Maneuvering at High Angles and Angular Rates

Robert Stengel, Aircraft Flight Dynamics
MAE 331, 2018

Learning Objectives

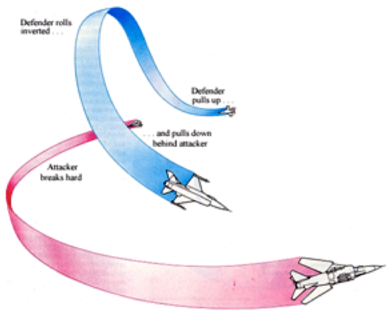
- High angle of attack and angular rates
- Asymmetric flight
- Nonlinear aerodynamics
- Inertial coupling
- Spins and tumbling

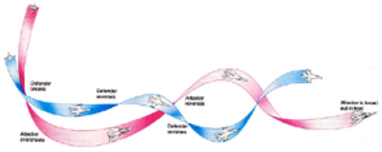
Flight Dynamics
681-785
Airplane Stability and Control
Chapter 8

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

Tactical Airplane Maneuverability

- Maneuverability parameters
  - Stability
  - Roll rate and acceleration
  - Normal load factor
  - Thrust/weight ratio
  - Pitch rate
  - Transient response
  - Control forces
- Dogfights
  - Preferable to launch missiles at *long range*
  - Dogfight is a *backup tactic*
  - Preferable to have an *unfair advantage*
- Air-combat sequence
  - Detection
  - Closing
  - Attack
  - Maneuvers, e.g.,
    - *Scissors*
    - *High yo-yo*
  - Disengagement





Coupling of Longitudinal and Lateral-Directional Motions

Longitudinal Motions can Couple to Lateral-Directional Motions

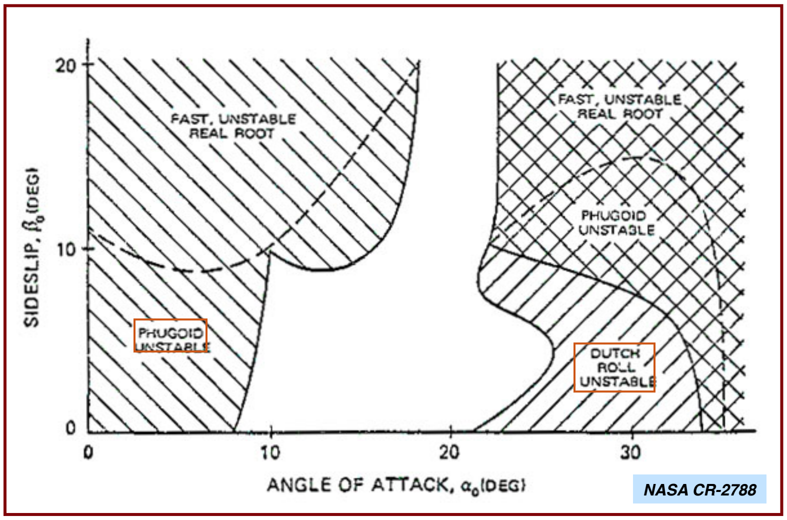
- **Linearized equations have limited application to high-angle/high-rate maneuvers**
  - Steady, non-zero sideslip angle (Sec. 7.1, *FD*)
  - Steady turn (Sec. 7.1, *FD*)
  - Steady roll rate

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{Lon} & \mathbf{F}_{Lat-Dir}^{Lon} \\ \mathbf{F}_{Lon}^{Lat-Dir} & \mathbf{F}_{Lat-Dir} \end{bmatrix}$$

$$\boxed{\mathbf{F}_{Lat-Dir}^{Lon}, \mathbf{F}_{Lon}^{Lat-Dir} \neq 0}$$

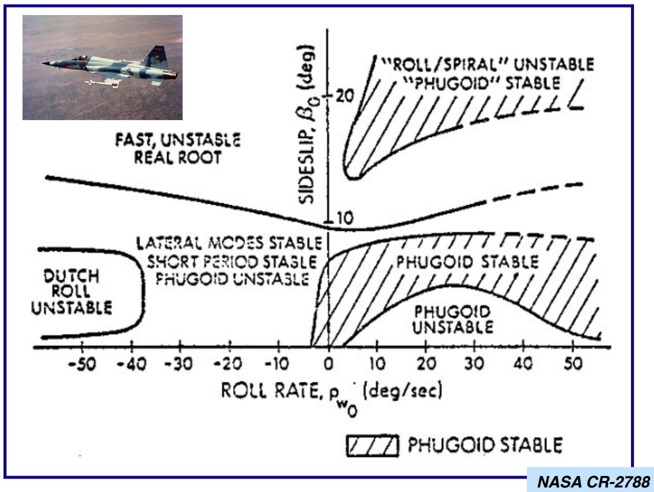
Stability Boundaries Arising From Asymmetric Flight


Northrop F-5E



NASA CR-2788

Stability Boundaries with Nominal Sideslip, β_o, and Roll Rate, p_o



NASA CR-2788

Pitch-Yaw Coupling Due To Steady Roll Rate, p_o

- Combine 2nd-order short period and Dutch roll modes
  - Body axes
  - Constant roll rate = p_o, rad/s

State vector

$$\Delta\mathbf{x}(t) = \begin{bmatrix} \Delta\mathbf{x}_{Lon} \\ \Delta\mathbf{x}_{LD} \end{bmatrix} = \begin{bmatrix} \Delta w \\ \Delta q \\ \Delta v \\ \Delta r \end{bmatrix} \begin{matrix} Normal\ velocity, m/s \\ Pitch\ rate, rad/s \\ Side\ velocity, m/s \\ Yaw\ rate, rad/s \end{matrix}$$

Control input vector

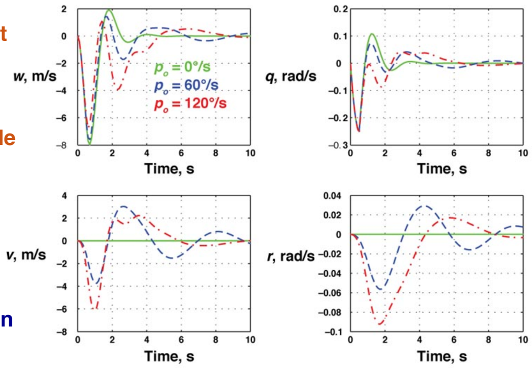
$$\Delta\mathbf{u}(t) = \begin{bmatrix} \Delta\delta E \\ \Delta\delta A \\ \Delta\delta R \end{bmatrix} \begin{matrix} Elevator, deg\ or\ rad \\ Ailerons, deg\ or\ rad \\ Rudder, deg\ or\ rad \end{matrix}$$

4th-order dynamic model

$$\begin{bmatrix} \Delta\dot{\mathbf{x}}_{Lon} \\ \Delta\dot{\mathbf{x}}_{LD} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{Lon} & \mathbf{F}_{LD}^{Lon} \\ \mathbf{F}_{Lon}^{LD} & \mathbf{F}_{LD} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x}_{Lon} \\ \Delta\mathbf{x}_{LD} \end{bmatrix} + \begin{bmatrix} \mathbf{G}_{Lon} \\ \mathbf{G}_{LD} \end{bmatrix} \Delta u$$

Time Response to Elevator Step Input

- When $p_o = 0°/s$
  - Elevator input produces longitudinal response but <u>no lateral-directional response</u>
- At $p_o = 60°/s$
  - Short-period (faster) mode dominates longitudinal response
  - Dutch-roll (slower) mode dominates lateral directional response
- At $p_o = 120°/s$
  - Both modes are evident in both responses
  - Fast mode is even faster
  - Slow mode is even slower



Pitch-Yaw Coupling Due To Steady Roll Rate, p_o

- 4^{th}-order stability matrix
  - Body axes
  - Neglible v_o, $u_o \sim V_N$
  - Negligible coupling aerodynamic effects
- Constant roll rate is only source of coupling

Short Period **Yaw-to-Pitch Coupling**

$$\left[\begin{array}{cc} \mathbf{F}_{Lon} & \mathbf{F}_{LD}^{Lon} \\ \mathbf{F}_{Lon}^{LD} & \mathbf{F}_{LD} \end{array}\right] = \left[\begin{array}{c:c} \left[\begin{array}{cc} Z_w & u_o \\ M_w & M_q \end{array}\right] & \left[\begin{array}{cc} -p_o & 0 \\ 0 & \dfrac{\left(I_{zz}-I_{xx}\right)}{I_{yy}} p_o \end{array}\right] \\ \hdashline \left[\begin{array}{cc} p_o & 0 \\ 0 & \dfrac{\left(I_{xx}-I_{yy}\right)}{I_{zz}} p_o \end{array}\right] & \left[\begin{array}{cc} Y_v & -u_o \\ N_v & N_r \end{array}\right] \end{array}\right]$$

Pitch-to-Yaw Coupling **Dutch Roll**

$$\Delta\mathbf{x}(t) = \left[\begin{array}{c} \Delta w \\ \Delta q \\ \Delta v \\ \Delta r \end{array}\right]$$

Pitch-Yaw Coupling Due To Steady Roll Rate, p_o

Characteristic Polynomial

$$\begin{aligned} \Delta_{rolling}(s) = & \left\{\left[(s-Z_w)(s-M_q)-u_o M_w\right]\left[(s-Y_v)(s-N_r)+u_o N_v\right]\right\} \\ & + p_o^2\left\{(s-M_q)(s-N_r)-(s-Z_w)(s-Y_v)\frac{\left(I_{zz}-I_{xx}\right)}{I_{yy}}\frac{\left(I_{xx}-I_{yy}\right)}{I_{zz}}-u_o M_w\frac{\left(I_{xx}-I_{yy}\right)}{I_{zz}}-u_o N_v\frac{\left(I_{zz}-I_{xx}\right)}{I_{yy}}\right\} \\ & - p_o^4\frac{\left(I_{zz}-I_{xx}\right)}{I_{yy}}\frac{\left(I_{xx}-I_{yy}\right)}{I_{zz}} \end{aligned}$$

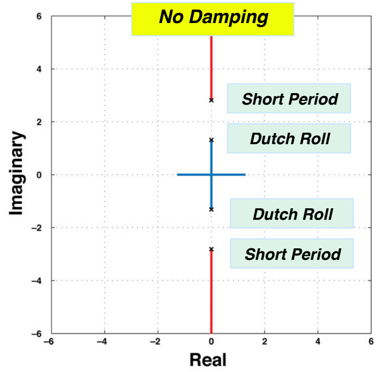
- Coupling effect is proportional to p_o^2 and p_o^4
- Effect on roots is independent of the sign of p_o
- Cannot use Evans' s root-locus rules with $k = p_o^2$, as k^2 also appears
- Can compute effect of p_o^2 on roots using MATLAB's *eig*

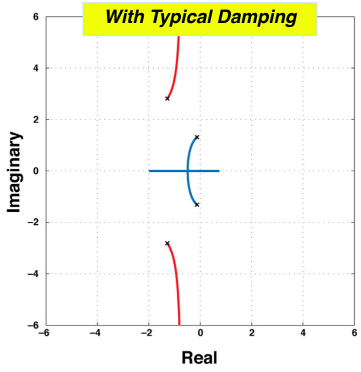
$$\Delta_{rolling}(s) = \left[\Delta_{SP}(s)\Delta_{DR}(s)\right] + p_o^2\left[fcn\left(s, M_q, N_r, Z_w, Y_v, I_{xx}, I_{yy}, I_{zz}, u_o, M_w, N_v\right)\right] - p_o^4\frac{\left(I_{zz}-I_{xx}\right)}{I_{yy}}\frac{\left(I_{xx}-I_{yy}\right)}{I_{zz}}$$

Thunderbird F-16 Barrel Roll
http://www.youtube.com/watch?v=ovSOStIncbU

Effect of Steady Roll Rate, p_o, on Pitching and Yawing Roots

- Factor $\Delta_{rolling}(s)$ for various values of p_o^2
- p_o^2 = root locus gain, k
- **Faster mode** gets faster
- **Slower mode** gets slower and may become unstable

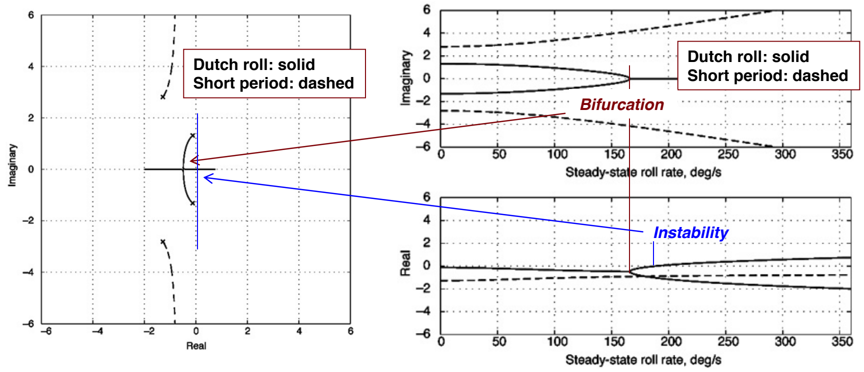




Steady Roll Rate, p_o, Effect Expressed by Root Locus or Parameter Plot

Divergent root at high roll rate

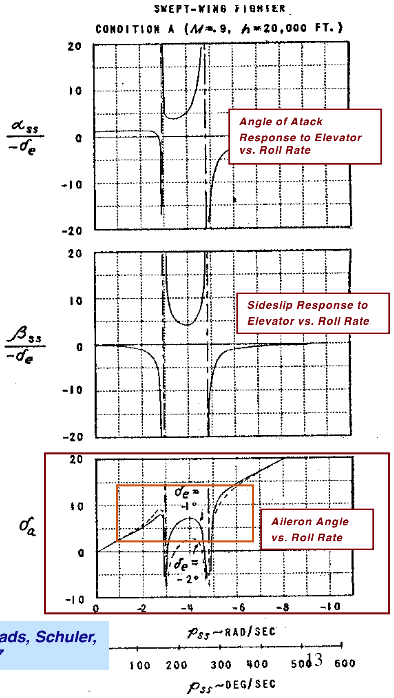
Parameter plot: variation of real and imaginary parts of roots vs. roll rate, p_o



Steady-State Response as Well as Stability is Affected by High Roll Rate

$$\mathbf{f}\left(v,w,p,q,r,\Delta\delta A,\Delta\delta E,\Delta\delta R\right)_{SS}=0$$

- Effects of steady roll rate on nonlinear equilibrium control response
  - Pitch-yaw coupling
  - "p jump" or "p acceleration"
- Multiple equilibria for same control settings
  - Up to 9 possible roll rates for one aileron setting
  - Sensitivity to elevator setting
    - *Flight Dynamics*, 7.3



Angle of Atack Response to Elevator vs. Roll Rate

Sideslip Response to Elevator vs. Roll Rate

Aileron Angle vs. Roll Rate

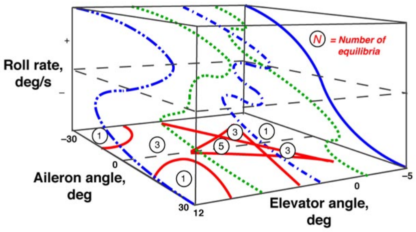
Rhoads, Schuler, 1957

The Butterfly Catastrophe*

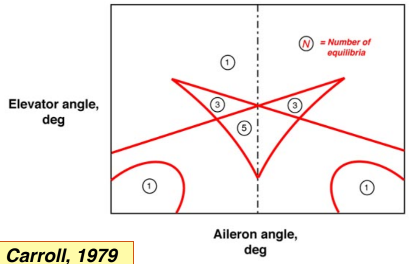
$$\mathbf{f}_1\left(v,w,p,q,r,\Delta\delta A,\Delta\delta E,\Delta\delta R\right)_{SS}=0$$
$$p_{SS}=\mathbf{f}_2\left(v,w,q,r,\Delta\delta A,\Delta\delta E,\Delta\delta R\right)_{SS}$$

- Surface of equilibrium solutions for roll rate
- Possibility of an unrecoverable spin

Roll Rate Equilibrium Surface



The Butterfly Catastrophe



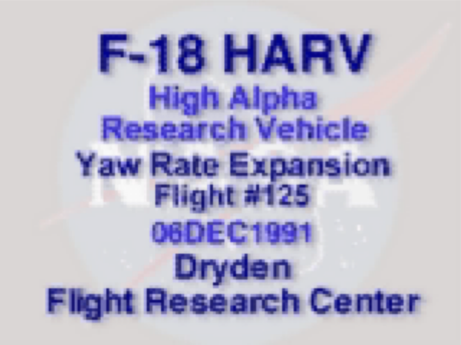
* René Thom, 1974

after Mehra, Carroll, 1979

Tumbling and Spins

Tumbling, Spins, and Recovery

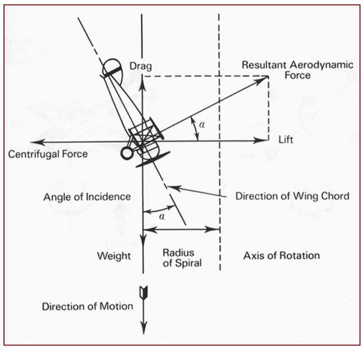
- Strong nonlinear effects
- Aircraft-specific control strategy for recovery

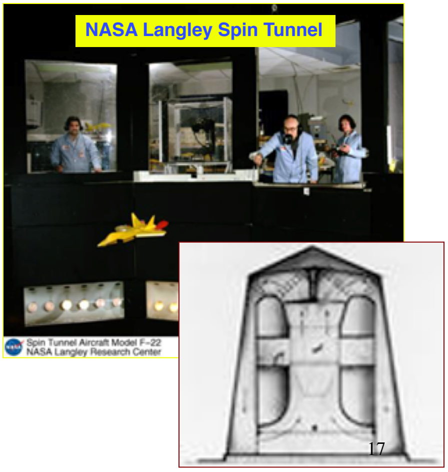


F-18 HARV
High Alpha
Research Vehicle
Yaw Rate Expansion
Flight #125
06DEC1991
Dryden
Flight Research Center

Wind Tunnel Spin Testing

- Sidney B. Gates, RAE: "The Spinning of Aeroplanes" (with L.W. Bryant, 1926), neutral and maneuver points, stick force per *g*
- Continued research on stalls and spins at NASA, USAF, and in many other countries





NASA Langley Spin Tunnel Testing

Parachute Recovery Tests on the 1/40-Scale Model of the B-58 Airplane.

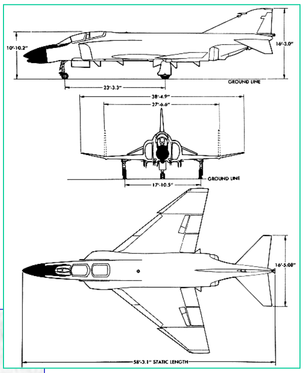
http://www.youtube.com/watch?v=u7FCqLpTgkk

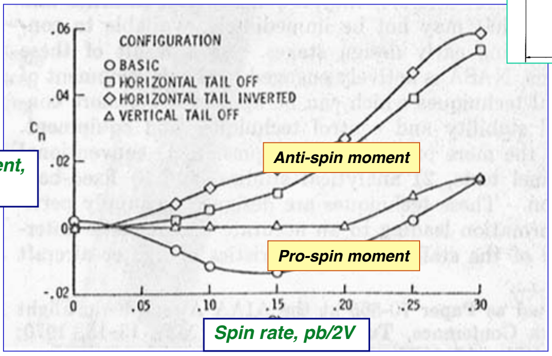
http://www.youtube.com/watch?v=tQwMCml55Q0

http://www.youtube.com/watch?v=VUKTBUY1RlI

Tails with Negative Dihedral (Anhedral)

- Horizontal tail below wing's wake
- May have adverse effect on spin characteristics
- F-4 model test



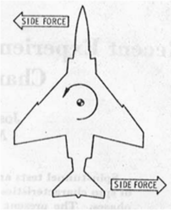


Yaw coefficient, C_n

Anti-spin moment

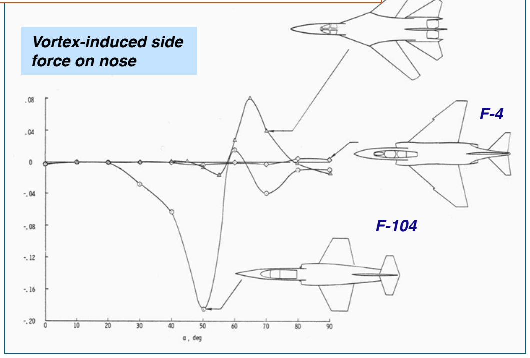
Pro-spin moment

Spin rate, pb/2V



Yawing Moment at High Angle of Attack

- Dynamic as well as static effects,e.g., *hysteresis*
- Random asymmetric yawing moments *(left or right)*
  - generated by slender nose at zero sideslip angle
  - may exceed rudder control power

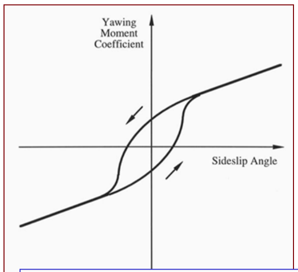


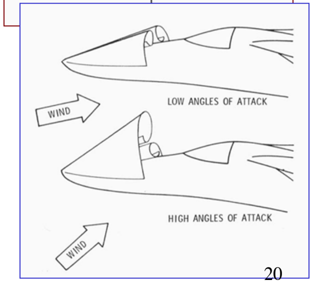
Vortex-induced side force on nose

F-111

F-4

F-104





Controlling Yawing Moment at High Angle of Attack

- *Sucking*, *blowing*, or *movable strakes* to control nose vortices
- *X-29, F/A-18 HARV*
- Vortex bursting effect on tail

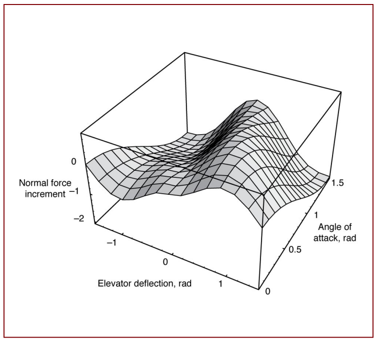

F-18 HARV
Smoke Test
late 1980's
Dryden
Flight Research Center


McDonnell-Douglas F/A-18

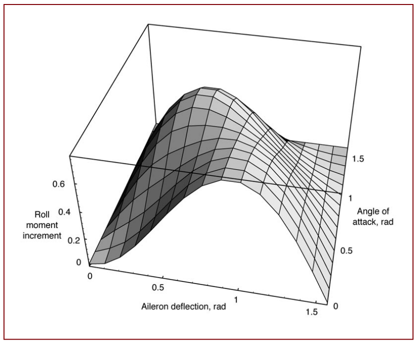
Control Effectiveness at High Angle of Attack and Deflection Angle

- Assumption of Newtonian flow

Elevator Effect



Aileron Effect



Control at High Aerodynamic Angles

Supermaneuverability

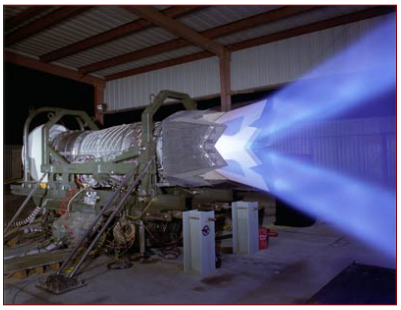
- Means of forcing opponent to *overshoot*
- *Pugachev's Cobra maneuver*, first done in *Sukhoi Su-27*
- Beneficial effect of thrust-vector control (*X-31*)
- *Mongoose maneuver* (*X-31*)
- Essentially low-speed maneuvers, not where you want to be in air combat (i.e., high energy-state)

Dryden
Flight
Research Center

Thrust Vector Control

Pitch and Yaw Control (X-31)

Pitch Control (F-22)



Next Time:
Aeroelasticity and Fuel Slosh

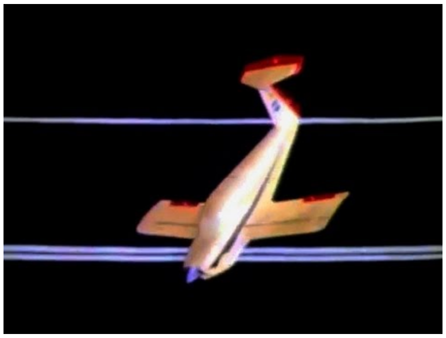
Flight Dynamics
418-419, 549-569, 665-678
Airplane Stability and Control
Chapter 19

Learning Objectives

- **Aerodynamic effects of bending and torsion**
- **Modifications to aerodynamic coefficients**
- **Dynamic coupling**
- **Fuel shift and sloshing dynamics**

Supplemental Material

Stall-Spin Studies of General Aviation Aircraft



http://www.youtube.com/watch?v=TmWB6oyJ9IE