Rashba and other spin-orbit interactions

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Work in progress in collaboration with M.I. Dykman

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Central Themes

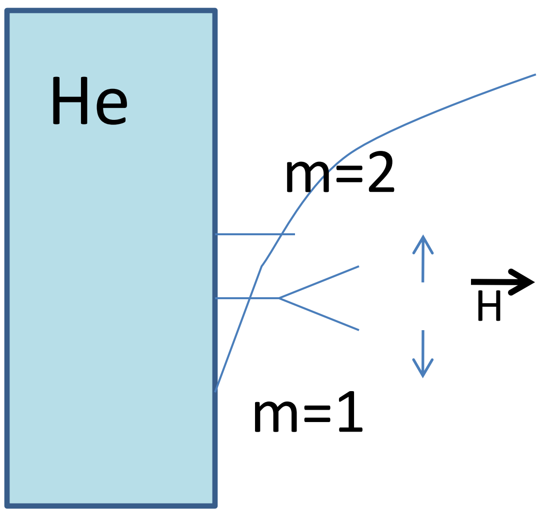
- Spin Qubit – spin-dependent interactions
- Spin-Orbit – free electrons
- Spin-Orbit – Confined Electrons (Rashba Interaction)

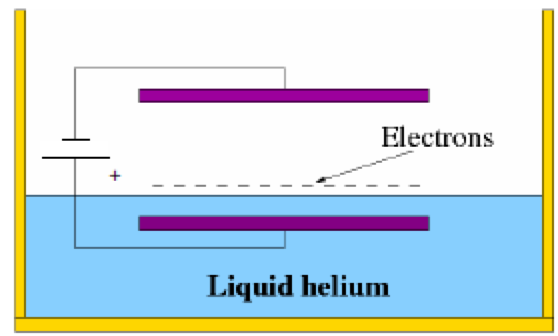
- Condensed Matter Systems
- Electrons on He – Rashba Interaction

- Spin-Orbit Coupling – ripplons

- Electron-Electron Interactions

Electron spin qubits on the He-4/vacuum interface





1D hydrogenic spectrum

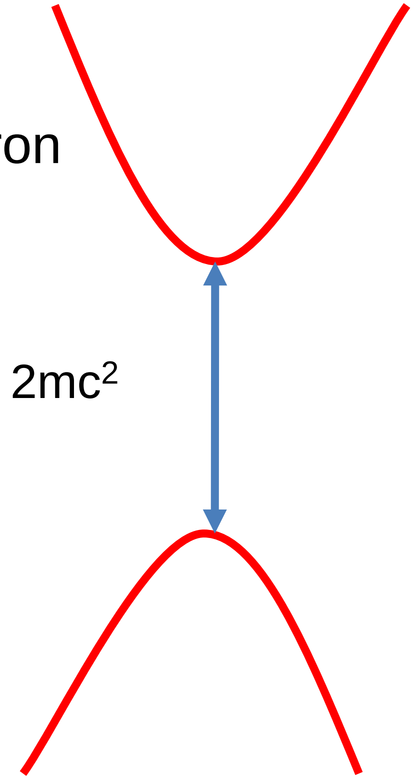
Decoherence ?

Spin-orbit effects ?

Spin-orbit interactions

$mc^2\, I+U(\mathbf{r})$	$c\boldsymbol{\sigma}\cdot\mathbf{p}$
$c\boldsymbol{\sigma}\cdot\mathbf{p}$	$-mc^2\, I+U(\mathbf{r})$

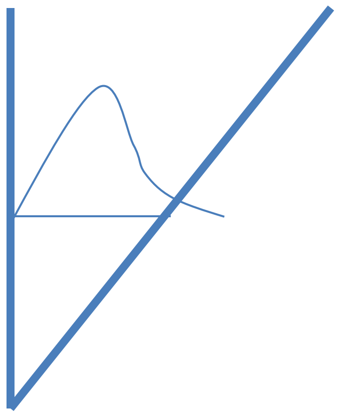
Free relativistic electron



Expansion in p/mc

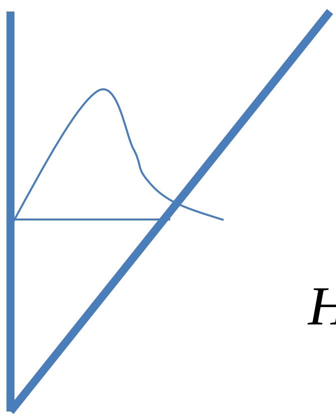
$$H=\frac{p^2}{2m}+U(\vec{r})+\frac{e\hbar}{2mc}\vec{H}\vec{\sigma}+\frac{\hbar\vec{\sigma}\cdot[\vec{p}\times\vec{\nabla}U]}{4m\cdot mc^2}$$

Spin-orbit interactions
Confined electrons



$mc^2\, I + U(\mathbf{z})$	$c\boldsymbol{\sigma}\cdot\mathbf{p}$
$c\boldsymbol{\sigma}\cdot\mathbf{p}$	$-mc^2\, I + U(\mathbf{z})$

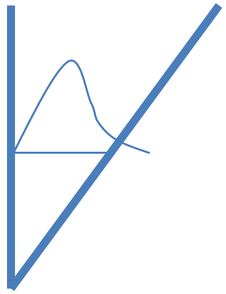
Dirac Hamiltonian

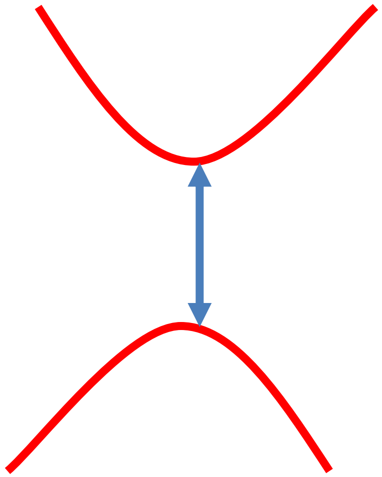


The same U acts on electrons and positrons

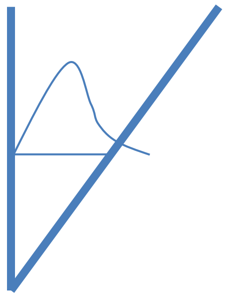
$$H = \frac{p^2}{2m} + U(\vec{r}) + \frac{e\hbar}{2mc}\vec{H}\vec{\sigma} + \int \Psi^*(z)\frac{\hbar\vec{\sigma}\cdot[\vec{p}\times\vec{\nabla}U]}{4m\cdot mc^2}\Psi(z)dz$$

Spin-orbit interactions
Confined electrons





$$\frac{[\vec{p}\times\vec{\sigma}]\hbar}{4m\cdot mc^{2}}\cdot\int\Psi^{*}(z)\,\vec{\nabla}U\,\Psi(z)\,dz$$



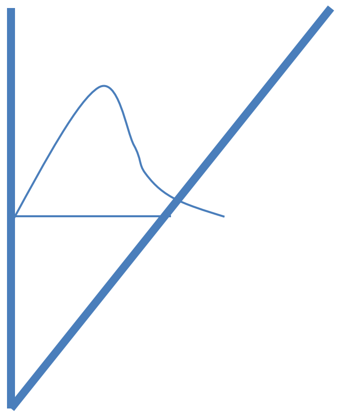
$$\nabla U=-i[H,p_z]=-i\left[U,-i\frac{\partial}{\partial z}\right]=-i\left(Hp_z-p_zH\right)$$

$$\int\Psi^{*}(z)\vec{\nabla}U\Psi(z)dz=-i\int\Psi^{*}(z)(Hp_z-p_zH)\Psi(z)dz$$

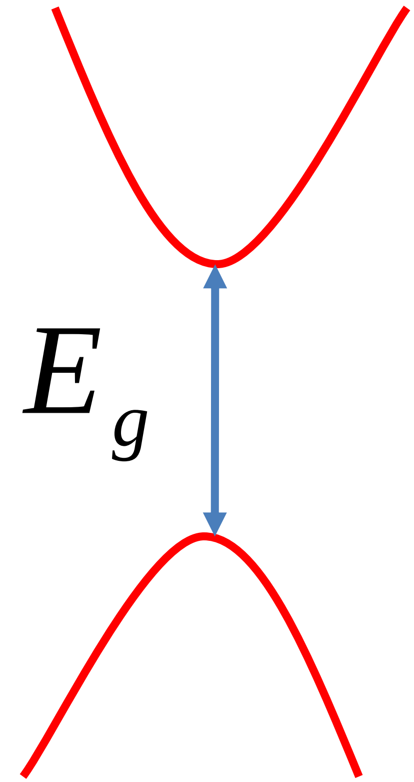
$$\Psi^{*}H=E\Psi^{*};\;\;H\Psi=E\Psi$$

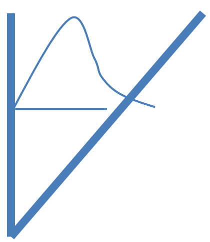
Ehrenfest theorem $$\frac{[\vec{p}\times\vec{\sigma}]\hbar}{4m\cdot mc^{2}}\int\Psi^{*}(z)\vec{\nabla}U\Psi(z)dz=0$$!!

Spin-orbit interactions
Condensed Matter systems



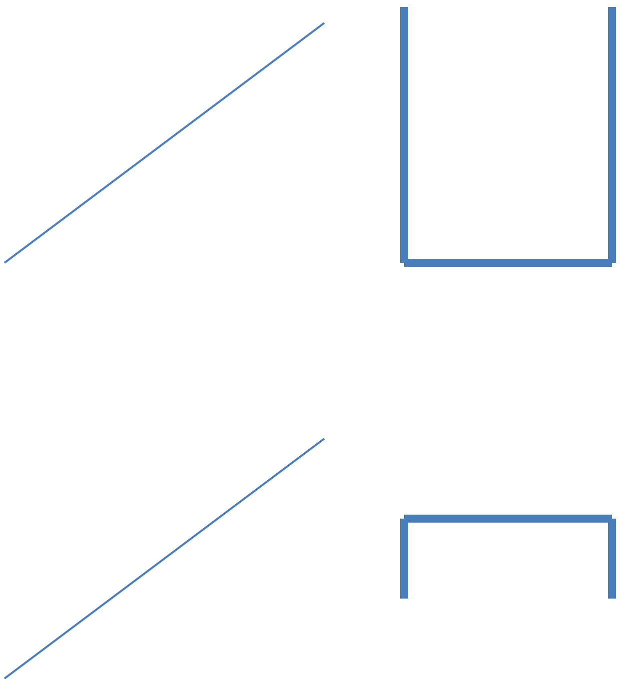
ms^2 I+$U_1(\mathbf{r})$+ eE**r**	s$\boldsymbol{\sigma}\cdot\mathbf{p}$
s$\boldsymbol{\sigma}\cdot\mathbf{p}$	-ms^2 I+$U_2(\mathbf{r})$ + eE**r**





Different offsets, different barriers
For conduction and valence electrons or ground s and excited p states

Spin-orbit interactions
Condensed Matter systems



U_c

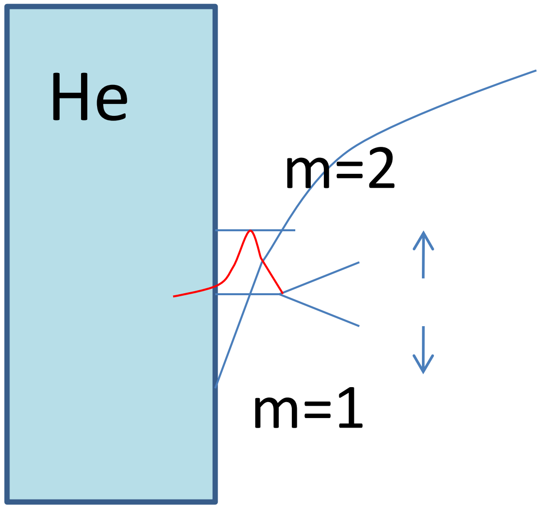
$$H_{so} = \frac{U_c + U_v}{U_c} \frac{\hbar \vec{\sigma} \cdot [\vec{p} \times eE]}{4m \cdot ms^2}$$

U_v

Different offsets in conduction and valence bands

Lassnig (1985)

Rashba type spin-orbit interaction He



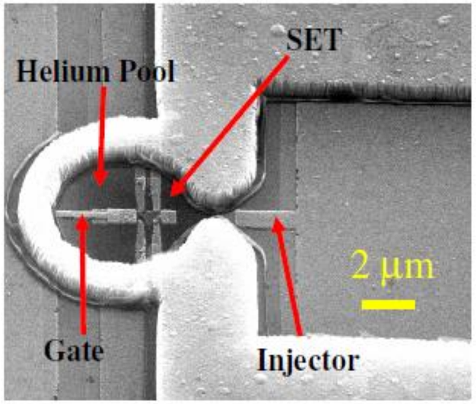
- Penetration into He
- Excited p-state
- Different parameters of quantized state in s and p-state
- E_g =50 eV , s= 2.7×10^8 cm/s
- m – free electron mass

$$H_{so} = \alpha \frac{\hbar \vec{\sigma} \cdot [\vec{p} \times e\overline{E}]}{4m \cdot ms^2}$$

Order of magnitude stronger than relativistic spin-orbit term.

T_1= 1s (τ=100ns)

Lateral confinement



All discrete levels; no k

Capillary waves - ripplons

Thermally excited height variations of the helium surface

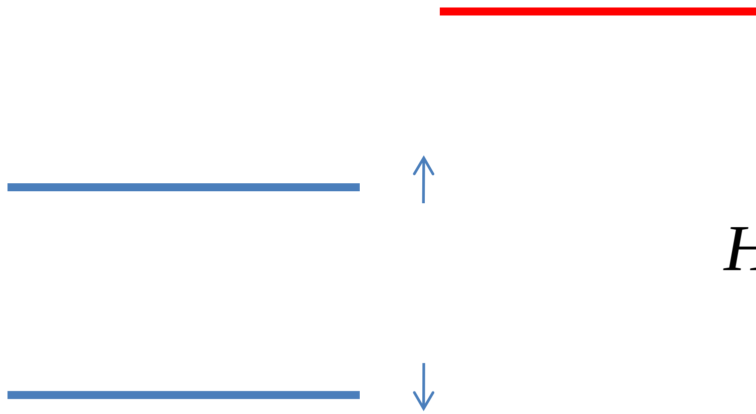
$$\delta(\vec{r},t)$$

$\vec{r}$ 2D in-plane coordinate

$$U(\vec{r}) = eE\delta(\vec{r},t)$$

Spectrum $\omega^2 = (\sigma / \rho)k^3$

Spin relaxation in a He surface quantum dot



$$H_{so} = \alpha \frac{\hbar\,(\sigma_x p_y - \sigma_y p_x)\, e\overline{E}_z}{4m \cdot ms^2}$$

Up and down spins are described by the same (blue) orbital wavefunction

Admixture by Rashba term to red state (different lateral orbital wavefunction

Ripplon

$T_1 = 10^6 s$

For inelastic [processes, acoustic phonons are more important

Other spin-orbit interactions

Capillary waves - ripplons

Thermally excited height variations of the helium surface

$$\delta(\vec{r},t)$$

$\vec{r}$ 2D in-plane coordinate

$$U(\vec{r}) = eE\delta(\vec{r},t)$$

Spectrum $\omega^2 = (\sigma/\rho)k^3$

$$H_{so} = \alpha \frac{\hbar\vec{\sigma}\cdot[\vec{p}\times\vec{\nabla}U]}{4m\cdot ms^2}$$

E – total electric field in z=direction

Spin-orbit interactions with ripplons

$ms^2\,I+U(\mathbf{r},t)$	$s\boldsymbol{\sigma}\cdot\mathbf{p}$
$s\boldsymbol{\sigma}\cdot\mathbf{p}$	$-ms^2\,I+\alpha U(\mathbf{r},t)$

$$H_{so} = \alpha e E \delta \frac{\hbar\vec{\sigma}\cdot[\vec{p}\times\vec{k}]}{4m\cdot ms^2}$$

Magnitude: δ is the average mean displacement

k is the transferred wavevector

Ripplon coupling limits mobility (μ= 10^8 cm^2 /V s)

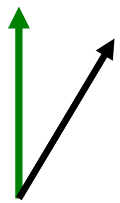
$$H_{so} = eE\delta \frac{\hbar\cdot\sigma_z[\vec{p}\times\vec{k}]_z}{4m\cdot ms^2}\alpha$$

Only in-plane wavevectors σ_z

Spin-orbit interactions with ripplons

$$H_{so} = \alpha e E \delta \frac{\hbar \cdot \sigma_z [\vec{p} \times \vec{k}]_z}{4m \cdot ms^2}$$

S_z is not affected , no spin relaxation



S_x , S_y are affected , hence spin dephasing (decoherence)

Spin dephasing times $0.1\alpha^{-2}$ s

Corresponds to momentum relaxation τ=100ns

Lateral confinement

$$H_{so} = \frac{\hbar \vec{\sigma} \cdot [\vec{p} \times \vec{\nabla} U]}{4m \cdot ms^2}$$

Direct spin-ripplon coupling to
Each of the excited level

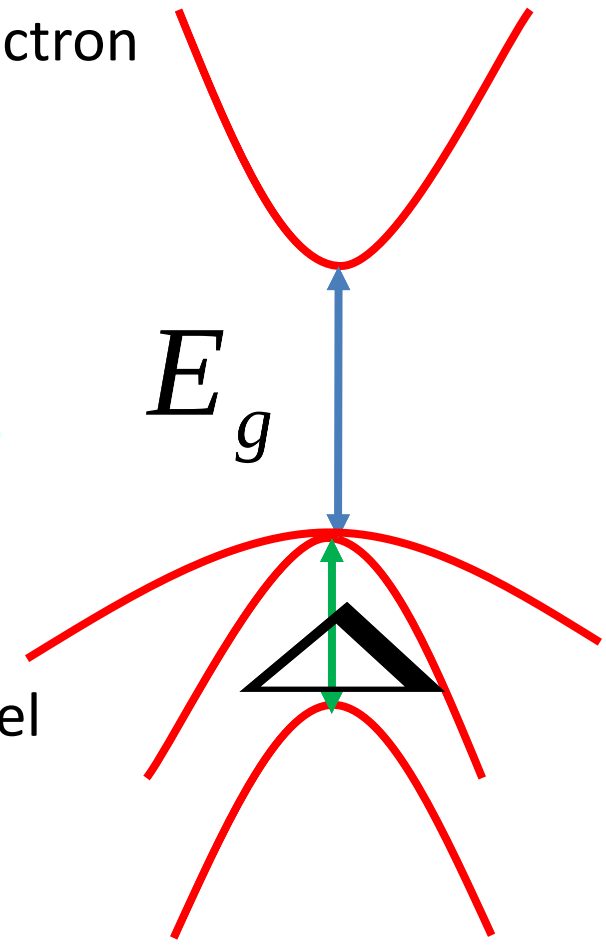
$T_2 = 10^2 s$

Virtual escape of electron from the dot

Other spin-orbit interactions

Solids : interaction of spin of one electron with an orbital motion of another electron

$$H_s^{(3)} = -\frac{e^2}{\kappa}\frac{2P^2}{3E_g^2}\frac{\Delta(2E_g+\Delta)}{(E_g+\Delta)^2} \times [(\mathbf{r}\times\hat{\mathbf{p}}_1)\cdot\mathbf{S}_1 - (\mathbf{r}\times\hat{\mathbf{p}}_2)\cdot\mathbf{S}_2]/\hbar^2 r^3.$$



Breit –Landau term in two-electron Dirac Model

Electron-electron Interactions contribute to relaxation and dephasing

Conclusions

- In 2D, Rashba interaction is present
- Coupling with ripplons is more important
- Spin dephasing times shorter than spin relaxation times
- Spin-orbit terms in electron-electron interactions