Collection of equations for atomic physics

Hydrogen $\Psi_{nlm} = R_{nl}Y_{lm}$ | $R_{10} = 2\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$ | $Y_{00} = \frac{1}{\sqrt{4\pi}}$

$a_0 = \frac{\hbar}{m_e c\alpha} = 0.53\text{Å}$ | $R_{20} = 2\left(\frac{Z}{2a_0}\right)^{3/2}\left(1-\frac{Zr}{2a_0}\right) e^{-Zr/2a_0}$ | $Y_{1\pm1} = \mp\sqrt{\frac{3}{8\pi}}e^{\pm i\phi}\sin\theta$

$Ry = \frac{mc^2(Z\alpha)^2}{2} = 13.6\text{eV}$ | $R_{21} = \frac{1}{\sqrt{3}}\left(\frac{Z}{2a_0}\right)^{3/2}\frac{Zr}{a_0}e^{-Zr/2a_0}$ | $Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$E_{nl} = -\frac{Ry}{(n-\delta_l)^2}$ | $\langle r\rangle = \frac{a_0}{2Z}[3n^2 - l(l+1)]$ | $Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$

$\langle 1/r\rangle = \frac{Z}{a_0 n^2}$ | $\langle r^2\rangle = \frac{a_0^2 n^2}{2Z^2}[5n^2 + 1 - 3l(l+1)]$ | $\langle 1/r^3\rangle = \frac{Z^3}{a_0^3 n^3 (l+1)(l+1/2)l}$

Two level system | $\frac{d\rho_{22}}{dt} = -\frac{d\rho_{11}}{dt} = -\frac{i\Omega}{2}(\rho_{12} - \rho_{21}) - 2\gamma_{sp}\rho_{22}$

$\gamma = \gamma_{sp} + \gamma_{coll}$ | $\frac{d\rho_{12}}{dt} = \frac{d\rho_{21}^*}{dt} = \frac{i\Omega}{2}(\rho_{11} - \rho_{22}) + [i(\omega_0 - \omega) - \gamma]\rho_{12}$

$2\gamma_{sp} = A = \frac{1}{\tau} = \frac{4\alpha\omega^3}{3c^2}|D|^2$ | $\rho_{22} = \frac{(\gamma/\gamma_{sp})\Omega^2/4}{(\omega_0-\omega)^2+\gamma^2+(\gamma/\gamma_{sp})\Omega^2/2}$ | $\Omega = e\,\langle 1|\mathbf{E}\cdot\mathbf{r}|2\rangle/\hbar$

$f = \frac{2m\omega|D|^2}{3\hbar}$ | $\rho_{12} = -\frac{(\omega_0-\omega-i\gamma)\Omega/2}{(\omega_0-\omega)^2+\gamma^2+(\gamma/\gamma_{sp})\Omega^2/2}$ | $\Omega^2 = \frac{4\pi\alpha f}{m\omega}I$

$\int\sigma(\omega)d\omega = 2\pi^2 r_0 c f$ | $\alpha(\omega) = \frac{fe^2}{2\varepsilon_0 m\omega_0}\frac{(\omega_0-\omega+i\gamma)}{(\omega_0-\omega)^2+\gamma^2+(\gamma/\gamma_{sp})\Omega^2/2}$ for $|\omega_0 - \omega| \ll \omega$

$r_0 = \frac{e^2}{4\pi\varepsilon_0 mc^2} = 2.8\times10^{-13}\text{cm}$ | $\sigma(\omega) = \text{Im}[\alpha]\frac{\omega_0}{c}$; $\sigma(\omega_0) = \frac{3\lambda_0^2}{2\pi}$ $(\gamma_{coll} = 0)$; $\phi = \text{Re}[\alpha]\frac{N}{V}\frac{\pi l}{\lambda}$

$\sum\limits_{n'l'} f_{nl\to n'l'} = 1$ (one electron) | $\alpha(\omega) = \frac{e^2}{\varepsilon_0 m}\sum\limits_k \frac{f_k}{\omega_k^2-\omega^2}$ for $|\omega_k - \omega| \gg \gamma$

$G(\omega-\omega_0) = \frac{c}{u\omega_0\sqrt{\pi}}e^{-\frac{c^2}{u^2}\left(\frac{w-\omega_0}{\omega_0}\right)^2}$ | $\alpha_G(\omega) = \int\alpha(\omega')G(\omega-\omega')d\omega'$ | $u = \sqrt{\frac{2k_BT}{M}}$

Quantum Fluctuations | Photons $\frac{\delta I}{I} = \frac{\delta N_{ph}}{N_{ph}} = \frac{1}{\sqrt{N_{ph}}}$ | $\delta\phi_{pol} = \frac{1}{\sqrt{2N_{ph}}}$

Spins | $\delta J_x \delta J_y \geq \hbar\,|J_z|\,/2$

$\delta\phi_x = \frac{\delta J_x}{J_z} = \frac{1}{\sqrt{N_s}}$ for N_s spins with $S = 1/2$ | $\delta\omega(T) = \frac{1}{\sqrt{N_s T\tau}}$ | T − measurement time
τ − coherence time

Liouville Equation | $\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho]$ | $\langle Q\rangle = Tr[Q\rho]$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ | $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ | $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$L_+\,|l,m\rangle = \sqrt{l(l+1) - m(m+1)}\,|l,m+1\rangle$; $L_-\,|l,m\rangle = \sqrt{l(l+1) - m(m-1)}\,|l,m-1\rangle$

Laser cooling | $T_D = \frac{\hbar A}{2k_B}$ | $T_r = \frac{\hbar^2 k^2}{k_B M}$

Gaussian beams | $\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi w^2}$ | $E(r) = \sqrt{\frac{P}{\pi\varepsilon_0 c}}\frac{2}{w}e^{-\frac{ir^2}{2\lambda q}}$

$q' = \frac{Aq+B}{Cq+D}$ | Space: $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ | Lens: $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

$w(z) = w_0\left[1 + \left(\frac{\lambda z}{\pi w_o^2}\right)^2\right]^{1/2}$ | $w_0^2 = \frac{\lambda}{2\pi}\sqrt{d(2R-d)}$ Mirrors with curvature R and distance d