Sensors and Actuators

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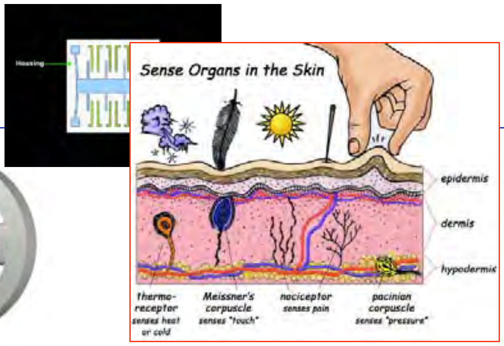
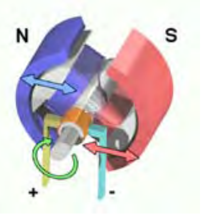
Robotics and Intelligent Systems, MAE 345,
Princeton University, 2017

- **Biological Antecedents**
- **Critical Elements for System Observation and Control**
- **Control Effecters**
- **Output Sensors**
- **Navigation**

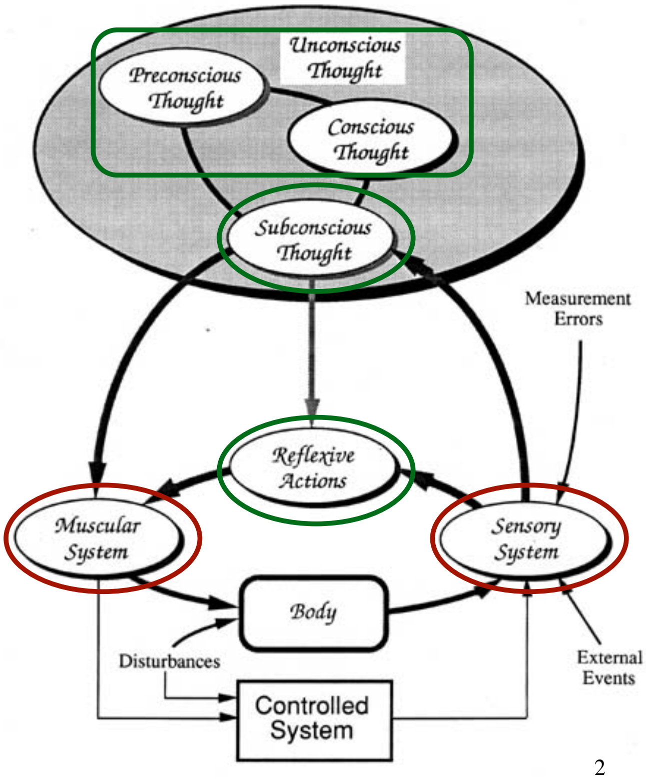




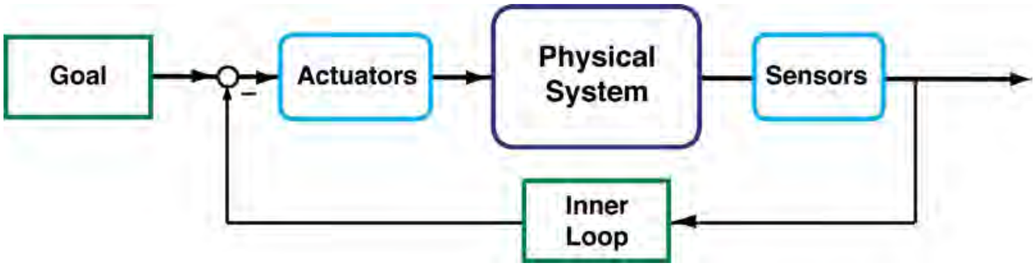
http://www.princeton.edu/~stengel/MAE345.html

Biologically Inspired Control

- **Declarative Planning**
- **Procedural Formatting**
- **Reflexive Control**
- **Sensory input**
- **Motor output**



Feedback Control Requires Sensors and Actuators



Goal

Actuators

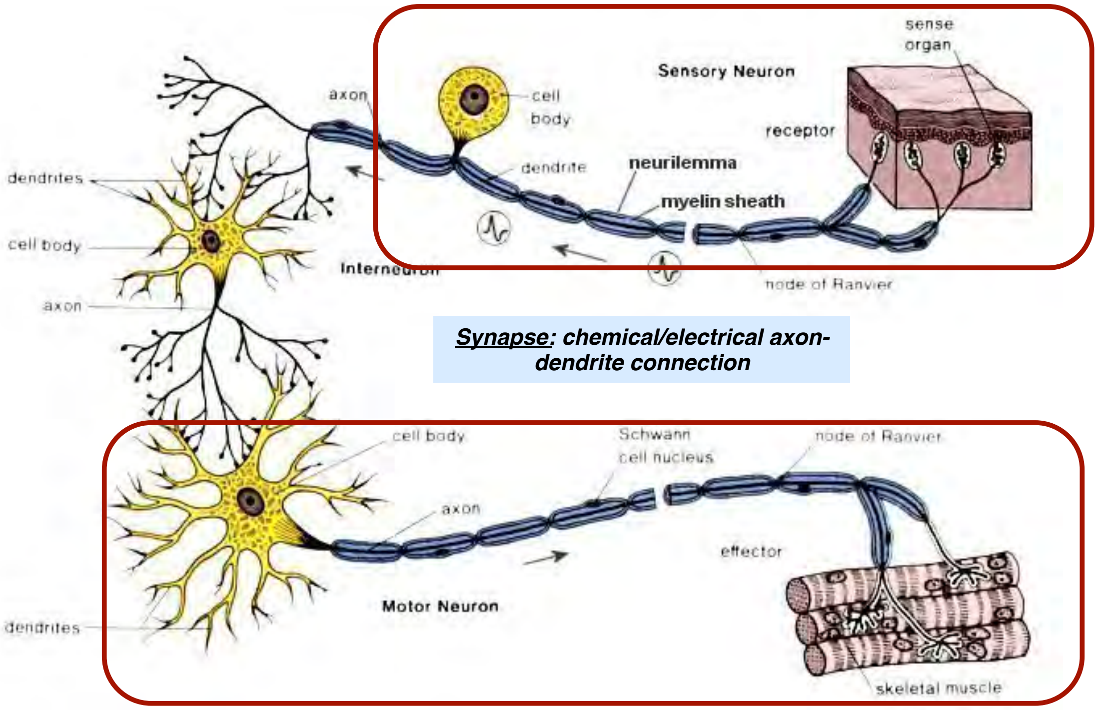
Physical System

Sensors

Inner Loop

- **Desirable properties of sensors and actuators**
  - High bandwidth (“faster” than system to be controlled)
  - Accuracy and Precision
  - Large dynamic range
  - Sufficient power for control
  - Reliability
  - Low cost

Peripheral Sensory and Motor Neurons

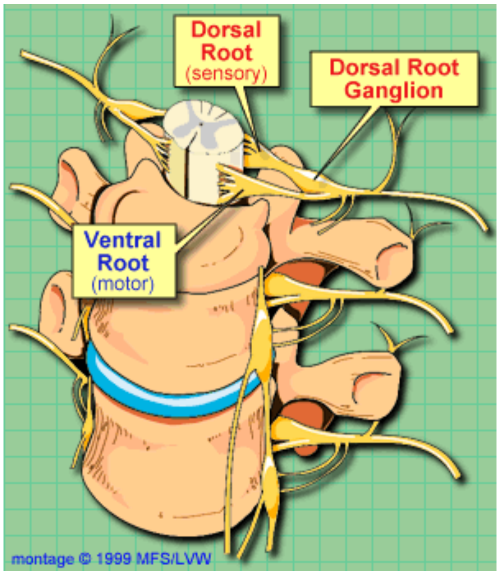


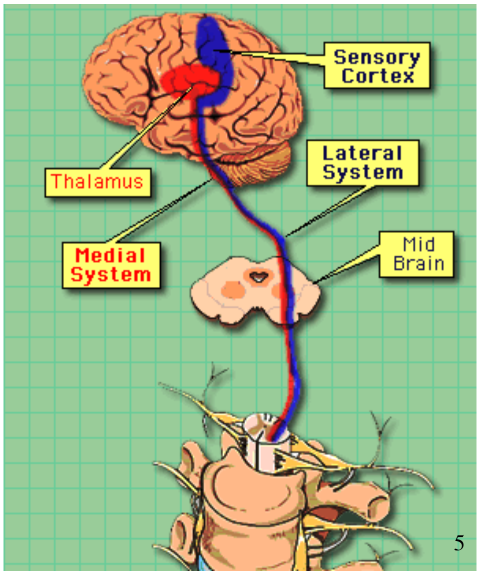
***Synapse*: chemical/electrical axon-dendrite connection**

Sensory and Motor Signal Paths to the Brain

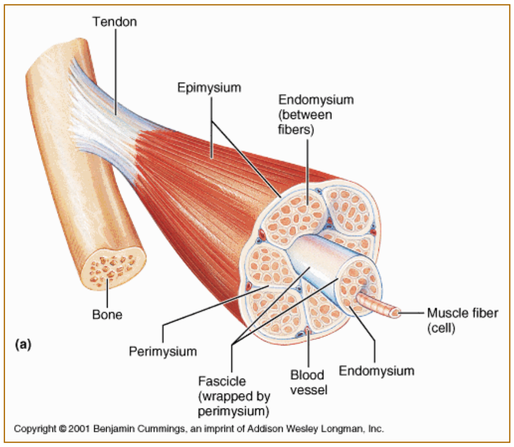
Reflexive response is processed in the spinal roots

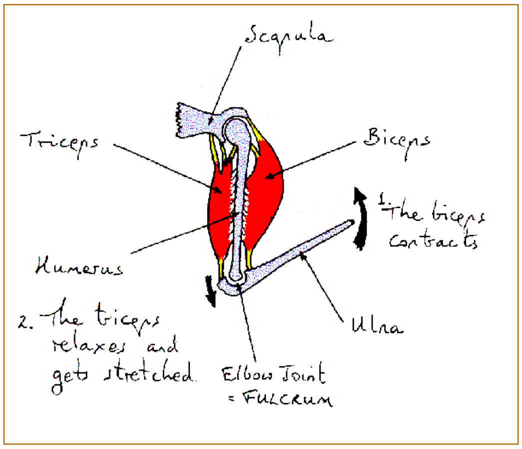
Declarative and procedural response is processed in the brain





Skeletal Muscle



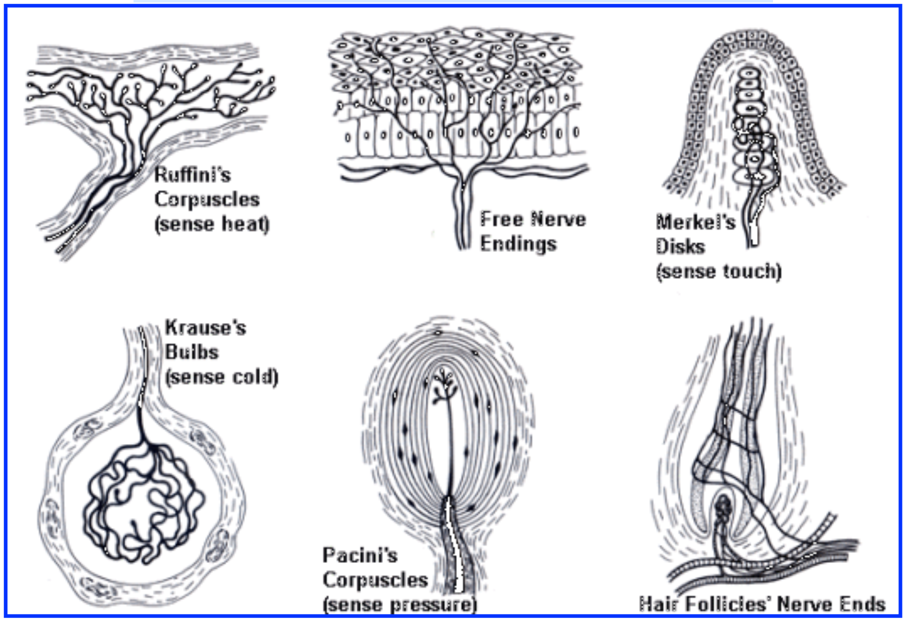


- **Attached to the skeleton** to produce motion of limbs, torso, neck, and head
- **Agonist-antagonist muscle pairs produce opposing motion (flexion and extension)**
- End-effecter strength depends on **lever arm** and varies with joint angle
- Voluntary (declarative) commands from **somatic central nervous system**

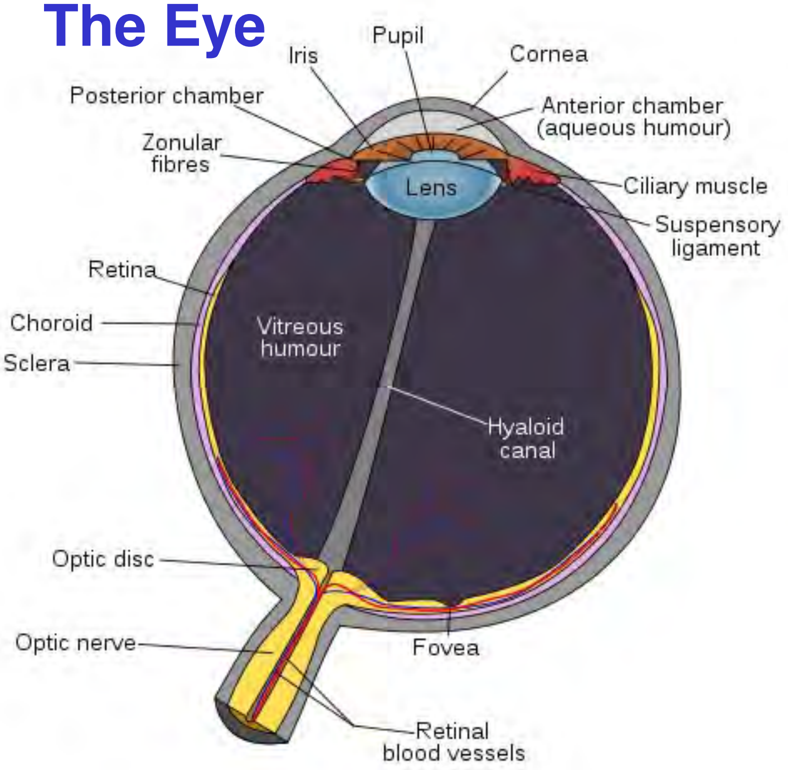
Sensory Neuron Receptors

Neuron Receptors (corpuscles, disks, cells, muscle spindles) generate action potentials that are transmitted to the spinal cord

Cutaneous and Sub-Cutaneous Receptors



The Eye

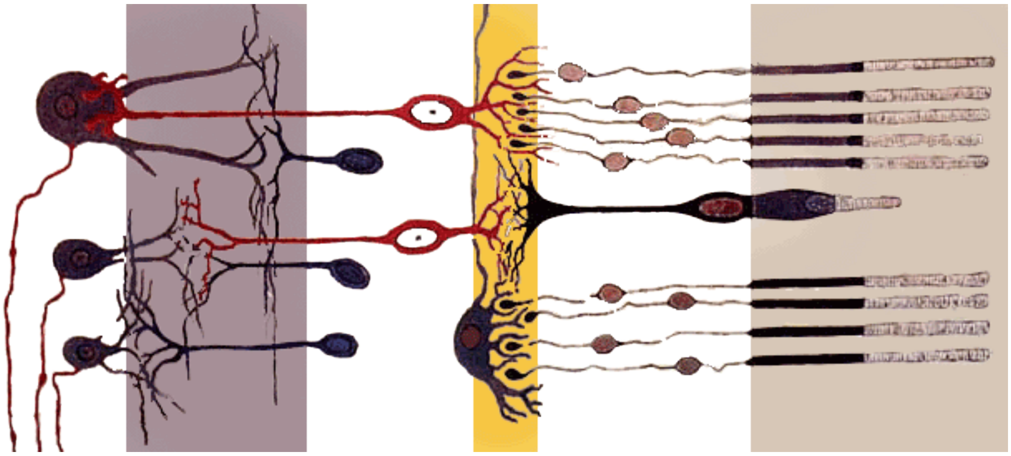


Retinal Cross Section

Retinal Ganglion Cells

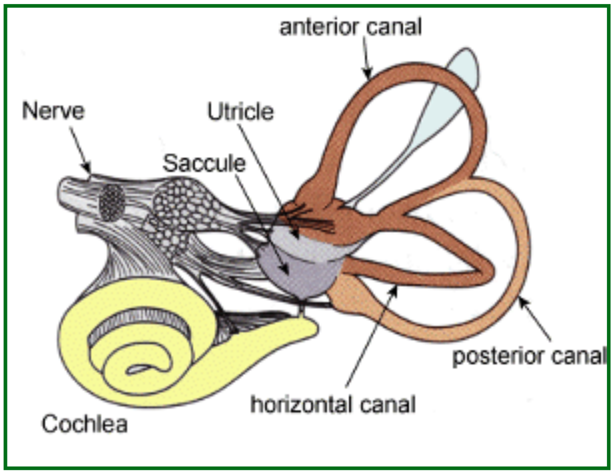
Amacrine and Horizontal Cells

Rod and Cone Cells

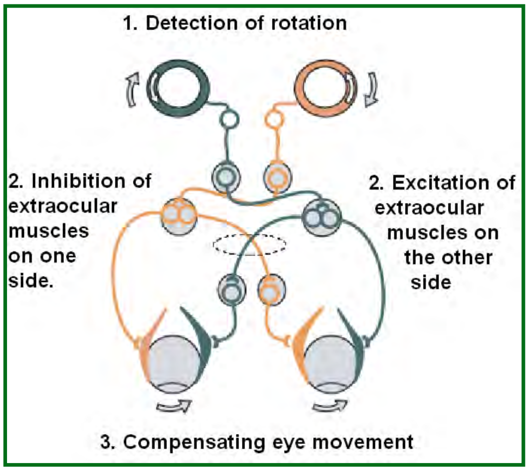


Biological Inertial Measurement: The Inner Ear

Vestibular system measures linear and angular acceleration



Integration with eye motion



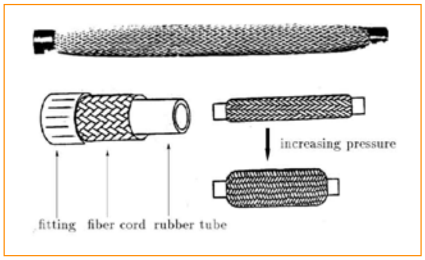
Actuators

Rubbertuator

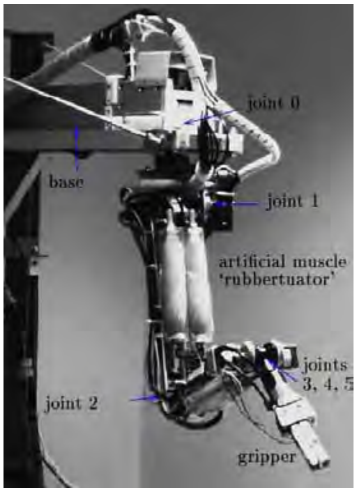

Princeton's SLIM Robot, 1993

Pneumatic analog of muscle

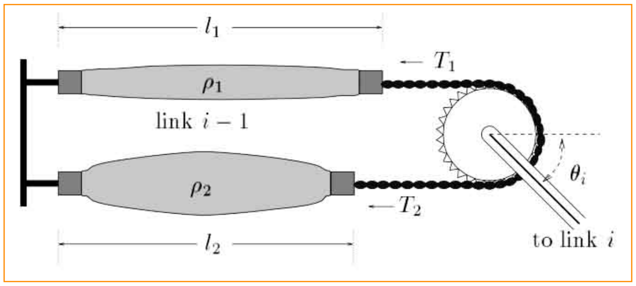
Contraction under pressure

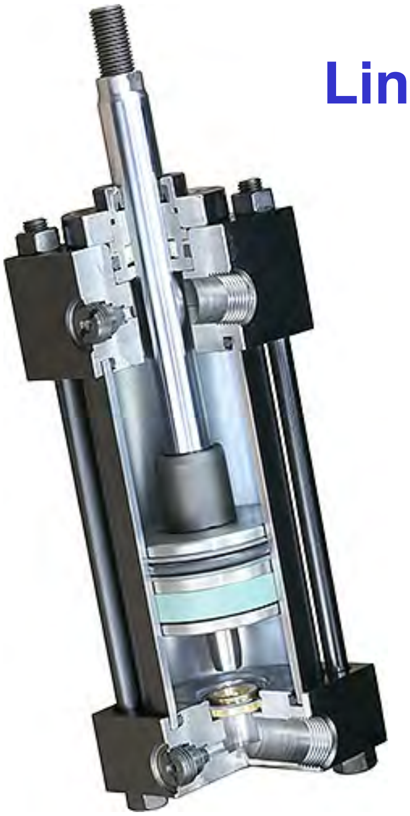


Robot arm

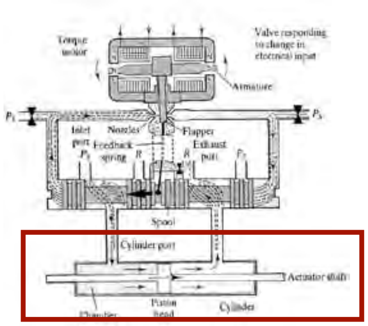


Agonist-antagonist action produces rotation

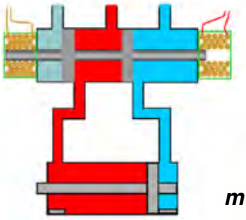




Linear Hydraulic Actuator

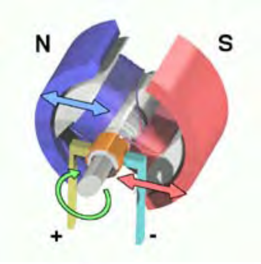


Electro/ mechanical transduction via small piston

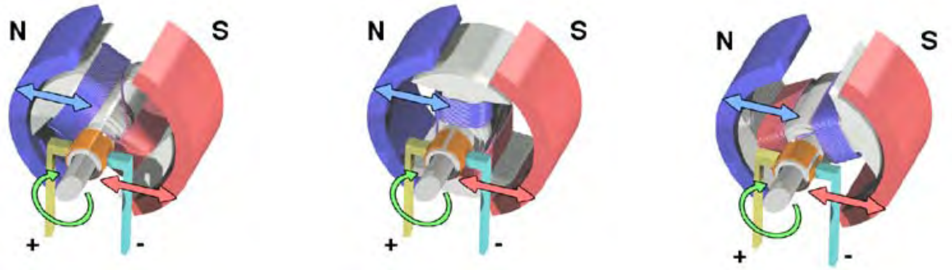


Force multiplication by large piston

Electric Actuator
Brushed DC Motor



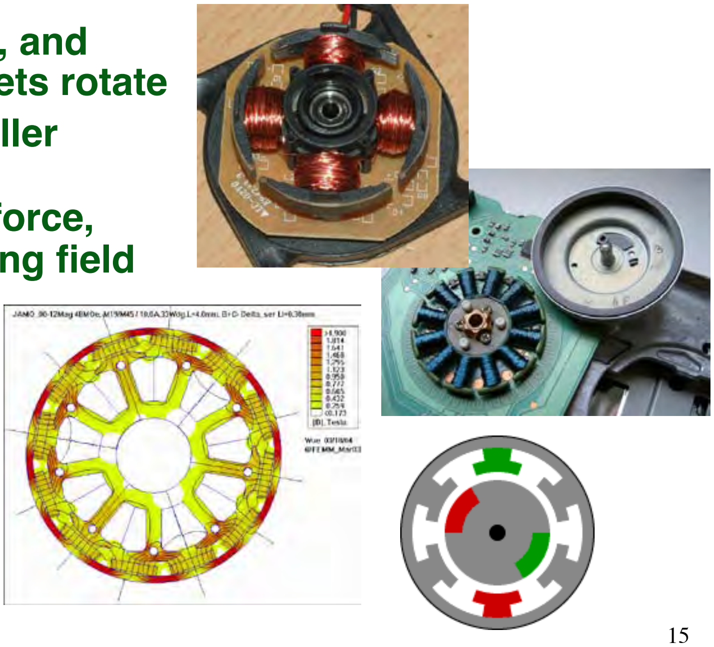
Two-pole DC Motor



- Current flowing through armature generates a magnetic field
- Permanent magnets torque the armature
- When armature is aligned with magnets, commutator ("brush") reverses current and magnetic field
- Multiple poles added to allow motor to smooth output torque and to start from any position

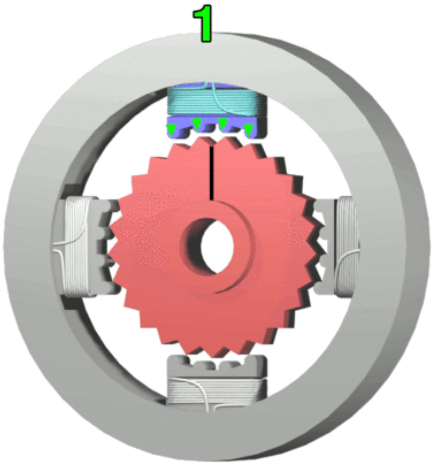
Electric Actuator
Brushless DC Motor

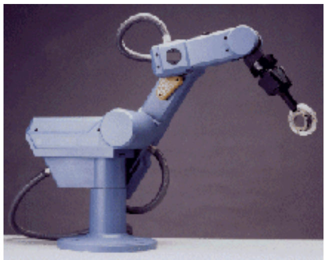
- **Armature is fixed, and permanent magnets rotate**
- **Electronic controller commutates the electromagnetic force, providing a rotating field**
- **Advantages**
  - **Efficiency**
  - **Noise**
  - **Lifetime**
  - **Reduced EMI**
  - **Cooling**
  - **Water-resistant**



Electric Actuator
Stepper Motor

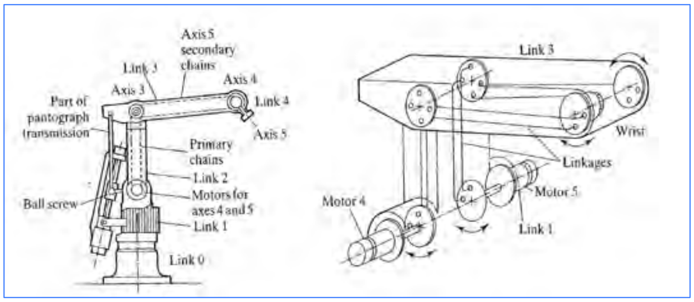
- **Brushless, synchronous motor that moves in discrete steps**
- **Precise, quantized control without feedback**
- **Armature teeth (outside of magnetic rotor) offset to induce rotary motion**





Actuation Linkages

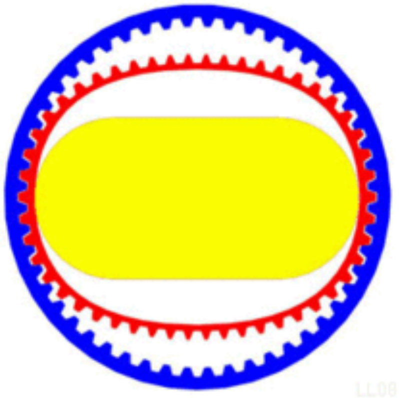
- Gearing, leverage
- Gears
- Belts, Chains, Cables, Tendons
- Bellcranks



Belt Linkage
http://www.youtube.com/watch?v=FV_P7GBAAgo

Harmonic Drive

- Strain wave gearing on motor output
- No backlash
- High gear ratios
- Good resolution and repeatability
- High torque



https://www.youtube.com/watch?v=8ATz0gSfOQ4

Ball/Roller Screw

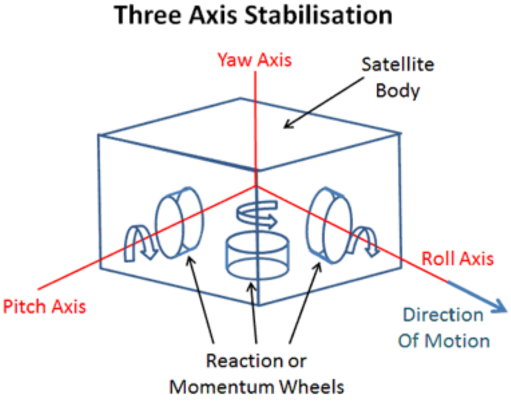
Transforms rotary to linear motion

Reaction Wheel

Flywheel on a motor shaft

Reaction wheel rpm is varied to trade angular momentum with a spacecraft for control

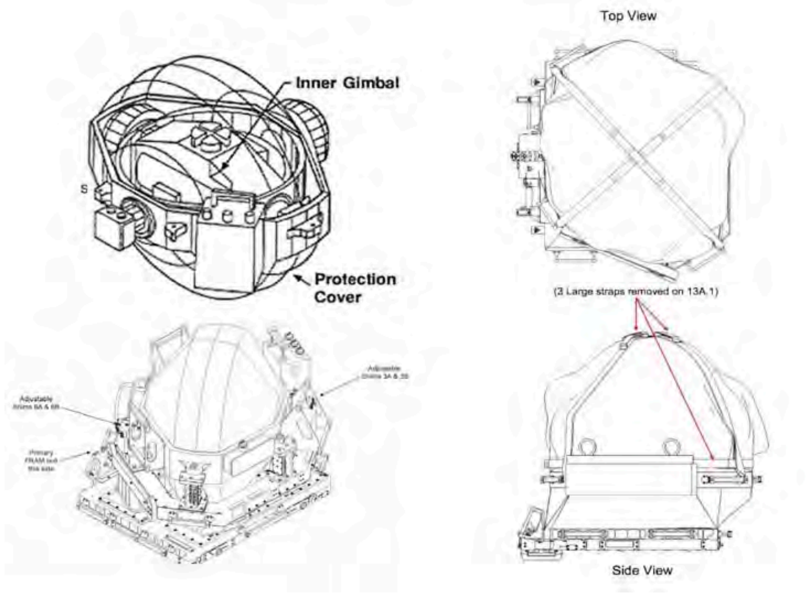
- Three orthogonal wheels vary all components of angular momentum
- Fourth wheel at oblique angle would provide redundancy



Control-Moment Gyro

Flywheel on a motor shaft

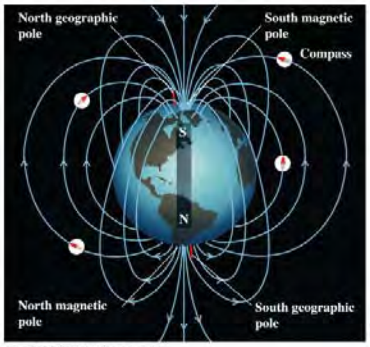
RPM is fixed, axis is rotated to impart torque



Sensors

Magnetometer

- Flux gate "compass"
  - Alternating current passed through one coil
  - Permalloy core alternately magnetized by electromagnetic field
  - Corresponding magnetic field sensed by second coil
  - Distortion of oscillating field is a measure of one component of the Earth's magnetic field
- Three magnetometers required to determine Earth's magnetic field vector



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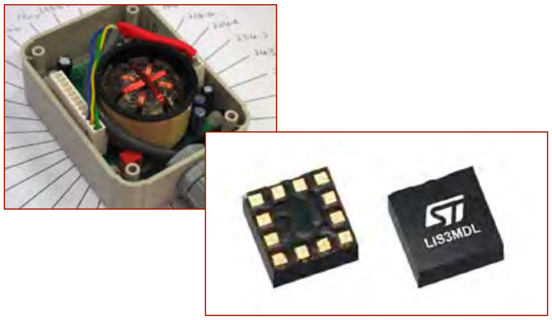
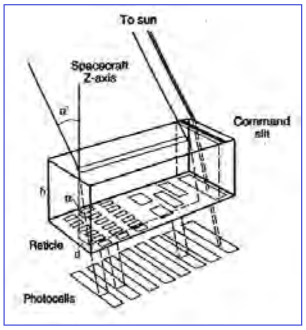
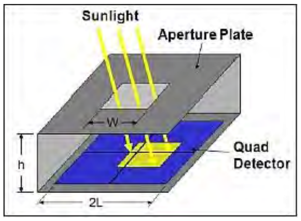


Sun Angle Sensor

- Distance from centerline measured by sensed pattern, which determines angle, α
- With index of refraction, *n*, angle to sun, α', is determined
- Reticle digitizes slit of light in 1st example
- Photodetectors may provide digital (coarse) or analog (fine) outputs





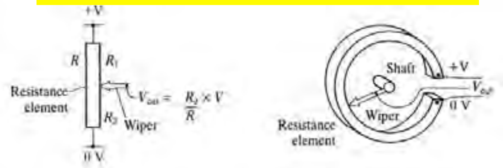
$$\tan\alpha = d/h$$

$$\sin\alpha' = n\sin\alpha \quad (Snell's\ law)$$

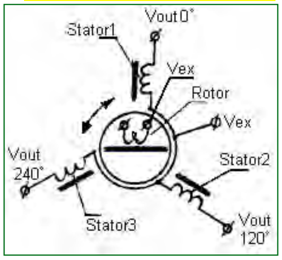
$$n = index\ of\ refraction$$

Potentiometer, Synchro, and Tachometer

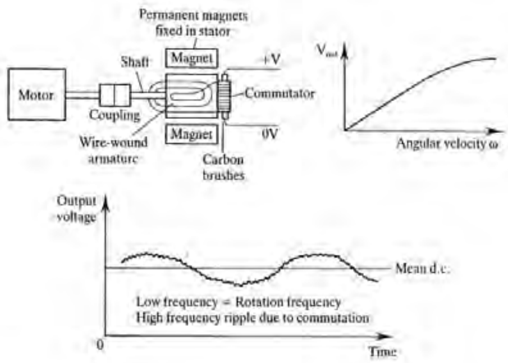
Potentiometer (displacement)



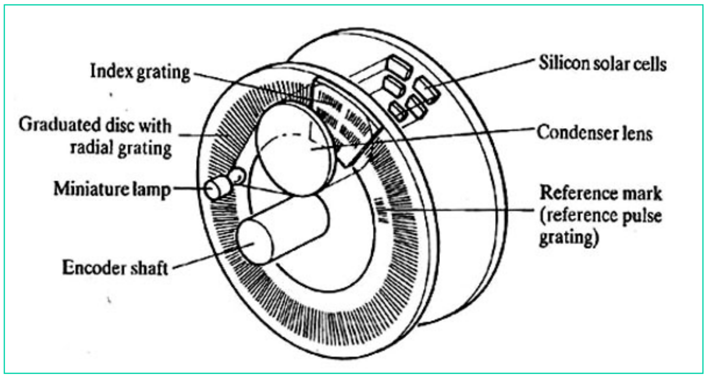
Synchro (angle)

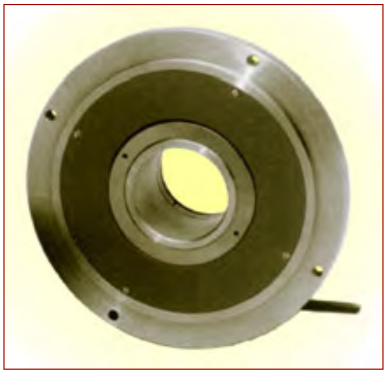


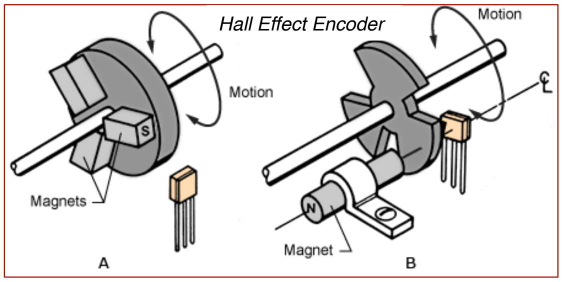
Tachometer (rate)



Angular Encoder

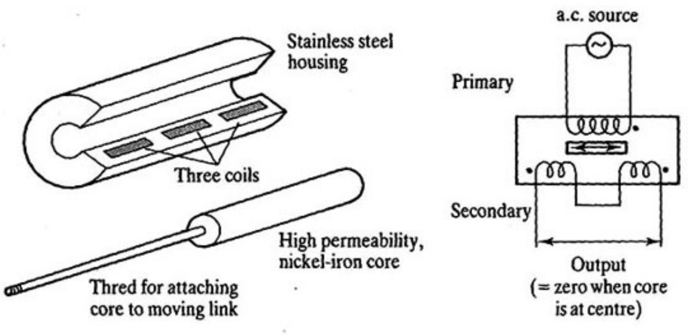


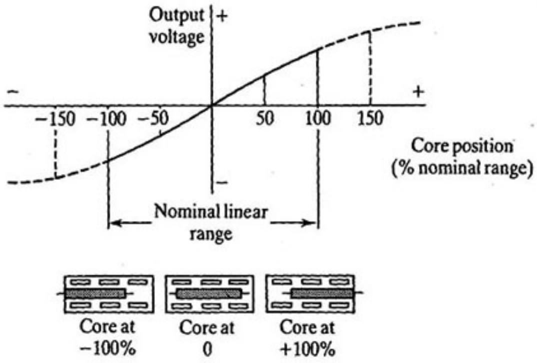




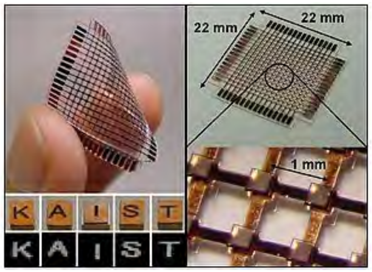
Transverse electric field induced by magnet

Linear Variable Differential Transformer

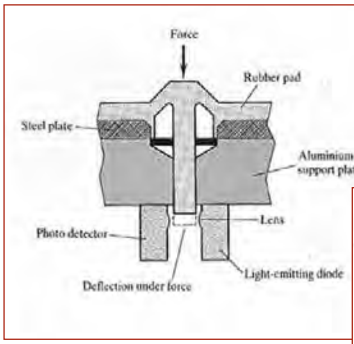




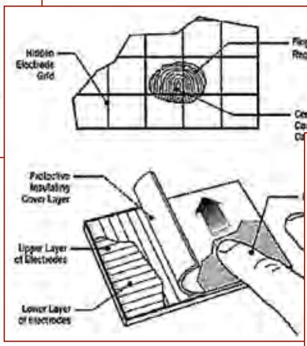
Tactile Sensors



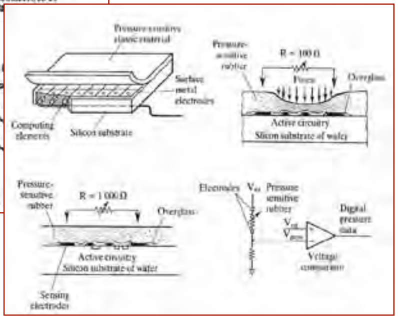
Photoelectric Key



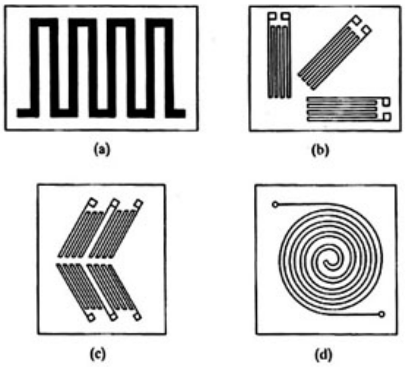
Capacitive Touchpad



Pressure-Sensitive Touchpad

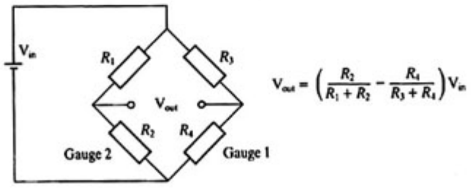


Strain Gauge

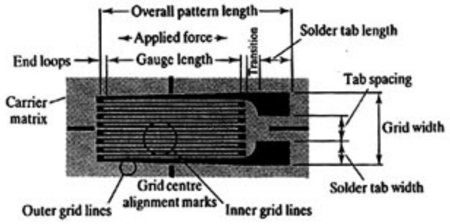


Resistance varies as material is stretched

Wheatstone Bridge

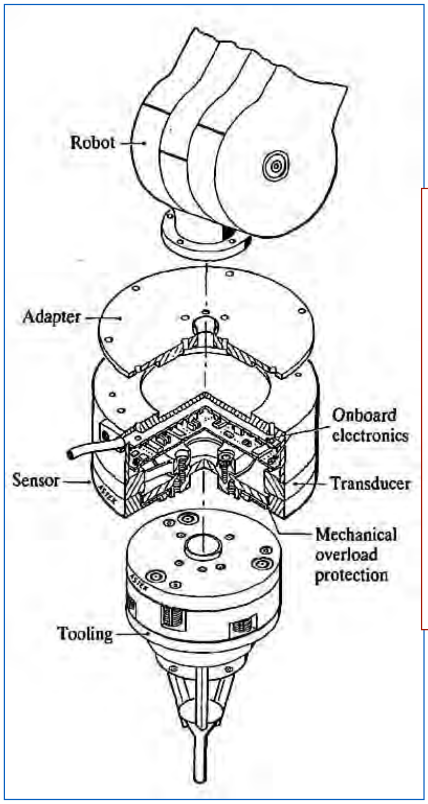


$$V_{out} = \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right)V_{in}$$

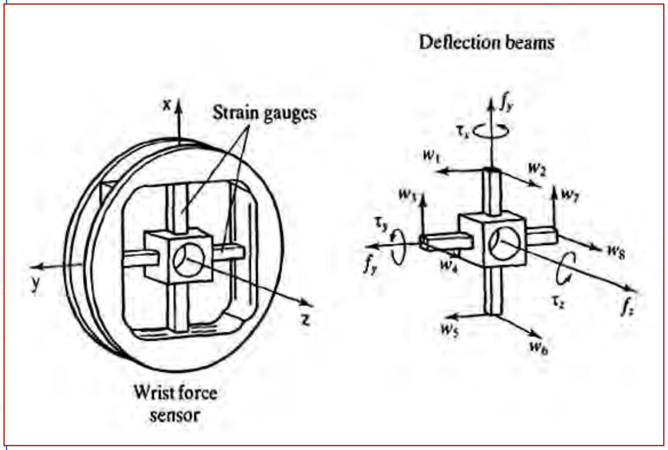


$$\varepsilon = \left(\frac{\Delta R}{R_o}\right) \Big/ \mathit{Gauge\ Factor}$$

Force Sensors



Examples: arrays of strain gauges

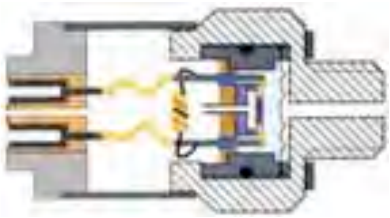


Force ∝

Stiffness x Displacement(Strain)

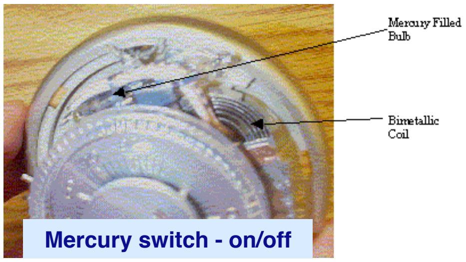
Pressure and Temperature Sensors

Deflection of Diaphragm Between Chambers at Different Pressure



Variation in Capacitance or Resistance

Deflection of Bi-Metallic Element

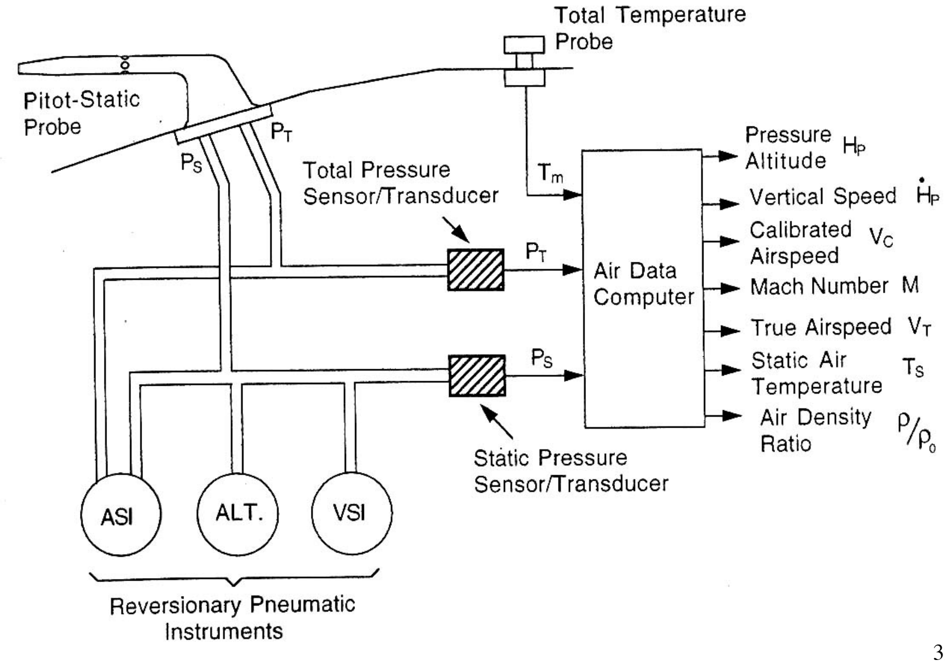


Mercury switch - on/off

Thermistors

Variation in Resistance

Air Data Sensors



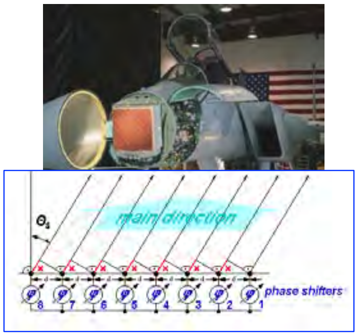
Radar and Sonar

Tracking (Pulse) RADAR

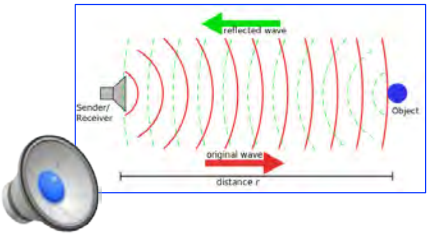
http://www.youtube.com/watch?v=LOqRBtbEuig

(Doppler) Radar Gun

Electronically Steered Array Tracking Radar

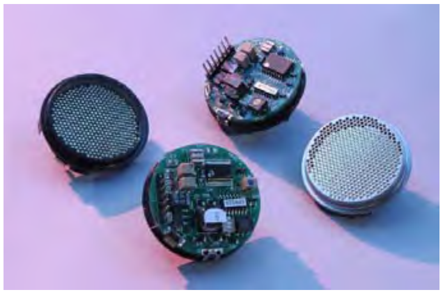


SONAR (SOund NAvigation and Ranging)

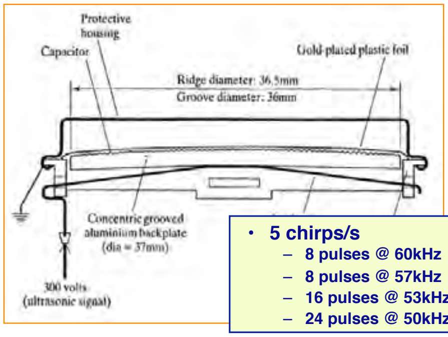


Ultrasonic (SONAR) Rangefinder

SensComp ("Polaroid") Devices

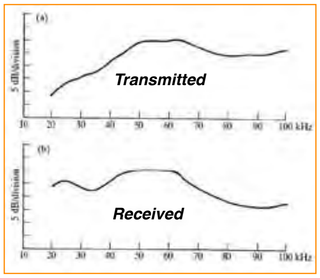


Transmit/Receive Unit

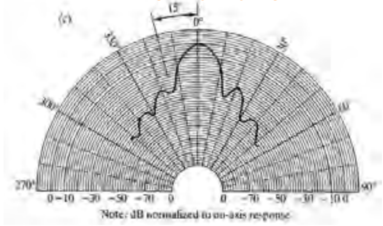


- 5 chirps/s
  - 8 pulses @ 60kHz
  - 8 pulses @ 57kHz
  - 16 pulses @ 53kHz
  - 24 pulses @ 50kHz

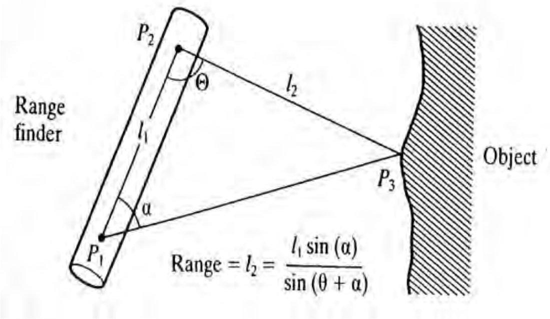
Chirp Spectrum

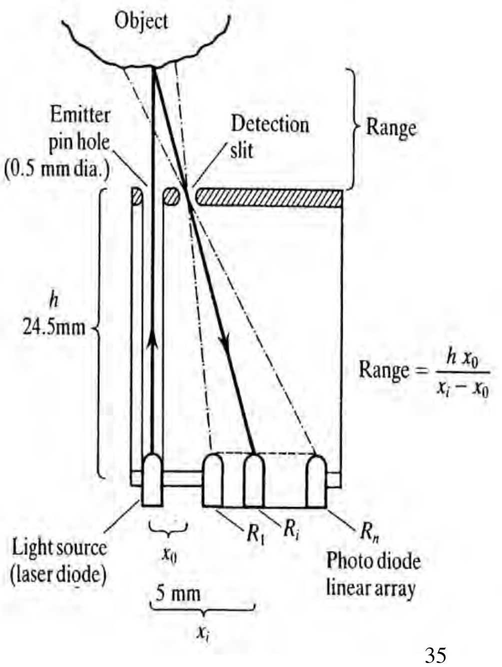


Antenna Pattern

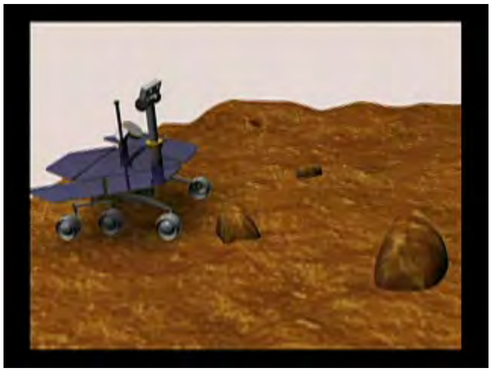


Triangulation Rangefinders

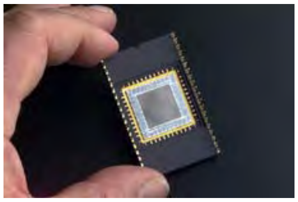




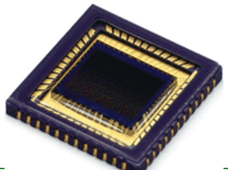
Video and Computer Vision



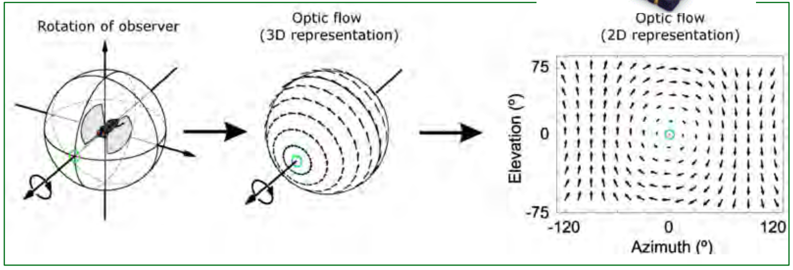
CCD Sensor



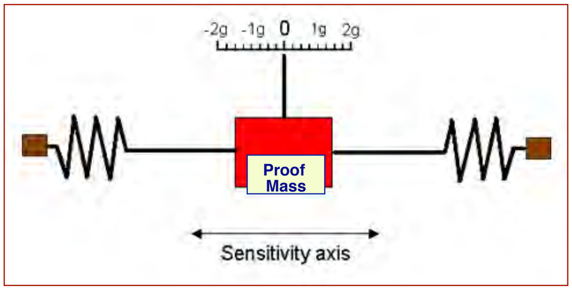
CMOS Device



Optic Flow



Spring Deflection Accelerometer

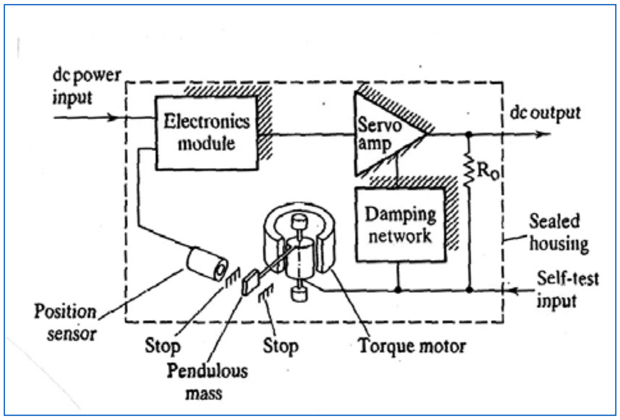


$$\Delta\ddot{x} = -k_s \Delta x / m$$

$$\Delta x = \frac{m}{k_s}\Delta\ddot{x}$$

Deflection is proportional to acceleration
Damping required to reduce oscillation

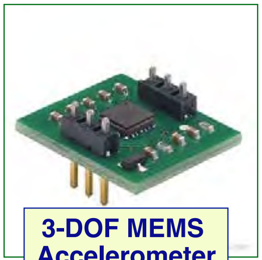
Force Rebalance Accelerometer



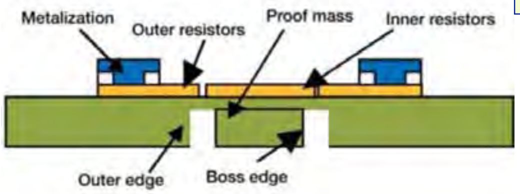
$$\Delta\ddot{x} = f_x / m = \frac{torque/moment\ arm}{m} \Rightarrow \Delta x \simeq 0$$

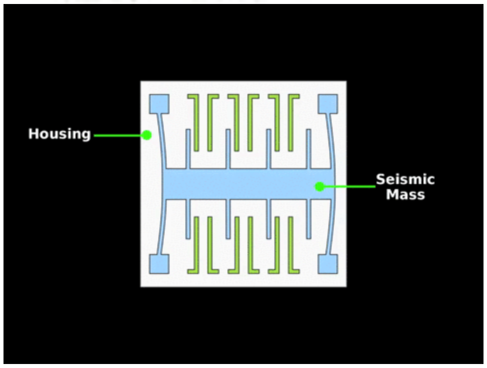
Torquer voltage required to re-center the proof mass becomes the measure of acceleration
Example of closed-loop control

MicroElectroMechanical System (MEMS) Accelerometer



3-DOF MEMS Accelerometer





Vibrating Piezoelectric Crystal Angular Rate Sensor

- “Tuning fork” principle
- 4 piezoelectric crystals
  - 2 active, oscillating out of phase with each other
  - 2 sensors, mounted perpendicular to the active crystals
- With zero rate along the long axis, sensors do not detect vibration
- Differential output of the sensors is proportional to angular rate

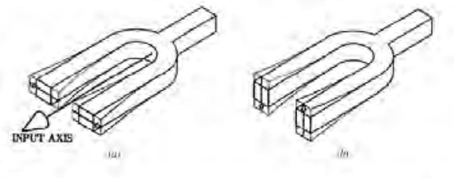
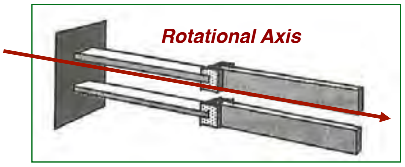
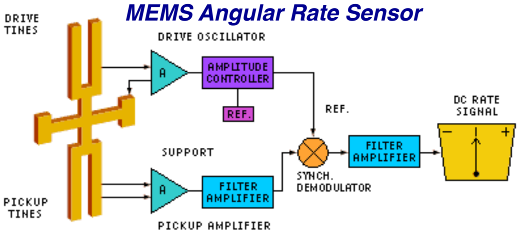


Fig. 9.11 Vibration modes of tuning fork gyroscope: (a) input mode; (b) output mode.



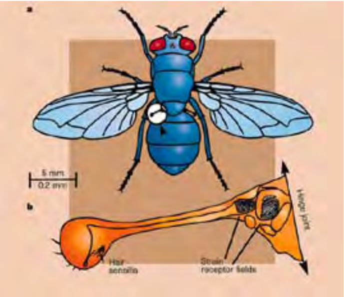
MEMS Angular Rate Sensor

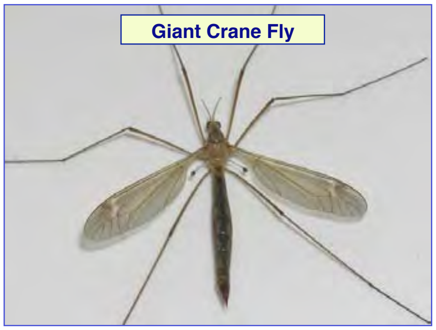


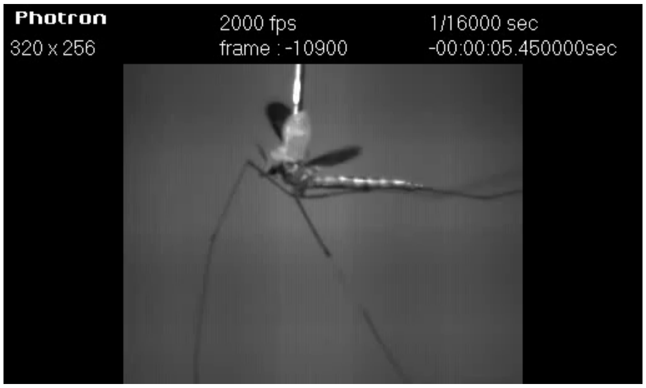
Halteres:

Biological Angular Rate Sensors

Vestigal second pair of wings





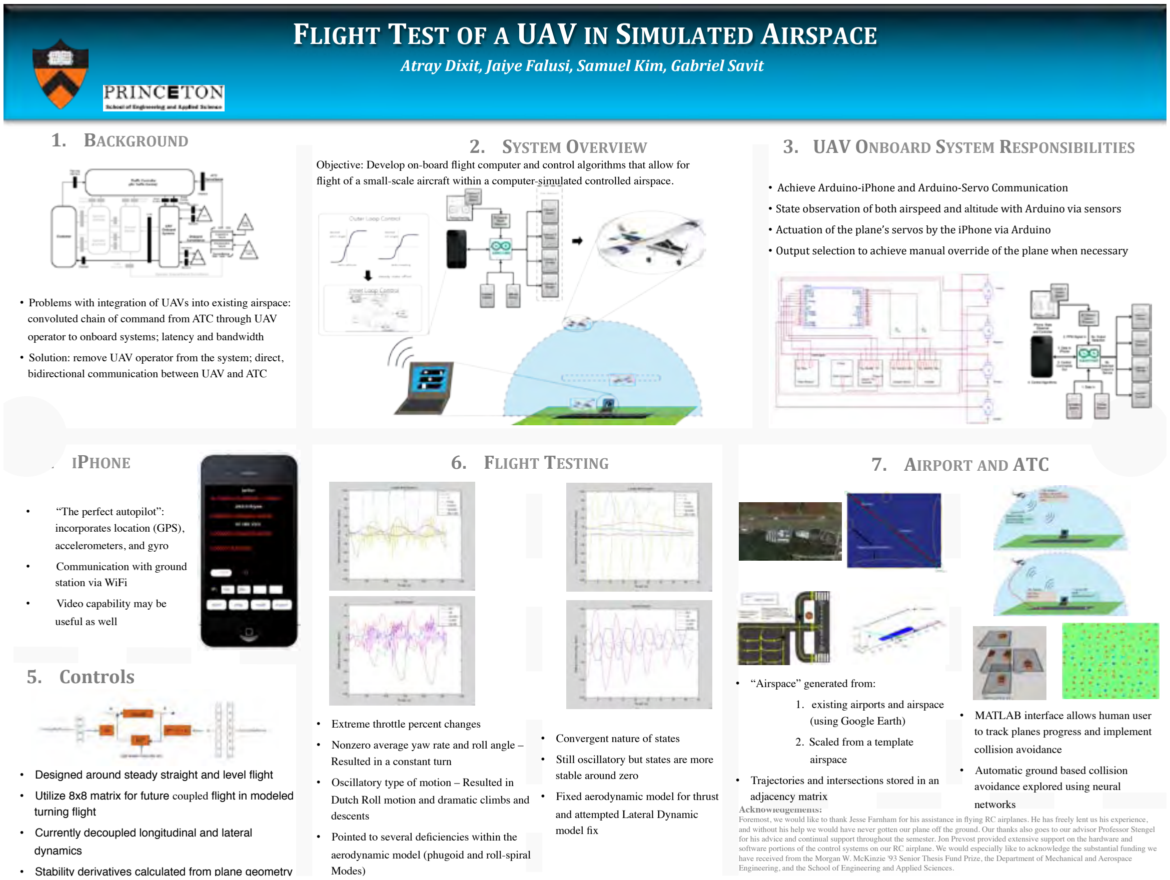


All in Your Pocket

iPhone 6s

- **3-axis accelerometer**
- **3-axis angular rate**
- **2-axis magnetometer compass**
- **GPS position measurement**
- **64-bit, 1.8 GHz processor**
- **2 GB RAM**
- **128 GB flash memory**
- **2 cameras, mic, speakers**

$$\mathbf{z} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ p \\ q \\ r \\ \varepsilon_{horizontal} \\ \varepsilon_{vertical} \\ L \\ \lambda \\ h \end{bmatrix}$$



FLIGHT TEST OF A UAV IN SIMULATED AIRSPACE

Atray Dixit, Jaiye Falusi, Samuel Kim, Gabriel Savit

PRINCETON
School of Engineering and Applied Science

1. BACKGROUND

- Problems with integration of UAVs into existing airspace: convoluted chain of command from ATC through UAV operator to onboard systems; latency and bandwidth
- Solution: remove UAV operator from the system; direct, bidirectional communication between UAV and ATC

2. SYSTEM OVERVIEW

Objective: Develop on-board flight computer and control algorithms that allow for flight of a small-scale aircraft within a computer-simulated controlled airspace.

3. UAV ONBOARD SYSTEM RESPONSIBILITIES

- Achieve Arduino-iPhone and Arduino-Servo Communication
- State observation of both airspeed and altitude with Arduino via sensors
- Actuation of the plane's servos by the iPhone via Arduino
- Output selection to achieve manual override of the plane when necessary

IPHONE

- "The perfect autopilot": incorporates location (GPS), accelerometers, and gyro
- Communication with ground station via WiFi
- Video capability may be useful as well

5. Controls

- Designed around steady straight and level flight
- Utilize 8x8 matrix for future coupled flight in modeled turning flight
- Currently decoupled longitudinal and lateral dynamics
- Stability derivatives calculated from plane geometry

6. FLIGHT TESTING

- Extreme throttle percent changes
- Nonzero average yaw rate and roll angle – Resulted in a constant turn
- Oscillatory type of motion – Resulted in Dutch Roll motion and dramatic climbs and descents
- Pointed to several deficiencies within the aerodynamic model (phugoid and roll-spiral Modes)

- Convergent nature of states
- Still oscillatory but states are more stable around zero
- Fixed aerodynamic model for thrust and attempted Lateral Dynamic model fix

7. AIRPORT AND ATC

- "Airspace" generated from:
    1. existing airports and airspace (using Google Earth)
    2. Scaled from a template airspace
- Trajectories and intersections stored in an adjacency matrix

- MATLAB interface allows human user to track planes progress and implement collision avoidance
- Automatic ground based collision avoidance explored using neural networks

Acknowledgements:
Foremost, we would like to thank Jesse Farnham for his assistance in flying RC airplanes. He has freely lent us his experience, and without his help we would have never gotten our plane off the ground. Our thanks also goes to our advisor Professor Stengel for his advice and continual support throughout the semester. Jon Prevost provided extensive support on the hardware and software portions of the control systems on our RC airplane. We would especially like to acknowledge the substantial funding we have received from the Morgan W. McKinzie '93 Senior Thesis Fund Prize, the Department of Mechanical and Aerospace Engineering, and the School of Engineering and Applied Sciences.

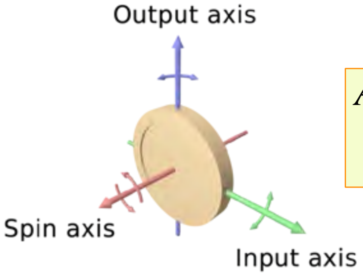
Parrot AR.Drone 2.0

- 1GHz 32 bit ARM Cortex A8 processor with 800MHz video DSP TMS320DMC64x
- Linux 2.6.32
- 1Gbit DDR2 RAM at 200MHz
- USB 2.0 high speed for extensions
- Wi-Fi b,g,n
- 3 axis gyroscope 2000°/second precision
- 3 axis accelerometer +-50mg precision
- 3 axis magnetometer 6° precision
- Pressure sensor +/- 10 Pa precision
- Ultrasound sensors for ground altitude measur
- 60 fps vertical ground speed m

- 4 brushless inrunner motors. 14.5W 28,500 RMP
- Micro ball bearing
- Low noise Nylatron gears for 1/8.75 propeller reductor
- Tempered steel propeller shaft
- Self-lubrificating bronze bearing
- Specific high propelled drag for great maneuverability
- 8 MIPS AVR CPU per motor controller
- 3 elements 1000 mA/H LiPo rechargeable battery (Autonomy: 12 minutes)
- Emergency stop controlled by software
- Fully reprogrammable motor controller
- Water resistant motor's electronic controller

- HD Camera. 720p 30fps
- Wide angle lens : 92° diagonal
- H264 encoding base profile
- Low latency streaming
- Video storage on the fly with the remote device
- JPEG photo
- Video storage on the fly with Wi-Fi directly on your remote device or on a USB key

Mechanical Gyroscope



Angular momentum

$$\mathbf{h}_B = \boldsymbol{I}_B \boldsymbol{\omega}_B$$

Body-axis moment equation

$$\boxed{\mathbf{M}_B = \dot{\mathbf{h}}_B + \tilde{\boldsymbol{\omega}}_B \mathbf{h}_B = \mathbf{I}_B \dot{\boldsymbol{\omega}}_B + \tilde{\boldsymbol{\omega}}_B \mathbf{h}_B}$$

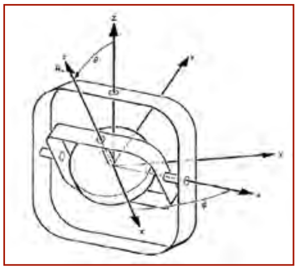
$$\boxed{\dot{\boldsymbol{\omega}}_B = \boldsymbol{I}_B^{-1}\left(M_B - \tilde{\boldsymbol{\omega}}_B \boldsymbol{I}_B \boldsymbol{\omega}_B\right)}$$

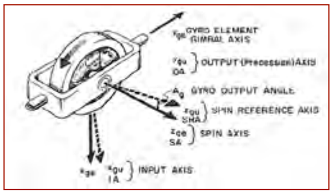
Constant nominal spin rate, *n*, about *z* axis

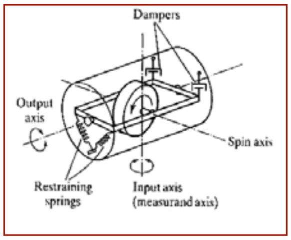
$I_{xx} = I_{yy} \ll I_{zz}$

Small perturbations in ω_x and ω_y

Types of Mechanical Gyroscope



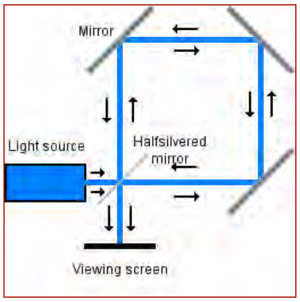




- **Two-degree-of-freedom gyro**
  - Free gyro mounted on a gimbaled platform
  - Gyro "stores" reference direction in space
  - Angle" pickoffs" (encoders) on gimbal axes measure pitch and yaw angles
  - Drift due to friction in bearings
  - Pendulum to maintain vertical over time
- **Single-degree-of-freedom gyro**
  - Gyro axis constrained to rotate in its case with respect to the output axis, ***y***, only
  - "Synchro" measures axis rotation, and "torquer" keeps θ small
  - Torque applied is a measure of the input about the ***x*** axis
- **Rate and integrating gyros**
  - Large angle feedback produces a **rate gyro**
  - Large rate feedback produces an **integrating gyro**

Optical “Gyroscope”

- **Sagnac interferometer** measures rotational rate, ω
  - $\omega = 0$, photons traveling in opposite directions complete the circuit in the same time
  - $\omega \neq 0$, travel length and time are different
- On a circular path of radius R:



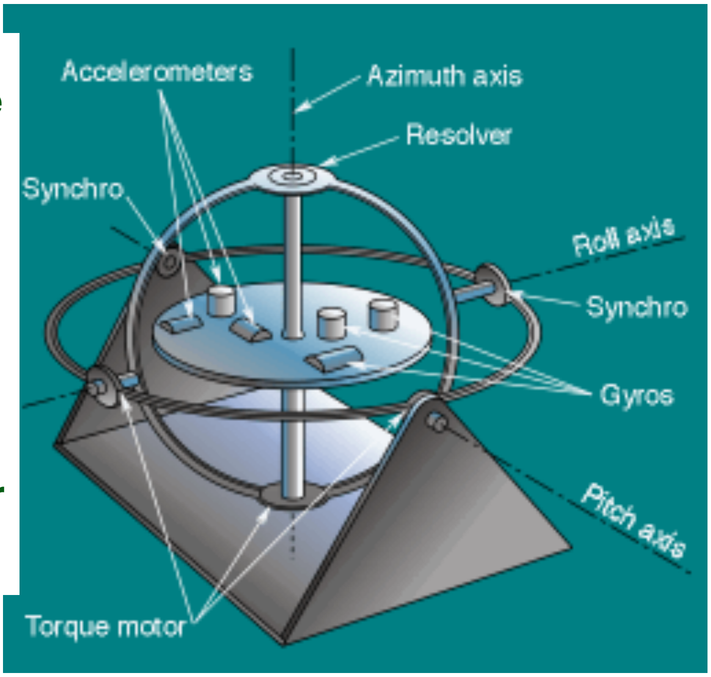
$$t_{CCW} = \frac{2\pi R}{c}\left(1-\frac{R\omega}{c}\right); \quad t_{CW} = \frac{2\pi R}{c}\left(1+\frac{R\omega}{c}\right)$$

$$\Delta t = t_{CW} - t_{CCW} = \frac{4\pi R^2}{c^2}\omega = \frac{4A}{c^2}\omega$$

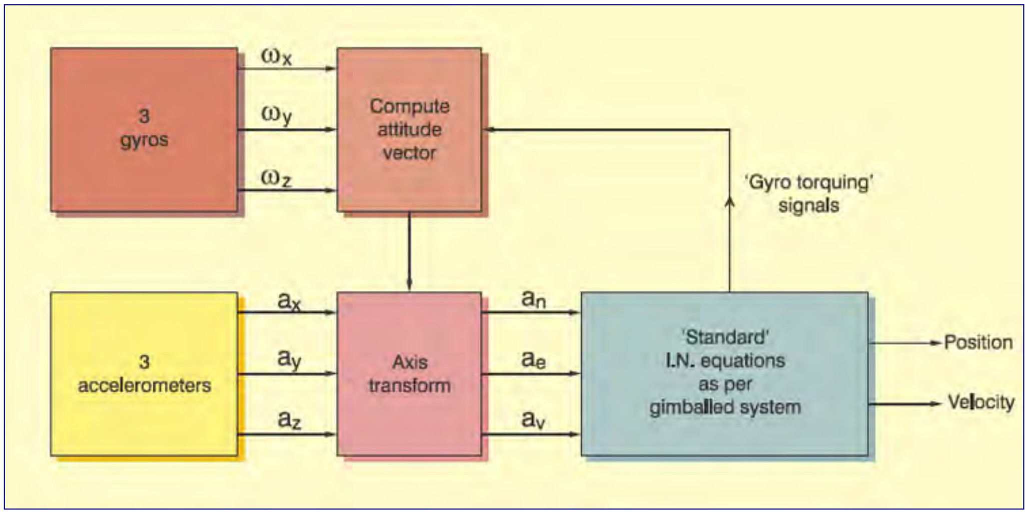
c : speed of light
R : radius
A : area

Physical Platform Inertial Reference Unit

- Servo-driven to maintain reference orientation
  - Instrument feedback
  - Schuler pendulum
  - Gyro-compassing
  - Star trackers
  - GPS
- 3 Accelerometers
- 3 Angle or Angular Rate Gyros



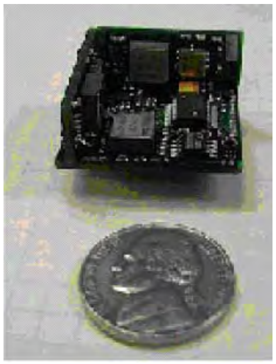
Strapdown Inertial Measurement Unit



- Rate gyros and accelerometers rotate with the vehicle
  - High dynamic range of instruments is required
  - Inertial reference frame is computed rather than physical

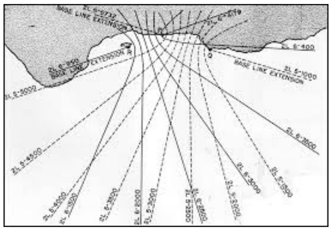
MicroElectroMechanical (MEMS) Strapdown Inertial Measurement Unit

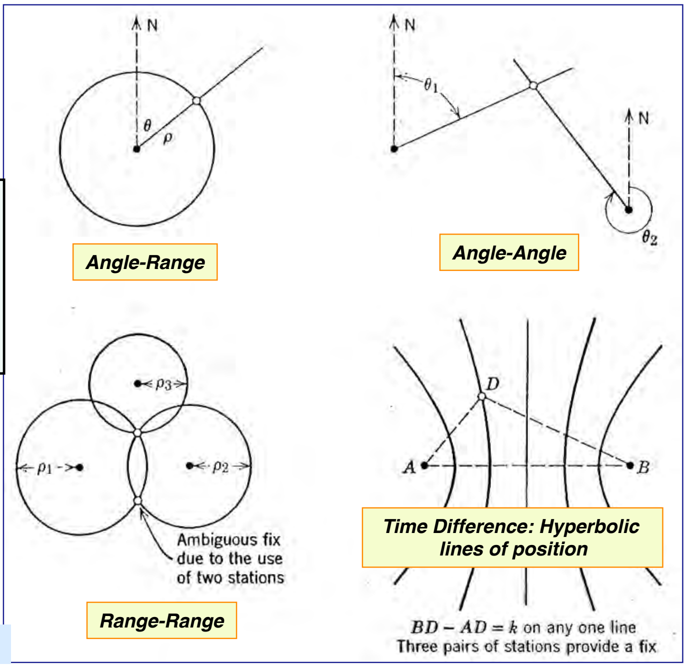
- 3 linear accelerometers, 3 angular rate sensors
  - High drift rates produce worsening navigation accuracy
  - Short-term accuracy sufficient for many applications
  - Inexpensive
  - GPS position updating counters the drift rate



Position Fixing for Navigation (2-D Examples)

- **Lines of position**
  - Straight line
  - Circle
  - Hyperbola



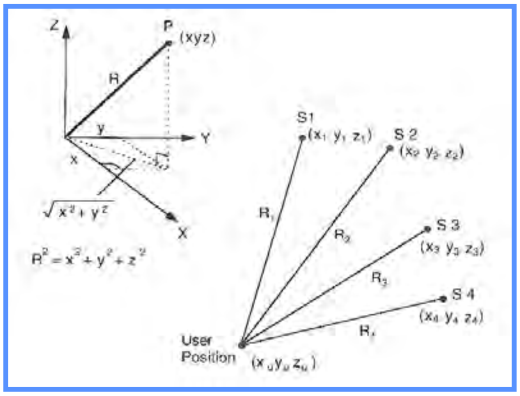


Kayton & Fried

Global Positioning System (GPS)

- Six orbital planes with four satellites each
  - Altitude: 20,200 km (10,900 nm)
  - Inclination : 55 deg
  - Constellation planes separated by 60 deg
- Each satellite contains an atomic clock and broadcasts a 30-sec message at 50 bps
  - Ephemeris
  - ID
  - Clock data
- Details of satellite signal at *http://en.wikipedia.org/wiki/Gps*
- *https://www.youtube.com/watch?v=FU_pY2sTwTA*

Position Fixing from Four GPS Satellites



- **Pseudorange estimated from *speed of light* and *time required* to receive signal**

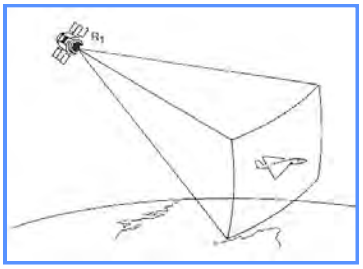
$$\Delta t_i = \left(t_{received} - t_{sent}\right)_{Satellite\,\#i}$$

$$Satellite\ \#1: R_{1_p} = c\Delta t_1$$
$$Satellite\ \#2: R_{2_p} = c\Delta t_2$$

$$Satellite\ \#3: R_{3_p} = c\Delta t_3$$
$$Satellite\ \#4: R_{4_p} = c\Delta t_4$$

User clock inaccuracy* produces error, *C_u

$$C_u = c\Delta t_{user\ clock\ error}$$



Position Fixing from Four GPS Satellites

$$\text{Satellite position: } \left(x_i, y_i, z_i\right)$$
$$\text{User position: } \left(x_u, y_u, z_u\right)$$

- **Satellite transmits transmit time and position via *ephemeris***

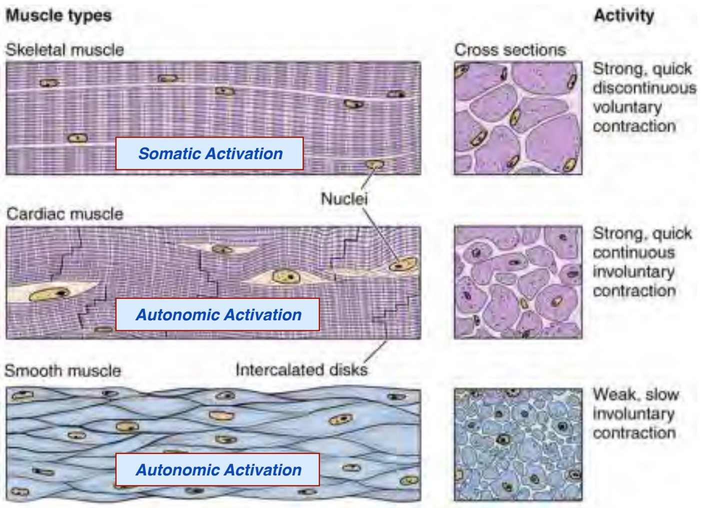
$$R_1 = \sqrt{\left(x_1 - x_u\right)^2 + \left(y_1 - y_u\right)^2 + \left(z_1 - z_u\right)^2} = R_{1_p} + C_u$$
$$R_2 = \sqrt{\left(x_2 - x_u\right)^2 + \left(y_2 - y_u\right)^2 + \left(z_2 - z_u\right)^2} = R_{2_p} + C_u$$
$$R_3 = \sqrt{\left(x_3 - x_u\right)^2 + \left(y_3 - y_u\right)^2 + \left(z_3 - z_u\right)^2} = R_{3_p} + C_u$$
$$R_4 = \sqrt{\left(x_4 - x_u\right)^2 + \left(y_4 - y_u\right)^2 + \left(z_4 - z_u\right)^2} = R_{4_p} + C_u$$

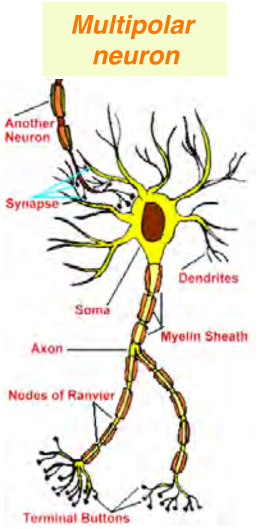
- **Four equations and four unknowns (*x_u, y_u, z_u, C_u*)**
- **Accuracy improved using data from more than 4 satellites**

Next Time:
Introduction to Optimization

Supplementary Material

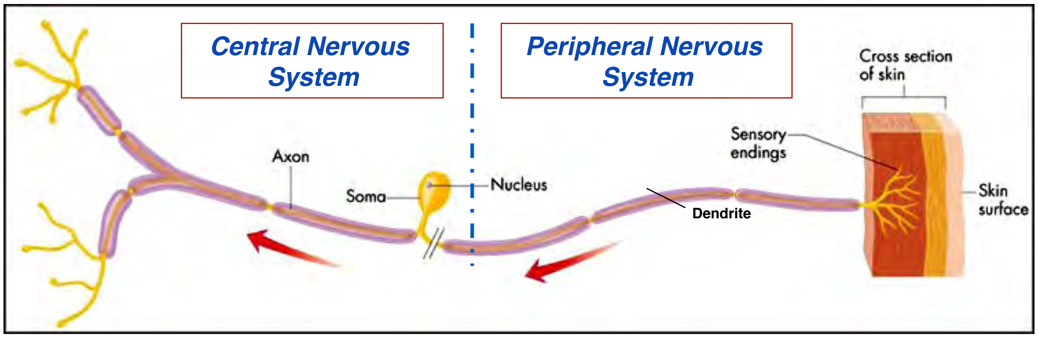
Muscle and Motor (*Efferent*) Neurons





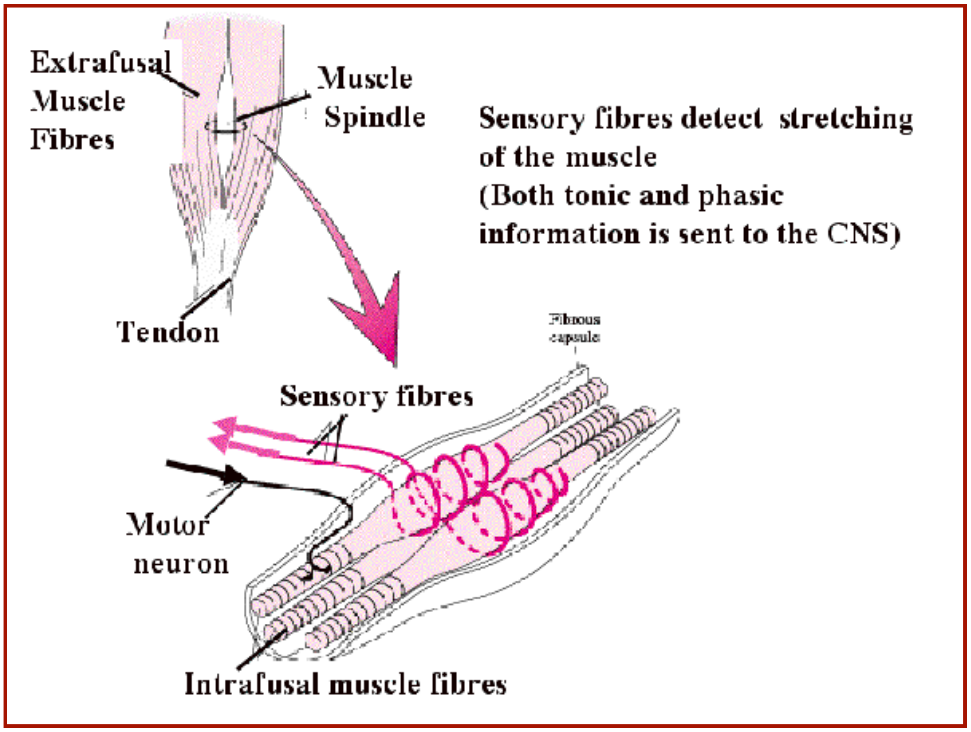
- **Force is produced by contraction of individual muscle cells**
- **Motor neurons command muscles**
- **Each muscle cell is innervated by many overlapping neurons**
- **Motor neuron soma are in ventral root ganglia of the spine**

Sensory (*Afferent*) Neurons



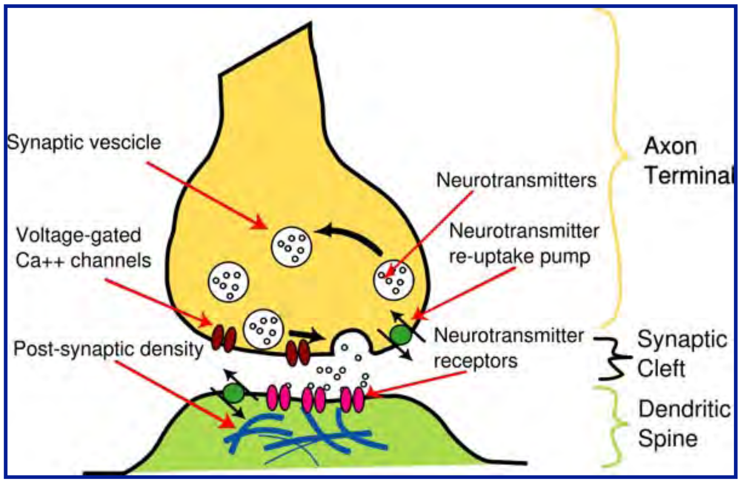
- **Components of the peripheral nervous system that measure *pressure*, *temperature*, *vibration*, etc.**
- **Neuron Soma located in the dorsal root at the base of the spine**
- **The sensory neuron is pseudo-unipolar**
  - **Input from a single receptor's axon**
  - **Output to a single axon to synapses in the spinal column**

Motor Neuron Receptors



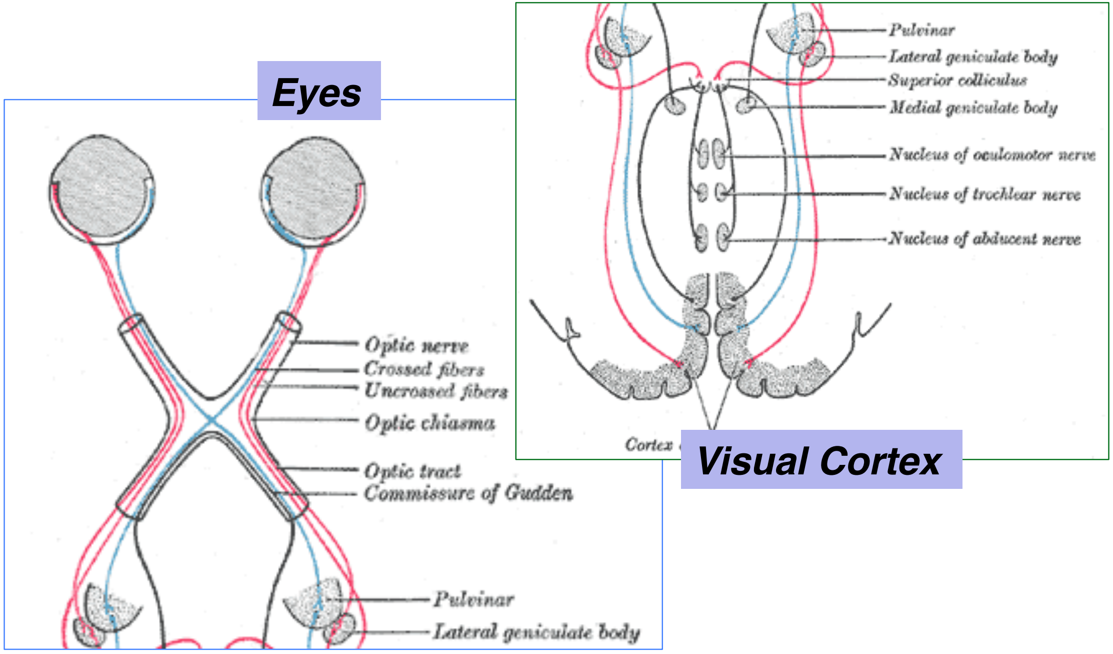
Sensory fibres detect stretching of the muscle
(Both tonic and phasic information is sent to the CNS)

Synapses Excite or Inhibit Downstream Cellular Activity



Post-synaptic cell can be a neuron, a muscle, or a gland

Optic Schema



ADVANTAGES AND DISADVANTAGES OF HYDRAULIC ACTUATORS (from McKerrow)

Hydraulic actuators

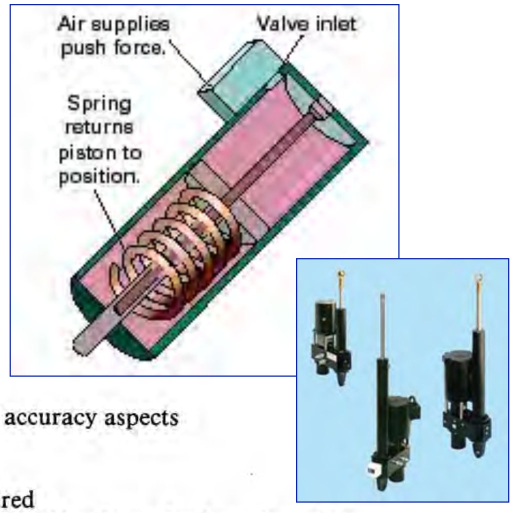
Advantages

- Large lift capacity
- High power to weight ratio
- Moderate speeds
- Oil is incompressible, hence once positioned joints can be locked to a stiff structure
- Very good servo control can be achieved
- Self lubricating and self cooling
- Operate in stalled condition with no damage
- Fast response
- Intrinsically safe in flammable and explosive atmospheres
- Smooth operation at low speeds

Disadvantages

- Hydraulic systems are expensive
- Maintenance problems with seals causing leakage
- Not suitable for high speed cycling
- Need for a return line
- Hard to miniaturize because high pressures and flow rates
- Need for remote power source which uses floor space
- Cannot back drive links against valves

ADVANTAGES AND DISADVANTAGES OF PNEUMATIC ACTUATORS (from McKerrow)



Pneumatic actuators

Advantages

- Relatively inexpensive
- High speed
- Do not pollute work area with fluids
- Can be used in laboratory work
- No return line required
- Common energy source in industry
- Suits modular robot designs
- Actuator can stall without damage

Disadvantages

- Compressibility of air limits control and accuracy aspects
- Noise pollution from exhausts
- Leakage of air can be of concern
- Additional drying/filtering may be required
- Difficulties with control of speeds, take up of loads, and exhausting of lines

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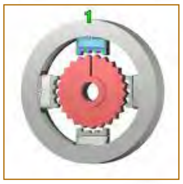
ADVANTAGES AND DISADVANTAGES OF ELECTRIC ACTUATORS (DC MOTOR AND STEPPER MOTOR) (from McKerrow)



Electric actuators (DC motors and stepper motors)

Advantages

- Actuators are fast and accurate
- Possible to apply sophisticated control techniques to motion
- Relatively inexpensive
- Very fast development times for new models
- New rare earth motors have high torques, reduced weight, and fast response times

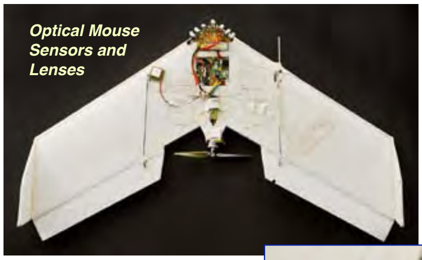
Disadvantages

- Inherently high speed with low torque, hence gear trains or other power transmission units are needed
- Gear backlash limits precision
- Electrical arcing may be a consideration in flammable atmospheres
- Problems of overheating in stalled condition
- Brakes are needed to lock them in position

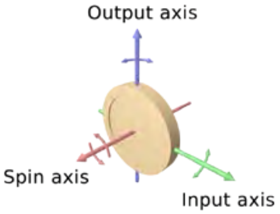
Autonomous Control of Miniature Aircraft Using Optical Flow

http://www.youtube.com/watch?v=F7QxDJiZHwI&feature=related

Swinglet, Ecole Polytechnique, Lausanne



Gyroscope Equations of Motion

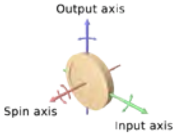


Linearized equations of angular rate change

$$\begin{bmatrix}\Delta\dot{\omega}_x\\ \Delta\dot{\omega}_y\\ 0\end{bmatrix} = \begin{bmatrix} I_{xx}&0&0\\0&I_{yy}&0\\0&0&I_{zz}\end{bmatrix}^{-1}\left[\begin{bmatrix}M_x\\M_y\\0\end{bmatrix} - \begin{pmatrix}0&-n&\Delta\omega_y\\ n&0&-\Delta\omega_x\\ -\Delta\omega_y&\Delta\omega_x&0\end{pmatrix}\begin{pmatrix}I_{xx}&0&0\\0&I_{yy}&0\\0&0&I_{zz}\end{pmatrix}\begin{pmatrix}\Delta\omega_x\\ \Delta\omega_y\\ n\end{pmatrix}\right]$$

$$\begin{bmatrix}\Delta\dot{\omega}_x\\ \Delta\dot{\omega}_y\\ 0\end{bmatrix} = \begin{bmatrix}\left[M_x - n\left(I_{zz}-I_{yy}\right)\Delta\omega_y\right]/I_{xx}\\ \left[M_y - n\left(I_{xx}-I_{zz}\right)\Delta\omega_x\right]/I_{yy}\\ 0\end{bmatrix}$$

$$\begin{bmatrix}\Delta\dot{\omega}_x\\ \Delta\dot{\omega}_y\end{bmatrix} = \begin{bmatrix}0 & n\left(I_{yy}-I_{zz}\right)/I_{xx}\\ n\left(I_{zz}-I_{xx}\right)/I_{yy} & 0\end{bmatrix}\begin{bmatrix}\Delta\omega_x\\ \Delta\omega_y\end{bmatrix} + \begin{bmatrix}M_x/I_{xx}\\ M_y/I_{yy}\end{bmatrix}$$



Gyroscope Natural Frequency

Laplace transform of dynamic equation

$$\begin{bmatrix} s & -n\left(I_{yy}-I_{zz}\right)/I_{xx} \\ -n\left(I_{zz}-I_{xx}\right)/I_{yy} & s \end{bmatrix}\begin{bmatrix} \Delta\omega_y(s) \\ \Delta\omega_y(s) \end{bmatrix}=\begin{bmatrix} M_x(s)/I_{xx} \\ M_y(s)/I_{yy} \end{bmatrix}$$

- Characteristic equation

$$\Delta(s)=s^2+n^2\left(\frac{I_{zz}}{I_{xx}}-1\right)^2=0$$

- Natural frequency, ω_n, of small perturbations

$$\omega_n=n\left(\frac{I_{zz}}{I_{xx}}-1\right) \quad rad/\sec$$

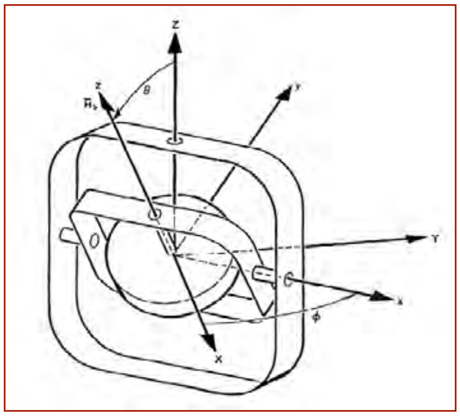
Example

$$n=36{,}000\ rpm=3{,}770\ rad/sec$$

$$\text{Thin disk: } \frac{I_{zz}}{I_{xx}}=2$$

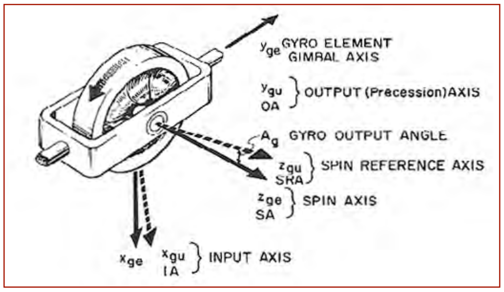
$$\omega_n=3{,}770\ rad/sec=600\ Hz$$

Two-Degree of Freedom Gyroscope



- Free gyro mounted on a gimbaled platform
- Gyro "stores" reference direction in space
- Angle "pickoffs" (encoders) on gimbal axes measure pitch and yaw angles
- Direction can be precessed by applying a torque

Single-Degree-of-Freedom Gyroscope



Gyro axis constrained to rotate in its case with respect to the output axis, *y*, only

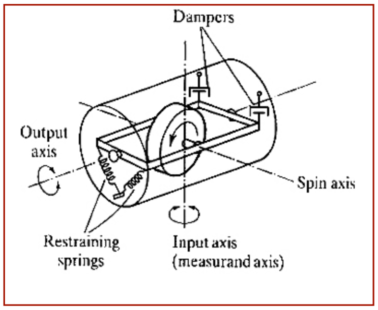
$$\boxed{\begin{bmatrix} \Delta\dot{\theta} \\ \Delta\dot{\omega}_y \end{bmatrix} = \begin{bmatrix} \Delta\omega_y \\ \left(h_{rotor}\Delta\omega_x + M_{y_{control}}\right)/I_{yy} \end{bmatrix}}$$

"Synchro" measures axis rotation, and "torquer" keeps *θ* small

Torque applied is a measure of the input about the *x* axis

$$\boxed{M_{y_{control}} = k_\theta\Delta\theta + k_\omega\Delta\omega_y + k_c\Delta u_c}$$

Rate and Integrating Gyroscopes



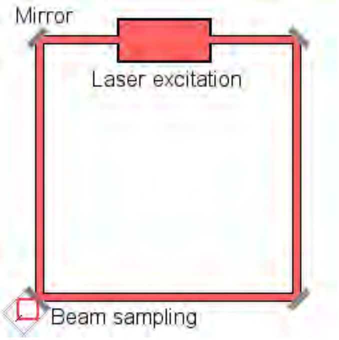
- **Large angle feedback produces a rate gyro**
  - **Analogous to a mechanical <u>spring</u> restraint**

$$\boxed{\begin{gathered} \Delta\dot{\omega}_{y_{SS}} = 0 = \left(h_{rotor}\Delta\omega_{x_{SS}} + k_\theta\Delta\theta_{SS}\right)/I_{yy} \\ \Delta\theta_{SS} = -\frac{h_{rotor}}{k_\theta}\Delta\omega_{x_{SS}} \end{gathered}}$$

- **Large rate feedback produces an integrating gyro**
  - **Analogous to a mechanical <u>damper</u> restraint**

$$\boxed{\begin{gathered} \Delta\dot{\omega}_{y_{SS}} = 0 = \left(h_{rotor}\Delta\omega_{x_{SS}} + k_\omega\Delta\omega_{y_{SS}}\right)/I_{yy} \\ \Delta\omega_{y_{SS}} = -\frac{h_{rotor}}{k_\omega}\Delta\omega_{x_{SS}} \\ \Delta\theta_{SS} = \Delta\phi_{SS} \end{gathered}}$$

Ring Laser Gyro



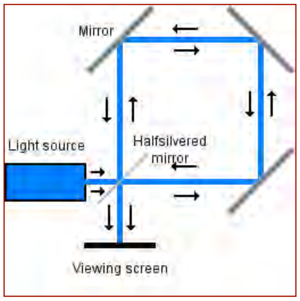
- Laser in optical path creates photon resonance at wavelength λ
- Frequency change in cavity is proportional to angular rate

$$\Delta f = \frac{4A}{\lambda P}\omega$$

$$P: \quad perimeter\ length$$

- Three RLGs needed to measure three angular rates

Fiber Optic Gyro



- Long length of fiber cable wrapped in a circle
- Photon source and sensor are external to the fiber optics
- Length difference for opposite beams is

$$\Delta L = \frac{4AN}{c}\omega$$

$A: \quad included\ area$

$N: \quad number\ of\ turns$

- Phase difference is proportional to angular rate

$$\Delta\varphi = \frac{8\pi AN}{\lambda c}\omega$$