Global Games and Illiquidity

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The Credit Crisis of 2008

- Bad news and uncertainty triggered market freeze
- Real bank runs (Northern Rock, Bear Stearns, Lehman Brothers....)
- Run-like behavior in markets (the "Repo Run")
Bank Runs and Multiple Equilibria

- If I expect everyone else to withdraw deposits from a bank, I have an incentive to do so.
- If everyone withdraws deposits from the bank, the bank collapses (even if assets are fine)
- Two equilibria: all stay, all run
Strategic Complementarities

- An increase in your strategy gives me a greater incentive to increase my strategy, e.g.,
  - If many others withdraw their deposits from a bank, I have an incentive to withdraw.
  - If many others are selling the "pegged" Brazilian real, I have an incentive to dump reals.
  - If others are hoarding liquidity, I hoard liquidity too
- Sufficiently strong strategic complementarities imply multiple equilibria (self-fulfilling beliefs)
Problems with Multiple Equilibrium Modelling

- Weak predictions
  - do not explain when a run occurs
  - do not explain correlation of runs with bad fundamentals

- Hard to do policy analysis
  - what is the impact of increased liquidity (cash on hand) requirements for financial institutions?

- Key unreasonable implicit assumption
  - Complete Information (= common knowledge of payoffs) and equilibrium implies common knowledge of everyone’s actions
Global Games

- Carlsson - van Damme 93, Morris-Shin 98, 03.
- Relax the assumption of common knowledge of payoffs in an intuitive and small way in order to
  1. Generate natural "strategic uncertainty" (uncertainty in equilibrium about others' actions)
  2. Unique equilibrium
  3. Economic impact of strategic complementarities (small changes in "fundamentals" lead to large changes in outcomes)
  4. Natural comparative statics and policy analysis
This Talk

1. Review Global Games Modelling
2. Discuss an application to quantifying and regulating liquidity risk:

Morris-Shin 09. "The Illiquidity Component of Credit Risk."
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- Slides, More Detailed References, this and other papers available on my website http://www.princeton.edu/~smorris/ (google "Stephen Morris")
- All based on joint work with Hyun Song Shin (Princeton University)
Example

- Two Players: Alice and Bob
- Two Actions: Stay or Run
- Payoffs depend on state of "Fundamentals" $\theta \in \mathbb{R}$
- Payoff Matrix:

| Alice | Bob \n|------|---|---|
| Stay |
| Run | $0, \theta - 1$ | $0, 0$ |
Complete Information

- Suppose $\theta$ is common knowledge to Alice and Bob
- Three regions:
  1. If $\theta < 0$, running is strictly dominant strategy and thus (run,run) is the unique Nash equilibrium
  2. If $\theta > 1$, staying is strictly dominant strategy and thus (stay,stay) is the unique Nash equilibrium
  3. If $0 \leq \theta \leq 1$, then there are multiple Nash equilibria: (run,run) and (stay,stay)
$\theta$

- RUN
- RUN OR STAY
- STAY
Incomplete Information: the Uniform-Uniform Case

- $\theta$ is uniformly distributed on the interval $[\theta, \bar{\theta}]$, where $\theta \ll 0$ and $1 \ll \bar{\theta}$
- Alice and Bob observe signals $x_A$ and $x_B$ respectively, where
  - $x_A = \theta + \sigma \varepsilon_A$
  - $x_B = \theta + \sigma \varepsilon_B$
  - $\varepsilon_A, \varepsilon_B$ are independently uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$
  - $\sigma > 0$ measures the accuracy of signals
Best Responses

- Each player $i$’s expectation of $\theta$ is $x_i$.
- Each player will run if he assigns probability greater than $x_i$ to his opponent running.
- Each player will stay if he assigns probability less than $x_i$ to his opponent running.
Deleting Dominated Strategies

- **Round 1**
  - Each player has a dominant strategy to run if $x_i < 0$
  - Each player has a dominant strategy to stay if $x_i > 1$
Deleting Dominated Strategies

- **Round 1**
  - Each player has a dominant strategy to run if \( x_i < 0 \)
  - Each player has a dominant strategy to stay if \( x_i > 1 \)

- But now a player observing signal 0 will assign probability at least \( \frac{1}{2} \) to his opponent running.

- Thus he will have a strict incentive to run if he observes signal 0 or (by continuity) a signal in an interval above 0.

- Symmetrically, a player has a strict incentive to stay if he observes signal 1 or (by continuity) a signal in an interval below 1.
Deleting Dominated Strategies

- **Round 1**
  - Each player has a dominant strategy to run if $x_i < 0$
  - Each player has a dominant strategy to stay if $x_i > 1$

- **Round 2:** there exist $0 < x_2^2 < \frac{1}{2} < \bar{x}^2 < 1$ such that
  - Each player has a dominant strategy to run if $x_i < x_2^2$
  - Each player has a dominant strategy to stay if $x_i > \bar{x}^2$
Deleting Dominated Strategies

- Round 2: there exist $0 < \underline{x}^2 < \frac{1}{2} < \bar{x}^2 < 1$ such that
  - Each player has a dominant strategy to run if $x_i < \underline{x}^2$
  - Each player has a dominant strategy to stay if $x_i > \bar{x}^2$

- Round 3: there exist $\underline{x}^2 < \underline{x}^3 < \frac{1}{2} < \bar{x}^3 < \bar{x}^2$ such that
  - Each player has a dominant strategy to run if $x_i < \underline{x}^3$
  - Each player has a dominant strategy to stay if $x_i > \bar{x}^3$
Deleting Dominated Strategies

- Round $k$: there exist $0 < \underline{x}^k < \frac{1}{2} < \bar{x}^k < 1$ such that
  - Each player has a dominant strategy to run if $x_i < \underline{x}^k$
  - Each player has a dominant strategy to stay if $x_i > \bar{x}^k$

- Round $k+1$: there exist $\underline{x}^k < \underline{x}^{k+1} < \frac{1}{2} < \bar{x}^{k+1} < \bar{x}^k$ such that
  - Each player has a dominant strategy to run if $x_i < \underline{x}^{k+1}$
  - Each player has a dominant strategy to stay if $x_i > \bar{x}^{k+1}$
RUN OR STAY

0 \quad x^2 \quad x^3 \quad \ldots \quad x^n \quad \bar{x}^n \quad \ldots \quad \bar{x}^3 \quad \bar{x}^2 \quad 1

\theta
There is an (essentially) unique strategy for each player surviving iterated deletion of strictly dominated strategies:

\[ s_i^* (x_i) = \begin{cases} 
  \text{Stay, if } x_i \geq \frac{1}{2} \\
  \text{Run, if } x_i < \frac{1}{2} 
\end{cases} \]

There is an (essentially) unique equilibrium where both players follow this strategy.

This is true independent of the size of \( \sigma > 0 \)
Key Assumption

- When Alice observes signal $x_A$, she assigns probability $\frac{1}{2}$ to Bob observing a lower signal and thus probability $\frac{1}{2}$ to Bob observing a higher signal; this is true independent of $x_A$.
- Thus Alice faces maximal strategic uncertainty exactly when she is at the margin between the two actions.
Incomplete Information: General Distributions

- $\theta$ is distributed with smooth density $g(\cdot)$ on the real line
- Alice and Bob observe signals $x_A$ and $x_B$ respectively, where
  - $x_A = \theta + \sigma \varepsilon_A$
  - $x_B = \theta + \sigma \varepsilon_B$
  - $\varepsilon_A, \varepsilon_B$ are independently distributed with smooth density $f(\cdot)$ on the real line
- $\sigma > 0$ measures the accuracy of signals
Key Assumption

If Alice observes signal \( x_A \), she assigns probability

\[
\psi_{\sigma}(x_A) = \text{Prob}_{\sigma}(x_B \geq x_A | x_A)
\]

\[
= \int_{\theta=-\infty}^{\infty} g(\theta) f\left(\frac{x_A-\theta}{\sigma}\right) \left(\int_{\varepsilon_B=\frac{x_A-\theta}{\sigma}}^{\infty} f(\varepsilon_B) \, d\varepsilon_B\right) \, d\theta
\]

\[
= \int_{\theta=-\infty}^{\infty} g(\theta) f\left(\frac{x_A-\theta}{\sigma}\right) \, d\theta
\]
Key Assumption

As \( \sigma \to 0 \),

\[
\psi_\sigma (x_A) = \text{Prob}_\sigma (x_B \geq x_A | x_A)
\]

\[
\rightarrow \int_{\theta = -\infty}^{\infty} f \left( \frac{x_A - \theta}{\sigma} \right) \left( \int_{\varepsilon_B = \frac{x_A - \theta}{\sigma}}^{\infty} f \left( \varepsilon_B \right) d\varepsilon_B \right) d\theta
\]

\[
= \int_{\varepsilon_A = -\infty}^{\infty} f \left( \varepsilon_A \right) \left( \int_{\varepsilon_B = \varepsilon_A}^{\infty} f \left( \varepsilon_B \right) d\varepsilon_B \right) d\theta
\]

\[
= \frac{1}{2}
\]
For all $\sigma > 0$, let $s^\sigma$ be an equilibrium of the $\sigma$ incomplete information game. As $\sigma \to 0$,

$$s_i^\sigma (x_i) \to s_i^* (x_i) = \begin{cases} 
\text{Stay}, & \text{if } x_i \geq \frac{1}{2} \\
\text{Run}, & \text{if } x_i < \frac{1}{2}
\end{cases}$$
Incomplete Information: General Distributions

In words: if information is accurate but there is not common knowledge, each player will run if and only if running is the best response to a 50/50 belief about the opponent’s action.
Many Player Binary Action Games

- Continuum of Players
- Two Actions: Stay or Run
- Payoffs depend on state of "Fundamentals" $\theta \in \mathbb{R}$
- Payoff to Running: $r^*$
- Payoff to Staying:
  - $\mathring{r}(\theta)$ if proportion running less than $\lambda(\theta)$
  - $\mathring{r}(\theta)$ and $\lambda(\theta)$ increasing in $\theta$
  - 0, if the proportion running is more than $\lambda(\theta)$
Suppose $\theta$ is common knowledge

Three regions:

1. If $\tilde{r}(\theta) < r^*$ or $\lambda(\theta) < 0$, running is strictly dominant strategy and thus "all run" is the unique Nash equilibrium.

2. If $\tilde{r}(\theta) > r^*$ and $\lambda(\theta) > 0$, staying is strictly dominant strategy and thus "all stay" is the unique Nash equilibrium.

3. Otherwise, then there are multiple Nash equilibria: "all run" and "all stay"
RUN  \quad \theta \quad \text{RUN OR STAY} \quad \bar{\theta} \quad \text{STAY} \quad \theta
Incomplete Information: General Distributions

- $\theta$ is distributed with smooth density $g(\cdot)$ on the real line
- Player $i$ observes signal $x_i = \theta + \sigma \varepsilon_i$
  - $\varepsilon_i$ are independently distributed with smooth density $f(\cdot)$ on the real line
  - $\sigma > 0$ measures the accuracy of signals
Laplacian Beliefs

- For small $\sigma$, there is an (essentially) unique equilibrium with each player following a threshold strategy:

$$s_i^* (x_i) = \begin{cases} 
\text{Stay, if } x_i \geq x^* \\
\text{Run, if } x_i < x^* 
\end{cases}$$

- For small $\sigma$,
  - an agent observing signal $x^*$ has a uniform belief over the proportion of other players with signals higher than $x^*$
  - the probability of "no run" is $\lambda (x^*)$
  - $\lambda (\theta^*) \tilde{r} (\theta^*) = r^*$
In words: if information is accurate but there is not common knowledge, a run occurs if the proportion running need to trigger a run \( \lambda(x^*) \) is less that the outside option ratio \( \frac{r^*}{\tilde{r}(x^*)} \).
Liquidity versus Solvency

Christopher Cox, (then) SEC chairman, on Bear Stearns in March 2008.

“[T]he fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. When the tumult began last week, and at all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard.

Specifically, even at the time of its sale on Sunday, Bear Stearns’ capital, and its broker-dealers’ capital, exceeded supervisory standards. Counterparty withdrawals and credit denials, resulting in a loss of liquidity - not inadequate capital - caused Bear’s demise.”
Liquidity versus Solvency

- An old and important distinction
- But hard to disentangle in practice
  - Did the run hasten failure of an already insolvent bank?
  - Or, did the run scupper an otherwise sound bank?
- Total credit risk is sum of *insolvency risk* and *illiquidity risk*
  - but the two are jointly determined
  - need to look at how they interact
  - need for a theory...
Fix a bank’s balance sheet.

- Let $\theta$ measure the quality of assets on the balance sheet.
- There will be an innovation in $\theta$ equal to $\tau \eta$, where $\eta \sim G(\cdot)$ and $\tau > 0$ measures the amount of uncertainty about solvency.
- The bank is solvent only if $\theta + \tau \eta$ exceeds some critical threshold $\theta^{**}$.
  - i.e. if $\eta \leq \frac{\theta^{**} - \theta}{\tau}$
- The bank will be insolvent if $\theta + \tau \eta \leq \theta^{**}$, where $\eta \sim G(\cdot)$.
- Thus the probability of insolvency is $G\left(\frac{\theta^{**} - \theta}{\tau}\right)$.
- And the probability of solvency is $1 - G\left(\frac{\theta^{**} - \theta}{\tau}\right)$. 
Sketch of the Theory: Insolvency Risk

In words, the probability of insolvency is a decreasing function of the quality of assets.
Thus if there is not a run, and \( r_S \) is the face return on short term debt, the expected return is \( \tilde{r} (\theta) = r_S \left( 1 - G \left( \frac{\theta^* - \theta}{\tau} \right) \right) \).

Suppose the bank’s balance sheet includes:

- Liquid assets: \( A \)
- Short term debt: \( S \)

Thus the threshold running to cause a run is \( \lambda (\theta) = \frac{A}{S} \).

Run point \( \theta^* \) solves (from the formula above):

\[
\lambda (\theta^*) \tilde{r} (\theta^*) = r^*
\]

Substitute for \( \lambda (\theta^*) \) and \( \tilde{r} (\theta^*) \):

\[
\frac{A}{S} r_S \left( 1 - G \left( \frac{\theta^{**} - \theta^*}{\tau} \right) \right) = r^*
\]

Re-arranging gives:

\[
\theta^* = \theta^{**} - \tau G^{-1} \left( 1 - \frac{r^* S}{r_S A} \right)
\]
Sketch of the Theory: Runs

In words, a run occurs only if the probability of insolvency is greater than

$$1 - \frac{r^* S}{r_s A}$$
Implications

- Comparative statics: Illiquidity risk...
  - decreases in the liquidity of the balance sheet, i.e., the "liquidity ratio" $\lambda = \frac{A}{S}$
  - decreases fastest when the level of liquidity is low
  - increases in the opportunity cost of funds, i.e., the "outside option ratio" $\frac{r^*}{r_S}$
  - increases in uncertainty about solvency $\tau$

- Policy
  - tells when the financial stability returns of liquidity requirements are highest and lowest
Generalizations and Related Work

- Analysis extends to general games with strategic complementarities [Frankel-Morris-Pauzner 03].
- "Calvo" frictions deliver similar results [e.g., Guimaraes 04].
- Higher Order Belief Foundations [e.g., Morris-Shin 07].
- Experiments [Heinemann, Nagel and Ockenfels 04]
Conclusions

- **Global Games**
  - multiplicity is - to some extent - an artifact of modelling choices
  - possible to get unique theoretical prediction building on the economics of self-fulfilling beliefs
  - sensible comparative statics / policy analysis

- **Illiquidity**
  - theory of how coordination is resolved allows model based distinction between insolvency and illiquidity risk
  - insolvency and illiquidity risk are inextricably linked
  - nonetheless, separate policies to address illiquidity risk make sense