Introduction to Econometrics (4th Edition)

by

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Solutions to Odd-numbered End-of-Chapter Exercises: Chapter 4

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4.1. (a) The predicted average test score is

$$\widehat{TestScore} = 520.4 - 5.82 \times 22 = 392.36$$

(b) The predicted change in the classroom average test score is

The change is

$$\Delta\widehat{TestScore} = (-5.82 \times 23) - (-5.82 \times 19) = -23.28$$

that is, the regression predicts that test scores will fall by 23.28 points.

(c) Using the formula for $\hat{\beta}_0$ in Equation (4.8), we know the sample average of the test scores across the 100 classrooms is

$$\overline{TestScore} = \hat{\beta}_0 + \hat{\beta}_1 \times \overline{CS} = 520.4 - 5.82 \times 21.4 = 395.85.$$

(d) Use the formula for the standard error of the regression (SER) in Equation (4.19) to get the sum of squared residuals:

$$SSR = (n-2)SER^2 = (100-2) \times 11.5^2 = 12961.$$

Use the formula for R^2 in Equation (4.16) to get the total sum of squares:

$$TSS = \frac{SSR}{1-R^2} = \frac{12961}{1-0.08} = 14088.$$

The sample variance is $s_Y^2 = \frac{\text{TSS}}{n-1} = \frac{14088}{99} = 142.3.$ Thus, standard deviation is $s_Y = \sqrt{s_Y^2} = 11.9.$

4.3 (a) The coefficient 9.6 shows the marginal effect of *Age* on *AWE*; that is, *AWE* is expected to increase by \$9.6 for each additional year of age. 696.7 is the intercept of the regression line. It determines the overall level of the line.

(b) *SER* is in the same units as the dependent variable (*Y*, or *AWE* in this example). Thus *SER* is measures in dollars per week.

(c) R^2 is unit free.

(d) (i) $696.7 + 9.6 \times 25 = \$936.7$;

(ii) $696.7 + 9.6 \times 45 = \$1{,}128.7$

(e) No. The oldest worker in the sample is 65 years old. 99 years is far outside the range of the sample data.

(f) No. The distribution of earning is positively skewed and has kurtosis larger than the normal.

(g) $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}$, so that $\overline{Y} = \hat{\beta}_0 + \hat{\beta}_1 \overline{X}$. Thus the sample mean of *AWE* is 696.7 □ 9.6 × 41.6 □ \$1,096.06.

4.5. (a) u_i represents factors other than time that influence the student's performance on the exam including amount of time studying, aptitude for the material, and so forth. Some students will have studied more than average, other less; some students will have higher than average aptitude for the subject, others lower, and so forth.

(b) Because of random assignment u_i is independent of X_i. Since u_i represents deviations from average $E(u_i) = 0$. Because u and X are independent $E(u_i|X_i) = E(u_i) = 0$.

(c) (2) is satisfied if this year's class is typical of other classes, that is, students in this year's class can be viewed as random draws from the population of students that enroll in the class. (3) is satisfied because $0 \leq Y_i \leq 100$ and X_i can take on only two values (90 and 120).

(d) (i) $49 + 0.24 \times 90 = 70.6$; $49 + 0.24 \times 120 = 77.8$; $49 + 0.24 \times 150 = 85.0$

(ii) $0.24 \times 10 = 2.4$.

4.7. The expectation of $\hat{\beta}_0$ is obtained by taking expectations of both sides of Equation (4.8):

$$E(\hat{\beta}_0) = E(\bar{Y} - \hat{\beta}_1\bar{X}) = E\left[\left(\beta_0 + \beta_1\bar{X} + \frac{1}{n}\sum_{i=1}^{n} u_i\right) - \hat{\beta}_1\bar{X}\right]$$

$$= \beta_0 + E(\beta_1 - \hat{\beta}_1)\bar{X} + \frac{1}{n}\sum_{i=1}^{n} E(u_i)$$

$$= \beta_0$$

where the third equality in the above equation has used the facts that $E(u_i) = 0$ and $E[(\hat{\beta}_1 - \beta_1)\bar{X}] = E[\bar{X} \times (E(\hat{\beta}_1 - \beta_1) | \bar{X})] = 0$ because $E[(\beta_1 - \hat{\beta}_1) | \bar{X}] = 0$ (see text equation (4.31).)

4.9. (a) With $\hat{\beta}_1 = 0, \hat{\beta}_0 = \bar{Y}$, and $\hat{Y}_i = \hat{\beta}_0 = \bar{Y}$. Thus ESS = 0 and $R^2 = 0$.

(b) If $R^2 = 0$, then ESS = 0, so that $\hat{Y}_i = \bar{Y}$ for all i. But, using the formula for $\hat{\beta}_0$, $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = \bar{Y} - \hat{\beta}_1(X_i - \bar{X})$, Thus $\hat{Y}_i = \bar{Y}$ for all i implies that $\hat{\beta}_1 = 0$, or that X_i is constant for all i. If X_i is constant for all i, then $\sum_{i-1}^{n}(X_i - \bar{X})^2 = 0$ and $\hat{\beta}_1$ is undefined (see equation (4.7)).

4.11. (a) The least squares objective function is $\sum_{i=1}^{n}(Y_i - b_1X_i)^2$. Differentiating with respect to b_1 yields $\frac{\partial \sum_{i=1}^{n}(Y_i - b_1X_i)^2}{\partial b_1} = -2\sum_{i=1}^{n} X_i(Y_i - b_1X_i).$ Setting this zero, and solving for the least squares estimator yields $\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_iY_i}{\sum_{i=1}^{n} X_i^2}$.

(b) Following the same steps in (a) yields $\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_i(Y_i - 4)}{\sum_{i=1}^{n} X_i^2}$

4.13. The answer follows the derivations in Appendix 4.3 in "Large-Sample Normal Distribution of the OLS Estimator." In particular, the expression for ν_i is now $\nu_i = (X_i - \mu_X)\kappa u_i$, so that $\text{var}(\nu_i) = \kappa^3\text{var}[(X_i - \mu_X)u_i]$, and the term κ^2 carry through the rest of the calculations.

4.15

(a.i) Because (Y^{oos}, X^{oos}) are from some population as the in-sample observation,

$$E(Y^{oos}|X^{oos} = x^{oos}) = E(Y|X = x^{oos}) = \beta_0 + \beta_1 x^{oos}.$$

(a.ii)

$$\begin{aligned} &E(\hat{Y}^{oos} \mid X^{oos} = x^{oos}) = E(\hat{\beta}_0 + \hat{\beta}_1 x^{oos} \mid X^{oos} = x^{oos}) \\ &= E(\hat{\beta}_0 + \hat{\beta}_1 x^{oos}) = E(\hat{\beta}_0) + E(\hat{\beta}_1) x^{oos} \\ &= \beta_0 + \beta_1 x^{oos} \end{aligned}$$

where the second line follows because X^{oos} is independent of $(\hat{\beta}_0, \hat{\beta}_1)$ because they depend only on the in-sample observations.

(a.iii) From appendix 4.4:

$$\hat{u}^{oos} = u^{oos} - \left[\left(\hat{\beta}_0 - \beta_0\right) + \left(\hat{\beta}_1 - \beta_1\right) x^{oos}\right]$$

and the result follows by noting that the two terms on the right hand side of the equation are independent, and therefore uncorrelated with each other.

(b.i) No. Because (Y^{oos}, X^{oos}) are drawn from distribution different that the sample data, then in general $E(Y^{oos}|X^{oos} = x^{oos}) \neq E(Y|X = x^{oos}) = \beta_0 + \beta_1 x^{oos}$.

(b.ii) Yes. Same reason as (a.ii).