

Signal Recovery Techniques

PHYS 210

In the last lab you experimented with generating sounds using an audio card and recording it with a microphone. You measured the response of the audio system as you stepped through a series of frequencies. You might have noticed that the resulting data were fairly noisy and easily affected by the ambient noise. In this lab we will learn several more sophisticated measurement techniques that will allow you to separate the signal from the noise. These techniques are used in a wide variety of physics experiments.

The general idea is to use as much prior information about the signal as possible. In our experiments with the sound system we generated a beep of a known frequency and measured the amplitude of the response. In the first experiment we simply calculated the root mean square of the microphone data, which did not take into account the fact that the signal had a single specific frequency. The r.m.s. was equally affected by the noise in the data at all frequencies. We want now to use the prior information about the frequency of the signal to extract it from the noise.

1. Filters

The simplest way to select the signal at a specific frequency is to use a frequency filter. Frequency filters for electrical signals can be easily constructed using simple electronic components, like resistors and capacitors. They can also be implemented in the software at the data analysis stage.

Filters can be divided into low-pass, high-pass, band-pass and band-stop types. As the names imply, low pass filters allow all frequencies below a certain cut-off frequency to pass through, high-pass filters pass all frequencies above a cut-off frequency. Band-pass filters pass signals in a certain frequency band and band-stop filters reject signals in a particular band, allowing all others to pass.

Filters are never perfect. That is, they can not pass signals in one frequency region undistorted and completely reject signals at other frequencies. The transition from the pass to the reject region is usually gradual. Filters can also introduce other undesirable effects, such as a delay and distortion of the signals.

Filters are useful for rejecting noise that is far away in frequency from the signal of interest. Filters are easy to construct and they work in real time, giving a filtered signal without the need for numerical analysis.

2. Fourier Transforms

Fourier transform is a mathematical operation that separates signals into different frequency components. If $S(t)$ is the signal as a function of time, then the Fourier transform is given by $F(\omega) = \int S(t)e^{-i\omega t} dt$. The Fourier transform can also be performed on a discrete set of data points by replacing the integral with a sum. If the number of data points is equal to an integer power of 2, the transform is numerically efficient and is called a Fast Fourier Transform (FFT). Since the Fourier transform is a complex number, one often looks at the Fourier Transform Power given by $F(\omega)F^*(\omega)$, which is proportional to the power of an electrical signal S as a function of frequency.

Fourier transforms are convenient for looking at multiple frequency components contained in a signal. They also allow one to measure the strength of the signal at a particular frequency and the noise in the signal as a function of frequency. One can also implement a filter by multiplying the Fourier transform by a certain function of frequency and then performing an inverse Fourier transform. Fourier filters have better properties than real-time filters, but they require numerical analysis and cannot be implemented in real time.

The conversion of analog signals to digital form is usually performed by integrated circuits called A/D converters. The A/D conversion is characterized by the sampling rate and the digital resolution in number of bits. If the signal is sampled at a rate equal to R samples/sec, one can show that the digital form will accurately represent signals with frequencies below $f_c = R/2$. Signals with a frequency f above f_c will appear at a frequency $2f_c - f$. This phenomenon is called aliasing. To avoid this problem, A/D conversion is usually preceded by an analog low-pass filter with a cut-off frequency below f_c . The digital resolution determines the minimum change in the signal that can be recorded. For example, a 12-bit A/D converter has a resolution of 1 part in 4096. Appearance of discrete steps in the recorded signal is an indication that the digital resolution is not sufficient.

3. Lock-in Amplifier

Lock-in amplifier is a device that “locks-in” on a signal of a particular frequency and amplifies it, rejecting all other frequencies. It was invented by Bob Dicke, a professor at Princeton, in 1946. To illustrate its operation, let's assume that the signal contains two frequencies, $S(t) = A_1 \sin(\omega_1 t + \phi) + A_2 \sin(\omega_2 t)$. In the lock-in amplifier the signal is multiplied by a reference function, $R(t) = \sin(\omega_r t)$ and the result is filtered using a low-pass filter. With simple trigonometric identities one can show that

$$X(t) = S(t)R(t) = A_1(\cos[(\omega_1 - \omega_r)t + \phi] - \cos[(\omega_1 + \omega_r)t + \phi])/2 + A_2(\cos[(\omega_2 - \omega_r)t + \phi] - \cos[(\omega_2 + \omega_r)t + \phi])/2$$

If the reference frequency ω_r is close to ω_1 , then $\omega_1 - \omega_r$ is a low frequency that will pass through the low-pass filter, while the rest of the terms will be rejected by the filter. In this way the lock-in can select the frequencies close to the reference frequency. If $\omega_1 = \omega_r$ and $\phi = 0$, then the lock-in simply measures the amplitude of the signal at ω_1 . If the two frequencies are slightly different, it measures the beats between them.

One can also determine the phase of the signal relative to the reference. If we multiply the signal by a reference function $R_1(t) = \cos(\omega_r t)$ then we obtain $Y(t) = A_1 \sin[(\omega_1 - \omega_r)t + \phi]/2 +$ (high frequency terms). If $\omega_1 = \omega_r$, then $Y/X = \tan(\phi)$. Sometimes it's also convenient to look at $R = \sqrt{X^2 + Y^2}$, which does not depend on the phase of the signal.

The cut-off frequency of the low-pass filter determines the selectivity of the amplifier. It's more common to refer to the time constant of the lock-in amplifier $\tau = 1/(2\pi f_c)$. A long time constant (a small cut-off frequency) is better for rejecting the noise but slows down the response of the lock-in to changes in the signal. A small time constant (high cut-off frequency) increases the speed of the response but also makes the signal noisier.

Lock-in amplifiers can be constructed using analog components, but it's a fairly complicated piece of equipment. However, it is easy to implement a lock-in in the computer by performing

the mathematical operations discussed above. Lock-in amplifiers are often used in combination with modulation techniques to detect a very weak signal. The idea is to modulate a small signal on and off at a known frequency and then detect the resulting response with a lock-in referenced to the same frequency.

4. Laboratory Instrumentation

Most signal measurements in a research lab can be performed with a few standard pieces of equipment:

Oscilloscope: Commonly used for diagnosis of signals and electronic circuits. Most modern scopes also have the ability to digitize the signal and transfer it to a computer. Oscilloscopes usually sample signals at a high rate, 10^9 samples/sec or higher, but the digital resolution is not very high, usually just 8 bits (1 part in 256). They are appropriate for recording of fast signals.

Multi-meters: Commonly used for measurements of DC voltage, current, resistance. Some multi-meters can be interfaced with a computer and can record signals with very high precision (1 part in 10^6 or even higher). They are usually not very fast and record only a few samples/sec.

Function generators: Used for generating voltage waveforms of various types. Most generators can generate sine, square, triangular and maybe a few other types of waveforms. The frequency, amplitude, and DC offset of the waveforms can be easily adjusted.

Lock-in amplifiers: Used for measurements of small signals using the technique discussed above. Most lock-in amplifiers operate in the range from 1 Hz to 100 KHz and have nanovolt sensitivity.

Pre-amplifiers: Used for amplifying small DC or AC signals for detection. Often include analog filters to reject unwanted signals at other frequencies.

Power supplies: A variety of power supplies is used in the lab to power electronics or generate large currents for magnetic field coils and other pieces of equipment.

Computer control: Computers are now widely used to acquire signals, output voltages and control other instruments. Plug-in cards can be inserted into regular PCs to perform A/D and D/A conversions. These cards usually have 12 bit or 16 bit resolution and can sample signals at a rate on the order of 10^6 samples/sec. Other pieces of lab equipment can often be computer controlled through so-called GPIB protocol, using a special plug-in card. Some equipment can now be also controlled using USB or Ethernet connections.

5. Audio lock-in amplifier

You have on your computer a lock-in amplifier program implemented in LabView. The program takes the input from the microphone and generates a single frequency tone with the speakers. It allows you to see the microphone signal in real time and to experiment with filters, Fourier transforms, and lock-in detection.

Here is a list of suggested experiments to perform with the lock-in program. Feel free to experiment on your own as well.

1. Signal filtering.

With the filter turned on, look at the real time window display and the Fourier transform display. Adjust the value of the cut-off frequency and note how it affects the signal, its r.m.s,

and the frequency spectrum of the signal. You can explore different types of filters and the filter order. A higher filter order gives a sharper cut-off of the frequencies outside of the pass band.

2. Fourier Transform.

With the speaker turned on, observe the peaks in the Fourier spectrum at the beep frequency and its harmonics. Change the shape of the beep waveform to see higher harmonics. Speak into the microphone to see the frequency spectrum of speech.

3. Lock-in Amplifier.

Set the lock-in amplifier reference frequency close to the beep frequency. Observe the behavior of the X, Y, R and ϕ signals for different values of the reference frequency and the time constant τ . Find a value of the reference frequency that is exactly equal to the beep frequency and gives a constant ϕ .

4. Doppler Effect.

Move the microphone relative to the speaker and observe changes in the phase of the signal. Study this effect as a function of the beep frequency (don't forget to change the reference frequency as well). Try to explain what you see in terms of the Doppler effect. Make a rough measurement of the velocity of sound by moving the microphone a known distance.

5. Limits of Signal Detectability

Find a minimum volume of the sound at which the lock-in amplifier can still reliably detect the beep tone. How does it depend on the time constant of the lock-in? Is it more or less sensitive than your ears?

6. Signal Detection in a Noisy Environment

Get together with another group using a computer on the other end of the room and agree on a certain frequency for communication. You can communicate with each other by turning the beep on and off, using Morse code for example. Two-way communication is also possible using different frequencies for beep and reference. You may need to compensate for the fact that the clocks on the two sound cards run at slightly different rates.

7. Mapping the response of the sound system.

Repeat the measurements of the sound system response done in the last lab by stepping through a range of frequencies. You will have to add a few things to the LabView program to automatically step through the frequencies and plot the response. You can plot different ways to measure the signal: r.m.s value, r.m.s. after band-pass filtering, peak of the Fourier transform, and lock-in signal output. Find which type of measurement gives you the best signal/noise ratio. Try to perform a "silent" measurement by setting the volume so low that you can't hear it, yet the computer can still reliably detect the amplitude of the signal.