Point-Mass Dynamics and Aerodynamic/Thrust Forces

Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2018

Learning Objectives

- Properties of atmosphere
- Frames of reference
- Velocity and momentum
- Newton's laws of motion
- Airplane axes
- Lift and drag
- Simplified equations for longitudinal motion
- Powerplants and thrust

Reading:
Flight Dynamics
Introduction, 1-27
The Earth's Atmosphere, 29-34
Kinematic Equations, 38-53
Forces and Moments, 59-65
Introduction to Thrust, 103-107

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

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The Atmosphere

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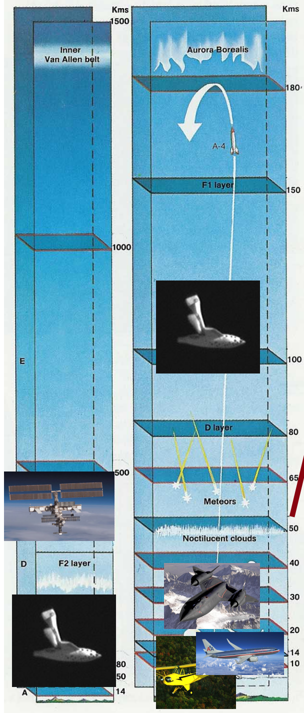
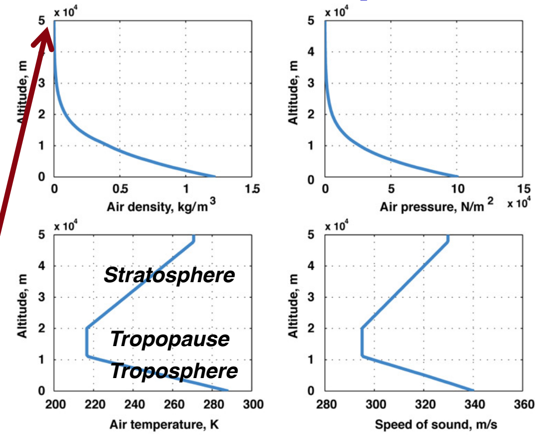


*Properties of the Lower Atmosphere**



- **Density and pressure decay exponentially with altitude**
- **Temperature and speed of sound are piecewise-linear functions of altitude**

** 1976 US Standard Atmosphere*

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Air Density, Dynamic Pressure, and Mach Number

$$\rho = \textit{Air density}, \text{ function of height}$$
$$= \rho_{sea\,level} e^{-\beta h} = \rho_{sea\,level} e^{\beta z}$$

$$\rho_{sea\,level} = 1.225\,\text{kg/m}^3; \quad \beta = 1/9{,}042\,\text{m}$$

$$V_{air} = \left[v_x^2 + v_y^2 + v_z^2\right]_{air}^{1/2} = \left[\mathbf{v}^T\mathbf{v}\right]_{air}^{1/2} = \textit{Airspeed}$$

$$\textit{Dynamic pressure} = \bar{q} = \frac{1}{2}\rho(h)V_{air}^2 \triangleq \text{"qbar"}$$

$$\textit{Mach number} = \frac{V_{air}}{a(h)}; \quad a = \text{speed of sound, m/s}$$

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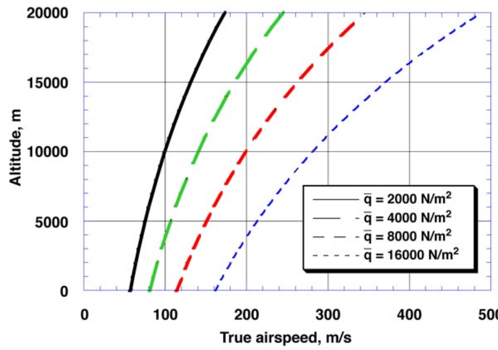
Contours of Constant Dynamic Pressure, $\bar{q}$

- In steady, cruising flight, $Weight = Lift = C_L \frac{1}{2} \rho V_{air}^2 S = C_L \bar{q} S$



True airspeed must increase as altitude increases

to maintain constant dynamic pressure

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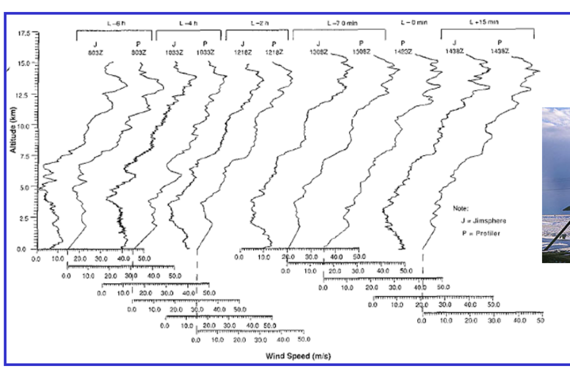
Wind: Motion of the Atmosphere

Zero wind at Earth's surface = Rotating air mass

Wind measured with respect to Earth's <u>rotating</u> surface

Wind Velocity Profiles vary over Time



Airspeed = Airplane's speed with respect to air mass

Earth-relative velocity = Wind velocity ± True airspeed *[vector]*

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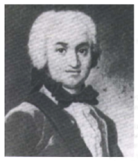
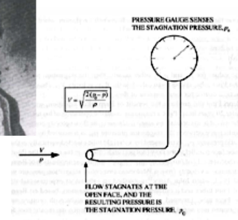
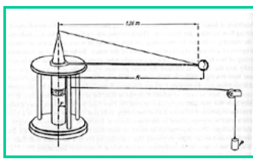
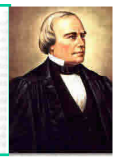
Historical Factoids

- **Henri Pitot: Pitot tube (1732)**

- **Benjamin Robins: Whirling arm "wind tunnel" (1742)**

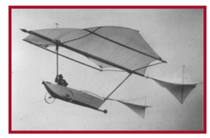



NASA SP-440, Wind Tunnels of NASA

Sir George Cayley

- **Sketched "modern" airplane configuration (1799)**
- **Hand-launched glider (1804)**

- **Applied aerodynamics (1809-1810)**
- **Triplane glider carrying 10-yr-old boy (1849)**
- **Monoplane glider carrying coachman (1853)**
  - Cayley's coachman had a steering oar with cruciform blades

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Equations of Motion for a Particle (Point Mass)

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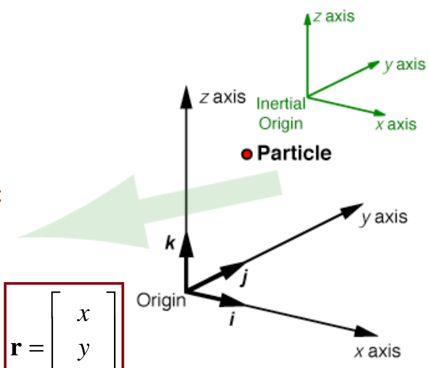
Newtonian Frame of Reference

- Newtonian (Inertial) Reference Frame
  - Unaccelerated Cartesian frame: origin referenced to inertial (non-moving) frame
  - Right-hand rule
  - Origin can translate at constant linear velocity
  - Frame cannot rotate with respect to inertial origin
- **Position: 3 dimensions**
- **What is a non-moving frame?**

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



Translation changes the position of an object

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Velocity and Momentum

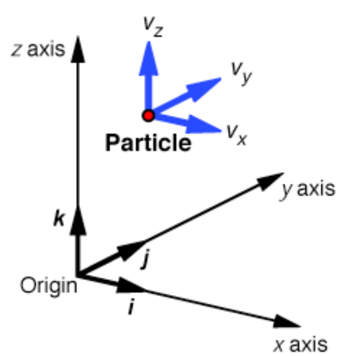
- **Velocity of a particle**

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

- **Linear momentum of a particle**

$$\mathbf{p} = m\mathbf{v} = m \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$where \quad m = mass\ of\ particle$$



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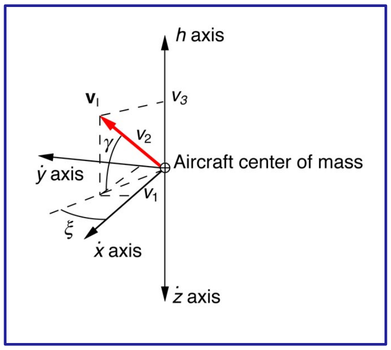
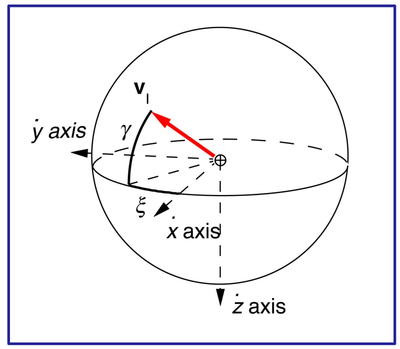
Inertial Velocity Expressed in Polar Coordinates

Polar Coordinates



Projected on a Sphere



γ : Vertical Flight Path Angle, rad or deg

ξ : Horizontal Flight Path Angle (Heading Angle), rad or deg

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Newton's Laws of Motion: Dynamics of a Particle

- **First Law: If no force acts on a particle,**
  - **it remains at rest or**
  - **continues to move in a straight line at constant velocity, as observed in an inertial reference frame**
  - **Momentum is conserved**

$$\frac{d}{dt}(m\mathbf{v}) = 0 \quad ; \quad m\mathbf{v}\big|_{t_1} = m\mathbf{v}\big|_{t_2}$$

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Newton's Laws of Motion: Dynamics of a Particle

- **Second Law**: Particle of fixed mass acted upon by a force
  - changes velocity with acceleration proportional to and in direction of force, as observed in inertial frame
  - Mass is ratio of force to acceleration of particle:

$$\mathbf{F} = m\mathbf{a}$$

$$\frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = \mathbf{F} \quad ; \quad \mathbf{F} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

$$\therefore \quad \frac{d\mathbf{v}}{dt} = \frac{1}{m}\mathbf{F} = \frac{1}{m}\mathbf{I}_3\mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

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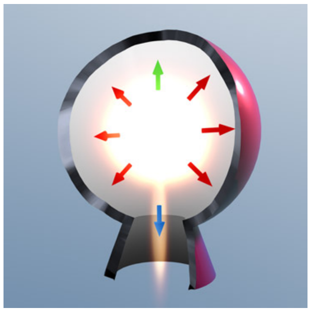
Newton's Laws of Motion: Dynamics of a Particle

- **Third Law**
  - For every action, there is an equal and opposite reaction



Force on Rocket = Force on Exhaust Gasses

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Equations of Motion for a Particle: Position and Velocity

Force vector

$$\mathbf{F}_I = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_I = \left[\mathbf{F}_{gravity} + \mathbf{F}_{aerodynamics} + \mathbf{F}_{thrust}\right]_I$$

Rate of change of velocity

$$\frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix}_I = \frac{1}{m}\mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_I$$

Rate of change of position

$$\frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_I = \mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_I$$

Integration for Velocity with Constant Force

Differential equation

$$\frac{d\mathbf{v}(t)}{dt} = \dot{\mathbf{v}}(t) = \frac{1}{m}\mathbf{F} = \begin{bmatrix} f_x/m \\ f_y/m \\ f_z/m \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Integral

$$\mathbf{v}(T) = \int_0^T \frac{d\mathbf{v}(t)}{dt}dt + \mathbf{v}(0) = \int_0^T \frac{1}{m}\mathbf{F}\,dt + \mathbf{v}(0) = \int_0^T \mathbf{a}\,dt + \mathbf{v}(0)$$

$$\begin{bmatrix} v_x(T) \\ v_y(T) \\ v_z(T) \end{bmatrix} = \int_0^T \begin{bmatrix} f_x/m \\ f_y/m \\ f_z/m \end{bmatrix} dt + \begin{bmatrix} v_x(0) \\ v_y(0) \\ v_z(0) \end{bmatrix} = \int_0^T \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} dt + \begin{bmatrix} v_x(0) \\ v_y(0) \\ v_z(0) \end{bmatrix}$$

Integration for Position with Varying Velocity

Differential equation

$$\frac{d\mathbf{r}(t)}{dt} = \dot{\mathbf{r}}(t) = \mathbf{v}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{bmatrix}$$

Integral

$$\mathbf{r}(T) = \int_0^T \frac{d\mathbf{r}(t)}{dt} dt + \mathbf{r}(0) = \int_0^T \mathbf{v}(t) dt + \mathbf{r}(0)$$

$$\begin{bmatrix} x(T) \\ y(T) \\ z(T) \end{bmatrix} = \int_0^T \begin{bmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{bmatrix} dt + \begin{bmatrix} x(0) \\ y(0) \\ z(0) \end{bmatrix}$$

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Gravitational Force: Flat-Earth Approximation

- Approximation
  - Flat earth reference assumed to be inertial frame, *e.g.*,
    - North, East, Down
    - Range, Crossrange, Altitude (–)
- g is gravitational acceleration
- *m*g is gravitational force
- Independent of position
- *z* measured down

$$\left(\mathbf{F}_{gravity}\right)_I = \left(\mathbf{F}_{gravity}\right)_E = m\mathbf{g}_E = m\begin{bmatrix} 0 \\ 0 \\ g_o \end{bmatrix}_E$$

$$g_o \simeq 9.807 \text{ m/s}^2 \text{ at earth's surface}$$

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Flight Path Dynamics, Constant Gravity, No Aerodynamics

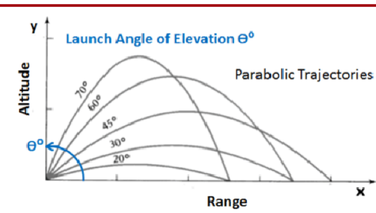
$$v_x(0)=v_{x_0}$$
$$v_z(0)=v_{z_0}$$
$$x(0)=x_0$$
$$z(0)=z_0$$

Differential equation

$$\dot{v}_x(t)=0$$
$$\dot{v}_z(t)=-g \quad (z \text{ positive } \textbf{up})$$
$$\dot{x}(t)=v_x(t)$$
$$\dot{z}(t)=v_z(t)$$

Integral

$$v_x(T)=v_{x_0}$$
$$v_z(T)=v_{z_0}-\int_0^T g\,dt=v_{z_0}-gT$$
$$x(T)=x_0+v_{x_0}T$$
$$z(T)=z_0+v_{z_0}T-\int_0^T gt\,dt=z_0+v_{z_0}T-gT^2/2$$



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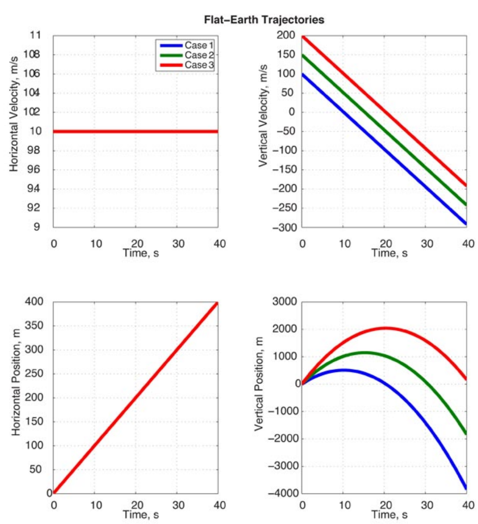
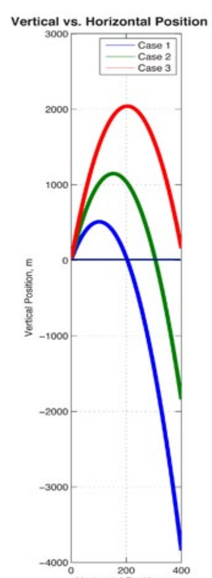
Flight Path with Constant Gravity and No Aerodynamics





Aerodynamic Force on an Airplane

Earth-Reference Frame	Body-Axis Frame	Wind-Axis Frame
$\mathbf{F}_I = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_E = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_E \frac{1}{2}\rho V_{air}^2 S = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_E \bar{q} S$	$\mathbf{F}_B = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_B \bar{q} S$	$\mathbf{F}_V = \begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} \bar{q} S$
Referenced to the Earth, not the aircraft (e.g., North, East, Down)	Aligned with the aircraft axes (e.g., Nose, Wing Tip, Down)	Aligned with and perpendicular to the velocity vector (Forward, Sideward, Down)

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Non-Dimensional Aerodynamic Coefficients

Body-Axis Frame	Wind-Axis Frame
$\begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_B = \begin{bmatrix} \text{axial force coefficient} \\ \text{side force coefficient} \\ \text{normal force coefficient} \end{bmatrix}$	$\begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} = \begin{bmatrix} \text{drag coefficient} \\ \text{side force coefficient} \\ \text{lift coefficient} \end{bmatrix}$

- Functions of flight condition, control settings, and disturbances, e.g., $C_L = C_L(\delta, M, \delta E)$
- Non-dimensional coefficients allow application of sub-scale model wind tunnel data to full-scale airplane

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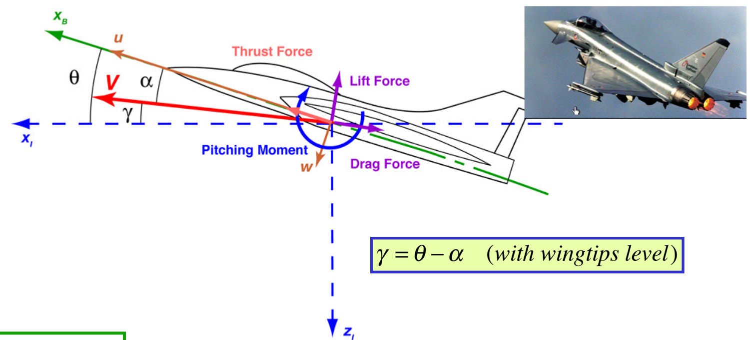
Longitudinal Variables



$$\gamma = \theta - \alpha \quad (\textit{with wingtips level})$$

- $u(t)$: axial velocity
- $w(t)$: normal velocity
- $V(t)$: velocity magnitude
- $\alpha(t)$: angle of attack
- $\gamma(t)$: flight path angle
- $\theta(t)$: pitch angle

- along vehicle centerline
- perpendicular to centerline
- along net direction of flight
- angle between centerline and direction of flight
- angle between direction of flight and local horizontal
- angle between centerline and local horizontal

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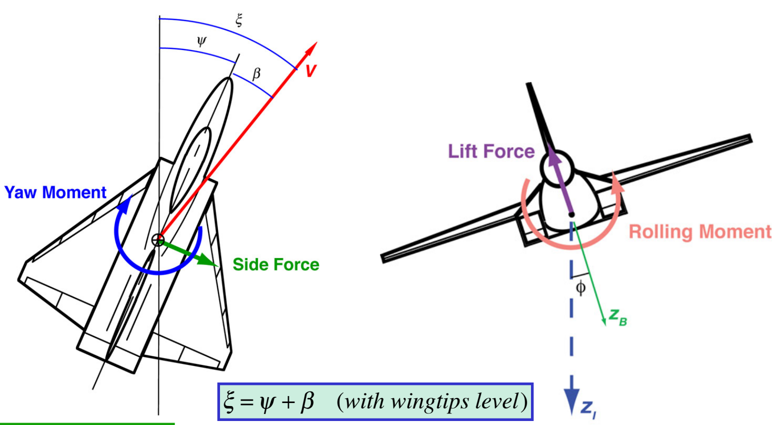
Lateral-Directional Variables



$$\xi = \psi + \beta \quad (\textit{with wingtips level})$$

- $\beta(t)$: sideslip angle
- $\psi(t)$: yaw angle
- $\xi(t)$: heading angle
- $\phi(t)$: roll angle

- angle between centerline and direction of flight
- angle between centerline and local horizontal
- angle between direction of flight and compass reference (e.g., north)
- angle between true vertical and body *z* axis

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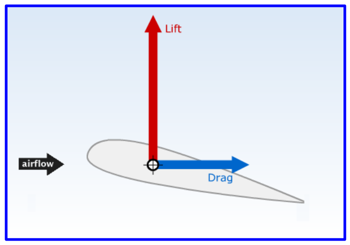
Introduction to Lift and Drag



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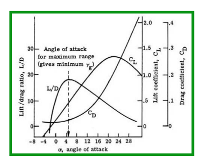
Lift and Drag are Referenced to Velocity Vector



$$Lift = C_L \frac{1}{2}\rho V_{air}^2 S \approx \left[C_{L_0} + \frac{\partial C_L}{\partial \alpha}\alpha\right]\frac{1}{2}\rho V_{air}^2 S$$

- **Lift components sum to produce total lift**
  - **Perpendicular to velocity vector**
  - **Pressure differential between upper and lower surfaces**

$$Drag = C_D \frac{1}{2}\rho V_{air}^2 S \approx \left[C_{D_0} + \varepsilon C_L^2\right]\frac{1}{2}\rho V_{air}^2 S$$

- **Drag components sum to produce total drag**
  - **Parallel and opposed to velocity vector**
  - **Skin friction, pressure differentials**

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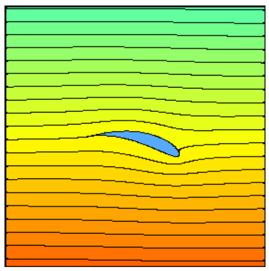
2-D Aerodynamic Lift

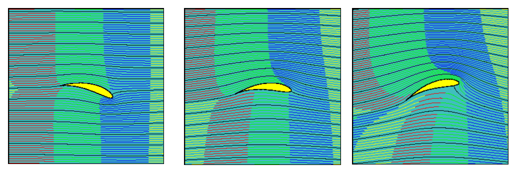
$$Lift = C_L \frac{1}{2}\rho V_{air}^2 S \approx \left(C_{L_{wing}} + C_{L_{fuselage}} + C_{L_{tail}}\right)\frac{1}{2}\rho V_{air}^2 S \approx \left[C_{L_0} + \frac{\partial C_L}{\partial \alpha}\alpha\right]\bar{q}S$$

- **Upper/lower speed difference proportional to angle of attack**
- **Stagnation points at leading and trailing edges**
  - **<u>Kutta condition</u>: Aft stagnation point at sharp trailing edge**

<u>Streamlines</u>
Instantaneous tangent to velocity vector



<u>Streaklines</u>
Locus of particles passing through points
Dye injected at fixed points



http://www.diam.unige.it/~irro/profilo_e.html

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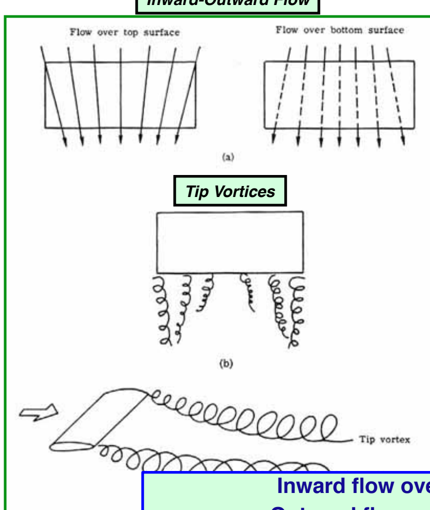
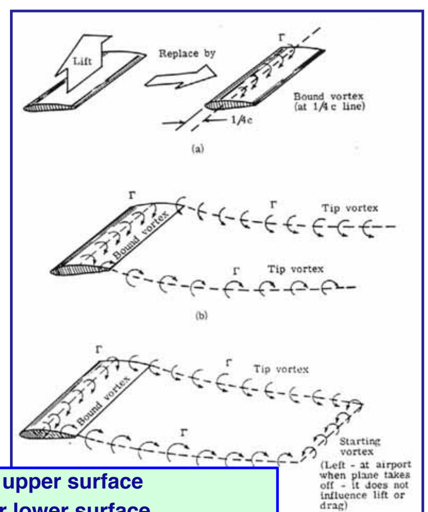
3-D Lift

Inward-Outward Flow





Inward flow over upper surface
Outward flow over lower surface
Bound vorticity of wing produces tip vortices

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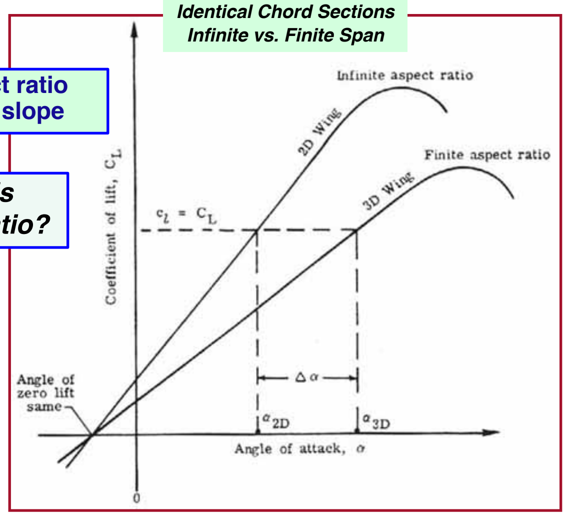
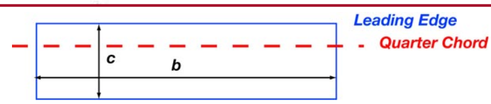
2-D vs. 3-D Lift

Finite aspect ratio reduces lift slope

What is aspect ratio?





Talay, NASA-SP-367

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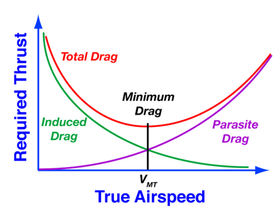
Aerodynamic Drag

$$Drag = C_D \frac{1}{2}\rho V_{air}^2 S \approx \left(C_{D_p} + C_{D_w} + C_{D_i}\right)\frac{1}{2}\rho V_{air}^2 S \approx \left[C_{D_0} + \varepsilon C_L^{\;2}\right]\bar{q}S$$

- **Drag components**
  - **Parasite drag (friction, interference, base pressure differential)**
  - **Wave drag (shock-induced pressure differential)**
  - **Induced drag (drag due to lift generation)**

- **In steady, subsonic flight**
  - **Parasite (form) drag increases as V^2**
  - **Induced drag(due to lift) proportional to $1/V^2$**
  - **Total drag minimized at one particular airspeed**



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2-D Equations of Motion with Aerodynamics and Thrust

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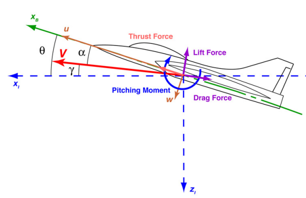
2-D Equations of Motion for a Point Mass



- Motions restricted to vertical plane
- Inertial frame, wind = 0
- *z* positive <u>down</u>, flat-earth assumption
- Point-mass location coincides with aircraft's center of mass

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_z \\ f_x / m \\ f_z / m \end{bmatrix}$$

$$\bar{q} = \frac{1}{2}\rho(z)\left(v_x^2 + v_z^2\right)$$

C_T = Thrust coefficient
θ = Pitch angle

$$= \left[\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} x \\ z \\ v_x \\ v_z \end{bmatrix} + \left[\begin{array}{c|c} 0 & 0 \\ 0 & 0 \\ \hline 1/m & 0 \\ 0 & 1/m \end{array}\right] \begin{bmatrix} \left(C_T \cos\theta + C_{X_I}\right)\bar{q}S \\ \left(-C_T \sin\theta + C_{Z_I}\right)\bar{q}S + mg_o \end{bmatrix}$$

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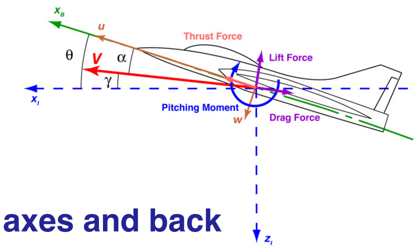
Transform Velocity from Cartesian to Polar Coordinates



Inertial axes -> wind axes and back

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_z \end{bmatrix} = \begin{bmatrix} V\cos\gamma \\ -V\sin\gamma \end{bmatrix} \Rightarrow \begin{bmatrix} V \\ \gamma \end{bmatrix} = \begin{bmatrix} \sqrt{\dot{x}^2+\dot{z}^2} \\ -\sin^{-1}\left(\frac{\dot{z}}{V}\right) \end{bmatrix} = \begin{bmatrix} \sqrt{v_x^2+v_z^2} \\ -\sin^{-1}\left(\frac{v_z}{V}\right) \end{bmatrix}$$

Rates of change of velocity and flight path angle

$$\begin{bmatrix} \dot{V} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \frac{d}{dt}\sqrt{v_x{}^2+v_z{}^2} \\ -\frac{d}{dt}\sin^{-1}\left(\frac{v_z}{V}\right) \end{bmatrix}$$

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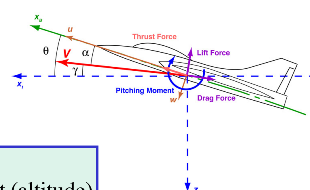
Longitudinal Point-Mass Equations of Motion



x : range
z : –height (altitude)
V : velocity
γ : flight path angle

$$\dot{x}(t) = v_x = V(t)\cos\gamma(t)$$

$$\dot{z}(t) = v_z = -V(t)\sin\gamma(t)$$

$$\dot{V}(t) = \frac{\left(C_T\cos\alpha - C_D\right)\frac{1}{2}\rho(z)V^2(t)S - mg_o\sin\gamma(t)}{m}$$

$$\dot{\gamma}(t) = \frac{\left(C_T\sin\alpha + C_L\right)\frac{1}{2}\rho(z)V^2(t)S - mg_o\cos\gamma(t)}{mV(t)}$$

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Steady, Level (*i.e.*, Cruising) Flight

In steady, level flight with cos $\alpha \sim 1$, sin $\alpha \sim 0$
Thrust = Drag, Lift = Weight

$$V_{cruise} = \dot{x}(t) = v_x$$

$$0 = \dot{z}(t) = v_z$$

$$0 = \frac{\left(C_T - C_D\right)\frac{1}{2}\rho(z)V_{cruise}^2 S}{m}$$

$$0 = \frac{C_L \frac{1}{2}\rho(z)V_{cruise}^2 S - mg(z)}{mV_{cruise}}$$

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Introduction to Aeronautical Propulsion

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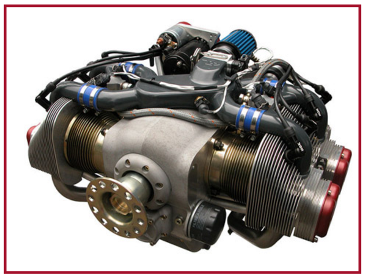
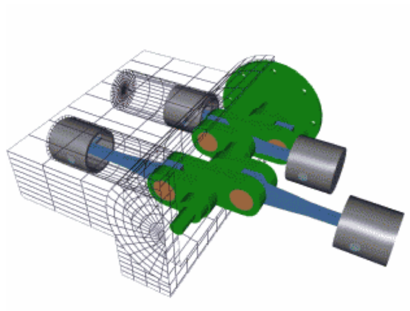
Internal Combustion Reciprocating Engine

Linear motion of pistons converted to rotary motion to drive propeller





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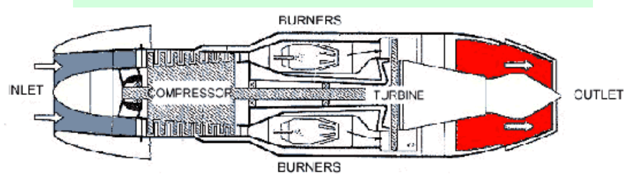
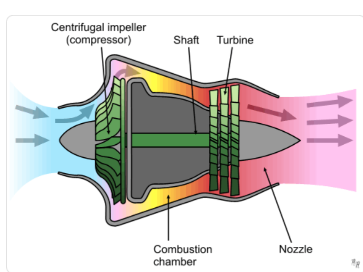
Turbojet Engines (1930s)

Thrust produced directly by exhaust gas

Axial-flow Turbojet (von Ohain, Germany)



Centrifugal-flow Turbojet (Whittle, UK)



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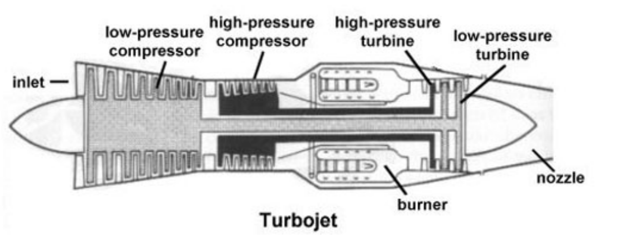
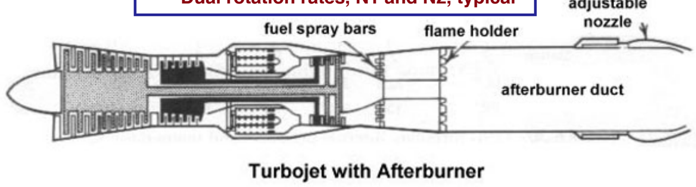
Birth of the Jet Airplane

Turbojet + Afterburner (1950s)



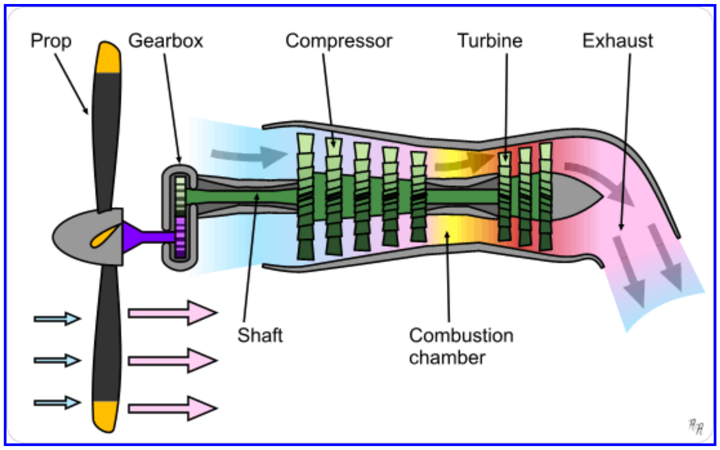
- Dual rotation rates, N1 and N2, typical



Fuel added to exhaust
Additional air may be introduced

Turboprop Engines (1940s)

Exhaust gas drives propeller to produce thrust



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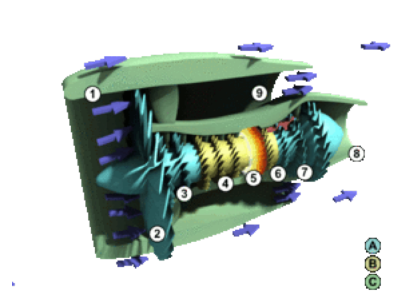
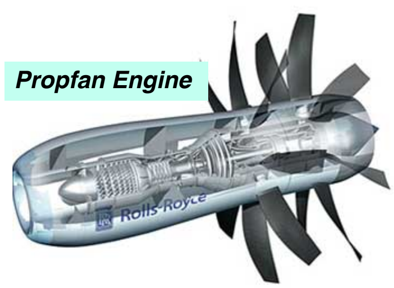
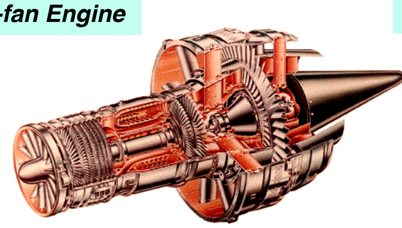
High Bypass Ratio Turbofan



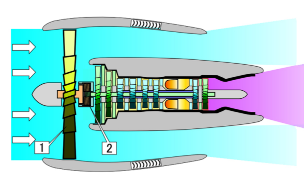
Propfan Engine



Aft-fan Engine



Geared Turbofan Engine



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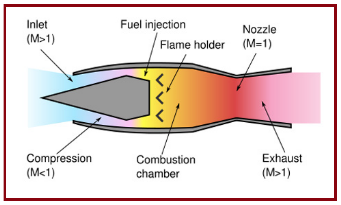
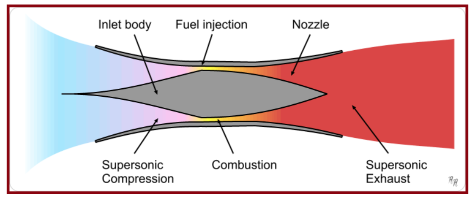
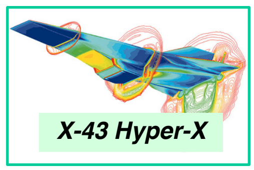
Ramjet and Scramjet

Ramjet (1940s)



Scramjet (1950s)





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Electric Propulsion

Available Energy	*Power Harvesting*	*Usable Power*	*Available Thrust*
Energy Storage	**Energy Conversion**	**Power Conditioning**	**Thrust Generation**
Batteries Fuel Sun	Solar Cells Fuel Cells Turbine/Reciprocating Engines	Transmission Inverters Regulators Generators Gearing	Motors Propellers Ducted Fans
	Conversion Efficiency < 1	Conditioning Efficiency < 1	Propulsive Efficiency < 1

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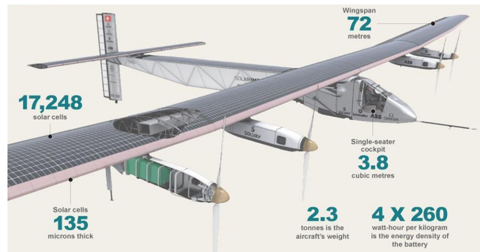
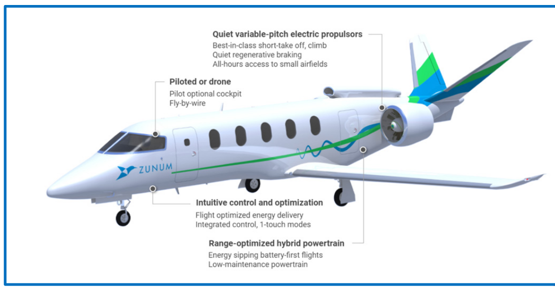
Electric Aircraft

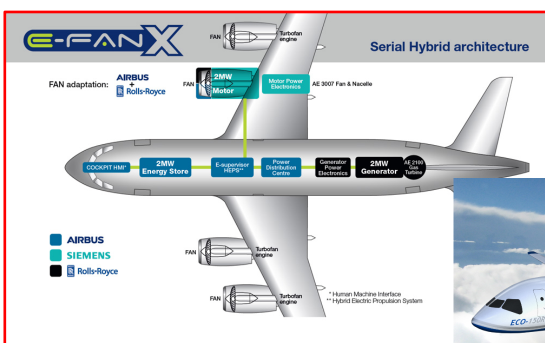
- Solar cells/batteries
- Fuel cells
- Batteries





Hybrid-Electric Aircraft

- Turbine engines
- Generators
- Thrust
- Drag reduction

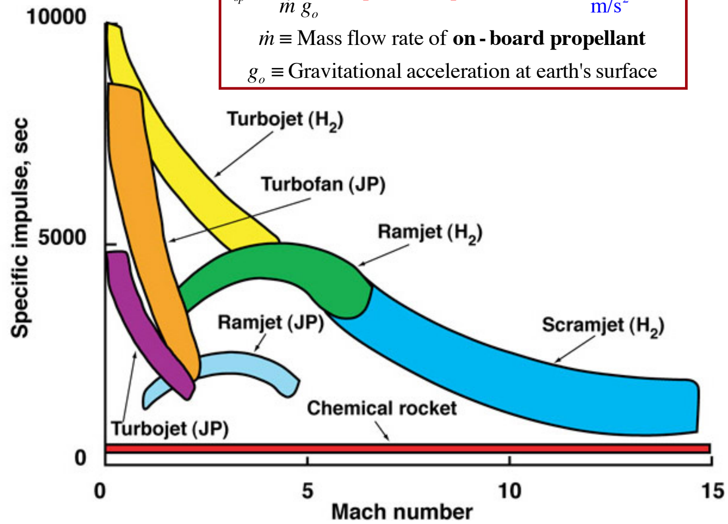


Thrust and Specific Impulse

$$I_{sp} = \frac{Thrust}{\dot{m}\, g_o} \triangleq \text{Specific Impulse}, \quad \text{Units} = \frac{\text{m/s}}{\text{m/s}^2} = \text{sec}$$

$\dot{m} \equiv$ Mass flow rate of **on-board propellant**

$g_o \equiv$ Gravitational acceleration at earth's surface



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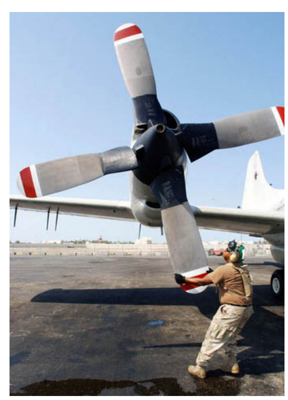
Thrust and Thrust Coefficient

$$Thrust \equiv C_T \frac{1}{2} \rho V^2 S$$

- **Non-dimensional thrust coefficient, C_T**
  - **C_T is a function of power/throttle setting, fuel flow rate, blade angle, Mach number, ...**
- **Reference area, S, may be**
  - **aircraft wing area,**
  - **propeller disk area, or**
  - **jet exhaust area**



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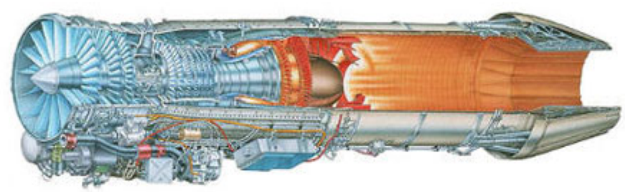
Sensitivity of Thrust to Airspeed

$$\text{Nominal Thrust} = T_N \equiv C_{T_N} \frac{1}{2} \rho V_N^{\,2} S$$

$$(.)_N = \text{Nominal (or reference) value}$$

Turbojet thrust is ~<u>independent</u> of airspeed over wide range



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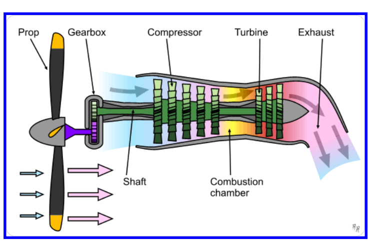
Power

Assuming thrust is aligned with airspeed vector

$$Power = P = Thrust \times Velocity \equiv C_T \frac{1}{2} \rho V^3 S$$

Propeller–driven power is ~<u>independent</u> of (subsonic) airspeed over a wide range

(reciprocating or turbine engine, with constant RPM or variable-pitch prop)



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Next Time:
Low-Speed Aerodynamics

Reading:
Flight Dynamics
Aerodynamic Coefficients, 65-84

Supplementary Material

MATLAB Scripts for Flat-Earth Trajectory, No Aerodynamics

Analytical Solution

```
g   =   9.8;
t   =   0:0.1:40;

vx0 =   10;
vz0 =   100;
x0  =   0;
z0  =   0;

vx1 =   vx0;
vz1 =   vz0 - g*t;
x1  =   x0 + vx0*t;
z1  =   z0 + vz0*t - 0.5*g*t.* t;
```

Numerical Solution

Calling Routine

```
tspan   = 40;
xo      = [10;100;0;0];
[t1,x1] = ode45('FlatEarth',tspan,xo);
```

Equations of Motion

```
function xdot = FlatEarth(t,x)
%   x(1)    =   vx
%   x(2)    =   vz
%   x(3)    =   x
%   x(4)    =   z
    g       =   9.8;
    xdot(1) =   0;
    xdot(2) =   -g;
    xdot(3) =   x(1);
    xdot(4) =   x(2);
    xdot    =   xdot';
end
```

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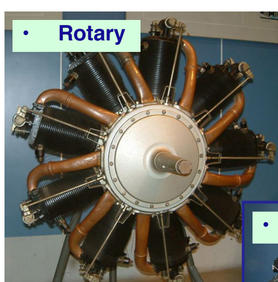
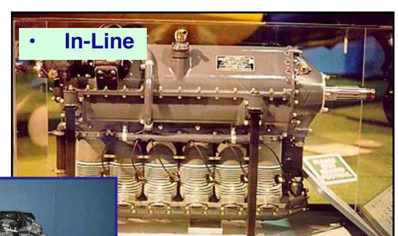
Early Reciprocating Engines

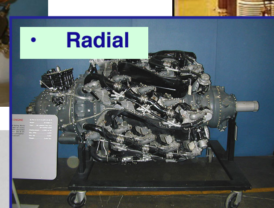
- **Rotary Engine:**
  - Air-cooled
  - Crankshaft fixed
  - Cylinders turn with propeller
  - On/off control: No throttle

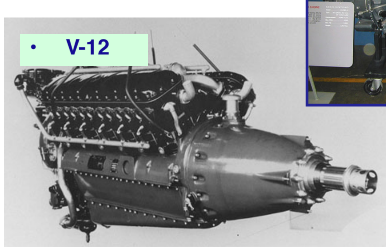
- **V-8 Engine:**
  - Water-cooled
  - Crankshaft turns with propeller

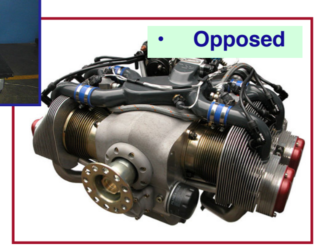
Reciprocating Engines





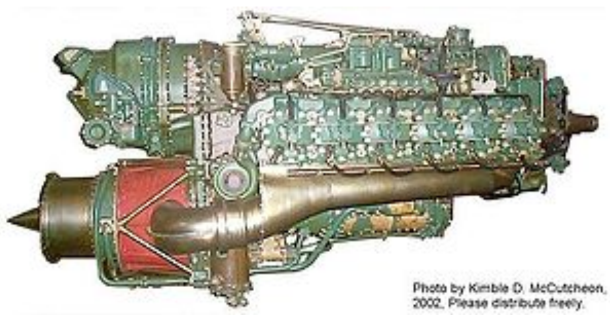






Turbo-compound Reciprocating Engine

- Exhaust gas drives the turbo-compressor
- Napier Nomad II shown (1949)



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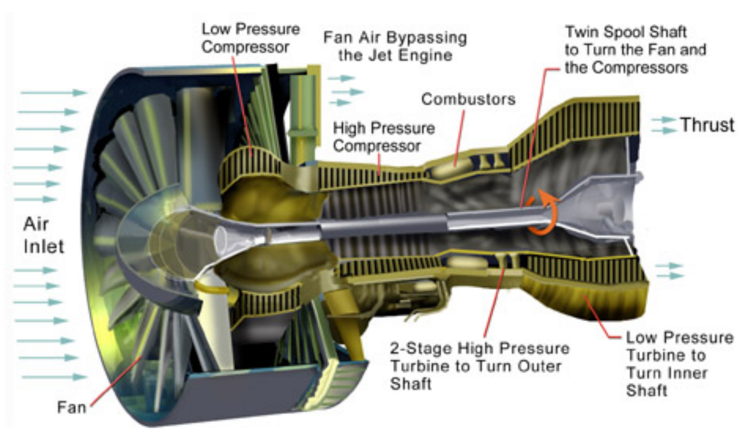
Turbofan Engine (1960s)



- Dual or triple rotation rates

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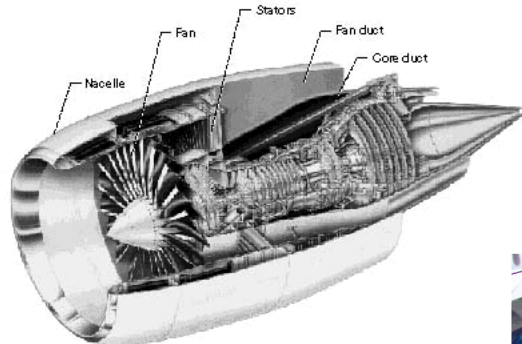
Jet Engine Nacelles



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Propeller-Driven Aircraft of the 1950s

Reciprocating Engines

Turboprop Engines

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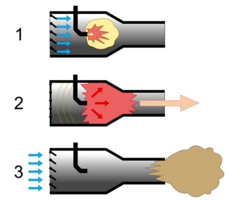
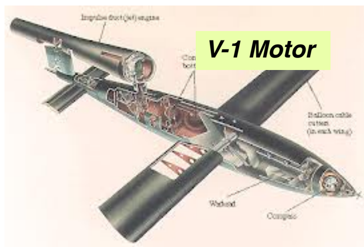
Pulsejet

Flapper-valved motor (1940s)



Dynajet Red Head (1950s)



V-1 Motor

Pulse Detonation Engine on Long EZ (1981)

http://airplanesandrockets.com/motors/dynajet-engine.htm

Jet Transports of the 2000s

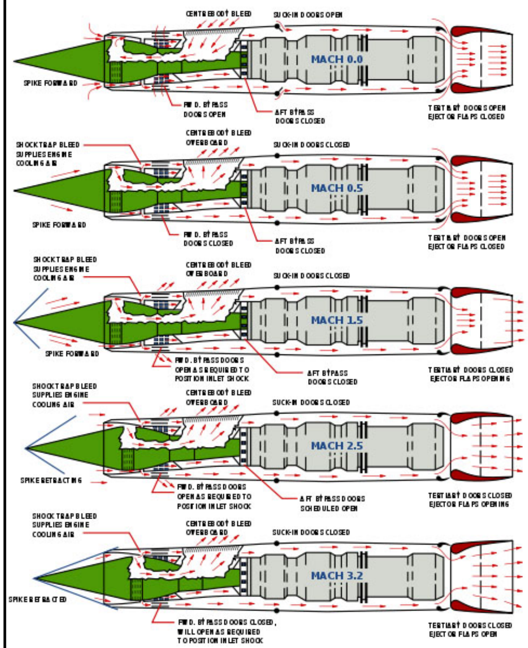
Airbus A380

Boeing 787

Embraer 195

Boeing 747-8F

SR-71: P&W J58 Variable-Cycle Engine (Late 1950s)



Hybrid Turbojet/Ramjet

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