.36963$

$b=1.8$  for  $E_c/k_B T_m=4.9$   
 (numerical calculation

by Fisher, et al, PRB **20**, 4692 ('79) )

$A=1.0, \Gamma_m^{2D}=130$

**Emergence of free dislocations in the channel causes the disappearance of the Bragg-Cherenkov scattering**



# Outline

---

## 1. Introduction

## 2. Device

## 3. Nonlinear transports of a Wigner crystal

- Bragg-Cherenkov scattering of ripplons
- decoupling of a Wigner crystal from the dimple lattice

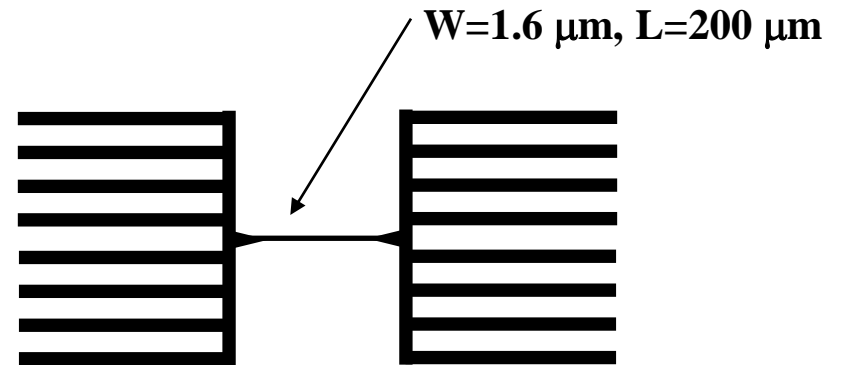
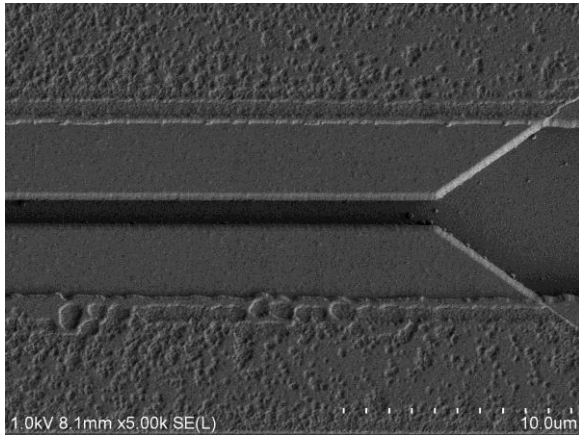
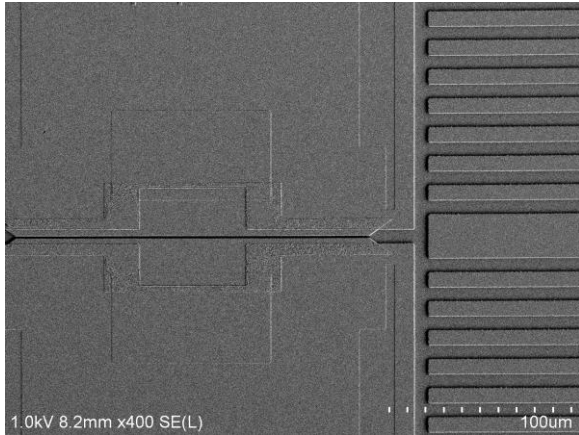
## 4. Melting of a Wigner crystal in quasi-1D geometry

- 1D-2D crossover

## 5. 1.6 $\mu\text{m}$ channel

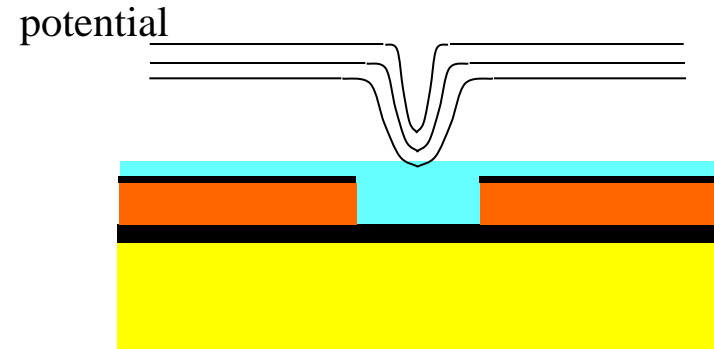
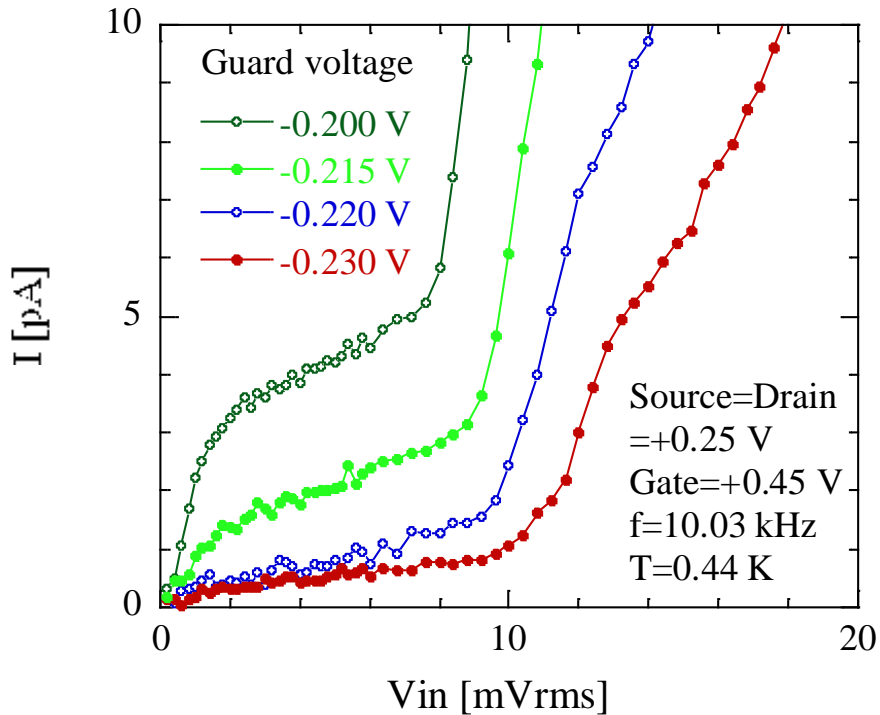
## 6. Summary

# 1.6 $\mu\text{m}$ Channel



# Nonlinear Transport in 1.6 $\mu\text{m}$ Channel

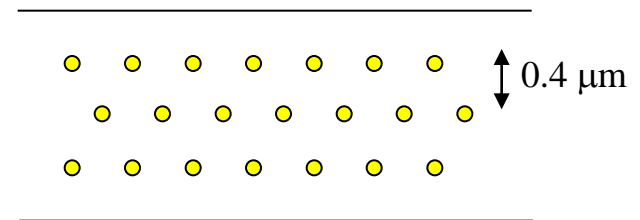
Current as a function of excitation



$$I = enWv_1$$

For three rows of electrons,

$$I = 5.3 \text{ pA}$$



**the Bragg-Cherenkov scattering occurs in the 1.6  $\mu\text{m}$ -channel**

# Summary

---

**We have investigated nonlinear transports of Wigner crystals in the quasi-1D geometry.**

## Nonlinear behaviors

- **Electron velocity is limited by the Bragg-Cherenkov scattering of ripplons.**
- **Mobility jumps at a large excitation due to the decoupling of the Wigner crystal from the dimple lattice.**
- **The decoupling takes place from the deepened dimple lattice.**

## Melting of a Wigner crystal in quasi-1D geometry

**The nonlinear behaviors disappear at a higher temperature for a fewer electrons across the channel.**

- **disappearance of the lattice structure**
- **caused by the emergence of free dislocations in the channel**