Laws and Authority

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Abstract

A law prohibiting a particular behavior does not directly change the payoff to an individual should he engage in the prohibited behavior. Rather, any change in the individual’s payoff, should he engage in the prohibited behavior, is a consequence of changes in other peoples’ behavior. If laws do not directly change payoffs, they are “cheap talk,” and can only affect behavior because people have coordinated beliefs about the effects of the law. Beginning from this point of view, we provide definitions of authority in a variety of problems, and investigate how and when individuals can have, gain, and lose authority.
1. Introduction

We will first argue a simple but powerful point: Laws by themselves cannot change the costs and benefits associated with any given actions people might take. Consider the prisoners’ dilemma game; the payoff matrix specifies outcomes yielding lower utility to the two players in the case that they both confess than in the case that neither confesses. In the standard analysis of the problem, these payoffs are taken as given, but in terms of the story motivating the game we might ask precisely why the payoffs are lower when they both confess? A simple answer is that they will be put in jail longer if they confess than if they do not. But why will they be put in jail longer? Arguably because the law specifies a longer sentence in this case.

There is, however, a difficulty with this simple explanation; under traditional view that an agent’s payoff is only determined by physical outcomes, the existence of a law doesn’t directly affect the utility that any player will get from any array of choices of the players in the game. A judge can order that players be put in jail, independent of what laws are on the books. Of course, it may be that the consequences of the judge ordering a person to be sent to jail differ depending on what laws, if any, are on the books. It may be that if there is no law, appellate courts may overturn the first judge’s order, jailors might refuse to carry out the order or voters might remove the judge from his position. But appellate courts can issue statements that they uphold or overturn the decision as easily when there is no law as when there is a law in effect. Similarly, the jailors can obey or refuse the order and the voters can vote to remove the judge or not independent of the law. It could be argued that both The People’s Republic of China and the USSR provide modern examples of societies where the law was ignored.

We typically analyze the prisoners’ dilemma game for the payoffs we have written in the matrix, and those numbers are primitives of the game, beyond the analysis at hand. In interpreting the game though, these numbers are induced by the behavior of other agents not included in the analysis. Implicitly, there are unmodelled players who are assumed to behave in a way that induces the matrix of payoffs for the modelled players. The question that motivates this work is “How is it that the payoffs of a game can be affected by the existence or nonexistence of laws?” If we’re willing to ignore the question of the incentives and interests of the unmodelled players, we can simply assume that they will follow the rules having to do with the enforcement of the laws. There is something unsatisfactory,
however, about ignoring these incentives if we are explicitly trying to understand how laws can alter agents’ behavior by changing incentives.

At the most general level, there is the “Game of Life.” This is the universal game that includes all people, present and future. Each player has a strategy set that includes all possible actions he or she can take in every imaginable circumstance. Any person can try to apprehend any other individual, and can exhort others to issue statements that the apprehended person should be incarcerated, and call upon still others to so incarcerate that person. Each individual can respond as he or she wishes to such exhortations. The outcome function for this game of life is simply the physical rules governing the universe: if sufficiently many people with sufficient strength try to lock an individual up, he will be locked up. If an individual has sufficient strength, weapons or allies, he will not.

There is one and only one such universal game; nothing can alter it. It cannot be altered, for example, by acquiring weapons, which would change the balance of power, since acquiring weapons is simply (a part of) a strategy for an individual. Assembling allies, studying one’s opponent, etc. are similarly incorporated into the strategy sets of the universal game. Although it is straightforward to see that there is a single universal game, we generally don’t analyze this game for obvious reasons: it is so complicated that it typically doesn’t admit analysis that is useful for problems we are interested in. Virtually all applications of game theory abstract from this universal game, focusing on a small subset of the players, taking as given the presumed behavior of the omitted players.

With the discussion above in mind, we return to the prisoners’ dilemma game. We see that the lower payoffs associated with “both confess” is based on the assumption that there is a different equilibrium among the unmodelled players when the players confess than when the players don’t confess. Why do these players play a different equilibrium in these cases? We start from the observation that laws have no effect on the actions available to any individual, nor on the outcome function in the universal game. A law prohibiting theft doesn’t alter any individual’s ability to take an object nor does it affect what physical activities (such as attempts to lock an individual in a cell) individuals are capable of. Nor does it affect the outcome function: my struggle to keep from being locked in a cage by you will result in an outcome that is determined by our relative strengths and martial arts abilities, but not by the existence or nonexistence of a law. Words written on a piece of paper don’t alter the laws of physics.

Yet laws do influence behavior. Our aim is to understand better how laws
affect behavior from a formal perspective.

Consider a law that specified that people must pay 12.27% of their income as a tax to the government, with horrific consequences for violations of the law. This could easily lead to behavior in which all individuals indeed paid 12.27% of their income in tax. How, one might ask, could this behavior arise as equilibrium in the absence of the law? The answer is that if everyone—tax authorities, judges, prison wardens, etc.—behaves as if there were such a law, the equilibrium behavior when the law exists is still an equilibrium when it is not. People who did not send in 12.27% of their income would be taken to court, judges would sentence them to jail and jailors would incarcerate them. Judges or jailors who did not behave in this way would be reprimanded, dismissed or sent to jail themselves.

A first response to this idea is that it is implausible that all people would behave as if there were a law in place when, in fact, it was not. There are, however, instances of precisely this. In the United States it has been a law, indeed a constitutional guarantee, that one’s property could not be taken away without due process of law. During the second world war, native born Japanese Americans had their property taken away and they were imprisoned for no reason other than their genetic makeup and the national origin of their ancestors. This was clearly unlawful, yet the entire court system, including the Supreme Court of the United States behaved as though there was an entirely different law, namely that one could not take away the property of an individual and imprison that individual unless there was a war with Japan and the individual in question had Japanese ancestry. The law specified something different from this, but the distinction was ignored by all, and ignoring it constituted equilibrium behavior.

This example is not unique. In the United States the law is clear that an individual may tell any other individual his opinion of him, and that murder in retaliation for doing so is against the law. For a large part of this century in a substantial part of this country, however, a black man who told a white man his views might well have been hanged for doing so. The hanging clearly constitutes murder, yet any policeman foolish enough to arrest the murderers would (in the unlikely event that he could find a prosecutor willing to consider this a crime) likely confront a jury that did not find it a crime. Observing the behavior, an outsider would reasonably have thought that the law was that confrontation of a white man by a black man was a capital offense.

A third example comes from the time when California was taken by force from Mexico by the United States. At the time it was taken from Mexico, Mexican law
immediately naturally ceased to be applicable. Nevertheless, for a long period subsequent to California’s becoming part of the US, Mexican law continued to be applied in California. Again, if all people involved in a particular problem behave as if the law is of a particular form, it may well be an equilibrium; no any individual deviating from behavior consistent with the law loses by doing so.

These examples illustrate how it may be an equilibrium for all people to behave as if there is a law in place when, in fact, it is not. The converse is clearly possible as well. There was for many years in Connecticut a law prohibiting the sale and use of contraceptive devices, which was eventually overturned on constitutional grounds by the Supreme Court. The law was ignored by virtually everyone. Merchants sold contraceptive devices openly; no law enforcement official intervened and presumably no jury would convict violations if brought to trial.

In summary, laws prescribing behavior and equilibrium behavior in a society are two quite distinct things. It isn’t at all obvious how and why the first can affect the second. Equivalently, how or why do certain individuals or groups (and not others) have the authority (in equilibrium) to pass or interpret a law? This question applies equally to authors of constitutions, legislatures, courts, policemen, etc. The formal version of our observation is that, in the absence of informational considerations, the set of Nash equilibria of the Game of Life is invariant with respect to the set of laws enacted: We argued above that there is a universal game, and that the strategy sets and outcome functions are not altered by laws. We further assumed that utilities were independent of laws, depending only on physical outcomes. But if passing a law doesn’t change strategy sets, the outcome function, information or the utilities associated with strategy combinations, it cannot affect the set of Nash equilibria.

It may be that laws affect behavior because they affect the utilities individuals derive from particular strategy choices. If, for example, an individual finds that his utility increases from taking an object from my yard, net of any consequences that result from taking it, optimal behavior on his part entails his taking the object. But perhaps by passing a law, that individual can be made to feel sufficiently guilty that the net gain is no longer positive. Alternatively, the individual may fear that God will punish him now or in the future for breaking the law; all that matters is that the act of passing a law could conceivably change the utilities associated with particular physical outcomes. This is undoubtedly correct to some extent. The ability to psychologically condition individuals so that their utilities associated with physical outcomes differ solely because their behavior
was sanctioned or not is one of the foundations of civilized human organizations. We focus in this paper, however, on the traditional case, and assume that utilities depend only on the physical outcomes, and are independent of the existence or nonexistence of laws.

This discussion points to three related but distinct questions: (i) What precisely does it mean for an individual or a group to have authority? (ii) How is authority acquired? (iii) How is authority maintained? The aim of this paper is to suggest approaches to answering these questions.

We proceed as follows. In section 2, we define what is meant by authority. An individual has authority over something, given the strategies pursued by others in the population, if by making some cheap talk statement, he can make that something happen. Crucially, our definition of authority makes no reference to payoffs or rational behavior. We want to separate our discussion of what it means to have authority from our equilibrium analysis. In section 3, we give a simple example of how authority might evolve in a simple evolutionary model. In section 4, we discuss what authority might emerge in equilibrium in an environment with incomplete information. In section 5, we present a preliminary examination of how courts and others might have strategic incentives to invest in acquiring and maintaining reputations. In each of these sections, we discuss some examples from the law to illustrate the type of questions that our approach might shed light on. We do not make any claim that the analytic approaches described are the only - or even the best - to address the questions, and we do not examine any one application in complete detail. Rather, our purpose is this paper is to set out a set of approaches that shed light on the three questions above.¹

¹Many of the themes in this paper have been extensively explored elsewhere. Subsequent to writing this paper, we became aware of Basu (2000), which contains a similar argument that laws are cheap talk that do not affect individuals’ payoffs. The “Game of Life” perspective was presented in Binmore (1997), where it was used as a foundation for a theory of justice. The role of social norms in determining the actual effect of the law is the subject of a large literature; see the books of Elickson (1994) and E. Posner (2000b). The evolutionary origins of the law and constitutions are studied in Hirsheifer (1982), Elliott (1985) and Posner (2000a). The fundamental role of coordination in all aspects of the law, and especially constitutions, is emphasized in the recent study of Hardin (1999). The role of cheap talk in altering the outcomes of coordination games is the subject of a large literature in game theory; see Farrell (1990), Kim and Sobel (1995), Farrell and Rabin (1996) and the references therein. Our contribution in this paper is to see what a strategic analysis of cheap talk has to contribute to our understanding of law and authority.
2. Defining Authority

By authority, we mean the ability of an individual or group to influence the actions of others by what they say. We start with a series of examples to make this notion more concrete, and subsequently, we develop formal definitions of authority.

- John proposes to a group of friends that they meet for dinner at a certain Italian restaurant. Each friend may have a different ranking over alternative restaurants, but the important thing is that each wants to go to the same restaurant the others go to. John’s choice may not be everyone’s favorite restaurant, but the group will surely meet there because each wants the group to be together, and each expects the others to follow John’s recommendation. Had John proposed a French restaurant, his friends would have met there instead. We may say that John has authority over the choice of restaurant because, given how the others will respond, he has the ability to determine which restaurant they meet at.

- An elderly Mafia don instructs a flunky to murder an informer. The flunky may not wish to carry out the murder (he would prefer that the don had asked someone else to do so). However, if he does not carry out the murder, he is confident that he himself will be murdered for violating the don’s orders. The flunky carries out the don’s order, not because he wishes to do so, nor because the frail don will hurt him if he does not, but because he (correctly) anticipates that the don’s instruction will influence others’ behavior. Had the don instructed his flunky to merely maim the informer, the flunky would have done that too. The don has authority over the flunky’s actions because, given the internal dynamic of the Mafia, the don has the ability to determine what the flunky does.

- Congress passes a law banning partial birth abortion. Most people would obey such a law, even though many people vocally oppose the law now and presumably would do so then. Presumably they would do so because they anticipate that they would be punished for violating the law. But why would the punishers (the legal system, the prison guards,) carry out that punishment? [The potential punishers may oppose partial birth abortion, but they are not punishing anybody now, before such a law has passed, so why should they do so after it is passed?] Presumably they would punish because they
receive some rewards for doing so, and might be punished for not doing so. Those punishers' expectations of their own rewards and punishments must also be linked to the votes of the legislators. The legislators, in voting, have changed nothing in the world (the act of voting is symbolic). Yet their majority votes would create expectations around the country that would shift the behavior of different groups (potential violators of the law, the police). The legislature has authority over partial birth abortion because so many people around the country would react to their apparently symbolic act of voting.

- After a period of prolonged and substantive public debate, a super majority of the citizens of the United States conclude that key principles of the U.S. constitution should be changed to reflect changed circumstances. Rather than follow the cumbersome legal rules in place for amending the constitution, all branches of government simply accept the change de facto and act as if the legal rules have been followed. Opponents as well as supporters of the change rapidly accept the change as irreversible in the light of the fact that a substantial majority supported the change after a prolonged and substantive public debate about the change. In this circumstance, we would say that the supermajority do indeed have authority over the constitution. Ackerman (1991) has argued that throughout the history of the United States, "we the people" have, on three occasions, exercised that authority in violation of established legal procedures: first, when the constitution was passed in a way that violated the Articles of Confederation; second, when the Reconstruction amendments were accepted as legitimate following the Civil War, in violation of the established amendment procedure; and third, during the New Deal, when economic provisions of the constitution were summarily reversed by the court in response to political pressure, without any legal amendment process. In Ackerman's account of the U.S. constitution, formal legal rules are ignored but there is a consistent understanding about where authority lies.

Each of these examples concerns a strategic coordination problem. John's friends wish to go to the same restaurant as the others. The Mafia flunky is concerned about what his Mafia colleagues might do to him (and they in turn are concerned about what their colleagues will do to them ...). Doctors and policeman alike are aware that other peoples' actions will respond to the passing of a partial
birth abortion ban. The most adamant critics of the Constitutional Convention, the Bill of Rights and new economic interpretation of the constitution during the New Deal understood that once most people and institutions had coordinated on accepting them, there was no point in arguing about them.

In each case, authority is exercised through cheap talk. John’s proposal, the don’s instruction, Congress’s law and the Constitutional Convention’s draft constitution do not change the set of feasible actions or rewards of the strategic players in these examples. But the cheap talk is effective nonetheless, by coordinating expectations.

Our purpose in this section is to provide formal definitions of what we mean by authority. We do not seek to describe the “Game of Life,” but rather try to define authority in a simple strategic environment. There are $N$ individuals, $1, \ldots, N$. Each individual $i$ observes a signal $s_i \in S_i$; we write $S = S_1 \times \cdots \times S_i$. After observing their signals, each individual $i$ has the opportunity to send a message $m_i \in M_i$; we write $M = M_1 \times \cdots \times M_i$. After observing all the messages, each individual $i$ has the opportunity to choose an action $a_i \in A_i$; we write $A = A_1 \times \cdots \times A_i$.

Later (in section 4), we will formally describe the strategic interaction among individuals in this environment. To do this, we must specify the individuals’ payoffs and analyze their talking strategies (what they say as a function of their private signals) and their action strategies (what they do as a function of their private signals and the public messages). For now, we are interested only in defining authority. Roughly speaking, we want to ask who has the ability to influence outcomes simply by what he says. Authority depends only on the individuals’ action strategies. Individual $i$’s action strategy is a mapping, $\alpha_i : S_i \times M \to A_i$; we write $\alpha$ for the collection $\{\alpha_i\}_{i=1}^N$.

We can illustrate the notation introduced so far with the restaurant choice example discussed in the introduction. There are three individuals, 1, 2 and 3 and three restaurants, a Bulgarian (B), a Catalan (C) and a Dutch (D). Each individual has some private information about his mood. For simplicity, suppose that each individual has a ranking of the three restaurants, so $S_1 = S_2 = S_3 = \{BCD, BDC, CBD, CDB, DBC, DCB\}$. Each individual sends a message proposing where to have dinner, so $M_1 = M_2 = M_3 = \{B, C, D\}$. Each individual then decides (simultaneously) which restaurant to go to.

**Example 1.** All individuals go to the restaurant proposed by individual 1. Thus $\alpha_i(s_i, m) = m_1$ for all $i, s_i, m$. 

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In this case, individual 1 can be said to have authority over which restaurant they will meet at. Notice that individual 1 does not (under these action strategies) have the absolute authority to decide where each individual will go. His authority is restricted to their common restaurant choice. More generally, we can introduce the following definitions.

**Definition 1.** Individual $k$ has absolute authority [under $\alpha$] if for each $a \in A$, there exists $m^*_k \in M_k$, such that $\alpha_i (s_i, (m^*_k, m_{-k})) = a_i$ for all $i = 1, \ldots, N$, $s \in S$ and $m_{-k} \in M_{-k}$.

In words, individual $k$ has absolute authority under the action strategy $\alpha$ if for any prespecified vector of actions $a$, there is a message that $k$ can send such that, regardless of what messages other agents might send, the action strategy $\alpha$ results in agents taking the prespecified actions $a$. An individual might have authority over some subset of actions agents might take, but not all:

**Definition 2.** Individual $k$ has authority [under $\alpha$] over $A^* \subset A$ if for all $a \in A^*$, there exists $m^*_k \in M_k$, such that $\alpha_i (s_i, (m^*_k, m_{-k})) = a_i$ for all $i = 1, \ldots, N$, $s \in S$ and $m_{-k} \in M_{-k}$.

In the above example, individual 1 does not have absolute authority, but he does have authority over $A^* = \{BBB, CCC, DDD\}$.

More generally, an individual might have authority over a subset of actions, but not the specific action within the subset.

**Example 2.** Restaurants $B$ and $C$ are downtown, while restaurant $D$ is nearby. Individual 1 has authority over where they eat, but individual 2 is the expert on downtown restaurants. Thus

$$
\alpha_i (s_i, m) = \begin{cases} 
B, & \text{if } m_1 = B \text{ or } C; \text{ and } m_2 = B, \\
C, & \text{if } m_1 = B \text{ or } C; \text{ and } m_2 = C, \\
D, & \text{if } m_1 = D,
\end{cases}
$$

for all $i, s_i, m$.

Thus individual 1 has authority over $DDD$, he can only ensure that either $BBB$ or $CCC$ occurs. Writing $X$ for a collection of subsets of $A$, an individual has authority over $X$ if for any $X \in X$, he can ensure that some element of $X$ occurs.
Definition 3. Individual \( k \) has authority over \( X \) [under \( \alpha \)] if for all \( X \in X \), there exists \( m^*_k \in M_k \), such that \( \{ \alpha_i (s_i, (m^*_k, m_{-k})) \}_{i=1}^N \in X \) for all \( s \in S \) and \( m_{-k} \in M_{-k} \).

Notice that under the action strategies of example 2, individuals 1 and 2 between them can determine everyone's actions. In this case, we say that authority is vested in the group of individuals \( \{1, 2\} \). More generally:

Definition 4. Group \( G \subseteq \{1, \ldots, N\} \) has authority (under \( \alpha \)) over \( A^* \) if for all \( a \in A^* \), there exists \( m^*_G \) such that \( \alpha_i (s_i, (m^*_G, m_{-G})) = a \) for all \( i = 1, \ldots, N \), \( s \in S \) and \( m_{-G} \in M_{-G} \).

This definition referred to a named group \( G \). Authority may also be dispersed among different groups who have the capacity to exercise authority, as under majority rule.

Example 3. All individuals go to the restaurant chosen by the majority (and if they all announce different restaurant, each person goes to the restaurant announced). Thus \( \alpha_i (s_i, m) = \begin{cases} X, & \text{if } \# \{ j \in \{1, 2, 3\} : m_j = X \} = 2 \\ m, & \text{otherwise} \end{cases} \), for all \( i, s_i, m \).

Definition 5. The majority has authority [under \( \alpha \)] over \( A^* \) if for all \( a \in A^* \) and all \( G \subseteq \{1, \ldots, N\} \) with \( \#G > \frac{N}{2} \), there exists \( m^*_G \in M_G \), such that \( \alpha_i (s_i, (m^*_G, m_{-G})) = a \) for all \( i = 1, \ldots, I \), \( s \in S \) and \( m_{-G} \in M_{-G} \).

An individual may also have authority most, but not all, of the time.

Example 4. All individuals follow individual 1's proposal all the time, except that if individual 3 has a yen for Bulgarian food, he will go to the Bulgarian restaurant regardless. Thus

\[ \alpha_i (s_i, m) = \begin{cases} B, & \text{if } i = 3 \text{ and } s_3 \in \{BCD, BDC\} \\ m, & \text{otherwise} \end{cases} \]

In this case, individual 1 has authority at all states \( s \), except those where \( s_3 \in \{BCD, BDC\} \).
**Definition 6.** Individual $k$ has authority [under $\alpha$] over $A^*$ in $S^*$ if for all $a \in A^*$, there exists $m_k^* \in M_k$, such that $\alpha_i(s_i,(m_k^*,m_{-k})) = a$ for all $i = 1, ..., I$, $s \in S^*$ and $m_{-k} \in M_{-k}$.

We have defined above what it means for a group to have authority over a subset of actions, and what it means for a majority to have authority. Institutions of interest such as courts and legislatures combine these two elements: the group as a whole has authority over some subset of actions, and within the group, a majority has authority.

**Example 5.** The members of an academic department at a university have an annual dinner. The restaurant will be decided upon by a majority of the executive committee of the department. Thus $\alpha_i(s_i,m) = \begin{cases} X, & \text{if } \# \{j \in E : m_j = X\} > \frac{\#E}{2} \\ m_i, & \text{otherwise} \end{cases}$ for all $i, s_i, m$, where $E \subseteq N$ is set of members of the department that are on the executive committee.

**Definition 7.** A group of individuals $L$ is a majority-rule legislature with authority over $X$ [under $\alpha$] if for all $X \in X$, and any $F \subseteq L$ with $\#F > \frac{1}{2}\#L$, there exists $m_F^* = (m_k^*)_{k \in F} \in \Pi_{k \in F} M_k$, such that $\{\alpha_i(s_i,(m_F^*,m_{-F}))\}_{i=1}^N \in X$ for all $s \in S$ and $m_{-F} \in M_{-F}$. We call $L$ a proper legislature with authority over $X$ if $L$ is a legislature and for any $L' \subsetneq L$, $L'$ is not a legislature with authority over $X$.

There are analogous definitions for supermajority legislatures, and legislatures with other decision rules.

The definitions of authority in this section are primarily to illustrate the notion of authority as a coordination mechanism, with a focus on the cheap talk nature of authority. We have provided some formal definitions of authority in a simple two stage environment (one round of simultaneous cheap talk, followed by a round of simultaneous actions). There are straightforward extensions of many of these definitions to dynamic settings with choices in a sequence of periods. Other natural extensions would involve generalizing the environment in other ways, some of which will be discussed in section 4.

3. The Evolution of Authority

"Even the simplest society has norms, tacit or explicit, that evolve from the needs of the society before there are judges or other officials."
When a customary norm is violated to someone’s injury in a simple, “prelegal” society, the instinct of the victim or his family to take revenge is activated. Tacit norms enforced by the threat of revenge are the rudimentary form of a legal system; or, if one prefers, a forerunner to it, for it is unimportant from a practical standpoint whether one calls a system of enforcing customary norms through revenge “law” or “prelaw.” What is important is that the grave drawbacks of a revenge system make it intolerable except in the smallest or most primitive societies. There are huge advantages to having specialists in the creation and enforcement of norms, and as soon as society can afford them these specialists emerge.

The history of law in forms recognizable to us (that is, of publicly declared and enforced norms) is to a significant degree one of increasing specialization in the performance of legal tasks. In the first stage after the pure system of private revenge, a chief or king, or perhaps even a popular assembly, will legislate and adjudicate as undifferentiated aspects of governing. Gradually these functions are hived off to specialists, but even before that happens, the system of social control will stand in marked contrast to that of a prelegal culture, where the victim is both the adjudicator and the enforcer.”


According to Posner’s account of the evolution of legal institutions, specialization arises because of the large efficiency benefits of specialization. One important specialized role in legal systems are the chiefs/judges/assemblies who may judge individual cases, summarize past cases into “common law,” and pronounce principles for future judgements (i.e., legislate). From our perspective of law as cheap talk, the question of who tends to become the chief or judge reduces to the question of whose cheap talk statements are going to have most authority in an evolving process. In this section, we give one example to illustrate how formal study of this question might proceed.

There are two possible actions $B$ and $C$ and three individuals $1$, $2$ and $3$. All receive payoff 0 if they do not all choose the same action, 1 if they co-ordinate on the less good action, and 2 if they co-ordinate on the best action. In each period, there is an independent draw of which is the best action. Each player $i$ observes a conditionally independent signal that is correct with probability $q_i > \frac{1}{2}$.
In each period $t$, we are in one of four states, $\{M, 1, 2, 3\}$ corresponding to majority rule and players 1, 2 and 3 respectively having authority. In state $M$, each player announces his signal of the best action and chooses the action recommended by the majority. If that action is indeed the best, or if everyone recommended that action, they stay in state $M$. If not, they switch to the state where the dissenting player has authority. In state $i$, each player announces his signal of the best action and chooses the action proposed by individual $i$. If that action is indeed the best, or if it wasn't but a majority recommended it, they stay in state $i$. But if the majority recommended the better action, they switch to state $M$.

The transition matrix between the four states, $\{M, 1, 2, 3\}$, is:

\[
\begin{pmatrix}
1 - q_1 (1 - q_2) (1 - q_3) \\
-q_2 (1 - q_1) (1 - q_3) \\
q_3 (1 - q_1) (1 - q_2) \\
q_1 (1 - q_2) (1 - q_3) \\
q_2 (1 - q_1) (1 - q_3) \\
q_3 (1 - q_1) (1 - q_2)
\end{pmatrix}
\begin{pmatrix}
(1 - q_1) q_2 q_3 \\
(1 - q_2) q_1 q_3 \\
(1 - q_3) q_1 q_2 \\
1 - (1 - q_1) q_2 q_3 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
1 - (1 - q_2) q_1 q_3 \\
0 \\
(1 - q_3) q_1 q_2
\end{pmatrix}
\]

Writing $q_i^* = \frac{q_i}{1 - q_i}$, the steady state distribution of this Markov process is:

\[
\frac{1}{q_1^* q_2^* q_3^* + (q_1^*)^2 + (q_2^*)^2 + (q_3^*)^2}
\begin{pmatrix}
q_1^* q_2^* q_3^* \\
(q_1^*)^2 \\
(q_2^*)^2 \\
(q_3^*)^2
\end{pmatrix}
\]

Thus well informed individuals develop authority more often than informed individuals. But individual authority emerges even when it would be more efficient to have majority rule.

4. Authority in Equilibrium

We spent much time defining authority in section 2, without reference to key features of the strategic environment, e.g., the individuals' payoffs and talking strategies of the individuals. This was deliberate: we wanted to define what it means to have authority before discussing who has it and why. In the previous section, we examined how authority would evolve in a simply dynamic setting.
In this section, we identify which authority structures are consistent with equilibrium. To do this, we must specify payoffs. Let \( \pi \in \Delta(S) \) be a common prior. Let individual \( i \) have a payoff function \( u_i : A \times S \to \mathbb{R} \); write \( u \) for the collection \( \{u_i\}_{i=1}^N \). Individual \( i \)’s talking strategy is a function \( \tau_i : S_i \to M_i \); write \( \tau \) for the collection \( \{\tau_i\}_{i=1}^N \). We also need to specify an individual’s beliefs about other individuals’ signals, given the messages they have received (we will later discuss where these beliefs come from). Thus let \( \mu_i : S_i \times M \to \Delta(S_{-i}) \); write \( \mu \) for the collection \( \{\mu_i\}_{i=1}^N \), and write \( \mu_i(s_{-i}|s_i,m) \) for the probability that individual \( i \) assigns to signal profile \( s_{-i} \), if he observes signal \( s_i \) and message profile \( m \).

**Definition 8.** \((\tau, \mu, \alpha) \) is a perfect Bayesian Nash equilibrium of the game \((\pi, u)\) if [1] actions are optimal given beliefs, i.e., for each \( i = 1, \ldots, N \) and each \( s_i \in S_i \), \( m \in M \) and \( a_i \in A_i \),

\[
\sum_{s_{-i} \in S_{-i}} \mu_i(s_{-i}|s_i,m)u_i\left((\alpha_j(s_j,m))_{j=1}^N, s\right) \\
\geq \sum_{s_{-i} \in S_{-i}} \mu_i(s_{-i}|s_i,m)u_i\left((a_i, (\alpha_j(s_j,m))_{j \neq i}), s\right);
\]

[2] messages are optimal, i.e., for each \( i = 1, \ldots, N \) and each \( s_i \in S_i \) and \( m_i \in M_i \),

\[
\sum_{s_{-i} \in S_{-i}} \pi_i(s_{-i}|s_i)u_i\left((\alpha_j(s_j,m_i, (\tau_k(s_k))_{k=1}^N))_{j=1}^N, s\right) \\
\geq \sum_{s_{-i} \in S_{-i}} \pi_i(s_{-i}|s_i)u_i\left((\alpha_j(s_j,m_i, (\tau_k(s_k))_{k \neq i}))_{j=1}^N, s\right);
\]

[3] beliefs are derived from talking strategies, i.e., for each \( i = 1, \ldots, N \) and \( m \in M \), if \( Z_i(m_{-i}) = \{s_{-i} : m_j = \tau_j(s_j) \text{ for all } j \neq i\} \neq \emptyset \),

\[
\mu_i(s_{-i}|s_i,m) = \frac{\pi(s_i, s_{-i})}{\sum_{s'_{-i} \in Z_i(m_{-i})} \pi(s_i, s'_{-i})}.
\]

**The Determinants of Authority** These formal conditions are rather involved, and so we consider several examples to make some simple points about how authority might arise in equilibrium,
Example 6. Coordination First. The most important objective of each individual is to coordinate with all the others, and he receives payoff 0 if they fail to coordinate. However, individuals' have preferences over which restaurant to meet in. Each individual gets payoff 1 if they coordinate on his least favorite restaurant, 2 if they coordinate on his second favorite restaurant, and 3 if they coordinate on his favorite restaurant. Thus,

\[ u_i(a, s) = \begin{cases} 
3, & \text{if } a_1 = a_2 = a_3 = s_1, \\
2, & \text{if } a_1 = a_2 = a_3 = s_2 \\
1, & \text{if } a_1 = a_2 = a_3 = s_3 \\
0, & \text{otherwise}
\end{cases} \]

We will be interested in authority over which restaurant the three individuals go to. If these are the underlying preferences, it is easy to see that there are no restrictions on how authority is allocated. For any player \( k \), there is an equilibrium where each player announces his most preferred restaurant, \( r_i(s_i) = s_1 \), and goes to the restaurant recommended by player \( k \), \( a_i(s_i, m) = m_k \).

We can express the idea that there are no restrictions on how authority is allocated by the following definition:

**Definition 9.** Authority over \( A^* \subset A \) is indeterminate if for any group \( G \), there is an equilibrium where group \( G \) has authority.

In the spirit of the restaurant example, we can describe simple sufficient conditions for indeterminate authority.

**Definition 10.** Action profile \( a^* \in A \) is a state-independent Nash equilibrium if for all \( s \in S \) and \( a_i \in A_i \),

\[ u_i(a^*, s) \geq u_i((a_i, a^*_{-i}), s) . \]

**Remark 4.1.** If \( A^* \) consists of state-independent Nash equilibria, then authority over \( A^* \) is indeterminate.

Any group can have authority over \( A^* \) when \( A^* \) consists of state-independent Nash equilibria because in a sense, there is nothing to distinguish groups. Regardless of which group suggests action profile \( a^* \in A^* \), it is an equilibrium for each player \( i \) to play \( a_i^* \) if all other players do. For authority not to be indeterminate,
players must be differentially informed. When a player has better (or different) information than other players, it may be an equilibrium for all players to follow that better informed player’s suggestions, but not to follow the suggestions of less informed players.

**Example 7.** Consider a game of common interests \( u_i(a, s) = v(a, s) \) for all \( i = 1, \ldots, N \) where player 1 is more informed than all other players in the sense that he knows the state \( s \) while others may not. There is a (ex ante) Pareto dominant equilibrium where player 1 has authority over

\[
\left\{ a : a \in \arg \max_{a' \in A} v(a', s) \text{ for some } s \right\}.
\]

Characteristics other than informativeness may affect who can have authority. Authority is the ability to affect, through messages, what actions should be taken. In our restaurant example, an individual who refuses to go to any restaurant except the one he suggests may have authority over which restaurant a group will go to, while his stubbornness prevents any other individual from having that authority.

**Example 8.** Suppose player 1’s payoffs depend on his own type alone \( (u_1(a, s) = \tilde{u}_1(a_1, s_1)) \) while all other types of all other players are more interested in coordinating than in the choice of action \( (u_i(a, s) > u_i(a', s') \text{ if } i \neq 1, a_1 = a_2 = \ldots = a_N \text{ and } a'_j \neq a'_i \text{ for some } j \text{ and } k) \). Then there is an equilibrium where player 1 has authority over \( A \).

5. Maintaining Authority

Courts often face a trade-off between doing what they think is right and “maintaining their authority.” The Supreme Court during the Second World War faced the difficult choice of declaring the internment unconstitutional, in which they might well have been ignored and “lost authority,” or investing in their authority by making a dubious decision. In the 1950’s, the Supreme Court’s positions on school desegregation were heavily influenced by the loss of authority the court would suffer if its decision were ignored. Commentators on the 2000 Florida vote count speculated that (whatever the legal niceties) the U.S. public would accept the authority of the U.S. Supreme Court as ultimate arbiter of the dispute but
would not accept the authority of the Florida Supreme Court. If this were true, there would be argument for the U.S. Supreme Court finding a pretext to enter the dispute (even if they did not have jurisdiction) in order to maintain the authority of the judiciary as a whole. Conventional formal accounts of authority have a hard time accounting for these arguments. Our equilibrium interpretation of authority offers a natural way of interpreting these arguments.

In this section, we attempt a preliminary analysis of the dynamics of authority over a sequence of decisions. For this section, we will consider a distinguished individual or group that we call a court. The basic message is simple: whether a court has authority depends on individuals' beliefs about whether other individuals will “respect” the authority of the court. A lack of common knowledge among individuals about the environment allows a rich discussion of court's authority. We will describe a simple environment that we will use in a sequence of examples to illustrate the central issues.

We consider a situation with a court and a continuum of people indexed by $i \in [0, 1]$. Society faces a sequence of problems. These problems are similar to coordination games (indeed, in our first example, the problem is precisely a coordination game). We think of there being (perhaps) returns to coordinated behavior. In the absence of a coordinating device such as law or edict, play cannot be coordinated (perhaps because there are too many possible ways to coordinate, or there is a lack of common knowledge). In the absence of an edict, players choose the default action $(D)$. If there is an edict indicating a possible coordinated behavior, then players have a choice between the default action and the indicated action, which we always denote $O$. In the case that an edict is issued, individual $i$'s utility depends on whether he chooses the indicated action or the default action, and in addition, on the proportion of other agents who choose the indicated action, which we designate $x$.

**Example 9.** The problem concerns driving. The default action is to drive fast (perhaps even dangerously so). An edict identifying a speed limit is passed. Each individual is affected by other individuals' choice of whether or not to speed in two distinct ways. First, as the proportion of those who don't speed increases, each individual's payoff increases because of the increased safety. On the other hand, if an individual chooses to speed, he is more likely to be caught himself when few others speed. In addition, if an individual speeds, he avoids the inconvenience of
driving within the speed limit. The payoff to individual } i \in [0,1] \text{ is}

\begin{align*}
\pi_i(O, x) &= (A + 1) x, \\
\pi_i(D, x) &= (A + 1) x + 1 - 2x.
\end{align*}

The term that is independent of whether } i \text{ speeds or not, } (A + 1) x, \text{ represents the increased safety that results from the proportion } x \text{ of people driving safely, where } A + 1 \text{ is the marginal value to individuals of safety. One can think of } (A + 1) \text{ as the marginal social value of safe driving. The second term in } \pi_i(D, x) \text{ represents the private returns to driving fast, } 1 \text{ is the value of avoiding the inconvenience of driving slowly, and } -2x \text{ is the expected cost of being caught when speeding, which is a decreasing function of the proportion speeding.}

It is clear that the return from driving safely is increasing in the proportion of others who drive safely, while the returns to speeding are decreasing. There is a threshold on the proportion of society who drive safely such that it is optimal for any individual to drive safely when the proportion is above this threshold and to speed if it is below this threshold. The threshold } x^* \text{ satisfies } \pi_i(O, x^*) = \pi_i(D, x^*), \text{ so that } x^* = 1/2. \text{ There are two symmetric equilibria in this example: in the first, all players speed, and in the second, no players speed.}

All individuals are identical in the example. In principle, different people have different payoff functions, and hence, different thresholds for driving safely. Furthermore, when there is a heterogeneous population, there may be uncertainty on an individual's part about how many other agents will speed.

**Example 10.** We modify the previous example by introducing a second type of individual for whom the value of avoiding the inconvenience of driving slowly strictly equals 3 (rather than 1). There are thus two types of people, prudent (P) and reckless (R). The payoffs to a prudent driver are as in the previous example,

\begin{align*}
\pi_i^P(O, x) &= (A + 1) x, \\
\pi_i^P(D, x) &= (A + 1) x + 1 - 2x,
\end{align*}

while the payoff to a reckless driver is

\begin{align*}
\pi_i^R(O, x) &= (A + 1) x, \\
\pi_i^R(D, x) &= (A + 1) x + 3 - 2x.
\end{align*}
There are two states of the world, bad (B) in which the proportion of reckless drivers in society is 2/3, and good (G), in which the proportion of reckless drivers is 1/3. Individuals do not know the state of the world, and observe only their own payoff function. The probability that the state of the world is good is $\frac{1}{4}$.

We note that for a reckless driver, speeding is a dominant strategy. Hence, either 1/3 or 2/3 of the population will speed for sure. If the state of nature is B, the maximal proportion of the population that would contemplate driving safely is 1/3, below the threshold for which prudent drivers would drive safely. Thus, if it were known that the state of nature is B, there can be no equilibrium with any individual driving safely. On the other hand, if the state of nature is known to be G, there can be an equilibrium in which some people drive safely, since prudent drivers, whose threshold is 1/2, constitute 2/3 of the population, thus making it optimal for them to drive safely if all prudent drivers do so.

But what happens when individuals don’t know whether the state is B or G? In this case each individual has his private signal, whether he is prudent or reckless, and can compute the posterior probability that the state of nature is G. The posterior probability that the state is G for an individual who has learned that he is a prudent is

$$\Pr(G|P) = \frac{\Pr(G \cap P)}{\Pr(P)} = \frac{\frac{2}{3} \left( \frac{1}{3} \right)}{\frac{2}{3} \left( \frac{1}{3} \right) + \frac{1}{3} \left( \frac{2}{3} \right)} = \frac{2}{5}.$$ 

Hence, an individual who finds himself prudent is relatively optimistic. The aggregate uncertainty about the state of nature makes it more likely that it is the good state of nature if one is prudent.

Suppose that individuals followed a strategy that they drive safely if they were prudent and speed if they were reckless. The expected utility of a prudent driver would then be

$$\frac{2}{5} \cdot (A + 1) \cdot \text{proportion of } P \text{'s given } G + \frac{3}{5} \cdot (A + 1) \cdot \text{proportion of } P \text{'s given } B = \frac{7}{15} (A + 1).$$

If a single prudent driver speeds, he receives a payoff of

$$\frac{2}{5} \cdot \{(A + 1) \cdot 2/3 + 1 - 4/3\} + \frac{3}{5} \cdot \{(A + 1) \cdot 1/3 + 1 - 2/3\}$$
\begin{equation}
\frac{7}{15} (A + 1) - \frac{1}{15}
\end{equation}

The unique equilibrium of this game has all drivers always speeding.

**Optimizing Courts** Now suppose that there is an expert court that is able to observe whether the state is good or bad. If the state is good, the court issues an edict declaring that drivers must drive prudently; if the state is bad, the court does not issue such an edict. With no changes in the drivers' payoffs, it is clear that prudent drivers will obey the edict (but drive recklessly if there is no edict) and reckless drivers will drive recklessly, whether or not there is an edict. By following this rule, the expert court is able to produce a Pareto-improvement: the reckless drivers benefit from having fewer reckless drivers on the road sometimes and the prudent drivers are able to use the edict to co-ordinate among themselves on the efficient outcome.

But would the court have an incentive to follow such a rule? Assume the court seeks to maximize the expected sum of the payoffs of prudent drivers (our argument would not be changed if the court also took into account the utility of reckless drivers). Also assume that the court has authority over prudent drivers: if the court issues an edict, prudent drivers will obey the edict; if they do not issue the edict, prudent drivers will drive recklessly. In this case, the court will clearly have an incentive to issue an edict if the state is good. But note that for a sufficiently large value of \( A \), the court will have an incentive to issue the edict even when the state is bad: if \( A > 2 \), prudent drivers receive a higher payoff when all prudent drivers follow the speed limit than they do if all speed. But a prudent driver then regrets having followed the speed limit. The problem is that if the state is bad and we restrict attention only to prudent drivers in the bad state \( B \), we see that for \( A > 2 \) they are playing a prisoners' dilemma game.

In this scenario, observing the behavior induced by court edicts reveals information about the composition of a society. The revelation of that information may be detrimental to a court's authority. Suppose we begin with the equilibrium in which the court has authority (that is, the equilibrium in which prudent drivers will follow speed limits if they are set when they do not know whether the state is \( G \) or \( B \)). Then issuing an edict specifying a speed limit reveals the state, and if it is \( B \), the court will cannot have authority in an identical problem in the future.\(^2\)

\(^2\)By identical, we mean that the payoffs are the same and the proportion of reckless drivers is the same, but now known by all.
Hence, a court that is concerned about its authority will understand that there is a cost to issuing edicts when it is uncertain as to whether it will be obeyed or not. In particular, consider the following example.

**Example 11.** A court is faced with an infinite sequence of problems, each exactly as in the previous example, example 10. In each problem, a new state is independently drawn and drivers are conditionally independently chosen to be prudent or reckless. The court discounts the future with discount rate $\delta < 1$.

Now consider the following strategies. The court bans speeding if and only if the state is good. Prudent drivers obey the court's edict if and only if the court has only issued an edict in the past when the state is good. If the court has ever issued an edict banning speeding and more than half the population has disobeyed it, the prudent drivers will never obey the court's edicts again (i.e., they will always speed in the future). Reckless drivers will always speed. For what values of $\delta$ does this strategy constitute an equilibrium? First note that since there are a continuum of prudent and reckless drivers, drivers will choose a static best response in each period. Reckless drivers have a dominant strategy to speed. Prudent drivers will speed if they expect other prudent drivers to speed. Thus drivers' strategies are equilibrium strategies for all values of $\delta$.

Now consider the incentives of the court. If the court maintains its authority by issuing a speeding ban only when the state is good, the expected utility of prudent drivers will be 0 if the state is bad, and $\frac{2}{3} (A + 1)$ if the state is good. Thus the ex ante expected payoff (before knowing the state) in any period is

$$\frac{1}{4} \left( \frac{2}{3} (A + 1) \right) = \frac{1}{6} (A + 1).$$

On the other hand, if the court has damaged its authority by issuing an edict that is obeyed by less than half the population, then everyone will always speed (whatever edicts the court announces) and the utility of all prudent drivers will be 0. Now consider the court's incentive to deviate from its strategy. This will arise if the state is bad and the court is tempted to issue the edict anyway. The utility of the prudent drivers in this circumstance will be $\frac{1}{3} (A + 1)$, instead of 0. Thus a current period gain from deviating of $\frac{1}{3} (A + 1)$ must be weighed against an expected gain in all future periods of $\frac{1}{6} (A + 1)$. The court has no incentive to deviate if

$$\frac{1}{3} (A + 1) (1 - \delta) \leq \frac{1}{6} (A + 1) \delta,$$
i.e., \( \delta \geq \frac{2}{3} \). Thus if the court is sufficiently concerned about the future, it will refrain from issuing edicts that will damage its authority (even if those edicts are optimal in terms of current outcomes). The court is thus investing in its authority by exercising restraint.

6. Conclusion

Announcing court opinions, passing legislation and endorsing constitutions are all symbolic actions that make no real changes in the world. They influence outcomes only to the extent that they coordinate peoples’ behavior, by conveying information or otherwise. Thus the relevant game theory to understand laws and authority is the theory of cheap talk in coordination games. We have sketched a number of ways in which this insight could be brought to bear on important legal issues. The analysis thus far is suggestive only, but we hope it points in the direction of interesting avenues of research.

References


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