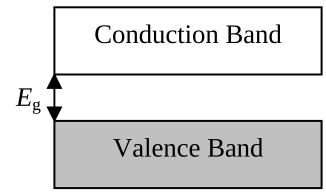
PHYS 312
Semiconductors and Optics

Semi-conductors probably had a larger impact on our society than any other materials discovery in the 20th century. The wide utility of semiconductors lies in the ability to tailor their electrical properties.

1. Undoped Semiconductors

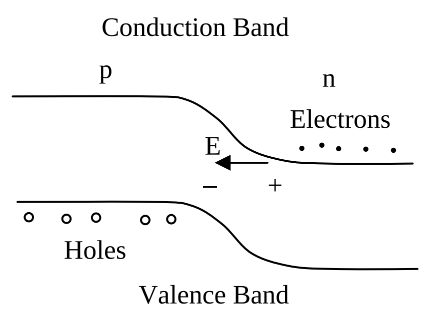


The energy levels of an un-doped semiconductor or an insulator are shown on the right. As in any solid, electron energy levels are merged into continuous bands. The valence band contains as many electrons as allowed by the Fermi exclusion principle while the conduction band is empty. Electrical conductivity results from electrons moving from occupied to unoccupied energy states, so in this state the material in insulating. The difference between semi-conductors and insulators is that in semi-conductors the energy gap E_g between the two bands is relatively small, about 1 eV. A small fraction of the electrons (about $\exp(-kT/ E_g)$) are thermally excited to the conduction band where they can participate in current flow. As a result, semi-conductors have an intermediate resistively that depends exponentially on temperature.

2. Doped semiconductors

Semiconductor materials, such as Si, are often doped with elements that have either one more or one less electron in the outer shell. For example, Ga has 3 valence electrons, one less than Si. When added to a Si crystal it produces "holes" – unoccupied electron states in the valence band. Similarly, As has 5 valence electrons and the extra electrons have to go into the conduction band. As a result, doped materials have much higher electrical conductivity than pure Si. The electric current can be carried either by electrons (n-type semiconductor) or holes (p-type semi-conductor).

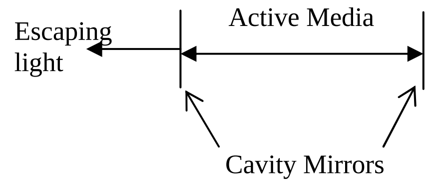
3. p-n-Junction



A p-n junction is formed by bringing two doped semi-conductors together. Initially, electrons and holes diffuse into the other material due to the gradient in their concentration. However that sets up a charge imbalance that creates an electric field and eventually stops diffusion of the charge carriers. An applied voltage can either increase or decrease this electric field. When the external voltage reduces the electric field, the p-n junction can conduct electric current. Thus, the p-n junction works as a diode, i.e. conducts current only in one direction. p-n junctions can also convert light to electricity and vise versa. If the electrons and holes recombine they release energy equal to the band gap. This energy can be emitted as a photon. Conversely, an electron-hole pair can be created by a photon, as used in photodiodes to detect light.

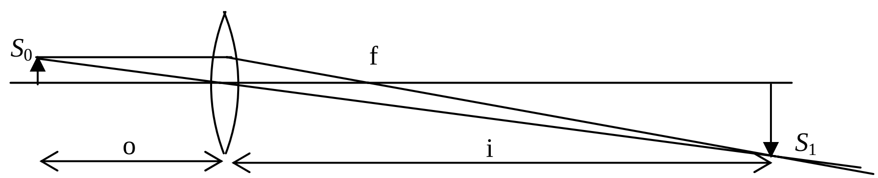
4. Diode lasers

As mentioned above, electron-hole recombination can result in emission of photons. Light-emitting diodes let the photons simply escape the semi-conductor. It is also possible to construct a diode laser by recycling the photons in a cavity. The basic layout of a laser is shown in the figure on the right. The photons bounce between the mirrors and stimulate additional photon emissions in the same direction. This is a consequence of Bose-Einstein condensation, photons prefer to occupy the same state. The light forms a standing wave in the cavity and some fraction of it escapes through the mirror, forming the output laser beam. For laser diodes the entire cavity is about 0.1-1mm long.



5. Geometric optics

Simple equations for the effects of thin lenses can be obtained following the rules of geometrical optics:



1) Rays going parallel to the axis on one side of the lens pass through the focal point on the other side
2) Rays going through the center of the lens are not deflected.

From these rules it follows from simple geometry that

$$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$$

where o is the distance from the object to a convergent lens, i is the distance from the lens to the image and f is the focal length of the lens. Similarly, one can show that the magnification is $S_1/S_0 = i/o$. For example, to obtain large magnification one needs $i >> o$ and therefore $o \sim f$.

6. ABCD law

Propagation of light through multiple lenses can be described by "ABCD" law. A light ray at a certain position z along the optic axis is described by two numbers: the distance from the optic axis r and its slope $r' = dr/dz$. The transformation of the ray by optical elements is given by

$$\begin{pmatrix} r_1 \\ r_1' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_0 \\ r_0' \end{pmatrix}$$

One can easily derive the "ABCD" matrix for simple cases:

Empty space of length d: $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ Thin lens with focal distance f: $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

For multiple elements the ABCD matrices are simply multiplied together. For example, consider a ray at a distance x from the optic axis initially propagating parallel to the axis, going through a lens of focal distance f and then empty space of length f. At that point it will have coordinates:

$$\begin{pmatrix}1 & f\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\-1/f & 1\end{pmatrix}\begin{pmatrix}x\\0\end{pmatrix}=\begin{pmatrix}0 & f\\-1/f & 1\end{pmatrix}\begin{pmatrix}x\\0\end{pmatrix}=\begin{pmatrix}0\\-x/f\end{pmatrix}$$

Hence, the ray will pass through the axis with a slope $r'=-x/f$, as expected. Arbitrary complicated optical systems, including mirrors, etc, can be modeled this way. However, ABCD law works only in the "paraxial" approximation when r' = tan(r') = sin(r') and does not include the effects of spherical aberration.

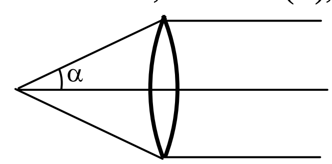
6. Wave optics

On length scales comparable to the wavelength of light its propagation is no longer governed by geometric optics. Light diffraction begins to play a large role, for example, allowing the light to pass around small obstruction. Diffraction also limits the minimum size of an object that can be seen with light to approximately the wavelength of light.

More precise calculations show that the spatial resolution of a microscope is given by

$$d_{\min} = 1.22\lambda / NA,$$

where the numerical aperture NA defines the light collection ability of the lens, NA= sin(α), where α is the opening angle, see picture to the right. Numerical aperture is larger for shorter focal length lenses, but is generally smaller than 1. Thus the maximum spatial resolution that can be obtained with visible light is on the order of 0.5 μm.



7. Gaussian Beams

More quantitative analysis of light propagation including diffraction can be obtained by considering Gaussian beams. Without going through detailed derivation, we can write $E \sim \exp\left[-ikz - i\frac{kr^2}{q(z)}\right]$, where z is the distance in the direction of propagation and r is the distance from the beam center. Hence the radial intensity profile of the light beam, given by EE^*, depends on the imaginary part of $1/q(z)$. For a beam propagating in empty space, one can show that $q(z) = \frac{i\pi w_0^2}{\lambda} + z$. For z = 0, it is easy to see that the radial intensity profile goes as $I(r) \sim EE^* \sim \exp(-2r^2 / w_0^2)$, so it is a Gaussian with a width given w_0. Away from z = 0 the width of the beam is given by $w(z) = w_0\sqrt{1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2}$. From this equation one can see that a tightly focused beam ($w_0 \sim \lambda$) will diverge very quickly, while a large beam ($w_0 >> \lambda$) will remain approximately the same size over a large distance z. Another advantage of describing the beam with the q parameter is that propagation through optical elements is still given by the "ABCD" matrix: $q_1 = \frac{Aq_0 + B}{Cq_0 + D}$. Using this law one can accurately describe light propagation in an optical system including diffraction, but still within the paraxial approximation valid only for small numerical apertures.