PRINCETON UNIVERSITY
Physics Department
Advanced Laboratory

POSITRON ANNIHILATION IN A METAL

Introduction

Certain radioactive isotopes decay by emitting positrons, and the positrons can be used to probe the structure of solid materials. For example, you can investigate the momentum distribution of conduction electrons in a metal. In the form in which we have implemented this experiment it is not possible recover the shape of the momentum distribution function in all its detail, but you can get a reliable estimate of the the maximum momentum—that is, the momentum corresponding to the Fermi energy.

Free Electron Theory of Metals

Many phenomena associated with the conduction electrons of a metal, such as electronic heat capacity, can be satisfactorily explained by treating the conduction electrons as nearly free particles moving in a container. In other words, the electrons have many of the properties of particles in an ideal gas. However, because of their high number densities and relatively long deBroglie wavelengths, you must use quantum statistical mechanics to analyze such a "Fermi gas." An approximate treatment of the problem, useful in the limit in which the Fermi energy ϵ_F is much greater than the thermal energy kT, leads to the conclusion that the electron kinetic energies should be distributed as

$$N(\epsilon) = \left[\frac{\epsilon^{1/2}}{e^{(\epsilon-\epsilon_F)/kT}+1}\right]\left(\frac{V}{2\pi^2}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2} ,$$

where m and ϵ are the electron mass and kinetic energy and V is the volume of the container (see references 1 and 2, for example). The normalization here is such that $\int_0^\infty N(\epsilon)d\epsilon = N$ is the total number of conduction electrons in the material, and the Fermi energy is

$$\epsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} .$$

Nature of the Experiment

In our experiment, positrons from a ^{22}Na source are allowed to strike an aluminum sheet. The kinetic energy of these positrons, initially in the range of 0–0.544 MeV, is rapidly lost to electrostatic interactions as the positrons move about in the solid aluminum. During this slowing down process, which is similar to what would be experienced by any charged particle, annihilation of the positron is unlikely. When a positron has become "thermalized" at the thermal energy level of the lattice (about 0.025 eV at room temperature), the probability of its annihilation becomes large.

The most likely annihilation process involves conversion of the total energy (mostly the rest mass energies) of a positron and an electron into two gamma-ray photons. [A much less likely process, with probability less than 1%, involves a conversion into *three* photons.] The thermalized positron interacts orimarily with the conduction electrons in the metal (why?), and allows you to determine a momentum distribution which in many metals agrees well with that predicted by the free electron theory.[1] Owing to conservation of energy and momentum, the photons emitted in the two-photon decay must have energies of approximately $mc^2 = 0.511$ MeV, and their momenta must be nearly opposite. Any departure from exactly oppositely directed momenta will correspond to the net momentum of the electron-positron pair before they annihilated.

The interaction of positrons with matter has many other interesting consequences not particularly relevant for this experiment. In some substances there is a substantial probability (of the order of 20%) for the formation of an exotic atom called "positronium," a hydrogen-like bound system consisting of an electron and a positron. Two states of this atom are known, with a mean lifetime of 1.4×10^{-7} s for the ${}^3\mathrm{S}_1$ state and 1.3×10^{-10} s for the ${}^1\mathrm{S}_0$ state before annihilation of the pair occurs. The energy separation of these states has been measured (see reference 3) to better than a part in 10^4—and serves as a detailed check of the predictions of quantum electrodynamics.

The Apparatus

A diagram of the experimental setup is presented in Figure 1. Fast particles entering the scintillation detectors produce pulses with voltage proportional to the particle energy. In our experiment the largest pulses come from the 1.274 MeV gamma ray emitted as part of the ^{22}Na decay process (see Figure 2). You will adjust the detector so that has good efficiency for the 0.511 MeV gamma rays created in positron annihilations. The other gamma rays will produce a background counting rate, but will otherwise have little effect on the experiment.

Signals from the scintillation detectors are fed to discriminators adjusted so that they accept pulses corresponding to the "photopeak" and part of the Compton continuum of the spectrum of 0.511 MeV gamma rays. They will reject any smaller pulses that come from assorted background events and noise. The discriminator outputs, in turn, are fed to a coincidence circuit which produces an output only when pulses occur at both of its inputs simultaneously (within about a microsecond). The probability of random coincidences of γ-rays at the two detectors is small. Therefore, to a good approximation, each count registered at the output of the coincidence circuit is due to a γ-ray pair originating from a single annihilating electron-positron pair.

The Positron Source

^{22}Na is a radioactive isotope which β-decays with a half-life of 2.6 years. Figure 2 illustrates the decay scheme. The emission of a positron is followed (in case of the strong branch) by a 1.274 MeV γ-ray as the excited ^{22}Ne nucleus relaxes to its ground state. Our sodium sample is in the form NaCl. The length and width of the salt deposit is $1/2 \times 1/8$ inch, and the thickness is small enough that most of the emitted positrons come out before annihilating internally. The source is sandwiched between two aluminum sheets, 0.005 inches in thickness. You should think about what considerations were used to determine this last dimension.

[1] Good experiments can also detect a "tail" on the momentum distribution, caused by positron annihilations with the more tightly bound "core" electrons.

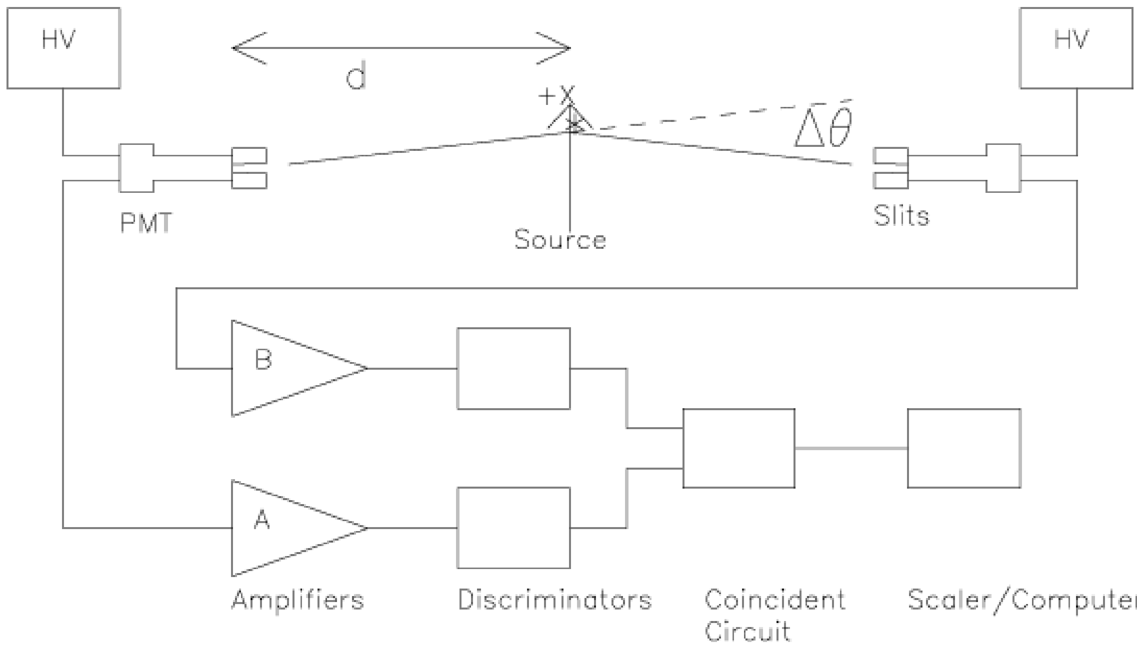

Figure 1: Block diagram of the positron annihilation experiment. The source is at the asterisk. HV stands for the high voltage supply and PMT stands for the photomultiplier tube.

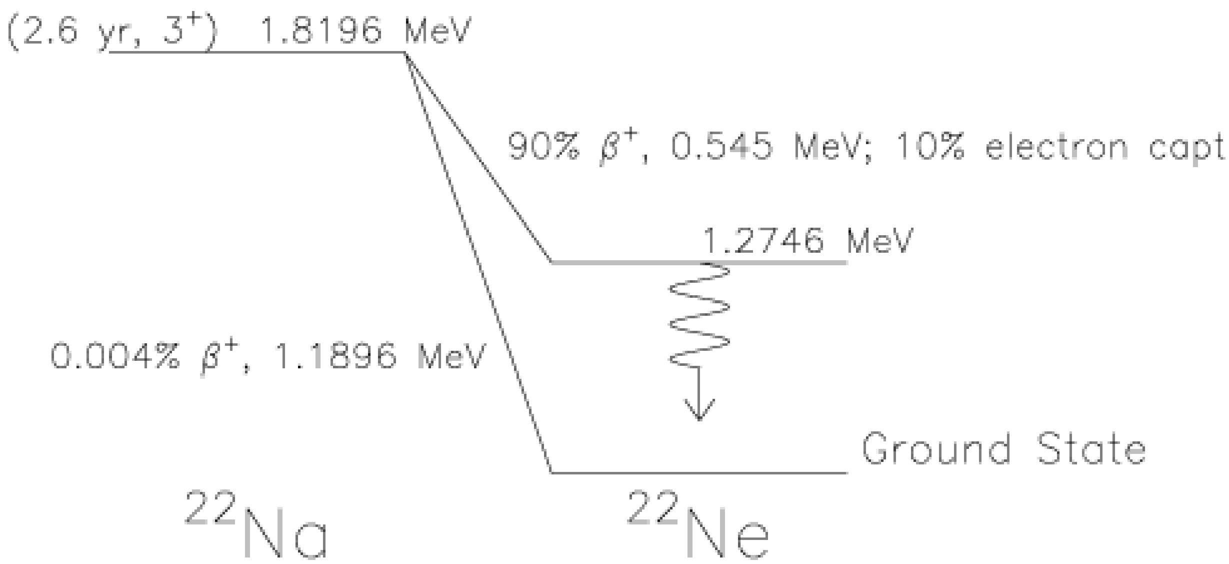

Figure 2: Decay scheme.

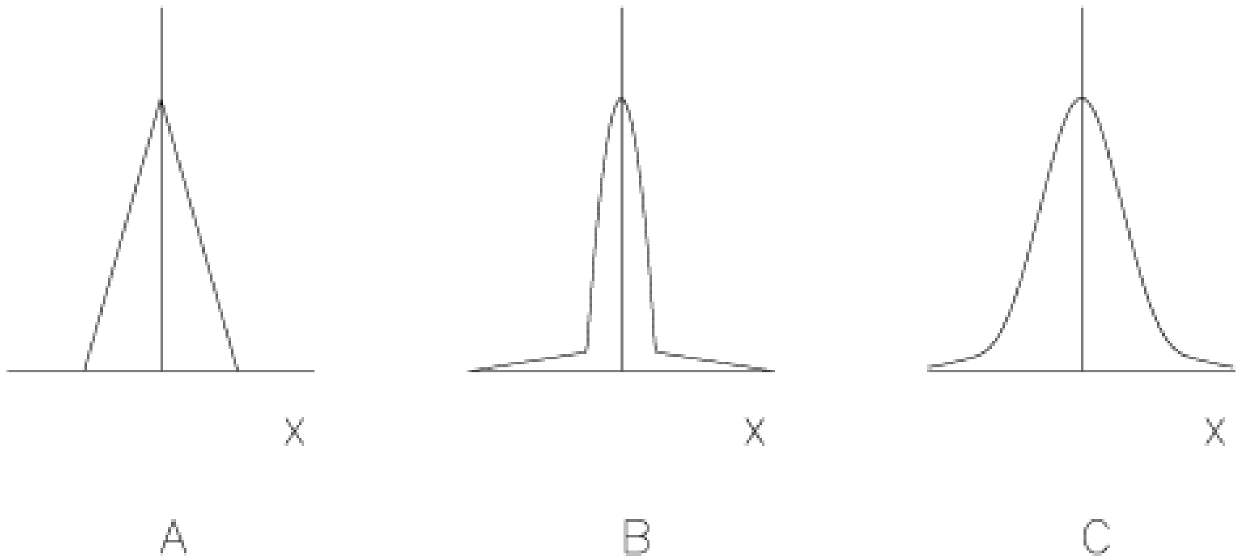

Figure 3: The x-axis is position along the scan direction and the y-axis is the count rate. Panel A shows the response to an isotropically radiating source. The width of the base is 0.4 cm is the width of the slit. Panel B shows the distribution of gamma rays from the source. Panel C shows the convolution of these 2 responses.

The strength of the radioactive source is about 20 microcuries. Your instructor will aquaint you with the hazards of handling radioactive sources.

Data Analysis

To make it possible to do this experiment in a reasonable time with a source of reasonable strength, the slits in front of the scintillation detectors must have widths w that are rather large. This choice is good for sensitivity, but bad for resolution: the slit widths significantly influence the form of the experimental data curve that you will measure. If a particular electron-positron pair had no transverse momentum, so that the annihilation photons were exactly colinear, and if the ^{22}Na source itself had negligible width, the count rate as a function of source position would have the triangular form shown in Figure 3. The finite source width and edge penetration effects at the slits combine to round the corners of this function somewhat. Let us call this instrumental smoothing function $f_w(x)$.

If there were no instrumental smooting, the rate of coincidence detections as a function of source position would have the same shape as the transverse momentum distribution of the gamma rays. We will call this distribution function $f_p(x)$. In the real experiment, the observed count rate will therefore be the convolution of f_p and f_w,

$$f(x) = \int_{-\infty}^{\infty} f_w(x') f_p(x - x') dx' \,.$$

It would be nice if you could mathematically "deconvolve" the results of an experiment so as to remove instrumental smoothing from the data and recover the unsmoothed curve and all of its glorious details. Unfortunately, this is seldom possible; in practice only some of the lost information can be regained. A useful relation to remember is that variances are additive under convolution, so that

$$\sigma^2 = \sigma_p^2 + \sigma_w^2 \, .$$

The σ's here are the widths of peaks in the distribution functions about their centroids, defined by

$$\sigma_f^2 = \frac{\int x^2 f(x) dx}{\int f(x) dx} - \left[\frac{\int x f(x) dx}{\int f(x) dx} \right]^2 \, ,$$

If the source is located at distance d from the detectors (see Figure 4), the expected variance of the momentum distribution is $\sigma_p^2 = d^2 \left\langle \Delta\theta^2 \right\rangle /4$. To separate this quantity from the smoothing due to finite slit widths, we can make use of the fact that σ_w is independent of the source-to-detector distance, while σ_p is not. If we repeat the experiment with the detectors a distance $2d$ from the source, we should find

$$\sigma'^2 = \sigma_w^2 + d^2 \left\langle \Delta\theta \right\rangle^2 \, .$$

Combining this with the expressions given above yields an estimate of the mean square angular deviation of the two photon directions,

$$\left\langle \Delta\theta^2 \right\rangle = \frac{4(\sigma'^2 - \sigma^2)}{3d^2} \, .$$

From this you can calculate the Fermi energy of aluminum. Note that in calculating variances of the experimental distributions, you must subtract the backgrounds and cut off the "tails" judiciously. The background can be estimated from the tails.

You can use similar arguments to those given above to calculate an effective width for the slits, assuming the triangular form for $f_w(x)$. You should compare this with the actual widths. Is the result reasonable? The slits are set about 0.5 cm apart, and a suggested value for d is 50 cm. Think about these choices of parameters. Do they make sense?

Once you obtain a value for the mean squared momentum of the electrons, $\langle p_x^2 \rangle$, you can compare it with the predictions of the free-electron model. The aluminum atom has three conduction electrons. The Fermi energy may be computed from their density and atomic weight. If you assume that the annihilating positron has negligible momentum, the mean square momentum of the gamma rays is the same as that of the electrons. The distribution of magnitudes of electron momenta is, in the free electron model at zero tmeperature, $n(p) \propto p^2$ for $p < p_F = \sqrt{2m\epsilon_F}$. [Why does this look different from the expression for $n(\epsilon)$?] This yields

$$\left\langle p^2 \right\rangle = \frac{3}{5} p_F^2$$

and therefore, in one dimension,

$$\left\langle p_x^2 \right\rangle \frac{1}{5} p_F^2 \, .$$

Setup Procedure

(0) Put a ^{22}Na calibration source between the collimator and detector A.

(1) Look at the output of the A with the scope. Adjust the high voltage so that the 1.27 MeV line is saturated, and the 0.511 MeV annihilation line is centered between 0.5 and 1 volt.

(2) Trigger the scope externally from output of the A discriminator, still looking at the signal from the A detector output. Set the discriminator level so that all pulses greater than about 0.4 volts survive.

(3) Repeat steps 1–3 for the B detector and its' discriminator.

(4)Now remove the calibration source, and insert the positron source in its holder. Make sure that the source height is centered well with the two detectors.

(5) Check to see that the counting rate has a sharp (but fairly slow) peak when the source is centered at $x = 0$. You should get at least 100 counts at the center in one minute.

(6) Run the LabView Positron program (icon is on the desktop).

(a). Fill in the parameters on the left-hand side of the panel and then click on the arrow icon.

(b) Be aware that ”acquisition time” is the integration time per position and thet ”carriage travel interval” and ”position” are in inches. Sorry for the English units.

(c) Be sure that the carriage moves (it WILL make a noise, LOOK at the position and make sure it is not the same).

(d) Be sure to record all the parameters you used to run the program.

You will probably want to run with integration times between 60 seconds and 120 seconds, with 0.01 inch steps. Make sure that you scan range backets the center peak. A 6 hour run should give you excellent statistics with center counts at least 400 counts.

References

1. Reif, *Statistical and Thermal Physics*.

2. Kittel, *Introduction to Solid State Physics*.

3. Theroit, Beers, and Hughes, 1967, “Precision Redetermination of the Hyperfine Structure Interval of Positronium,” it Phys. Rev. Lett., **18,** 767.

4. Stewart and Roelling, “Positron Annihilation.”

5. Seitz and Turnbull, *Solid State Physics*, vol. 10, pp. 1-46.

6. Green and Lee, *Positronium Chemistry*.

7. Melissinos, *Experiments in Modern Physics*.

Revised January, 1990
J. H. Taylor

Revised March, 2002
V. Pelletier/R. Austin