Coulomb's Law Lab Instructions

1 Introduction

The goal of this experiment is to test the validity of the Coulomb force law $F(r) = A/r^2$ between two charges. It servers as an example of a powerful "null measurement" technique for constraining hypothetical deviations from known laws of physics and introduces a number of experimental techniques used in precision measurements.

A general central force law which has no preferred distance scale is one with a radial force whose magnitude is given by $F(r) = A/r^{\alpha}$. Empirically one can ask if α is exactly equal to 2 or perhaps has a slight deviation: $\alpha = 2 + \epsilon$. Coulomb was the first to show that $|\epsilon| < 1$ using a torsion balance. Later Cavendish and Maxwell constrained $|\epsilon|$ to be less than 0.02 and 5×10^{-5} respectively. However, if one looks deeper into electromagnetic theory, it is easy to show that α being equal to 2 is a simple consequence of Gauss law and the fact that we live in 3 spatial dimensions. It may then appear pointless to test for small deviations in the power law. But in the case of gravitational interactions, which also have the same power dependence, such tests at short distances are being actively pursued because there are suggestions from string theory that gravitons live in a different number of dimensions at short distances.

Another way to formulate the problem, more applicable to electromagnetic interactions, is to ask if the photon is really massless. If it had a small but finite mass μ, Maxwell equations would be modified (known as Proca equations) and give for the Coulomb force

$$F(r) = \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\mu c}{\hbar r}\right) e^{-\frac{\mu c r}{\hbar}}. \tag{1}$$

Hence for a length scale on the order of the Compton wavelength of the photon $\lambda = \hbar/\mu c$ the Coulomb force would deviate from $1/r^2$ law and for $r \gg \lambda$ it would be dramatically suppressed. Since macroscopic objects are usually electrically neutral, its not obvious from everyday observations that λ is very large. (In contract, it is clear even from planetary motion that corresponding Compton wavelength of the graviton has to be large on astronomical scale). A modern way of interpreting the Coulomb force experiment is to set a limit on the mass of the photon since there are various theoretical proposals that couple photons to other hypothetical particles in a way that induces a small photon mass.

Most laboratory tests of the Coulomb law since Cavendish rely on well-known fact that the electric field inside a charged conductor is equal to zero, see Figure 1. However, it would not be the case if $\alpha \neq 2$ or $\mu \neq 0$. Most experiments use two concentric conductive spheres, the outer sphere is strongly charged and the potential difference between the inner and outer spheres is measured with high sensitivity. One can show that this potential difference is proportional to ϵ or μ. Hence we have a "null experiment" in the sense that if $\alpha = 2$ the measured signal is equal to zero. It is much easier to measure a small signal proportional to ϵ than something that depends on $2 + \epsilon$ because a whole class of experimental uncertainties associated with the absolute scale of the apparatus is eliminated.

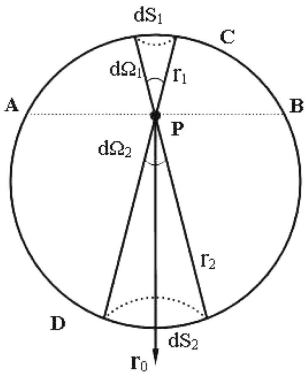

Figure 1: Consider forces on an electric charge P placed inside a charged sphere. The forces acting on the charge from areas dS_1 and dS_2 will cancel if the Coulomb force drops off exactly as $1/r^2$. (figure from Tu and Luo).

2 Basic Experimental Setup

The basic setup of this experiment is similar to that of Bartlett *et al* and Williams *et al.* Two concentric spheres reside in a shielded room. A large AC voltage (∼10 kV, 600 Hz) is applied to the outer sphere. As the experiment is a null experiment one can improve the sensitivity by amplifying the signal while reducing background noise as much as possible. We use a lock-in amplifier which measures the signals with the same frequency as the excitation voltage on the outer sphere. The reference signal for the lock-in amplifier is synchronized with the high voltage oscillation and is transmitted into the spheres using an analog fiber-optic link. Finally, the output of the lock-in amplifier is measured with a digital voltmeter and sent out of the sphere over another fiber-optic link using GPIB communication protocol.

3 Warnings

DURING OPERATION, THE SPHERE IS CHARGED TO A LETHAL VOLTAGE! The door has a built-in switching mechanism which only allows application of high voltage when the door is closed. DON'T BE IN THE ROOM WHEN THE DOOR IS CLOSED. Also, switches can and do stick on occasion. You should always monitor the voltage between the transformer terminals on the oscilloscope using the high voltage probe, and always turn off the high voltage before entering.

When entering the room after running the experiment you should:

1. Shut off the high voltage. This can be done easily by using the standby or power switches on the amplifier.
2. Check that the high voltage is off by looking at the signal on the high voltage probe.
3. *Only then* open the door.

4. Ground the sphere using the metal braid on the insulated wooden grounding rod. Don't touch the braid while doing this — it is not a resistance-less path to ground.
5. Remove the charging wire and plug it into the socket in the wall. Try to touch only the insulation.
6. Ground the sphere again.

CAR BATTERY USED INSIDE THE SPHERES CAN PRODUCE VERY LARGE CURRENTS. Accidental shorting of car battery terminals will result in huge sparks. When connecting the battery to the inverter or charger

1. Be careful to double check proper polarity: red - plus, black - minus.
2. Keep metal objects away from battery terminals

When done running the experiment for the day, open the spheres, turn off the inverter and disconnect the battery to avoid draining it.

4 Apparatus Description

Three distinct tasks must be performed in this experiment: 1) A high voltage must be generated, 2) A small voltage between the spheres must be measured, 3) The measured voltage must be transmitted out of the spheres, displayed and logged. Task (1) is performed by a group of equipment outside the spheres. A Wavetek signal generator produces a sine wave at a specified frequency. This signal is fed into a Marshall audio amplifier, which has a variable gain. The output of the Wavetek is also used as a reference for the lock-in amplifier after passing through a phase shifter (more on this later). The output from the Marshall amplifier is fed into the shielded room. It goes through a door interlock switch, which opens the circuit (stopping the current) when the door is open - this is a safety measure. The voltage then goes to a transformer, which steps it up to a lethal level of about 10 kV. One terminal of the transformer is connected to the outer sphere and the other terminal is connected to the wall of the room - forming a circuit with the room-sphere system acting as a capacitor. The output of the transformer is also connected to a 1:1000 voltage probe, the output of which is fed back out of the room to be observed by an oscilloscope (this is how we can measure the amplitude of the high voltage being applied to the sphere). The system outside the sphere is shown in Figure 2.

Task (2) is performed by a set of equipment located inside the inner sphere (and hence it is shielded from the effects of the high voltage). The equipment is powered by a car battery and a power inverter, which converts the 12V DC voltage from the battery to a standard household AC signal. The voltage between the spheres is measured from a BNC connector whose shield is connected to the inner sphere and the central conductor connected to the outer sphere. This signal is fed into an SRS preamplifier, which can amplify the signal by a a large factor and apply a band-pass filter with adjustable cut-off frequencies. The preamp output is connected to a lock-in amplifier which further filters out noise by only measuring waveforms that match a reference frequency. In our case, the reference is provided

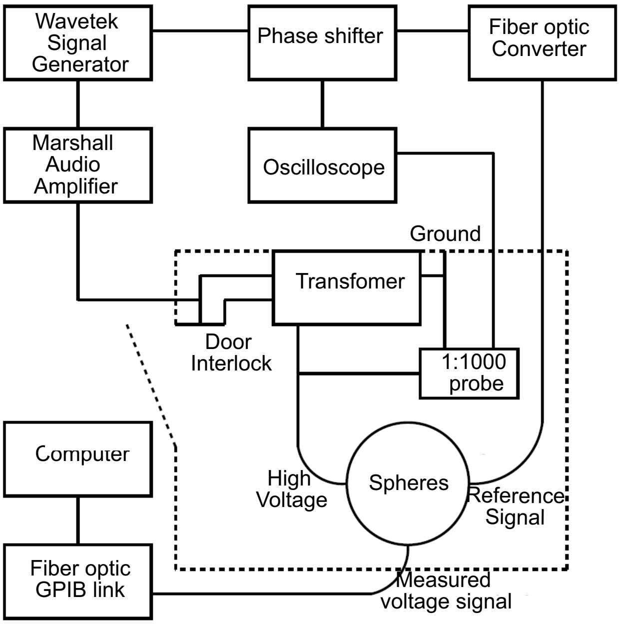

Figure 2: Experimental apparatus outside the spheres

by the output of the Wavetek generator, so that only a signal matching the frequency of the high voltage is detected (a signal voltage resulting from a violation of Coulombs law will alternate following the same waveform as the applied voltage). The lock-in produces a DC output that is proportional to the amplitude of the input AC signal at the reference frequency. This output is connected to a multimeter, which measures the DC voltage. Figure 3 shows the equipment inside the inner sphere.

Task (3) is performed by a computer which resides outside the spheres. The signal from the multimeter is transmitted to the computer, which records and displays the data. Communication between the systems inside and outside the spheres is accomplished by fiber optic cables, which are fed through small holes in both spheres. Fiber optic converters are required to translate analog signals (from the Wavetek generator) and communicate with the multimeter using a "GPIB" protocol.

5 Operating Instructions

5.1 Oscillator Setup

The (Wavetek) oscillator signal is used to drive the amplifier and to send a reference signal to the lock-in through the fiber-optic link. The resonant frequency of the transformer-sphere system is around 600 Hz. However, if the frequency of the driving signal is too close to harmonics of the power inverter frequency the lock-in will pick up noise from it, so you should choose a frequency between multiples of the fundamental frequency. The output of the Wavetek should be fed into the Marshall amp. It is also used to generate the reference signal, so the amplitude should be set to maximum. You can use the gain of Marshall

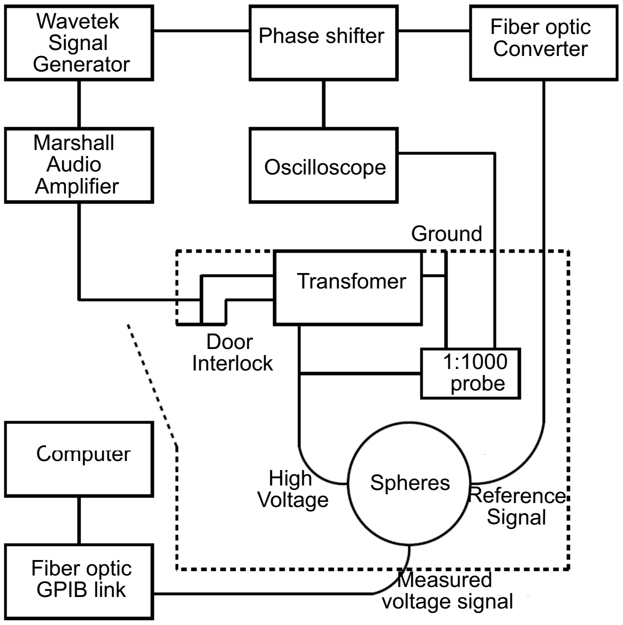

Figure 3: Experimental apparatus inside the spheres.

amplifier to adjust high voltage amplitude. The reference must be in phase with the input signal in order to maximize the output of the lock-in. The phase shift is generated by a third order passive filter and can be adjusted using the phase shift potentiometer.

5.2 Power Up Instruments inside the sphere

Inside the sphere are the following components:

- Car Battery
- Power inverter to generate 120 V
- Analog fiber-optic receiver (LuxLink INSR-3001) to receive reference signal
- Lock-in Amplifier (ThorLabs Model LIA100)
- Digital fiber-optic extender (GPIB-140) to communicate with the multimeter using GPIB
- Preamplifier (SRS Model SR560)
- Multimeter (HP 34401-A)

All the instruments inside the sphere are driven by a 120V AC signal generated by the inverter, which is hooked up to a car battery. If the battery is undercharged then the inverter will not produce a voltage. If this happens you will have to charge the battery using the battery charger. There are two batteries so you can always keep one charging. *Check the polarity before attaching the leads to the battery.* If the battery gets low the inverter will start beeping.

5.3 Computer Setup

Inside the Coulombs Law folder on the desktop you will find two LabVIEW files entitled Run Experiment and Test AC. These take, respectively, DC and AC readings from the multimeter. After making sure that fiber-optic extenders both inside and outside the sphere are turned on, run one of these programs (right-pointing arrow in the toolbar) and check to see if the computer is communicating with the multimeter. If it is communicating, the multimeter will begin taking data at the rate specified in the interface, and new readings will show up on the multimeter display at the same rate. If the multimeter remains on autotrigger mode and the program displays a steady signal of 0, try stopping the program, turning the multimeter and fiber-optic extenders off and on again, and then run the program once more. When the LabVIEW program is running successfully, the readings from the multimeter should be graphed on the screen. Note that Run Experiment displays its readings in millivolts. To begin storing data to a file, click Start Acquisition, and to stop, Stop Acquisition. The stored data is stored as read from the multimeter. It is in two columns; the first column is the time (in milliseconds) since the program was started; the second is the reading taken off the multimeter.

5.4 Background Measurement

Because there will be a signal due to a variety of factors such as the leakage of the reference signal and the penetration of magnetic fields into the outer sphere, this background signal should be subtracted out. To make a measurement of the background signal, use the dummy load, which is a capacitor with roughly the same capacitance as the sphere-room system. It is to be connected to the high voltage terminal on top of the outer sphere. The high voltage signal is then connected to the top of the dummy load. The sphere must be grounded when using the dummy load. Grounding the sphere completes the circuit with the transformer (since the negative terminal of the transformer is also connected to ground). This system simulates the current flow on the sphere, without actually putting a voltage on the sphere. Take two runs with the dummy load connected: one with the sphere grounded at the top (there are terminals on top of the sphere and in the rooms ceiling) and one using the grounding clip at the bottom of the sphere. The average of these approximates the current flow during actual measurements.

5.5 Calibration

In order to actually place bounds on the accuracy of Coulombs law, one must understand how much of a signal the apparatus would see if a signal were present; that is, how large the signal-to-high-voltage ratio v/V must be in order to be detectable by the equipment. To do this, feed a small but known signal into the preamplifier using the 1:1000 high voltage probe (with the spheres open) to find the relation between the applied voltage and the measured signal. It is important to check the calibration of the probe. The phase of the reference must be readjusted since the phase shift without using the high voltage transformer would be different. You can also calculate the conversion factor from known amplification factors of the pre-amp and the lock-in.

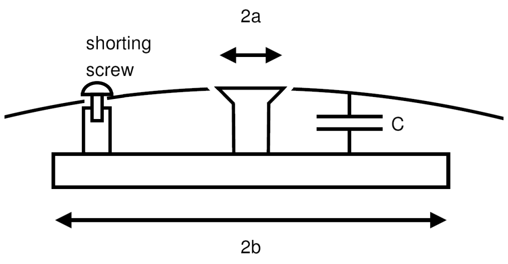

Figure 4: Schematic of the built-in calibration system

To calculate how a voltage on the sphere would be measured up by the equipment one should also take into account the input impedance of the preamplifier (100 MΩ), the capacitances of the input BNC cable (30 pF/foot) and the preamplifier (25 pF). The capacitance between the spheres is 280 pF which is the impedance of the voltage source.

In order to confirm your predictions for the equipment calibration it is important to measure a known signal and compare it with your predictions. The experiment has a calibration system built into it, consisting of a small metal plate suspended just inside the outer sphere by a screw, see Figure 4. Electrostatics tells us that the electric field is continuous across the screw-sphere join, so the surface charge on the screw must be the same as the surface charge on the sphere.

Between the plate and the sphere is a capacitor C, whose capacitance is 1080 pF. If R is the "radius" of the room and r the radius of the outer sphere, the charge on the outer sphere when it is at a given voltage V is

$$Q = \frac{rR}{R-r} V \, .$$

Thus, when the sphere holds a charge Q, the charge on the capacitor is

$$q = \frac{\pi a^2}{4\pi r^2}$$

where a is the radius of the screw head. The voltage across the capacitor, and so the voltage between the plate and the outer sphere, is given by

$$V_b = \frac{q}{C} = \frac{a^2 R}{4r(R-r)} \frac{V}{C} \, .$$

Finally the voltage at the sphere's center — and thus the voltage on the inner sphere — is found by averaging the potential over the outer sphere. Taking the reference voltage to be that on the outer sphere, this gives a signal of

$$v = \frac{\pi b^2}{4\pi r^2} = \frac{a^2 b^2 R}{16 r^3 (R-r)} \frac{V}{C} \, .$$

To find the expected calibration signal, all that is required is to measure/estimate the values of the lengths in this formula. (Be careful of the units!)

Calibration is done by removing the screw that shorts the capacitor (it is labeled). The calibration signal is read by turning on the high voltage and then adjusting the phase shift until the maximum signal is found.

5.6 Running the Experiment

As this is a null experiment, it is important to amplify the signals as much as possible. However, too high an amplification will saturate the output of the preamplifier or the input of the lock-in. Ensure this does not happen by starting at a low amplification and moving upward. This must be done with the spheres closed, as they act as an antenna and overload the preamplifier. Its best to start in the ”calibration mode” (with the shorting screw removed) so you can expect a definite signal proportional to the high voltage. Also, as you increase the amplification you must increase the integration time constant on the lock-in to keep the noise level low.

Another effect that must be taken into account is that the voltage is applied at a single point on the outer sphere, so magnetic effects will depend on the angle between the charging terminal and the point where the voltage is measured. There are five terminals for measuring voltages; the voltage should be measured at several of them to check for such dependance.

For each setup, take two sets of data: (1) background (using the dummy load) and (2) a data run. Try not to disturb the spheres between these steps, as doing so may change the contacts and affect the final result. For each of these also take data with the high voltage off (but with the reference signal on) to see how the signal compares to the signal without the high voltage. The signal without high voltage should be very small, it can be reduced by separating the reference cable as much as possible from the signal input cable.

At the end you might find that you have a finite signal due to ”deviations from Coulomb law”. Compare your result with prior limits on ϵ or μ. Since this is not a state-of-the-art experiment, any signal you see would be due to a systematic effect. You should investigate how it depends on various aspects of the experiment, for example, how tightly you close the spheres. If you find something that causes dramatic changes, it would confirm the hypothesis that you are measuring a systematic effect, even in the absence of prior limits. By improving that aspect of the experiment, you can reduce the problem and place a better limit.

6 References

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Written May 2005
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Modified Mike Romalis, February 2009.