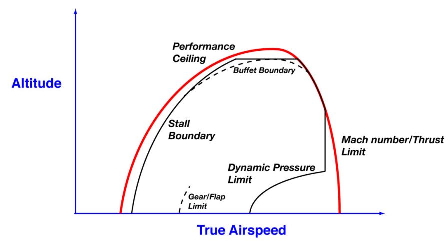
Cruising Flight Envelope

Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2018

Learning Objectives

- *Definitions of airspeed*
- *Performance parameters*
- *Steady cruising flight conditions*
- *Breguet range equations*
- *Optimize cruising flight for minimum thrust and power*
- *Flight envelope*



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http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

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The Flight Envelope

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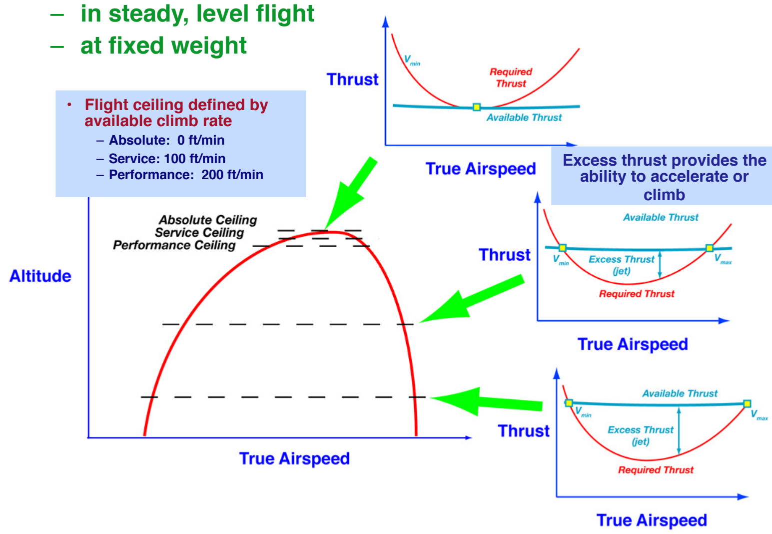
Flight Envelope Determined by Available Thrust

- All altitudes and airspeeds at which an aircraft can fly
  - in steady, level flight
  - at fixed weight



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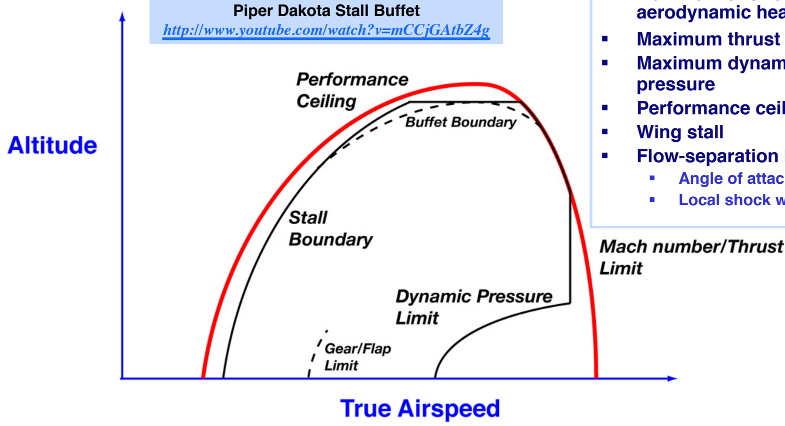
Additional Factors Define the Flight Envelope

Piper Dakota Stall Buffet
http://www.youtube.com/watch?v=mCCjGAtbZ4g



- Maximum Mach number
- Maximum allowable aerodynamic heating
- Maximum thrust
- Maximum dynamic pressure
- Performance ceiling
- Wing stall
- Flow-separation buffet
  - Angle of attack
  - Local shock waves

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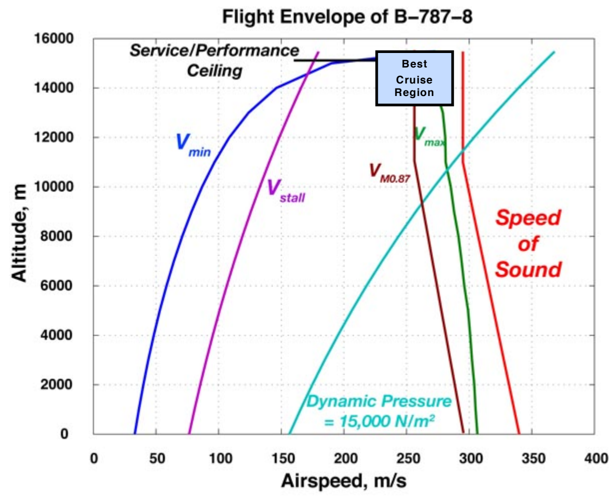
Boeing 787 Flight Envelope

(HW #5, 2008)



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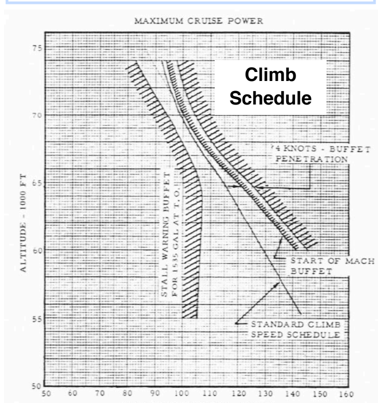
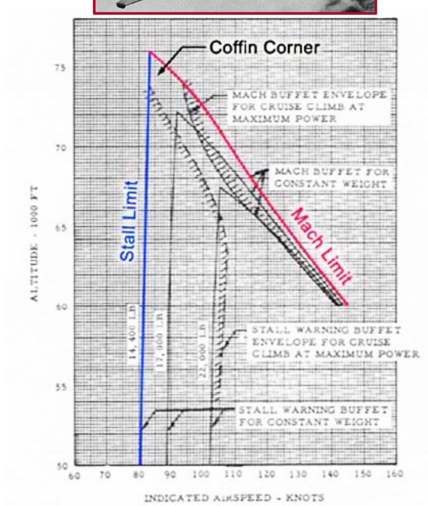
Lockheed U-2 "Coffin Corner"

Stall buffeting and Mach buffeting are limiting factors
Narrow corridor for safe flight





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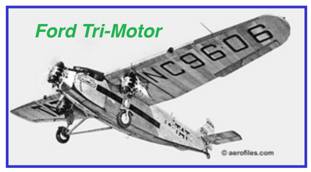
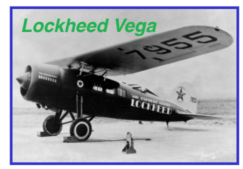
Historical Factoids

Air Commerce Act of 1926

- **Airlines formed to carry mail and passengers:**
  - **Northwest (1926)**
  - **Eastern (1927), bankruptcy**
  - **Pan Am (1927), bankruptcy**
  - **Boeing Air Transport (1927), became United (1931)**
  - **Delta (1928), consolidated with Northwest, 2010**
  - **American (1930)**
  - **TWA (1930), acquired by American**
  - **Continental (1934), consolidated with United, 2010**



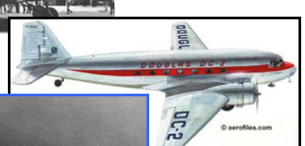


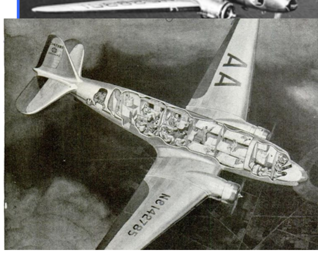
http://www.youtube.com/watch?v=3a8G87qnZz4

Commercial Aircraft of the 1930s

Streamlining, engine cowlings

Douglas DC-1, DC-2, DC-3





Lockheed 14 Super Electra, Boeing 247

Comfort and Elegance by the End of the Decade

***Boeing 307*, 1st pressurized cabin (1936), flight engineer, *B-17* pre-cursor, large dorsal fin (exterior and interior)**

Sleeping bunks on transcontinental planes (e.g., *DC-3*)

Full-size dining rooms on flying boats

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Optimal Cruising Flight

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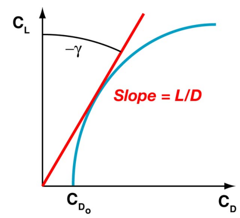
Maximum Lift-to-Drag Ratio



Lift-to-drag ratio

$$L/D = C_L/C_D = \frac{C_L}{C_{D_o} + \varepsilon C_L^2}$$

Satisfy necessary condition for a maximum

$$\frac{\partial\left(C_L/C_D\right)}{\partial C_L} = \frac{1}{C_{D_o} + \varepsilon C_L^2} - \frac{2\varepsilon C_L^2}{\left(C_{D_o} + \varepsilon C_L^2\right)^2} = 0$$

Lift coefficient for maximum *L/D* and minimum thrust are the same

$$\left(C_L\right)_{L/D_{max}} = \sqrt{\frac{C_{D_o}}{\varepsilon}} = C_{L_{MT}}$$

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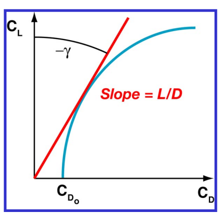
Airspeed, Drag Coefficient, and Lift-to-Drag Ratio for L/D_{max}



Airspeed

$$V_{L/D_{max}} = V_{MT} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{\varepsilon}{C_{D_o}}}}$$

Drag Coefficient

$$\left(C_D\right)_{L/D_{max}} = C_{D_o} + C_{D_o} = 2C_{D_o}$$

Maximum L/D

$$\left(L/D\right)_{max} = \frac{\sqrt{C_{D_o}/\varepsilon}}{2C_{D_o}} = \frac{1}{2\sqrt{\varepsilon C_{D_o}}}$$

Maximum *L/D* depends only on induced drag factor and zero-lift drag coefficient

Induced drag factor and zero-lift drag coefficient are functions of Mach number

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Cruising Range and Specific Fuel Consumption

- **Thrust = Drag**
- **Lift = Weight**

$$0 = (C_T - C_D)\frac{1}{2}\rho V^2 S \Big/ m$$

$$0 = \left(C_L \frac{1}{2}\rho V^2 S - mg\right)\Big/ mV$$

- **Level flight**

$$\dot{h} = 0$$
$$\dot{r} = V$$

- **Thrust specific fuel consumption, *TSFC* = c_T**
  - **Fuel mass burned per sec per unit of thrust**

$$c_T : \frac{kg/s}{kN} \qquad \dot{m}_f = -c_T T$$

- **Power specific fuel consumption, *PSFC* = c_P**
  - **Fuel mass burned per sec per unit of power**

$$c_P : \frac{kg/s}{kW} \qquad \dot{m}_f = -c_P P$$

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Breguet Range Equation for Jet Aircraft

Rate of change of range with respect to weight of fuel burned

$$\frac{dr}{dm} = \frac{dr/dt}{dm/dt} = \frac{\dot{r}}{\dot{m}} = \frac{V}{(-c_T T)} = -\frac{V}{c_T D} = -\left(\frac{L}{D}\right)\frac{V}{c_T mg}$$

$$dr = -\left(\frac{L}{D}\right)\frac{V}{c_T mg}dm$$

Range traveled

$$Range = R = \int_0^R dr = -\int_{W_i}^{W_f}\left(\frac{L}{D}\right)\left(\frac{V}{c_T g}\right)\frac{dm}{m}$$

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Maximum Range of a Jet Aircraft Flying at Constant Altitude

At constant altitude and *SFC*

$$V_{cruise}(t) = \sqrt{2W(t)/C_L \rho(h_{fixed})S}$$

$$Range = -\int_{W_i}^{W_f} \left(\frac{C_L}{C_D}\right)\left(\frac{1}{c_T g}\right)\sqrt{\frac{2}{C_L \rho S}}\frac{dm}{m^{1/2}}$$

$$= \left(\frac{\sqrt{C_L}}{C_D}\right)\left(\frac{2}{c_T g}\right)\sqrt{\frac{2}{\rho S}}\left(m_i^{1/2} - m_f^{1/2}\right)$$

Range is maximized when $\left(\frac{\sqrt{C_L}}{C_D}\right) = \text{maximum}$

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Breguet Range Equation for Jet Aircraft at Constant Airspeed

For constant true airspeed, $V = V_{cruise}$, and *SFC*

$$R = -\left(\frac{L}{D}\right)\left(\frac{V_{cruise}}{c_T g}\right)\ln(m)\Big|_{m_i}^{m_f}$$

$$= \left(\frac{L}{D}\right)\left(\frac{V_{cruise}}{c_T g}\right)\ln\left(\frac{m_i}{m_f}\right)$$

$$= \left(V_{cruise}\frac{C_L}{C_D}\right)\left(\frac{1}{c_T g}\right)\ln\left(\frac{m_i}{m_f}\right)$$

- $V_{cruise}(C_L/C_D)$ as large as possible
- $M \rightarrow M_{crit}$
- ρ as small as possible
- h as high as possible

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Maximize Jet Aircraft Range Using Optimal Cruise-Climb

$$\frac{\partial R}{\partial C_L} \propto \frac{\partial\left(V_{cruise} \, C_L / C_D\right)}{\partial C_L} = \frac{\partial\left[V_{cruise} \, C_L \Big/ \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = 0$$

$$V_{cruise} = \sqrt{2W / C_L \rho S}$$

$$\text{Assume } \sqrt{2W(t)/\rho(h)S} = \text{constant}$$

i.e., airplane **climbs at constant *TAS*** as fuel is burned

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Maximize Jet Aircraft Range Using Optimal Cruise-Climb

$$\frac{\partial\left[V_{cruise} \, C_L / \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = \sqrt{\frac{2W}{\rho S}} \frac{\partial\left[C_L^{1/2} / \left(C_{D_o} + \varepsilon C_L^2\right)\right]}{\partial C_L} = 0$$

Optimal values: (see Supplemental Material)

$$C_{L_{MR}} = \sqrt{\frac{C_{D_o}}{3\varepsilon}}$$: **Lift Coefficient for Maximum Range**

$$C_{D_{MR}} = C_{D_o} + \frac{C_{D_o}}{3} = \frac{4}{3} C_{D_o}$$

$$V_{cruise-climb} = \sqrt{2W(t) / C_{L_{MR}} \rho(h) S} = a(h) M_{cruise\text{-}climb}$$

$a(h)$: Speed of sound; $M_{cruise\text{-}climb}$: Mach number

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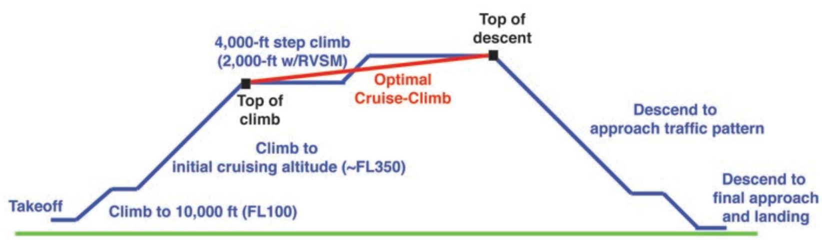
Step-Climb Approximates Optimal Cruise-Climb

- Cruise-climb usually violates air traffic control rules
- Constant-altitude cruise does not
- Compromise: Step climb from one allowed altitude to the next as fuel is burned



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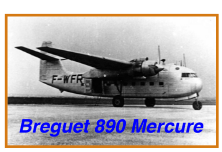
Historical Factoid

- Louis Breguet (1880-1955), aviation pioneer
  - Gyroplane (1905), flew vertically in 1907
  - Breguet Type 1 (1909), fixed-wing aircraft
  - Formed Compagnie des messageries aériennes (1919), predecessor of *Air France*
- Breguet Aviation: built numerous aircraft until after World War II; teamed with BAC in SEPECAT (1966)
- Merged with Dassault in 1971



SEPECAT Jaguar

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Next Time:
Gliding, Climbing, and Turning Flight

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Supplemental Material

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Seaplanes Became the First TransOceanic Air Transports

- **PanAm led the way**
  - **1st scheduled TransPacific flights(1935)**
  - **1st scheduled TransAtlantic flights(1938)**
  - **1st scheduled non-stop Trans-Atlantic flights (*VS-44*, 1939)**
- ***Boeing B-314, Vought-Sikorsky VS-44, Shorts Solent***
- **Superseded by more efficient landplanes (lighter, less drag)**

http://www.youtube.com/watch?v=x8SkeE1h_-A

Maximize Jet Aircraft Range Using Optimal Cruise-Climb

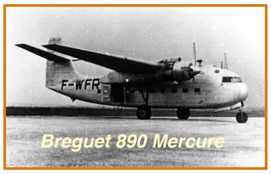
$$\frac{\partial\left[V_{cruise}\,C_L\Big/\left(C_{D_o}+\varepsilon C_L^2\right)\right]}{\partial C_L}=\sqrt{\frac{2w}{\rho S}}\,\frac{\partial\left[C_L^{1/2}\Big/\left(C_{D_o}+\varepsilon C_L^2\right)\right]}{\partial C_L}=0$$

$$\sqrt{\frac{2w}{\rho S}} = \text{Constant; let } C_L^{1/2}=x, \quad C_L=x^2$$

$$\frac{\partial}{\partial x}\left[\frac{x}{\left(C_{D_o}+\varepsilon x^4\right)}\right]=\frac{\left(C_{D_o}+\varepsilon x^4\right)-x\left(4\varepsilon x^3\right)}{\left(C_{D_o}+\varepsilon x^4\right)^2}=\frac{\left(C_{D_o}-3\varepsilon x^4\right)}{\left(C_{D_o}+\varepsilon x^4\right)^2}$$

Optimal values:

$$C_{L_{MR}}=\sqrt{\frac{C_{D_o}}{3\varepsilon}}\ :\ C_{D_{MR}}=C_{D_o}+\frac{C_{D_o}}{3}=\frac{4}{3}C_{D_o}$$



Breguet Range Equation for Propeller-Driven Aircraft

Rate of change of range with respect to weight of fuel burned

$$\frac{dr}{dw} = \frac{\dot{r}}{\dot{w}} = \frac{V}{\left(-c_P P\right)} = -\frac{V}{c_P T V} = -\frac{V}{c_P D V} = -\left(\frac{L}{D}\right)\frac{1}{c_P W}$$

Range traveled

$$Range = R = \int_0^R dr = -\int_{W_i}^{W_f} \left(\frac{L}{D}\right)\left(\frac{1}{c_P}\right)\frac{dw}{w}$$

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Breguet Range Equation for Propeller-Driven Aircraft

For constant true airspeed, $V = V_{cruise}$

$$R = -\left(\frac{L}{D}\right)\left(\frac{1}{c_P}\right)\ln\left(w\right)\Big|_{W_i}^{W_f}$$

$$= \left(\frac{C_L}{C_D}\right)\left(\frac{1}{c_P}\right)\ln\left(\frac{W_i}{W_f}\right)$$

Range is maximized when

$$\left(\frac{C_L}{C_D}\right) = maximum = \left(L/D\right)_{\max}$$

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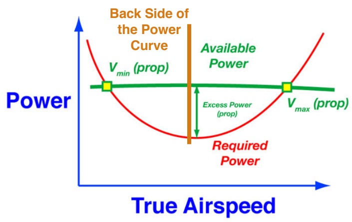


Achievable Airspeeds in Propeller-Driven Cruising Flight

Power = constant

$$P_{avail} = T_{avail} V$$

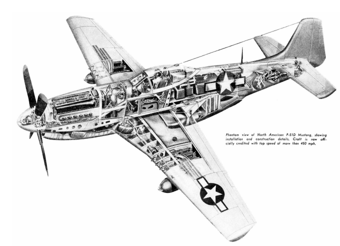
$$V^4 - \frac{P_{avail} V}{C_{D_o} \rho S} + \frac{4 \varepsilon W^2}{C_{D_o} (\rho S)^2} = 0$$

Solutions for *V* cannot be put in quadratic form; solution is more difficult, e.g., Ferrari's method

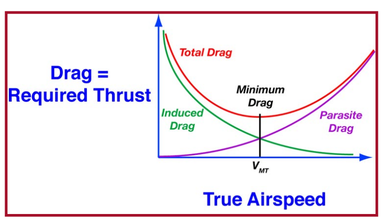
$$aV^4 + (0)V^3 + (0)V^2 + dV + e = 0$$

Best bet: *roots* in MATLAB

P-51 Mustang Minimum-Thrust Example



$$Wing\ Span = 37\ ft\ (9.83\ m)$$
$$Wing\ Area = 235\ ft^2\ (21.83\ m^2)$$
$$Loaded\ Weight = 9{,}200\ lb\ (3{,}465\ kg)$$
$$C_{D_o} = 0.0163$$
$$\varepsilon = 0.0576$$
$$W / S = 39.3\ lb / ft^2\ (1555.7\ N / m^2)$$



Airspeed for minimum thrust

$$V_{MT} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)}\sqrt{\frac{\varepsilon}{C_{D_o}}} = \sqrt{\frac{2}{\rho}(1555.7)}\sqrt{\frac{0.947}{0.0163}} = \frac{76.49}{\sqrt{\rho}}\ m/s$$

Altitude, m	Air Density, kg/m^3	V_{MT}, m/s
0	1.23	69.11
2,500	0.96	78.20
5,000	0.74	89.15
10,000	0.41	118.87

P-51 Mustang Maximum L/D Example

$$\left(C_D\right)_{L/D_{\max}} = 2C_{D_o} = 0.0326$$

$$\left(C_L\right)_{L/D_{\max}} = \sqrt{\frac{C_{D_o}}{\varepsilon}} = C_{L_{MT}} = 0.531$$

$$\left(L/D\right)_{\max} = \frac{1}{2\sqrt{\varepsilon C_{D_o}}} = 16.31$$

$$V_{L/D_{\max}} = V_{MT} = \frac{76.49}{\sqrt{\rho}}\ m/s$$

$$\textit{Wing Span} = 37\ ft\ (9.83\ m)$$
$$\textit{Wing Area} = 235\ ft\ (21.83\ m^2)$$
$$\textit{Loaded Weight} = 9{,}200\ lb\ (3{,}465\ kg)$$
$$C_{D_o} = 0.0163$$
$$\varepsilon = 0.0576$$
$$W/S = 1555.7\ N/m^2$$

Altitude, m	Air Density, kg/m^3	V_{MT}, m/s
0	1.23	69.11
2,500	0.96	78.20
5,000	0.74	89.15
10,000	0.41	118.87

P-51 Mustang Maximum Range (Internal Tanks only)

$$W = C_{L_{trim}} \bar{q} S$$

$$C_{L_{trim}} = \frac{1}{\bar{q}}(W/S) = \frac{2}{\rho V^2}(W/S) = \left(\frac{2\,e^{\beta h}}{\rho_0 V^2}\right)(W/S)$$

$$R = \left(\frac{C_L}{C_D}\right)_{\max}\left(\frac{1}{c_P}\right)\ln\left(\frac{W_i}{W_f}\right) = (16.31)\left(\frac{1}{0.0017}\right)\ln\left(\frac{3{,}465+600}{3{,}465}\right) = 1{,}530\ km\ ((825\ nm)$$