Natural Resources and Growth

We consider a model that replaces K, reproducible capital, with Z, an exhaustible resource, in our standard OG model. The consumer's problem is essentially the same as in the standard model. Even though Z is a very different thing from K from the point of view of firms who use them in production, for consumers they are both just means to allow trading consumption in the first period of life for consumption in the second period of life. The consumer maximizes

$$C_1(t)^a \, C_2(t+1)^{1-a} \tag{1}$$

subject to

$$C_1(t) + V(t)S(t) = W(t) \tag{2}$$

$$C_2(t+1) = V(t+1)S(t) \ , \tag{3}$$

where S is the amount of natural resources purchased on the market for natural resources when young, and then sold on that market when old. V is the price of the natural resource. The price variables W and V are taken as given by the consumer. First order conditions are

$$\frac{\P}{\P C_1}: \qquad\qquad a\left(\frac{C_1(t)}{C_2(t+1)}\right)^{a-1} = \lambda_1(t) \tag{4}$$

$$\frac{\P}{\P C_2}: \qquad\qquad (1-a)\left(\frac{C_1(t)}{C_2(t+1)}\right)^{a} = \lambda_2(t+1) \ . \tag{5}$$

$$\frac{\P}{\P S}: \qquad\qquad \lambda_1(t)V(t) = \lambda_2(t+1)V(t+1) \ . \tag{6}$$

We can solve these equations to eliminate the Lagrange multipliers, arriving at

$$\frac{V(t+1)}{V(t)} = \frac{a C_2(t+1)}{(1-a)C_1(t)} \ . \tag{7}$$

It is not hard to check that this leads to the same conclusion as in our previous model about how W and the rate of return on savings determine consumption choices:

$$C_1(t) = aW(t) \tag{8}$$

$$C_2(t+1) = (1-a)\frac{V(t+1)}{V(t)}W(t) \ . \tag{9}$$

The firm maximizes profits, given by

$$AZ(t)^g L(t)^{1-g} - V(t)Z(t) - W(t)L(t), \tag{10}$$

with respect to natural resource usage Z and labor input L. The firm takes the price V and W as given. Its first order conditions are

$$\frac{\partial}{\partial Z}: \qquad g A z(t)^{g-1} = V(t) \qquad (11)$$

$$\frac{\partial}{\partial L}: \qquad (1-g)Az(t)^{g} = W(t), \qquad (12)$$

where $z(t) = Z(t)/L(t)$ is per-worker resource usage.

With population at t given by $N(t)$ and the number of firms at time t given by $F(t)$, market-clearing conditions are

$$N(t)C_1(t) + N(t)V(t)S(t) + N(t-1)C_2(t)$$
$$= F(t)AZ(t)^{g}L(t)^{1-g} \qquad (13)$$

$$N(t-1)S(t-1) = F(t)Z(t) + N(t)S(t) \qquad (14)$$

$$N(t) = F(t)L(t) . \qquad (15)$$

Note that (14) is different in form from what was done in class. Here we assume that consumers sell natural resource stocks directly from old to young, with firms buying only the amount (Z) that they use up. In class we had the firms buying up the entire resource stock of the old and then reselling some of it to the young, which requires slightly more notation without changing the economics.

Using (12) in (8) gives us

$$C_1(t) = a \cdot (1-g)Az(t)^{g} . \qquad (16)$$

Using this result and (11) in (2) (and reusing (12)) gives us

$$g A z(t)^{(g-1)} S(t) = (1-a)(1-g)Az(t)^{g} , \qquad (17)$$

which reduces to

$$\frac{z(t)}{S(t)} = \frac{g}{(1-a)(1-g)} = \frac{S(t-1)}{(1+n)S(t)} - 1 , \qquad (18)$$

where the last equality uses (14) and our standard assumption that $N(t)/N(t-1) = 1+n$ for all t. Equation (18) tells us that the per capita holding S of the natural resource shrinks by the constant factor

$$\frac{(1+n)(1-a+ag)}{(1-g)(1-a)} \qquad (19)$$

each period.

Equation (19) implies that the rate of resource exhaustion depends on the relative importance of labor (which the young are endowed with) and the natural resource (which the old sell to feed themselves) in production. A high g implies a large role for the natural resource in production, more economic resources in the hands of the old, and therefore rapid exhaustion of the resource. As $g \to 1$, the rate of exhaustion goes to infinity. The rate of exhaustion goes to zero (the rate of

decline of per capita resources approaches the population growth rate) as $g \to 0$. The effect of increased taste for consumption late in life, $a \to 0$, i.e. higher savings rates, is to reduce the rate of resource exhaustion, but cannot reduce the rate of exhaustion to zero. Increased taste for consumption when young $(a \to 1)$, however, can push the exhaustion rate to infinity.

It is interesting to consider extending this model to a case where there is also capital in the technology, so that output is given by

$$Y(t) = Af(K(t), Z(t), L(t)) + (1-d)K(t) . \tag{20}$$

What results depends on how "essential" Z is to production. The CES family of production functions, which we have used previously, includes forms where Z is essential and where it is not. Recall that for this class

$$Y(t) = A\left(bK(t)^s + gZ(t)^s + (1-b-g)L(t)^s\right)^{1/s} + (1-d)K(t) . \tag{21}$$

The marginal product of capital is

$$bA\left(b + g\left(\frac{Z(t)}{K(t)}\right)^s + (1-g-b)\left(\frac{L(t)}{K(t)}\right)^s\right)^{\frac{1}{s}-1} + 1 - d . \tag{22}$$

For CES formula to yield a well-behaved production function we require $0 < s < 1$. When $0 < s < 1$, the isoquants cut the axes, as in the graph below. As can be seen from (22), there is in this case a lower bound on the marginal product of capital of $Ab^{1+(1/s)} + 1 - d$, which may exceed one. With such a technology, production can continue even when $Z(t) = 0$, and the marginal product of capital is not driven to zero as the resource is exhausted. Competitive equilibria will generally exhaust the resource in finite time but then converge to a steady state.

CES isoquants, sigma=.5
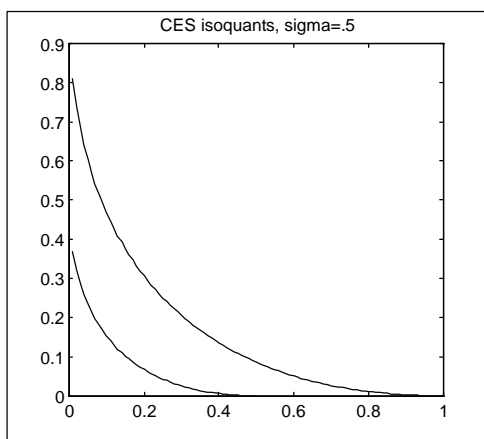
When instead $s < 0$, the isoquants have asymptotes parallel to the axes, as in the graph below. Then output and consumption are zero when Z is zero, and shrinkage of Z to zero requires that consumption shrink to zero also. This is a characteristic of the technology, not of competitive equilibrium alone.

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CES isoquants, sigma=-2
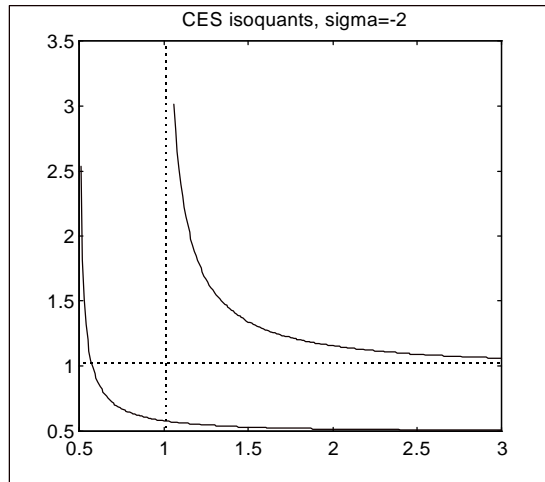
The borderline $s = 0$ case is Cobb-Douglas. It is an interesting curiosity that when $n = d = 0$, even though $Z = 0$ implies $Y = 0$, in this case it is technically feasible to prevent consumption from collapsing to zero as the resource is exhausted. This involves increasing K rapidly enough to offset a declining Z. Competitive equilibrium does not deliver this result, however. {}