PHYS 551 Homework 5 Solutions

Problem 1

The depth of the dipole trap is given by

$$U = -\frac{1}{4}\varepsilon_0 \alpha(\omega) E_0^2, \tag{1}$$

where E_0 is the amplitude of the oscillating electric field. The polarizability of Cs atoms can be calculated from

$$\alpha(\omega) = \frac{e^2}{\varepsilon_0 m}\sum_k \frac{f_k}{\omega_k^2 - \omega^2}. \tag{2}$$

The two strongest resonances are D1 and D2 lines at 894.3 and 852.1 nm respectively with oscillator strengths of 0.34 and 0.71 respectively. Since the sum of these oscillator strengths already slightly exceeds 1, the other resonances are very weak. Hence, the polarizability of Cs at 1064 nm is equal to $\alpha = 2.1 \times 10^{-27}$(MKS). From Eq. (1) to get a potential depth of 1 mK$= 1.38 \times 10^{-26}$J, we need electric field of $E_0 = 1.7 \times 10^6$V/m. In a gaussian beam with spot w the electric field is given by

$$|E(r)| = E_0 e^{-r^2/w^2} \tag{3}$$

The electric field at the cetner can be related to the total laser power

$$P = \int_0^\infty I(r) 2\pi r dr = \pi\varepsilon_0 c E_0^2 \int_0^\infty e^{-2r^2/w^2} r dr = \pi\varepsilon_0 c E_0^2 w^2/4 \tag{4}$$

Hence to get required electric field with $P = 10$W need $w = 41\mu$m.

We start with a collimated beam and focus it by a lens with focal distance f. The focal point is some distance d beyond the lens. Hence the ABCD matrix is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1-d/f & d \\ -1/f & 1 \end{pmatrix} \tag{5}$$

Initially the q parameter of the beam is equal to $q_0 = i\pi w_0^2/\lambda$. After the lens at distance d

$$q = \frac{Aq_0 + B}{Cq_0 + D} = \frac{\left[(1-d/f)\, i\pi w_0^2/\lambda + d\right](1 + i\pi w_0^2/\lambda f)}{1 + (\pi w_0^2/\lambda f)^2} \tag{6}$$

We want q to be purely imaginary at the focal spot after the lens, so the distance d is found by setting the real part of q to zero

$$d + (1 - d/f)\,(\pi w_0^2/\lambda)^2/f = 0 \tag{7}$$

$$d = \frac{f}{1 + f^2\lambda^2/\pi^2 w_0^4} \tag{8}$$

$$q(d) = \frac{f^2\lambda}{\pi w_0^2}\frac{i}{1 + f^2\lambda^2/\pi^2 w_0^4} \tag{9}$$

$$w(d) = \frac{f\lambda}{\pi w_0}\left(1 + f^2\lambda^2/\pi^2 w_0^4\right)^{-1/2} \tag{10}$$

From this equation we find $f = 121\,$mm. Note that $f^2\lambda^2/\pi^2 w_0^4 = 0.0015$, so the focal point is nearly at $d = f$.