



# Temperature Dependent Energy Levels of Electrons on Liquid Helium

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Khalil Harrabi, [Mike Lea](#).  
+ Eddy Collin, Grenoble

# Microwave absorption

CW microwaves  
(165 GHz - 220 GHz)

Sweep DC

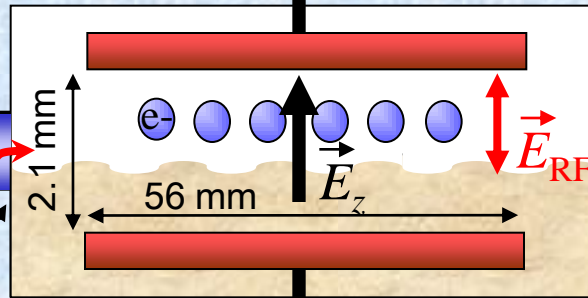
1 kHz

Modulation AC

Lock-in

Cell

Electrodes



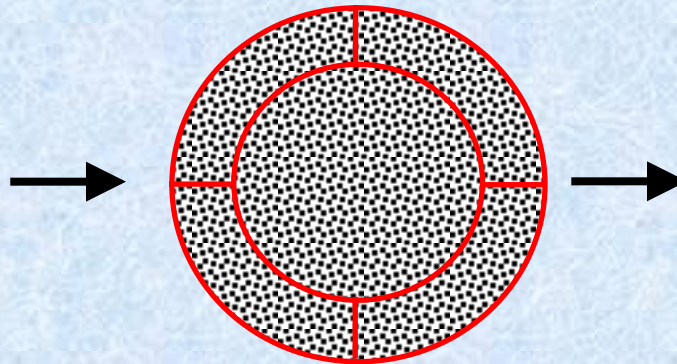
Putley detector  
(InSb bolometer)



Cell

Waveguide

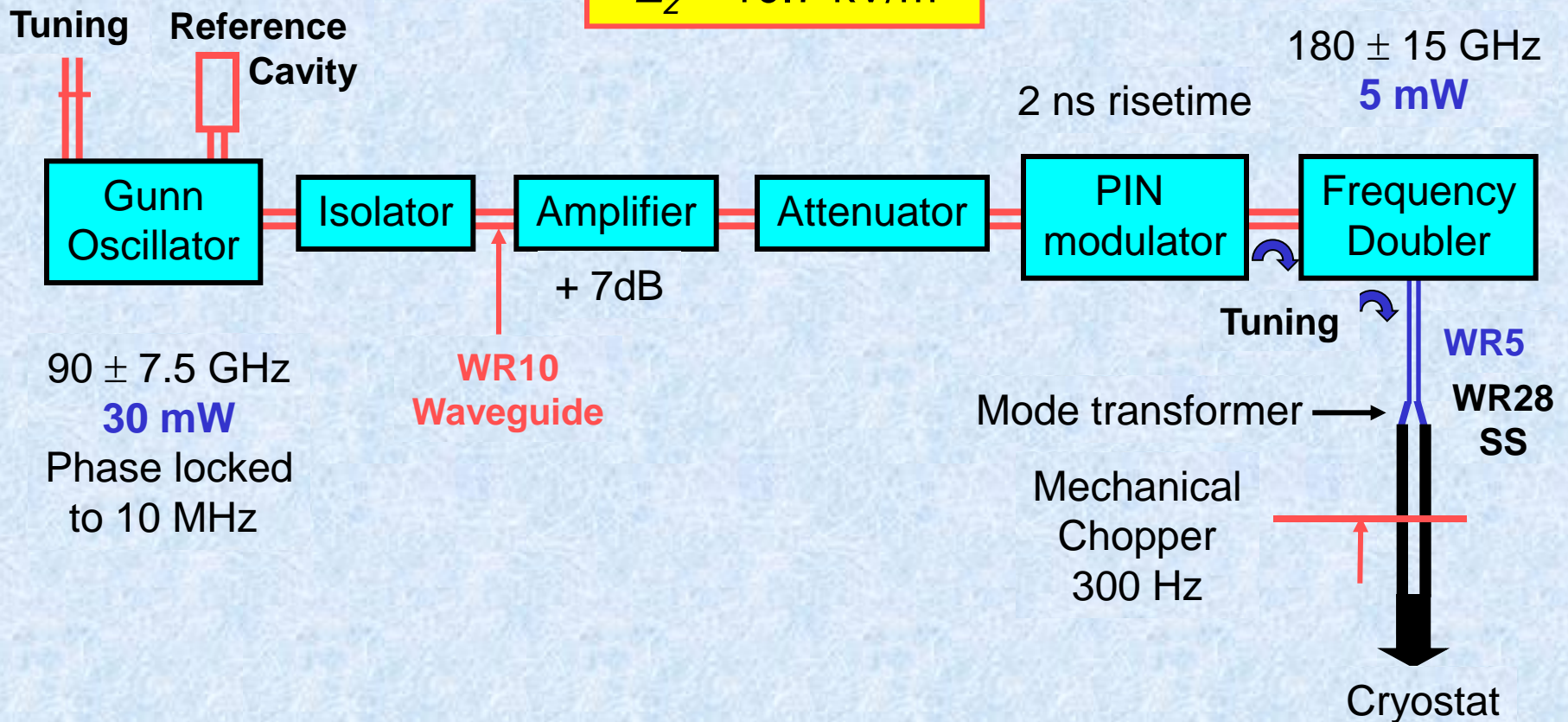
Polarising  
Grid



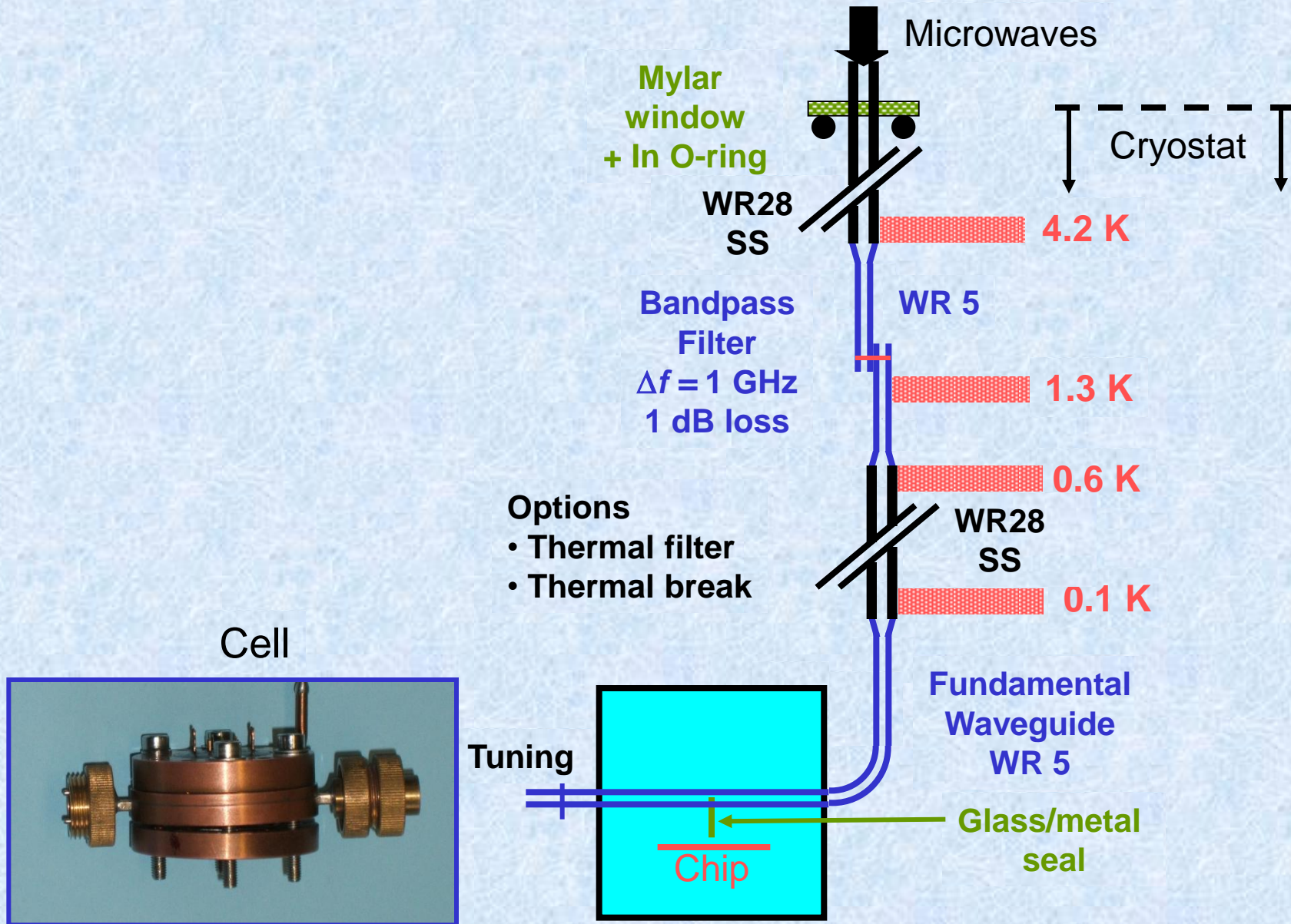
Segmented  
Corbino

Peter Frayne  
Royal Holloway

Rydberg  
resonance  
190 GHz  
 $E_z = 10.7 \text{ kV/m}$







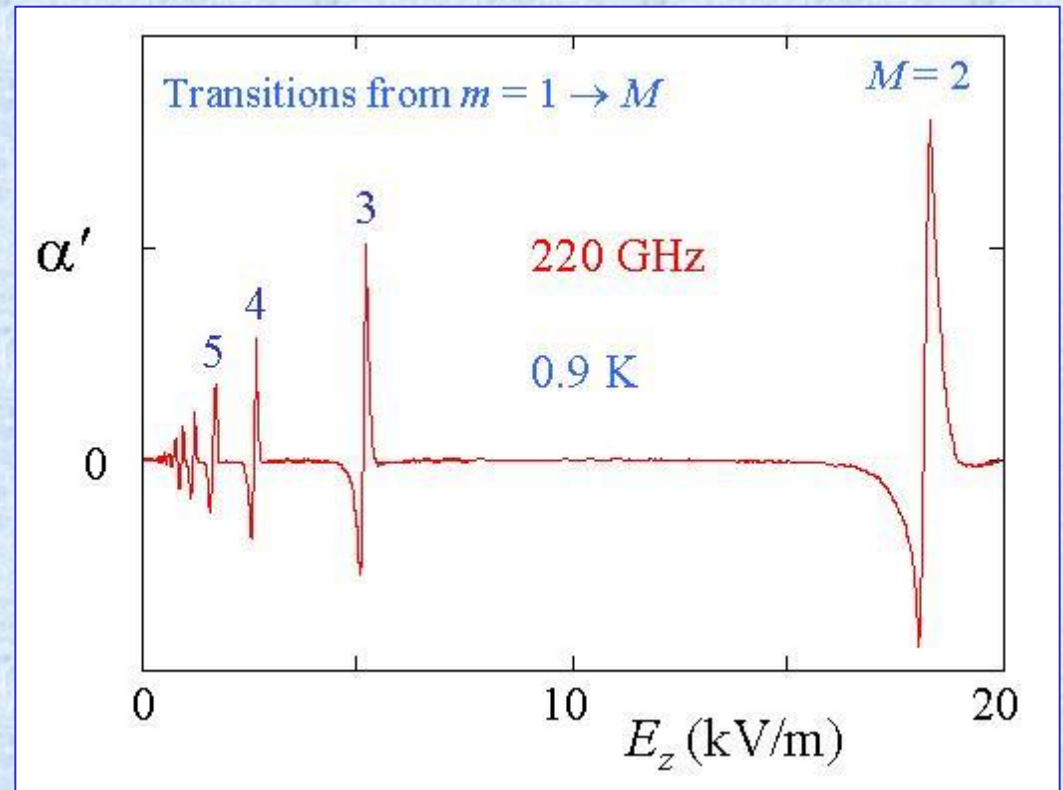
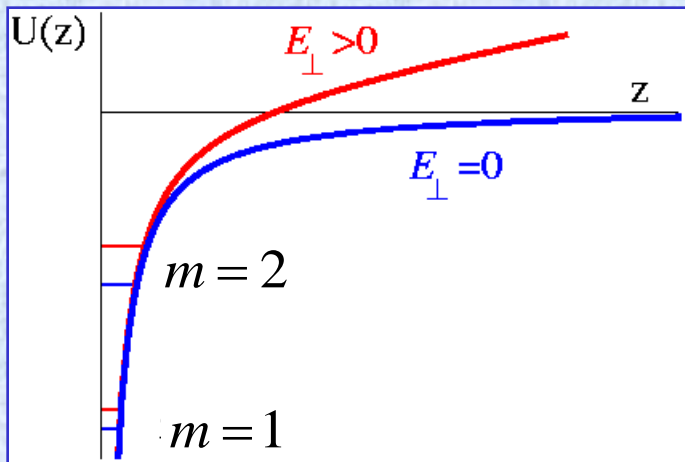
# Rydberg states – liquid $^4\text{He}$

**Image charge**  $U_0(z) = \frac{-Ze^2}{4\pi\epsilon_0 z}$  for  $z > 0$      $Z = \frac{\epsilon - 1}{4(\epsilon + 1)}$      $E_m = \frac{-R_e}{m^2}$      $f_{12} = 119.3 \text{ GHz}$

**Grimes et al:**  $U_0(z) = \frac{-Ze^2}{4\pi\epsilon_0(z+b)}$  for  $z > 0$   
 $= V_0$  for  $z \leq 0$

**Experiment for  $E_z = 0$ :**  
 $f_{12} = 125.9 \text{ GHz at } 1.2 \text{ K}$

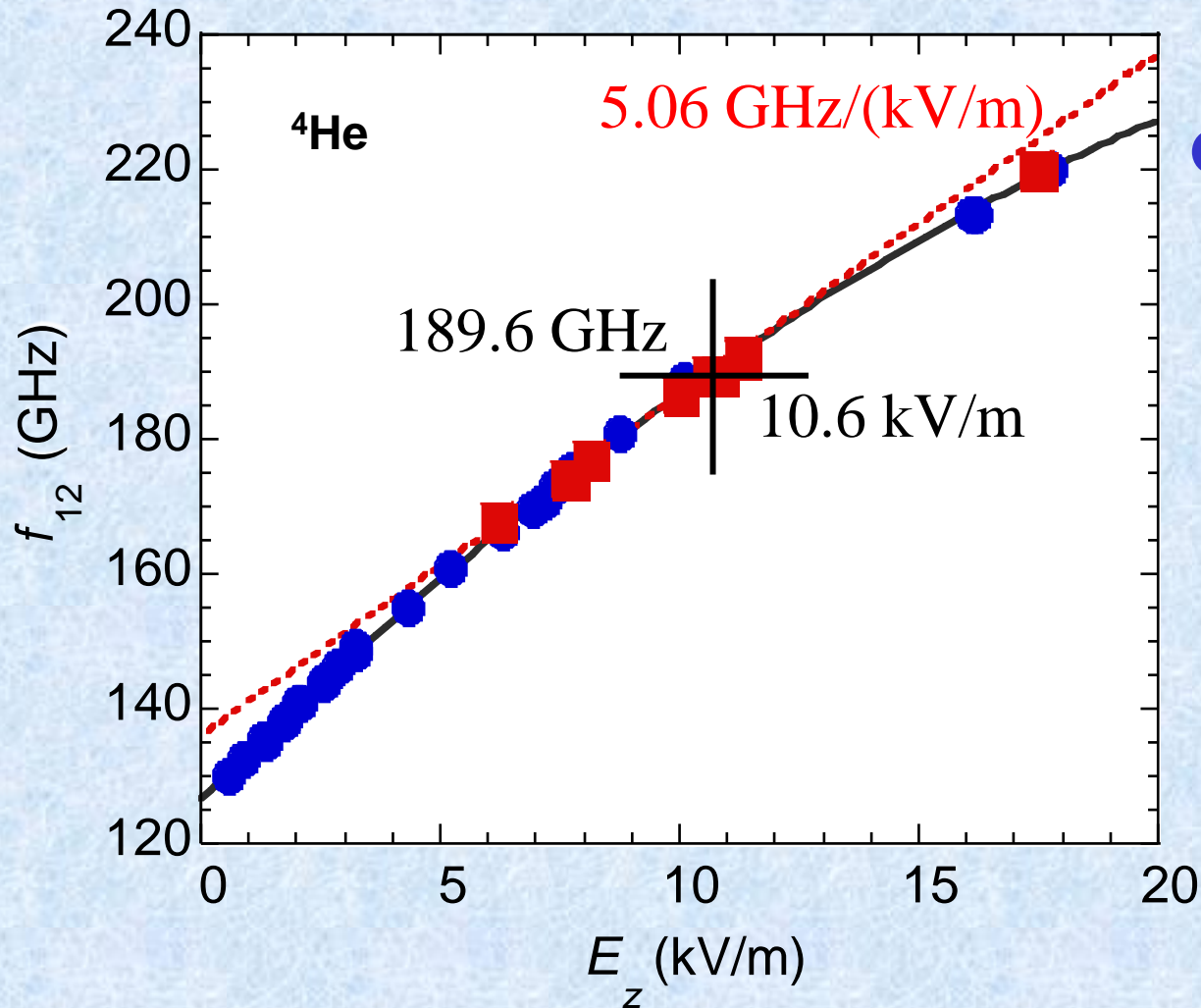
**Stark tuning**  $U(z) = U_0(z) + eE_z z$



# Stark Tuning Resonance

Ground state to first excited Rydberg state

Resonant frequency  $f_{12}$  increases with  $E_z$



● Brown, Grimes, Zipfel, 1976

1.5 K  
Low power

# Temperature dependent resonance

Low temperatures

Inhomogenous broadening

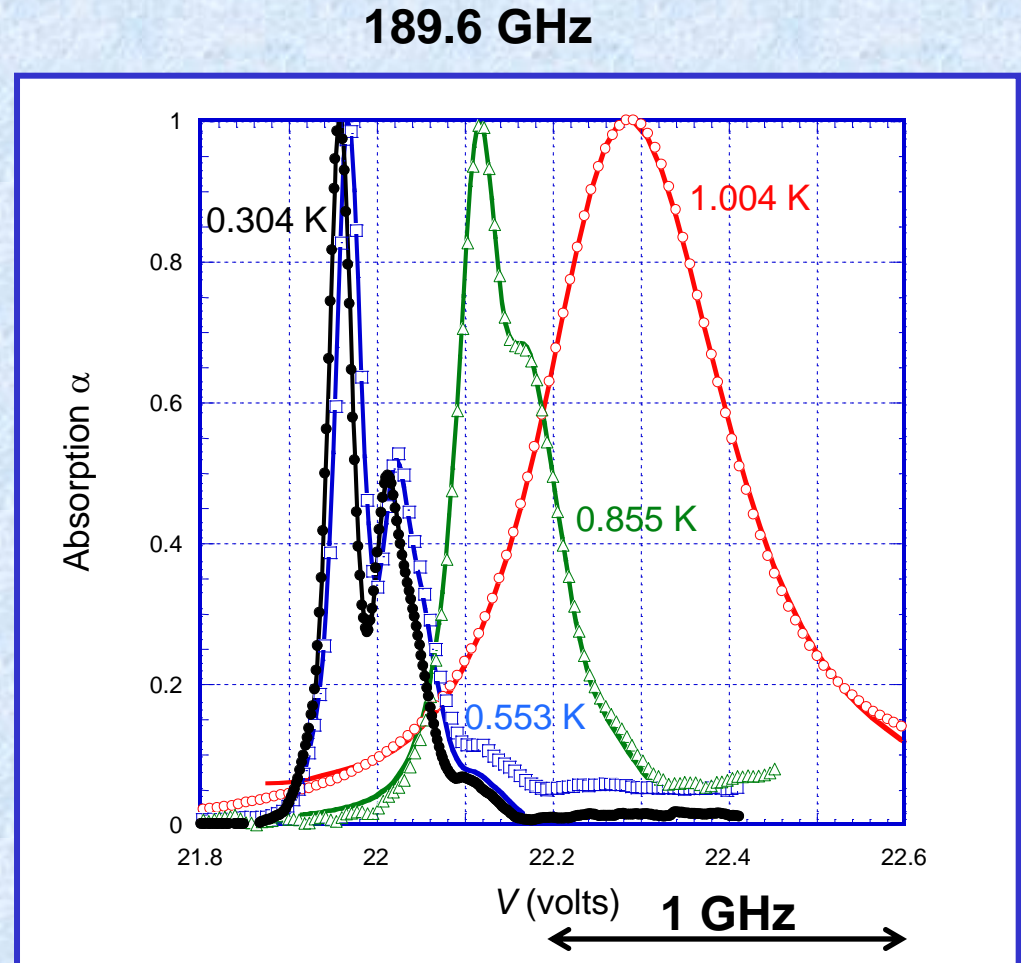
Medium temperatures

Inhomogenous broadening  
convoluted with a Lorentzian

High temperatures

Lorentzian

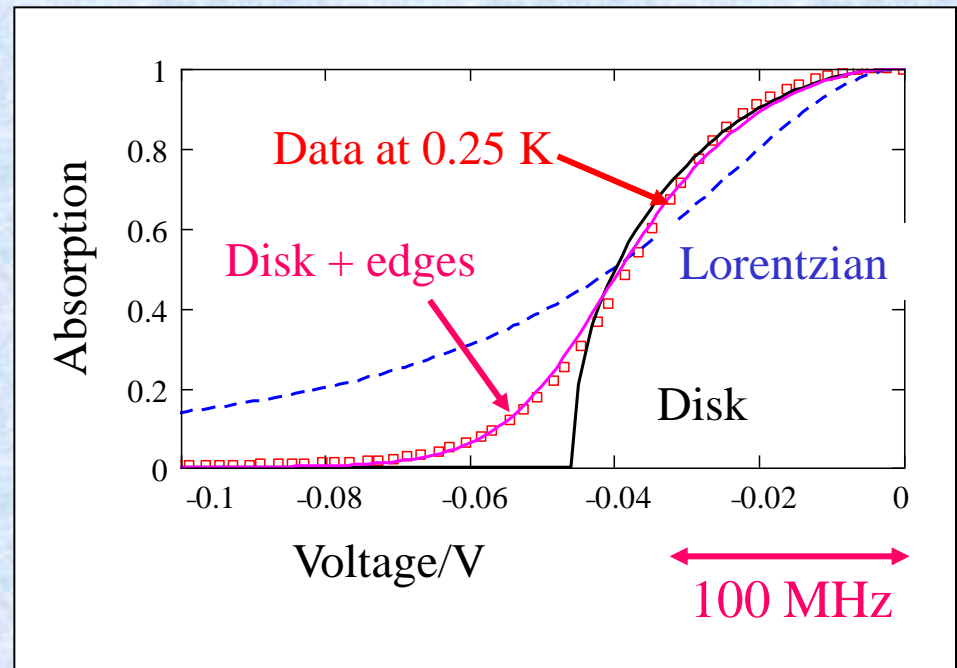
Resonance frequency *decreases*  
as the temperature *increases*





## Non-parallel electrodes

- Lineshape independent of  $T < 0.5$  K
- Inhomogeneous broadening Peaks from  $E_z$  variation (0.3%)?
- Non-parallel disk electrodes: Parabolic lineshape
- $8 \mu\text{m}$  across 50 mm
- $\theta = 0.17$  mrad =  $35''$  arc
- Lorentzian contribution small?





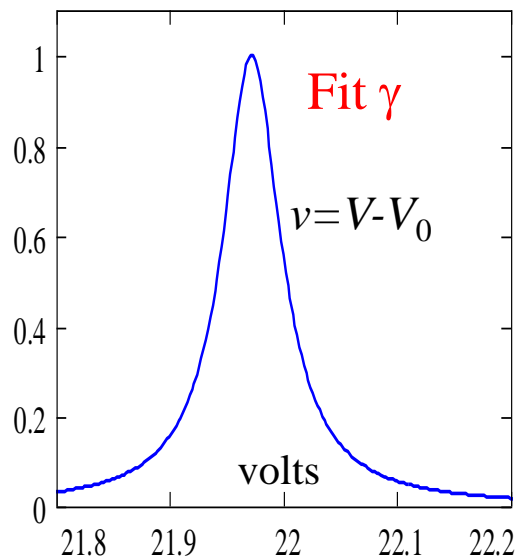
# Convolution

Use lineshape at 0.3 K as a template

Convolute with a Lorentzian

Fit linewidth  $\gamma$  to data  $\Rightarrow \gamma(T)$

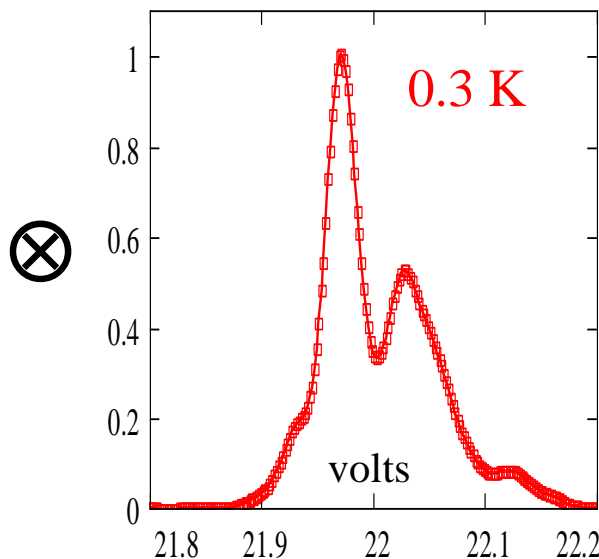
Lorentzian



$$L(v, \gamma) = \frac{\gamma / \pi}{v^2 + \gamma^2}$$

Intrinsic  
linewidth

Cell  
 $\equiv \delta$ -response

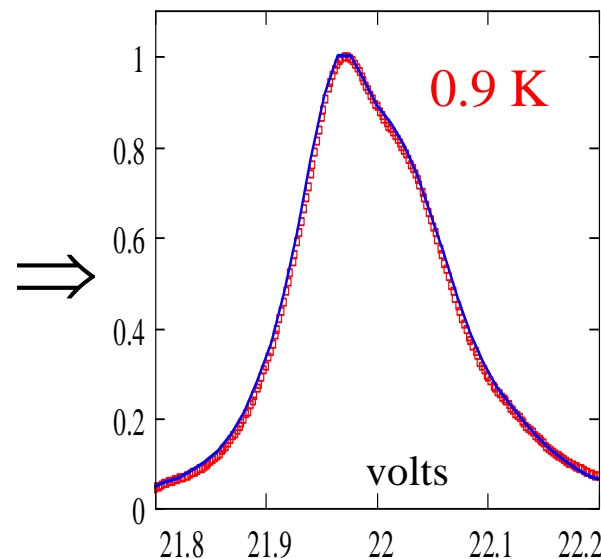


$G(V)$

Inhomogeneous  
broadening

Measured

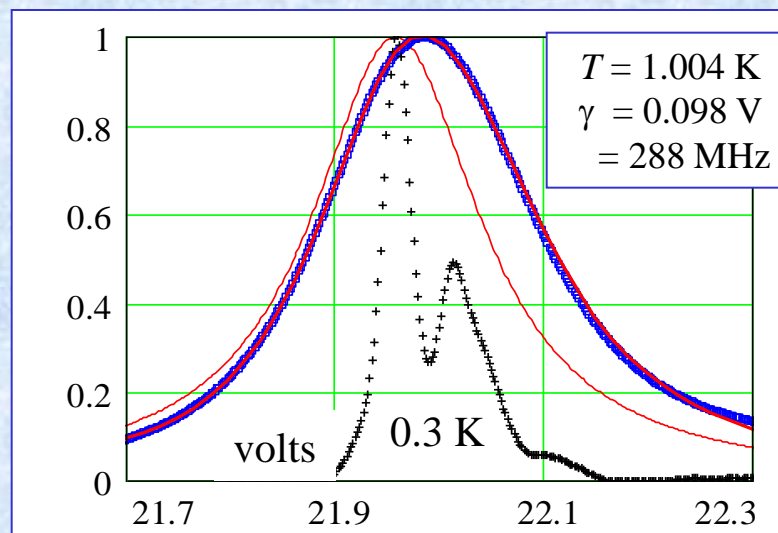
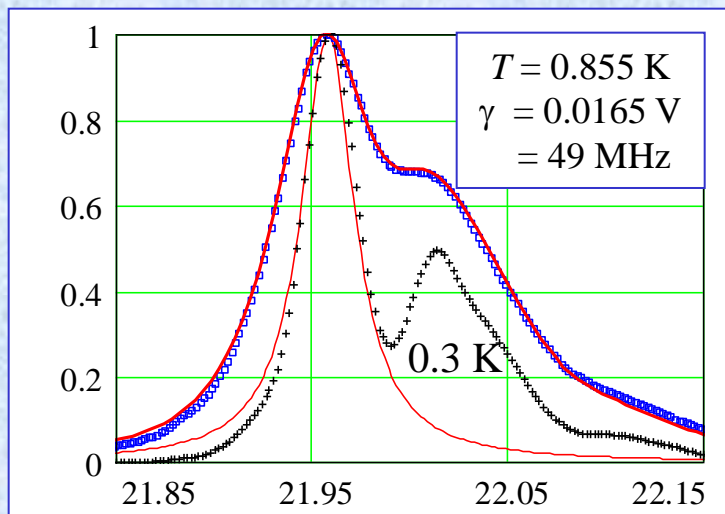
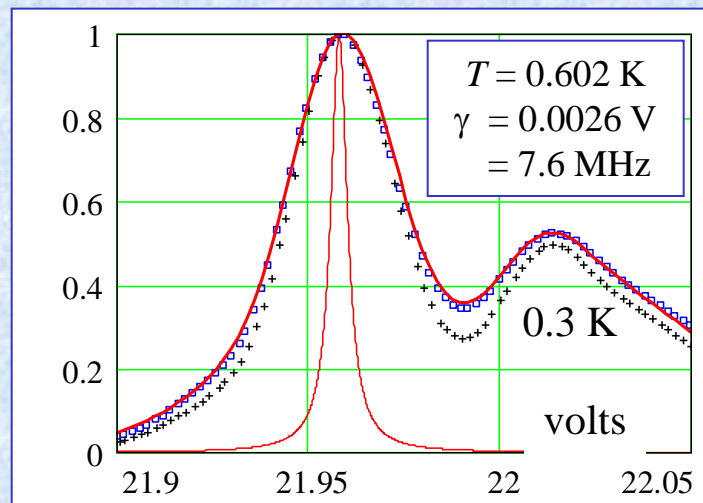
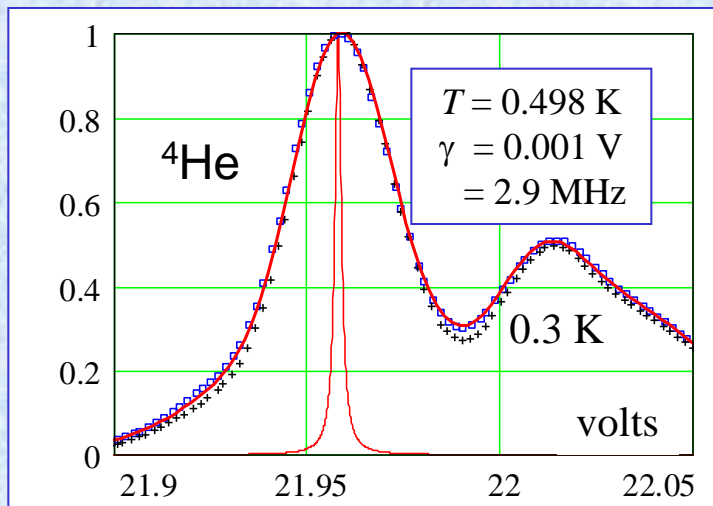
Fit  $\gamma = 106$  MHz



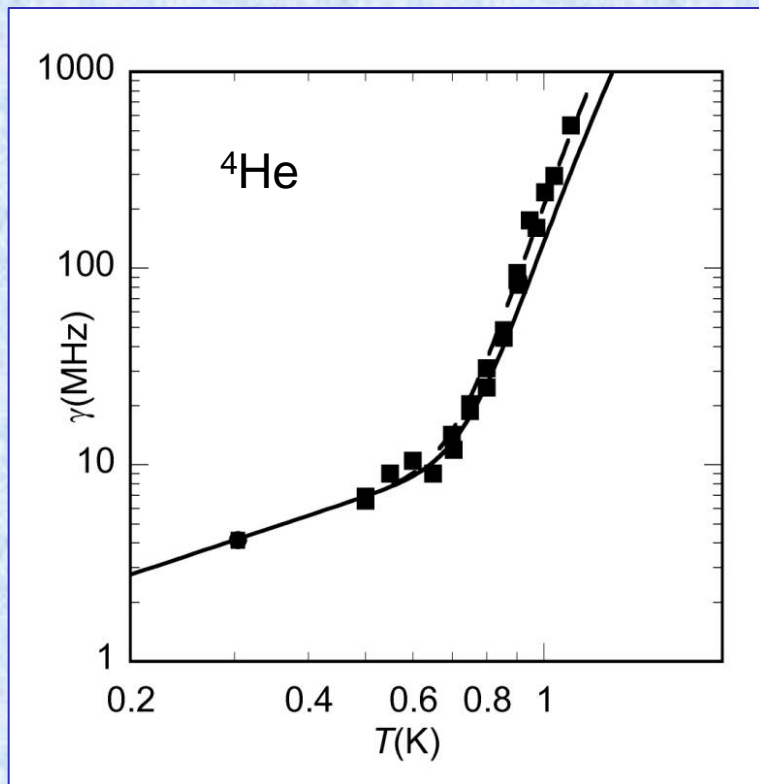
$$S(V) = \int G(V - v)L(v, \gamma)dv$$

Convolution

Output = Lorentzian  $L(V)$   $\otimes$  Cell response  $G(V)$



# Microwave linewidth $\gamma(T)$



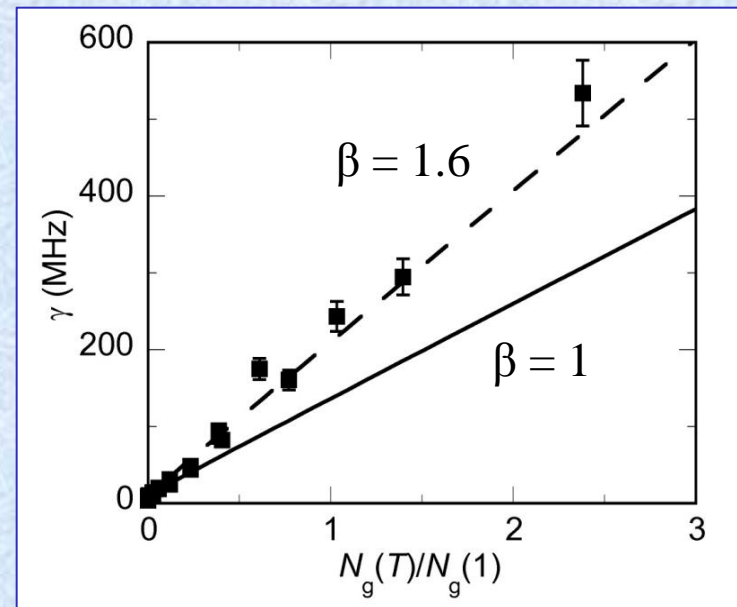
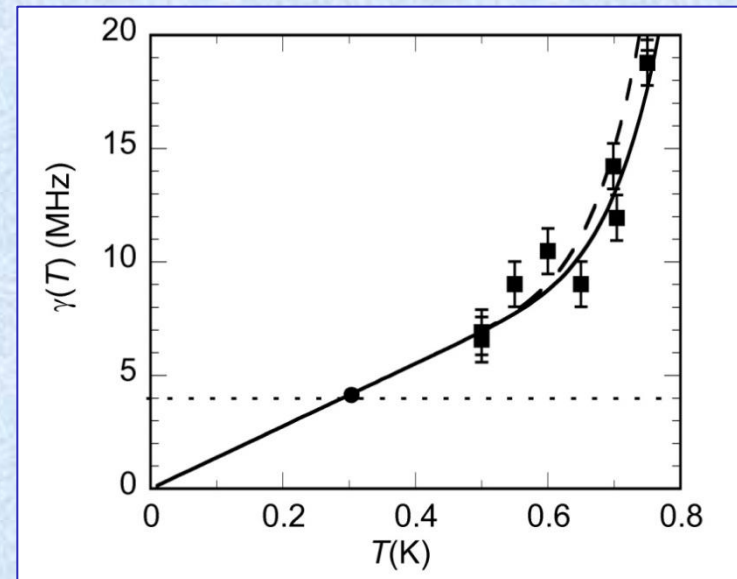
Theory:

T. Ando, J.Phys.Soc Japan, 44,765 (1976)

$$\gamma = aT + \beta b N_{\text{gas}}$$

Ripplon      Gas atom  
 Scattering

[H. Isshiki *et al.* J.Phys.Soc Japan (2007):  
 $\beta = 2.1$  ( $^3\text{He}$ );  $1.6$  ( $^4\text{He}$ )]

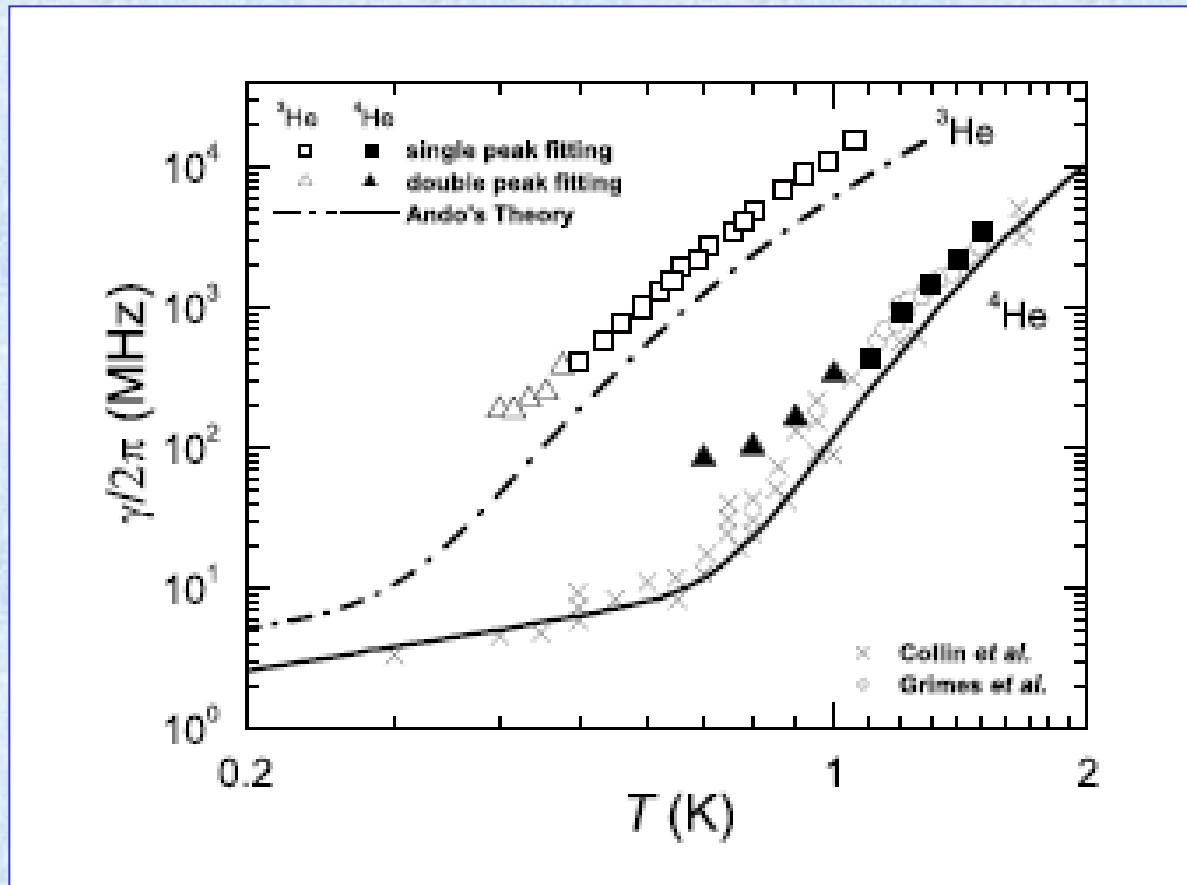


## Temperature dependent linewidth

H. Isshiki *et al.* J.Phys.Soc Japan 76, 094704 (2007)

$$\beta = 2.1 (^3\text{He}); 1.6 (^4\text{He})$$

$$\gamma = aT + \beta bN_{\text{gas}}$$



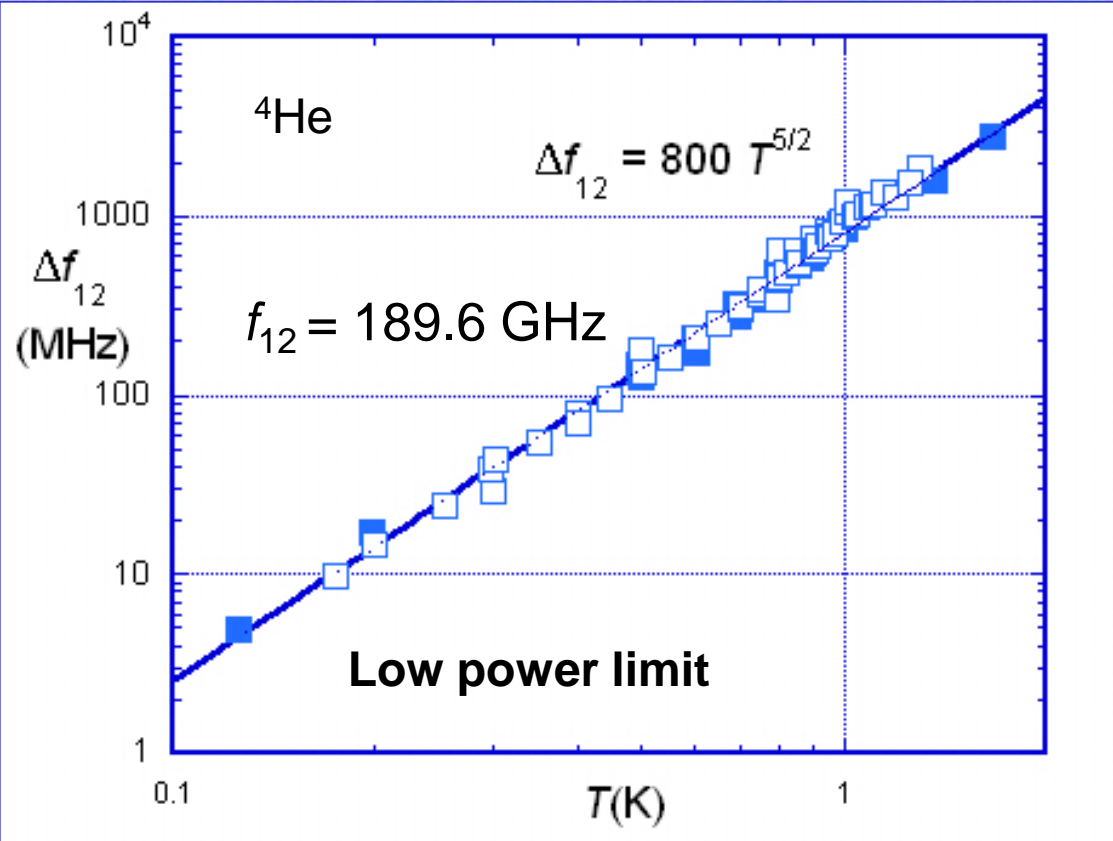


## Temperature dependent resonance $f_{12}$

$$\Delta f_{12}(T) = f_{12}(0) - f_{12}(T) \\ \approx 800 \text{ MHz at 1 K}$$

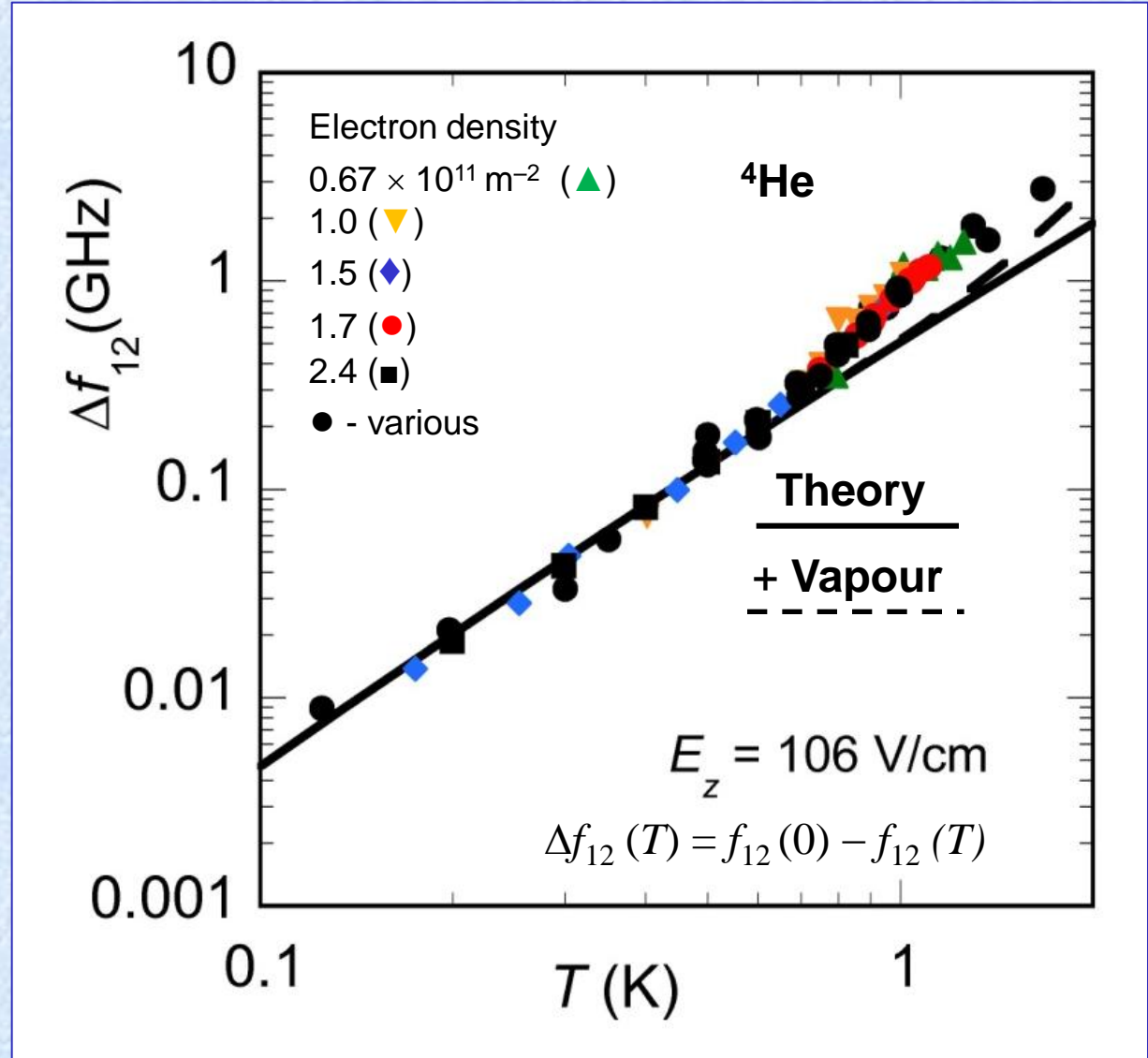
$$\Delta f_{12}(T) \propto T^{5/2} \text{ or} \\ \propto T^{7/3}$$

Ripplons?



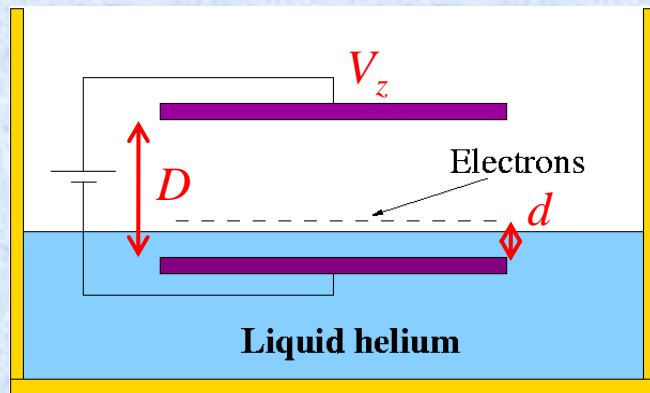
Theory:  
 Mark Dykman,  
 Denis Konstantinov  
*et al* (2010)

2-ripplon processes:  
 Lamb shift

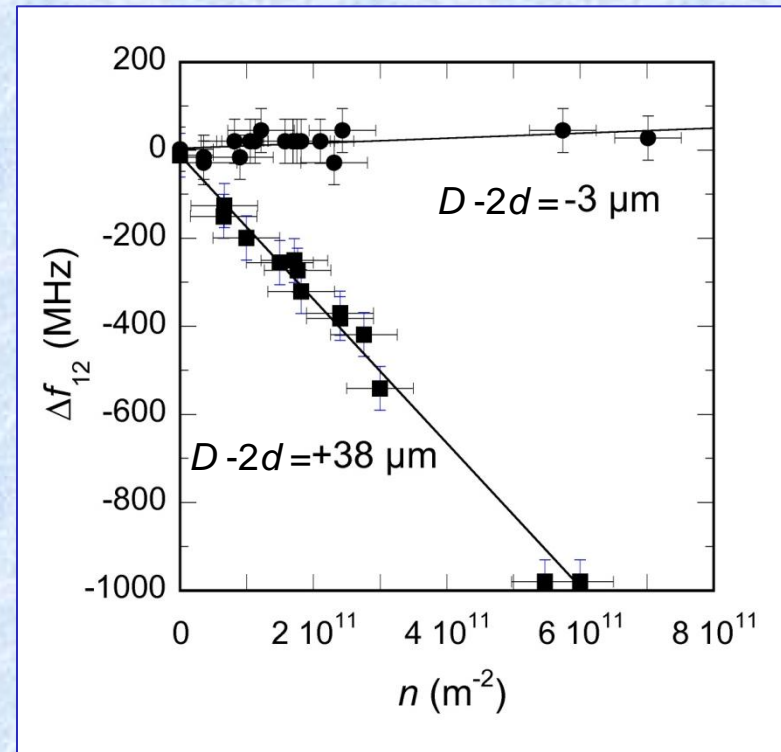


## Density dependence of holding field

$$E_z = \frac{-V_z}{(D-d+d/\epsilon)} + \frac{ne}{\epsilon_0(\epsilon+1)} \frac{(D-2d)}{(D-d+d/\epsilon)}$$



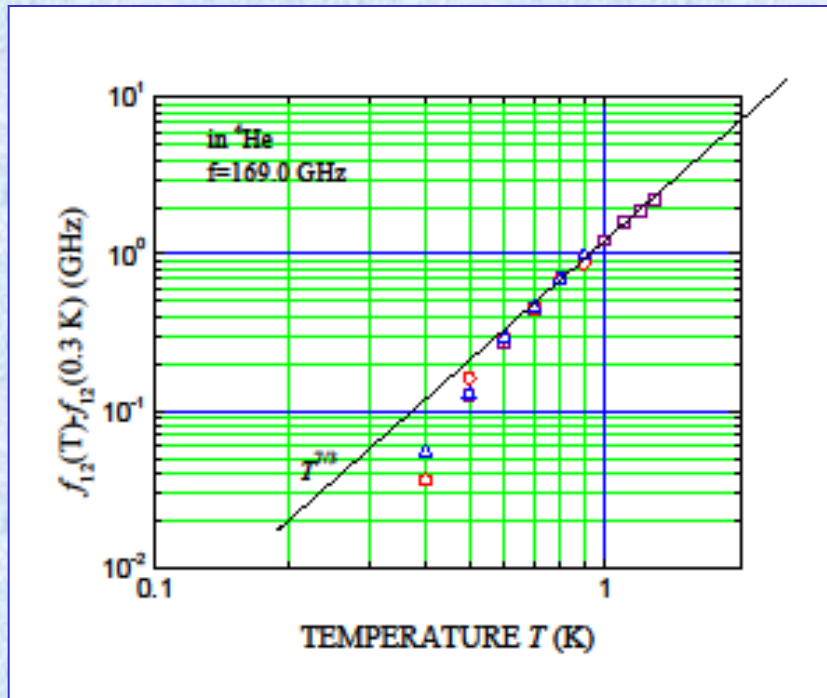
Extrapolated to  $T = 0$



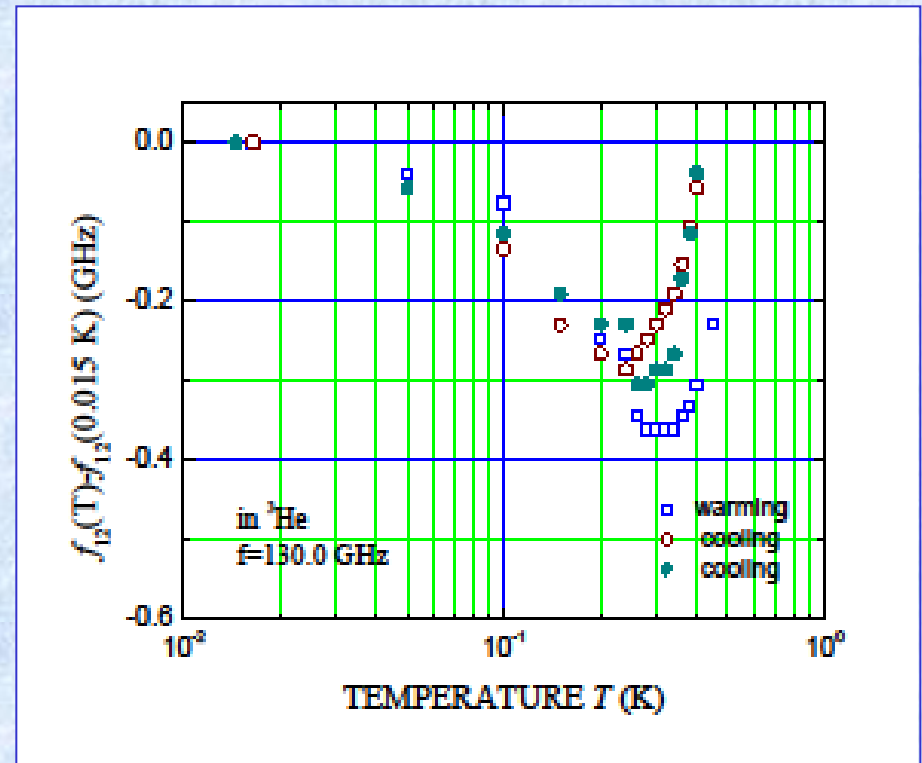
# Temperature dependence of $f_{12}$

RIKEN

$^4\text{He}$



$^3\text{He}$





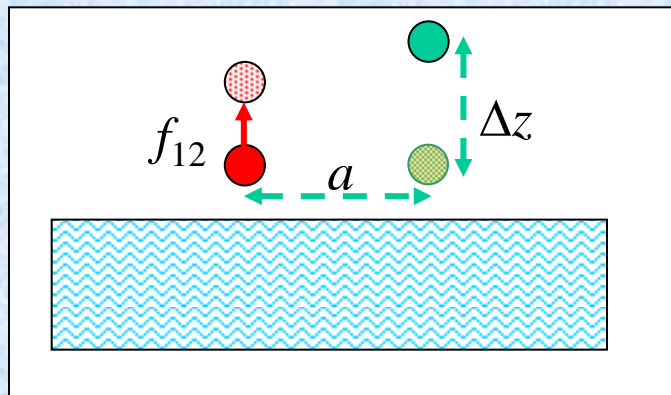
# Microwave absorption - Coulomb shift

D. Konstantinov *et al.* PRL 98, 235302 (2007)

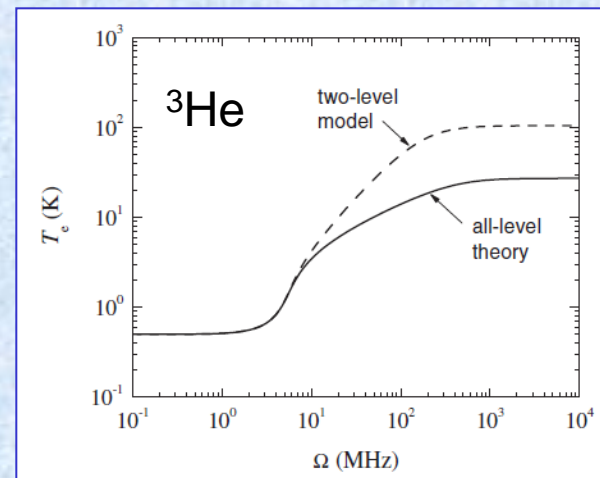
Ultra-hot Electrons on Liquid  $^3\text{He}$ :  $T_e < 27$  K

Resonance frequency shifts with

- Power absorbed
- Excited state population
- Electron temperature  $T_e$
- Electron density

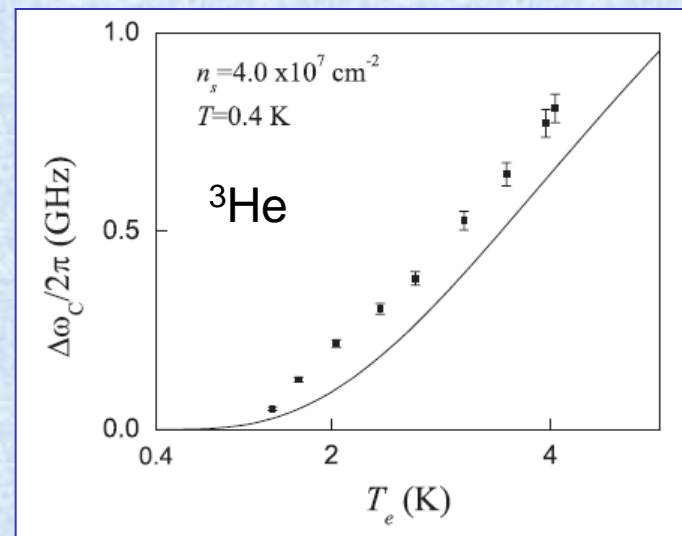


$$\Delta\omega_{21} = \frac{Fe^2n_s^{3/2}}{2\hbar} \left[ (z^2)_{11} - (z^2)_{22} - 2(z_{11} - z_{22}) \times \sum_l z_{ll}\rho_{ll} + 2|z_{12}|^2(\rho_{11} - \rho_{22}) \right]$$



Rabi frequency  $\Omega \propto \sqrt{\text{Power}}$

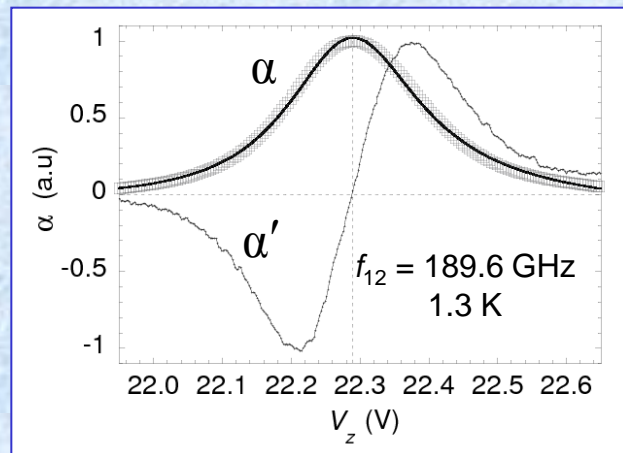
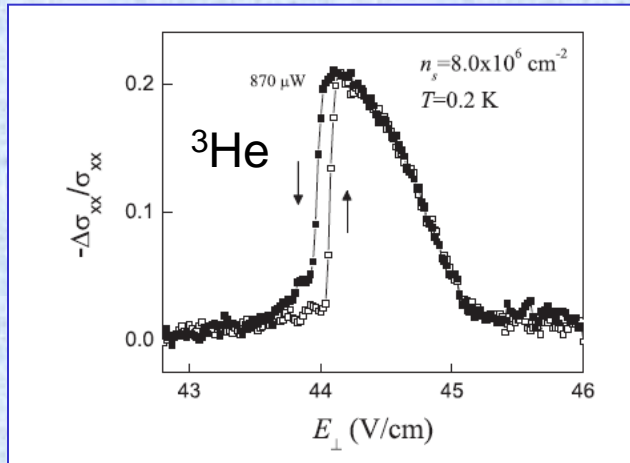
D. Konstantinov *et al.* PRL 103, 096801 (2009)



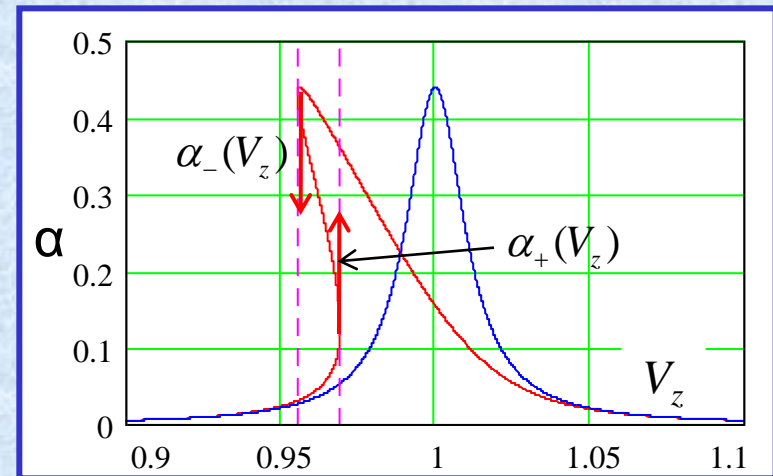
# Optical bistability in microwave absorption

D. Konstantinov *et al.* PRL 103, 096801 (2009)

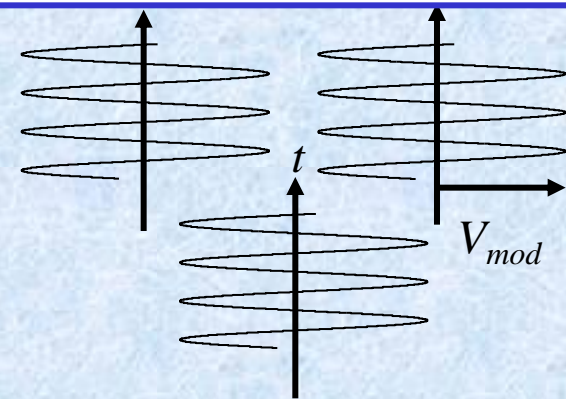
**High-powers:**  
Hysteresis in conductivity



**High-powers:**  
A.C. modulation (10 mV at 1 – 10 kHz)  
Complex microwave lineshape

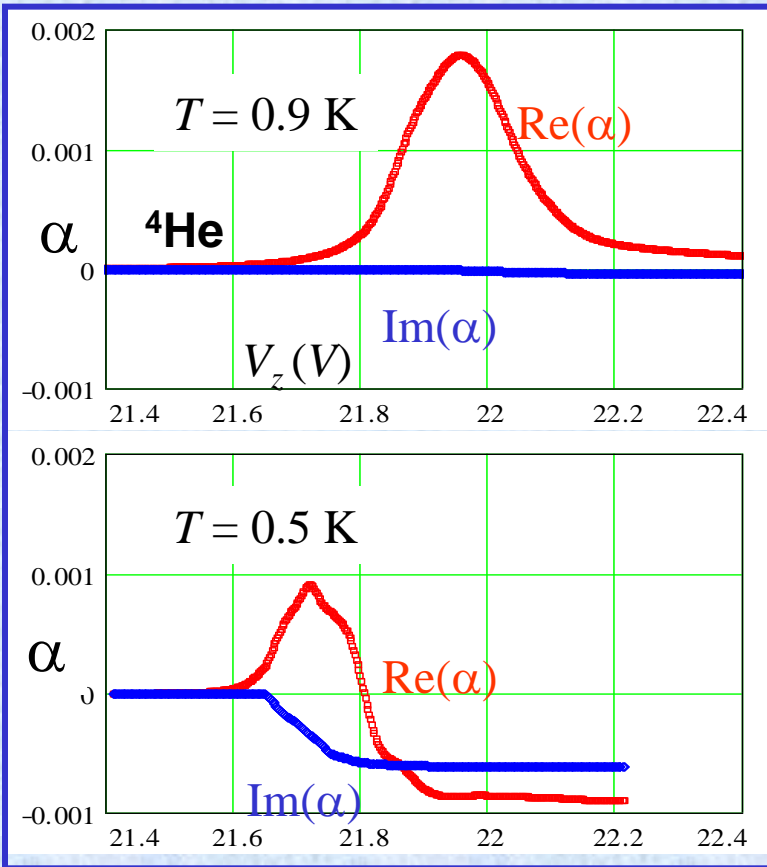


**Differential absorption**  
 $\alpha' = d\alpha/dV_z$



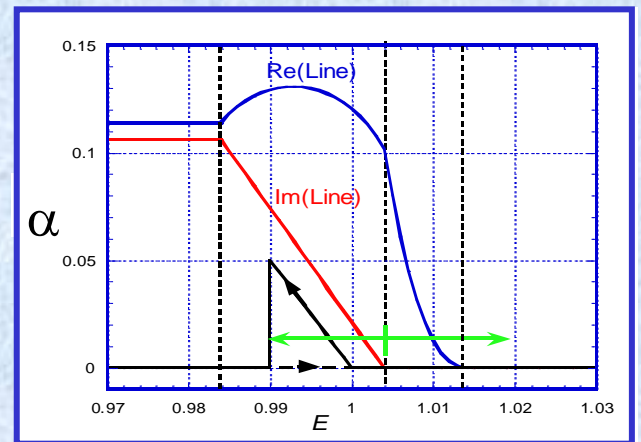
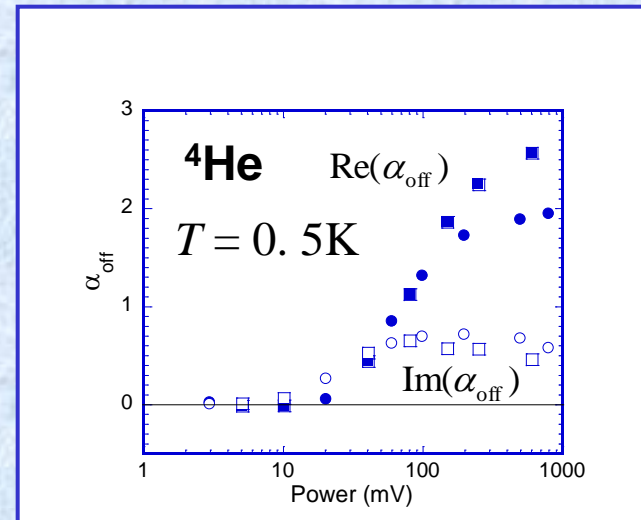
$$\text{Im}(\alpha') = \frac{1}{\pi V_m} \int (\alpha_-(V_z) - \alpha_+(V_z)) dV_z$$

# Hysteresis $\equiv$ Complex Lineshape



$$\text{Im}(\alpha_{\text{off}}) = \frac{2}{\pi} \left( 1 - \frac{\Delta V}{2V_m} \right) \int (\alpha_-(V_z) - \alpha_+(V_z)) dV_z$$

**Inhomogeneous power broadening**  
**Inhomogeneous Coulomb broadening**



$$\text{Im}(\alpha_{\text{off}}) = \frac{\alpha_{\text{max}} \Delta V}{\pi} \left( 1 - \frac{\Delta V}{2V_m} \right)$$

$$2V_m \approx 20 \text{ mV} \equiv 50 \text{ MHz}$$

## Conclusions

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- Temperature dependent Rydberg levels
- Inhomogeneous broadening
- Enhanced Ando linewidth
- Microwave absorption bistability (hysteresis)