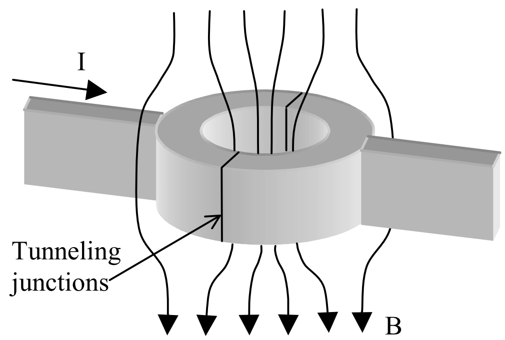
PHYS 210
Spring 2006
Condensed Matter Physics Experimental Techniques Part II

Quantum interference in superconductors

In the last lab we observed the superconducting phase transition by making resistance measurements as a function of temperature. However, the phenomenon of superconductivity is much richer. Today we will study purely quantum effects in a superconductor using a Superconducting Quantum Interference Device (SQUID). A proper explanation of these effects requires a fairly advanced knowledge of quantum mechanics. Here I will give only a hand-waving explanation.

a) Meissner Effect

This effect refers to the fact that a superconductor expels all magnetic fields from its interior volume. It is easy to see that the magnetic field inside a superconductor has to be constant. If the magnetic field is changing it will induce an EMF by Faraday's law, $\varepsilon = -d\Phi/dt$. Since the resistance of the superconductor is zero, any finite EMF can induce an infinite current that will generate a magnetic field opposing the initial change. As a result, the superconductor will always have a persistent current such that the magnetic field remains constant and $d\Phi/dt = 0$. It turns out that the magnetic field is not only constant but is exactly equal to zero inside a Type I superconductor. Thus, flux lines go around a superconductor as shown in the figure.



b) Interaction of electrons with magnetic field

In classical mechanics the interaction of electrons with magnetic field is given by the Lorentz force $F = -e\mathbf{v}\times\mathbf{B}/c$. In quantum mechanics the magnetic field interaction is included in the Hamiltonian through the vector potential $\mathbf{A}$ related to the magnetic field $\mathbf{B}$ by $\mathbf{B} = \nabla\times\mathbf{A}$,

$$H = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla + \frac{e}{c}\mathbf{A}\right)^2 + V$$

Note that even if the magnetic field is equal to zero, $\mathbf{A}$ does not have to be equal zero. For example, if $\mathbf{A}$ is equal to a gradient of a function $\mathbf{A} = \nabla f(\mathbf{r})$, $\mathbf{B} = \nabla\times\nabla f = 0$. For such choice of $\mathbf{A}$ it can be shown that the eigenstate of the Schrodinger equation $H\psi = E\psi$ is

$$\psi = e^{-(ie/\hbar c)f(r)}\psi_0$$

where ψ_0 is the eigenstate for $\mathbf{A} = 0$. Thus, the presence of a non-zero vector potential $\mathbf{A}$ with a zero magnetic field only changes the phase of the wavefunction, which is not directly observable in most cases.

However, consider electrons propagating around a ring. The phase of the wavefunction accumulated in going around the ring is

$$\Delta\phi = (e/\hbar c)f = (e/\hbar c)\oint\mathbf{A}(\mathbf{r})\cdot d\mathbf{r} = (e/\hbar c)\int_S \nabla\times\mathbf{A}\cdot d\mathbf{S} = (e/\hbar c)\int_S B\cdot d\mathbf{S} = (e/\hbar c)\Phi$$

If the wavefunction is to be single-valued, this phase has to be a multiple of 2π. Thus we conclude that the magnetic field flux through a closed path in zero magnetic field must be given by $\Phi = 2\pi\hbar cn/e$, where n is an integer.

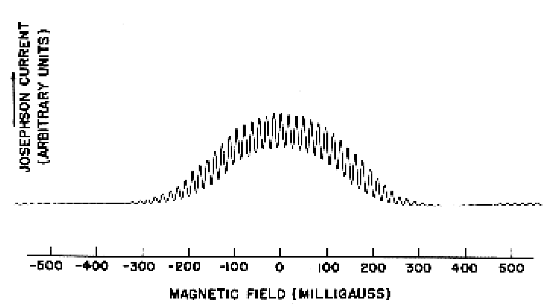
The effect of flux quantization can be observed in a superconducting ring shown above. The magnetic field inside the ring is equal to zero and all electrons are described by a common wavefunction since they are in a Bose-Einstein condensate. Measurements of the magnetic field flux through the ring showed that it is indeed quantized, but a single flux quantum is equal to

$$\Phi_0 = \frac{2\pi\hbar c}{2e}$$

These measurements confirm that superconducting phenomena are associated with Cooper pairs of electrons with a total charge equal to $2e$.

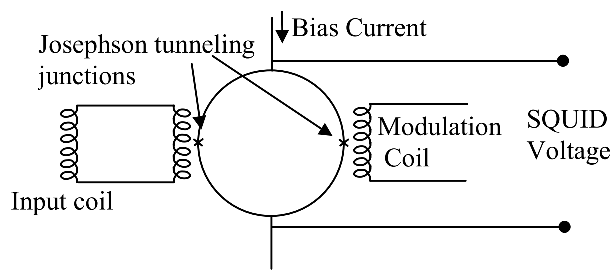
c) Quantum Tunneling

Even more interesting effects can be observed if the superconducting ring is split into two halves by *tunneling junctions*. A tunneling junction is made by inserting a very thin layer of an insulating material between two superconductors. The insulator presents an energy barrier for the flow of Cooper pairs. Even though the barrier is higher than the energy of the electrons, they can tunnel through it quantum-mechanically with a certain probability. Any amount of magnetic flux can go through the superconducting ring if it is broken by tunneling junctions. However, if the flux is equal to an integer number of flux quanta, the phase of the Cooper pair wavefunction is the same on both sides of the junction and that increases the tunneling probability. As a result, the current flowing through the SQUID is a periodic function of the magnetic field, as shown on the right.



d) SQUID circuit

We will use a commercial SQUID sensor whose circuit is shown below. The SQUID is made from Nb which superconducts below 9 K. The tunneling barrier is made from Al oxide. The SQUID is commonly used in liquid He at 4.2 K. The resistance of the SQUID is measured using a 4-wire measurement. The magnetic fields are coupled to the SQUID using two inductive superconducting coils. The SQUID can be used to measure the magnetic field going through the input coil. Such magnetometers are extremely sensitive and find a wide range of applications. SQUIDs can also be used to construct a very sensitive voltmeter or ampermeter.



The output of the SQUID is a periodic function of the magnetic field. To make unique measurements SQUIDs are usually used in a feedback mode where they are locked to one of the outout peaks. The modulation coil is used to generate a feedback signal that keeps the SQUID always locked to the same peak. The current in the modulation coil is then proportional to the external magnetic field.