Introduction to Econometrics (4th Edition)

by

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Solutions to Odd-Numbered End-of-Chapter Exercises: Chapter 16

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16.1. (a) See the table below. β_i is the dynamic multiplier. With the 25% oil price jump, the predicted effect on output growth for the *i*th quarter is $25\beta_i$ percentage points.

Period ahead (i)	**Dynamic multiplier (β_i)**	**Predicted effect on output growth ($25\beta_i$)**	**95% confidence interval $25 \times [\beta_i \pm 1.96SE(\beta_i)]$**
0	-0.006	-0.15	[-0.787,0.487]
1	-0.014	-0.35	[-0.889,0.189]
2	-0.020	-0.5	[-0.990,-0.010]
3	-0.024	-0.6	[-1.041,-0.159]
4	-0.036	-0.9	[-1.488,-0.312]
5	-0.013	-0.325	[-0.668,0.018]
6	0.005	0.125	[-0.365,0.615]
7	-0.007	-0.175	[-0.567,0.217]
8	0.005	0.125	-[0.267,0.517]

(b) The 95% confidence interval for the predicted effect on output growth for the *i*'th quarter from the 25% oil price jump is $25 \times [\beta_i \pm 1.96SE(\beta_i)]$ percentage points. The confidence interval is reported in the table in (a).

(c) The predicted cumulative change in GDP growth over eight quarters is

$$25 \times (-0.006 - 0.014 \ldots + 0.005) = 25\times(-0.110) = -2.75\%.$$

(d) The 1% critical value for the *F*-test is 2.407. Since the HAC *F*-statistic 5.45 is larger than the critical value, we reject the null hypothesis that all the coefficients are zero at the 1% level.

16.3. The dynamic causal effects are for experiment A. The regression in exercise 16.1 does not control for interest rates, so that interest rates are assumed to evolve in their "normal pattern" given changes in oil prices.

16.5. Substituting

$$\begin{aligned} X_t &= \Delta X_t + X_{t-1} = \Delta X_t + \Delta X_{t-1} + X_{t-2} \\ &= \cdots \\ &= \Delta X_t + \Delta X_{t-1} + \cdots + \Delta X_{t-p+1} + X_{t-p} \end{aligned}$$

into Equation (16.4), we have

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + \beta_3 X_{t-2} + \cdots + \beta_{r+1} X_{t-r} + u_t \\ &= \beta_0 + \beta_1(\Delta X_t + \Delta X_{t-1} + \cdots + \Delta X_{t-r+1} + X_{t-r}) \\ &\quad + \beta_2(\Delta X_{t-1} + \cdots + \Delta X_{t-r+1} + X_{t-r}) \\ &\quad + \cdots + \beta_r(\Delta X_{t-r+1} + X_{t-r}) + \beta_{r+1} X_{t-r} + u_t \\ &= \beta_0 + \beta_1 \Delta X_t + (\beta_1 + \beta_2)\Delta X_{t-1} + (\beta_1 + \beta_2 + \beta_3)\Delta X_{t-2} \\ &\quad + \cdots + (\beta_1 + \beta_2 + \cdots + \beta_r)\Delta X_{t-r+1} \\ &\quad + (\beta_1 + \beta_2 + \cdots + \beta_r + \beta_{r+1}) X_{t-r} + u_t. \end{aligned}$$

Comparing the above equation to Equation (16.7), we see

$\delta_0 = \beta_0$, $\delta_1 = \beta_1$, $\delta_2 = \beta_1 + \beta_2$, $\delta_3 = \beta_1 + \beta_2 + \beta_3$, … , and $\delta_{r+1} = \beta_1 + \beta_2 + \cdots + \beta_{r+1}$.

16.7. Write $u_t = \sum_{i=0}^{\infty} \phi_1^i \tilde{u}_{t-i}$

(a) Because $E(\tilde{u}_i|X_t) = 0$ for all i and t, $E(u_i|X_t) = 0$ for all i and t, so that X_t is strictly exogenous.

(b) $X_t = \tilde{u}_{t+1}$. Note that $E(u_t \mid X_t) = E(u_t \mid \tilde{u}_{t+1}) = 0$. But $E(u_t \mid X_t, X_{t-1}, X_{t-2}, \ldots) = E(u_t| \tilde{u}_{t+1}, \tilde{u}_t, \tilde{u}_{t-1}, ..) = u_t$, so X_t is not exogenous (and therefore not strictly exogenous).

16.9. (a) This follows from the material around equation (3.2).

(b) Quasi differencing the equation yields $Y_t - \phi_1 Y_{t-1} = (1 - \phi_1)\beta_0 + u_t$, and the GLS estimator of $(1 - \phi_1)\beta_0$ is the mean of $Y_t - \phi_1 Y_{t-1} = \frac{1}{T-1}\sum_{t=2}^{T}(Y_t - \phi_1 Y_{t-1})$. Dividing by $(1-\phi_1)$ yields the GLS estimator of β_0.

(c) This is a rearrangement of the result in (b).

(d) Write $\hat{\beta}_0 = \frac{1}{T}\sum_{t=1}^{T} Y_t = \frac{1}{T}(Y_T + Y_1) + \frac{T-1}{T}\frac{1}{T-1}\sum_{t=2}^{T-1} Y_t$, so that $\hat{\beta}_0 - \hat{\beta}_0^{GLS} = \frac{1}{T}(Y_T + Y_1) - \frac{1}{T}\frac{1}{T-1}\sum_{t=2}^{T-1} Y_t - \frac{1}{1-\phi}\frac{1}{T-1}(Y_T - Y_1)$ and the variance is seen to be proportional to $\frac{1}{T^2}$.

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(a) Follows directly from multiplying the terms.

(b) If $|\phi| \geq 1$, the coefficients in b(L) do not converge to zero.