Information, Search, and Expert Systems

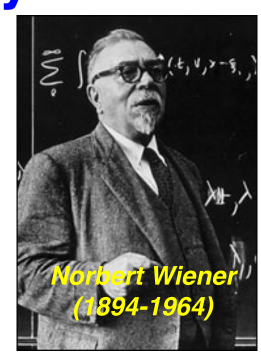
Robert Stengel
Robotics and Intelligent Systems MAE 345
Princeton University, 2017

- Communication/Information Theory
  - Wiener vs. Shannon
  - Entropy
- Finding Decision Rules in Data
  - ID3 Algorithm
- Graph and Tree Search
- Expert Systems
  - Forward and Backward Chaining
  - Bayesian Belief Network
  - Explanation

"Communication Theory" or "Information Theory"?

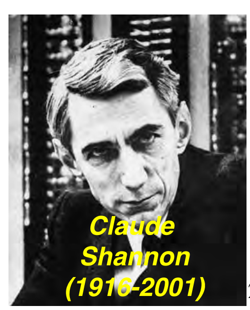
- **Prodigy at Harvard, professor at MIT**
  - Cybernetics
  - Feedback control
  - Communication theory

Dark Hero Of The Information Age: In Search of Norbert Wiener, the Father of Cybernetics, Flo Conway and Jim Siegelman, 2005. Basic Books


Norbert Wiener (1894-1964)

- **University of Michigan, MIT (student), Bell Labs, MIT (professor)**
  - Boolean algebra
  - Cryptography, telecommunications
  - Information theory

The Information: A History, A Theory, A Flood, James Gleick, 2011, Pantheon.

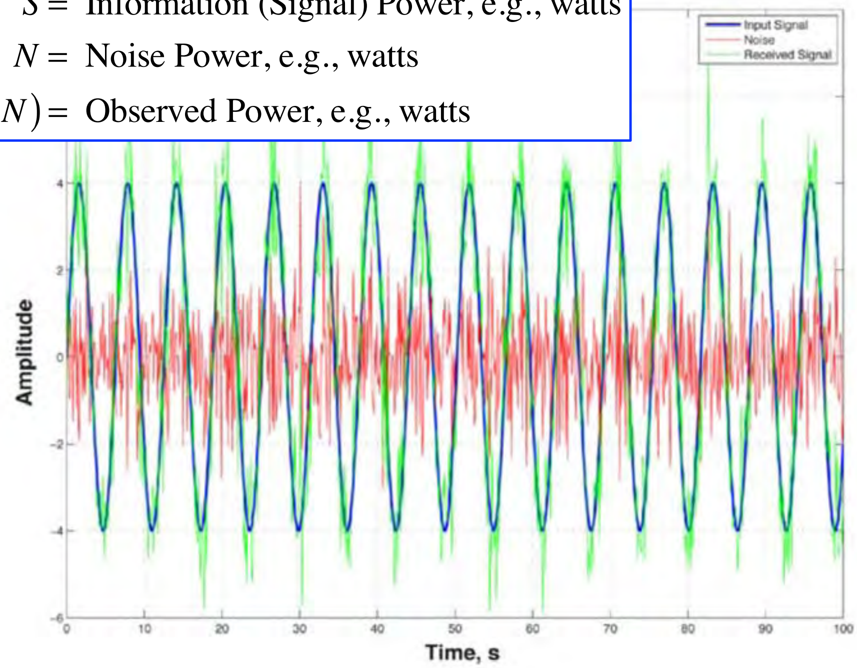

Claude Shannon (1916-2001)

Information, Noise, and Observation

$S =$ Information (Signal) Power, e.g., watts

$N =$ Noise Power, e.g., watts

$(S+N) =$ Observed Power, e.g., watts



Communication: Separating Signals from Noise

Signal-to-Noise Ratio, *SNR*

$$SNR = \frac{Signal\ Power}{Noise\ Power} \triangleq \frac{S}{N}$$

$$= \frac{\sigma^2_{signal}}{\sigma^2_{noise}} (zero\text{-}mean),\ \text{e.g.,}\ \frac{watts}{watts}$$

SNR often expressed in decibels

$$SNR(dB) = 10\log_{10}\frac{Signal\ Power}{Noise\ Power} = 10\log_{10}\frac{(Signal\ Amplitude)^2}{(Noise\ Amplitude)^2}$$

$$= 20\log_{10}\frac{Signal\ Amplitude}{Noise\ Amplitude} = S(dB) - N(dB)$$

Communication: Separating Analog Signals from Noise

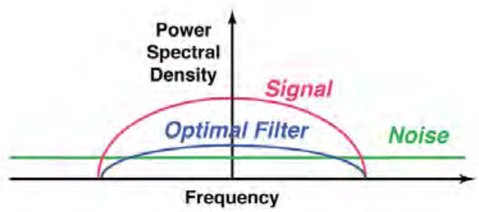
$$Power = \int_{-\infty}^{\infty} [Power\ Spectral\ Density(f)]df = \int_{-\infty}^{\infty} [PSD(f)]df; \quad f = \text{Frequency, Hz}$$

Signal-to-Noise Spectral Density Ratio, *SDR(f)*

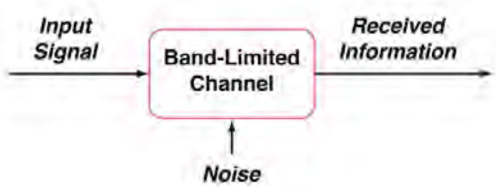
$$SDR\left(\frac{\omega}{2\pi}\right) = SDR(f) = \frac{Signal\ Power\ Spectral\ Density(f)}{Noise\ Power\ Spectral\ Density(f)} \triangleq \frac{PSD_{signal}(f)}{PSD_{noise}(f)}$$

Optimal (non-causal) Wiener Filter, *H(f)*

$$H(f) = \frac{PSD_{signal}(f)}{PSD_{signal}(f) + PSD_{noise}(f)} = \frac{SDR(f)}{SDR(f)+1}$$



Communication: Bit Rate Capacity of a Noisy Analog Channel



Shannon-Hartley Theorem, *C* bits/s

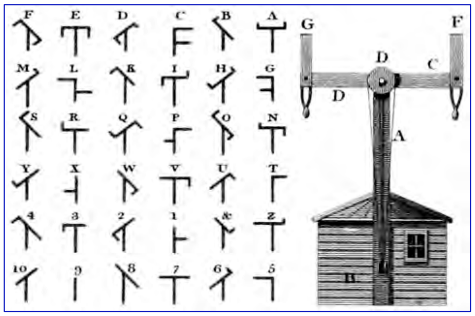
$$C = B\log_2\left(\frac{S+N}{N}\right) = B\log_2\left(\frac{S}{N}+1\right) = B\log_2(SNR+1)$$

$S =$ Signal Power, e.g., watts
$N =$ Noise Power, e.g., watts
$(S+N) =$ Observed Power, e.g., watts

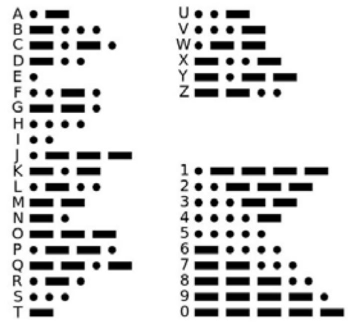
$B =$ Channel Bandwidth, Hz
$C =$ Channel Capacity, bits/s

Early Codes: How Many Bits?

Semaphore Line Code



Morse Code



- ~ (10 x 10) image = 100 pixels = 100 bits required to discern a character

ASCII encodes 128 characters in 7 bits (1 byte – 1 bit)

8th bit? | Parity check

- **Dot = 1 bit**
- **Dash = 3 bits**
- **Dot-dash space = 1 bit**
- **Letter space = 2 bits**
- **3 to 21 bits per character**

Information

Ralph Hartley's Definition of Information (1928)

$$H = \log_{10} S^n = n \log_{10} S \qquad S = \text{ \# possible denary symbols}, \quad n = \text{ \# transmitted symbols}$$

- **Claude Shannon, 1948: *Self-Information*, $\mathbb{I}(x_A)$, contained in observation of Event *A*, x_A, depends on probability of occurrence, $\Pr(x_A)$:**

$$\mathbb{I}(x_A) = fcn\left[\Pr(x_A)\right]$$

1) **Information increases as uncertainty decreases**
2) **$\mathbb{I}(x_A) \geq 0$: Information is positive or zero**
3) **If $\Pr(x_A) = 1$ or 0, $\mathbb{I}(x_A) = 0$: No information in observation if x_A is certain or not present**
4) **For observations of independent events, x_A and x_B, *Joint-Information* must be additive**

$$\mathbb{I}(x_A, x_B) = \mathbb{I}(x_A) + \mathbb{I}(x_B)$$

Information

- From (4),

$$\therefore fcn\left[\Pr(x_A, x_B)\right] = fcn\left[\Pr(x_A)\right] fcn\left[\Pr(x_B)\right] \\ = fcn\left[\Pr(x_A)\right] + fcn\left[\Pr(x_B)\right]$$

- What function has these properties?
- Shannon's answer: the logarithm
- From (1),

$$\mathbb{I}(x_i) = \log\left[1/\Pr(x_i)\right] = -\log\left[\Pr(x_i)\right]$$

- Mean value of self-information is the expected value

$$E\left[\mathbb{I}(x_A)\right] = -\lim_{N\to\infty}\left(\frac{n_A}{N}\right)\log\left(\frac{n_A}{N}\right) = -\Pr(x_A)\log\left[\Pr(x_A)\right]$$

Entropy as a Measure of Information

- Prior result true in any numbering system
- Expressing Self-Information in *bits* (or "shannons"),

$$E\left[\mathbb{I}(x_A)\right] = -\Pr(x_A)\log_2\left[\Pr(x_A)\right]$$

- Given *I* distinct events, the *entropy** of set of events is

$$H \triangleq \sum_{i=1}^{I} E\left[\mathbb{I}(x_i)\right] = -\sum_{i=1}^{I} \Pr(x_i)\log_2\left[\Pr(x_i)\right] \text{ bits}$$

  - Entropy indicates the *degree of uncertainty* associated with the process
  - The greater the uncertainty, the higher the required channel capacity for transmission

* *After thermodynamic definition of entropy*

Entropy of Two Events with Binary Frequencies of Occurrence

- **$-\Pr(i)\log_2 \Pr(i)$ represents the channel capacity (i.e., <u>average number of bits</u>) required to portray the i^{th} event**
- **Frequencies of occurrence estimate probabilities of each event (#1 and #2)**

$$\Pr(\#1) = \frac{n(\#1)}{N}$$

$$\Pr(\#2) = \frac{n(\#2)}{N} = 1 - \frac{n(\#1)}{N}$$

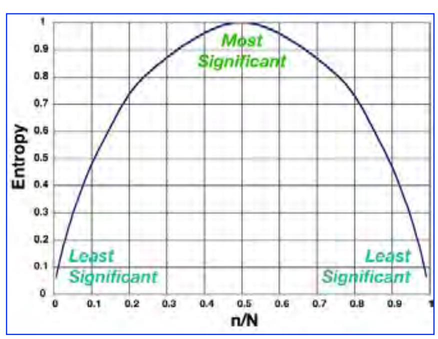
$$\log_2 \Pr(\#1 \text{ or } \#2) \le 0$$

- **Combined entropy**

$$H = H_{\#1} + H_{\#2}$$
$$= -\Pr(\#1)\log_2 \Pr(\#1) - \Pr(\#2)\log_2 \Pr(\#2)$$

Entropy of Two Events with Binary Frequencies of Occurrence

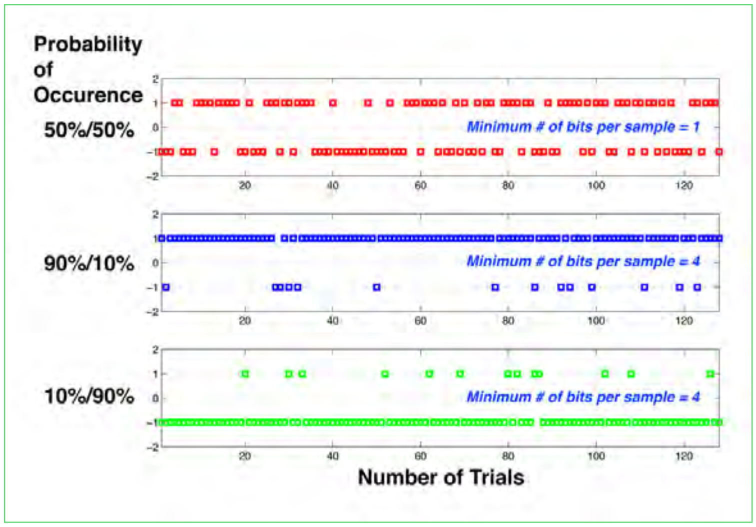
	Entropies for 128 Trials				
	Pr(#1)	- # of Bits(#1)	Pr(#2)	- # of Bits(#2)	Entropy
n	n/N	$\log_2(n/N)$	1 - n/N	$\log_2(1 - n/N)$	H
1	0.008	-7	0.992	-0.011	0.066
2	0.016	-6	0.984	-0.023	0.116
4	0.031	-5	0.969	-0.046	0.201
8	0.063	-4	0.938	-0.093	0.337
16	0.125	-3	0.875	-0.193	0.544
32	0.25	-2	0.75	-0.415	0.811
64	0.50	-1	0.50	-1	1
96	0.75	-0.415	0.25	-2	0.811
112	0.875	-0.193	0.125	-3	0.544
120	0.938	-0.093	0.063	-4	0.337
124	0.969	-0.046	0.031	-5	0.201
126	0.984	-0.023	0.016	-6	0.116
127	0.992	-0.011	0.008	-7	0.066



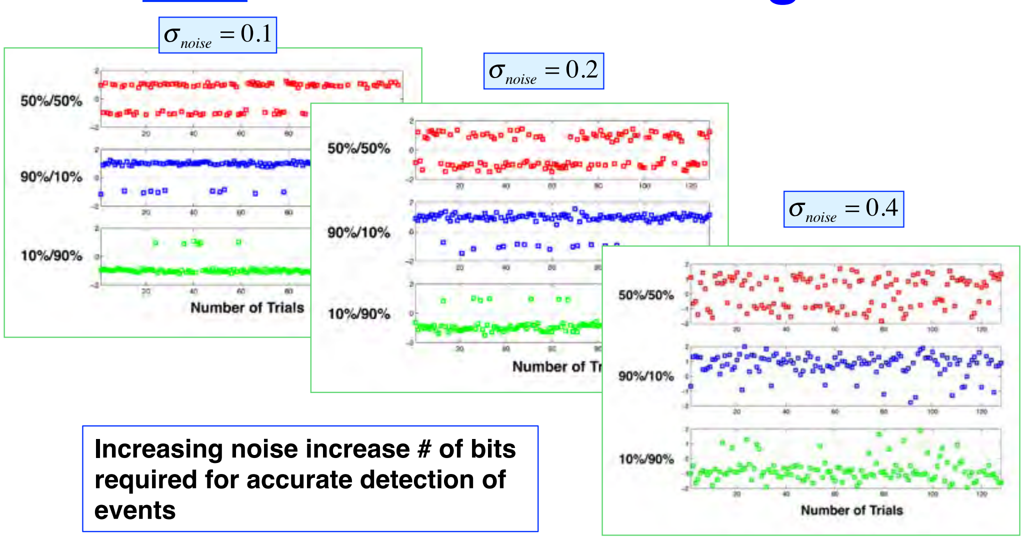
Entropy of a fair coin flip = 1

Accurate Detection of Events Depends on Their Probability of Occurrence

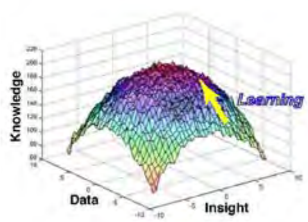
Signals Rounded to Their Intended Values



Accurate Detection of Events Depends on Their Probability of Occurrence – and the Noise in the Signal



Increasing noise increase # of bits required for accurate detection of events



Finding Efficient Decision Rules in Data

(Off-Line)

- **Choose most important attributes first**
- **Recognize when no result can be deduced**
- **Exclude irrelevant factors**

- **Iterative Dichotomizer*: the ID3 Algorithm**
  - **Build an efficient decision tree from a fixed set of examples** ***(supervised learning)***

***Dichotomy: Division into two (usually contradictory) parts or opinions**

Quinlan, 1986

Fuzzy Ball-Game Training Set

	Attributes				Decisions
Case #	**Forecast**	**Temperature**	**Humidity**	**Wind**	**Play Ball?**
1	Sunny	Hot	High	Weak	**No**
2	Sunny	Hot	High	Strong	**No**
3	Overcast	Hot	High	Weak	**Yes**
4	Rain	Mild	High	Weak	**Yes**
5	Rain	Cool	Low	Weak	**Yes**
6	Rain	Cool	Low	Strong	**No**
7	Overcast	Cool	Low	Strong	**Yes**
8	Sunny	Mild	High	Weak	**No**
9	Sunny	Cool	Low	Weak	**Yes**
10	Rain	Mild	Low	Weak	**Yes**
11	Sunny	Mild	Low	Strong	**Yes**
12	Overcast	Mild	High	Strong	**Yes**
13	Overcast	Hot	Low	Weak	**Yes**
14	Rain	Mild	High	Strong	**No**

Parameters of the ID3 Algorithm

- **Decisions, e.g., Play ball or don't play ball**
  - ***D*** **= Number of possible decisions**
    - **Decision:** **Yes, no**

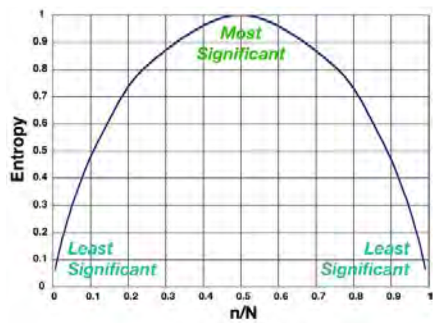
Parameters of the ID3 Algorithm

- **Attributes, e.g., Temperature, humidity, wind, weather forecast**
  - ***M*** **= Number of attributes to be considered in making a decision**
  - I_m **= Number of values that the** i^{th} **attribute can take**
    - **Temperature:** **Hot, mild, cool**
    - **Humidity:** **High, low**
    - **Wind:** **Strong, weak**
    - **Forecast:** **Sunny, overcast, rain**
- **Training trials, e.g., all the games attempted last month**
  - ***N*** **= Number of training trials**
  - $n(i)$ **= Number of examples with** i^{th} **attribute**

Best Decision is Related to Entropy and the Probability of Occurrence

$$H = -\sum_{i=1}^{I} \Pr(i) \log_2 \Pr(i)$$

- **High entropy**
  - **Signal provides low coding precision of distinct events**
  - **Differences can be coded with few bits**
- **Low entropy**
  - **More complex signal structure**
  - **Detecting differences requires many bits**
- **Best classification of events when *H* = 1...**
  - **but that may not be achievable**



Case #	Forecast	Temperature	Humidity	Wind	Play Ball?
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Low	Weak	Yes
6	Rain	Cool	Low	Strong	No
7	Overcast	Cool	Low	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Low	Weak	Yes
10	Rain	Mild	Low	Weak	Yes
11	Sunny	Mild	Low	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Low	Weak	Yes
14	Rain	Mild	High	Strong	No

Decision-Making Parameters for ID3

H_D = Entropy of all possible decisions

$$H_D = -\sum_{d=1}^{D} \Pr(d) \log_2 \Pr(d)$$

G_i = Information "gain" (or contribution) of i^{th} attribute

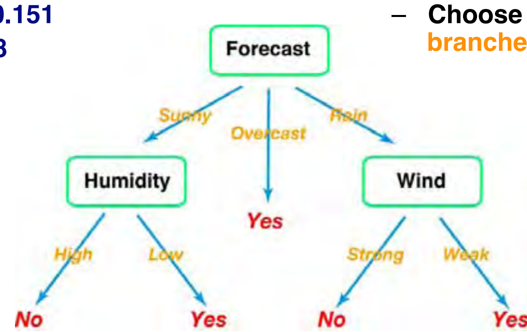
$$G_i = H_D + \sum_{i_m=1}^{M} \Pr(i_m) \sum_{i_d=1}^{D} \left[\Pr(i_d) \log_2 \Pr(i_d) \right]$$

$\Pr(i_d) = n(i_d) / N(d)$: Probability that i^{th} attribute depends on d^{th} decision

$\sum_{i=1}^{I_m} \Pr(i) \sum_{d=1}^{D} \left[\Pr(i_d) \log_2 \Pr(i_d) \right]$: **Mutual information** of i and d

Decision Tree Produced by ID3 Algorithm

- Typical Root Attribute gains, G_i
  - Forecast: 0.246
  - Temperature: 0.029
  - Humidity: 0.151
  - Wind: 0.048
- Therefore
  - Choose *Forecast* as root
  - Ignore *Temperature*
  - Choose *Humidity* and *Wind* as branches

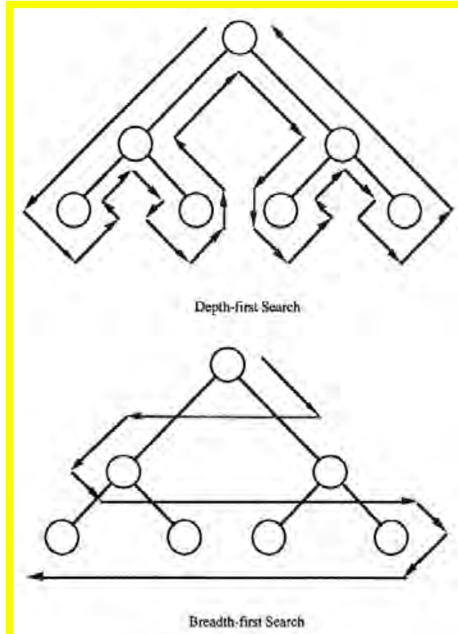


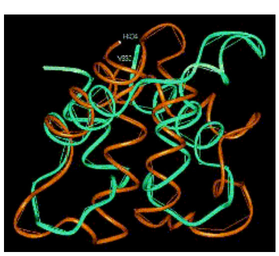
- Evaluating remaining gains,
  - *Sunny* branches to *Humidity*
  - *Overcast = Yes*
  - *Rain* branches to *Wind*

Graph and Tree Search

Search for Best Solution

- **Typical textbook problems**
  - **Prove theorem**
  - **Solve puzzle** *(e.g., Tower of Hanoi)*
  - **Find sequence of chess moves to win a game**
  - **Find shortest path between points** *(e.g., Traveling salesman problem)*
  - **Find sequence of symbolic transformations that solve problem** *(e.g., Mathematica)*



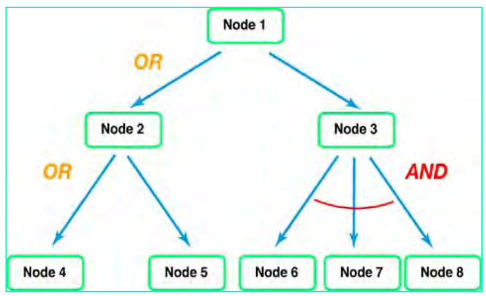


Curse of Dimensionality

- **Feasible search paths may grow without bound**
  - **Possible combinatorial explosion**
  - **Checkers: 5 x 10^{20} possible moves**
  - **Chess: 10^{120} moves**
  - **Protein folding: ?**
- **Limiting search complexity**
  - **Redefine search space**
  - **Employ heuristic (i.e., pragmatic) rules**
  - **Establish restricted search range**
  - **Invoke decision models that have worked in the past**

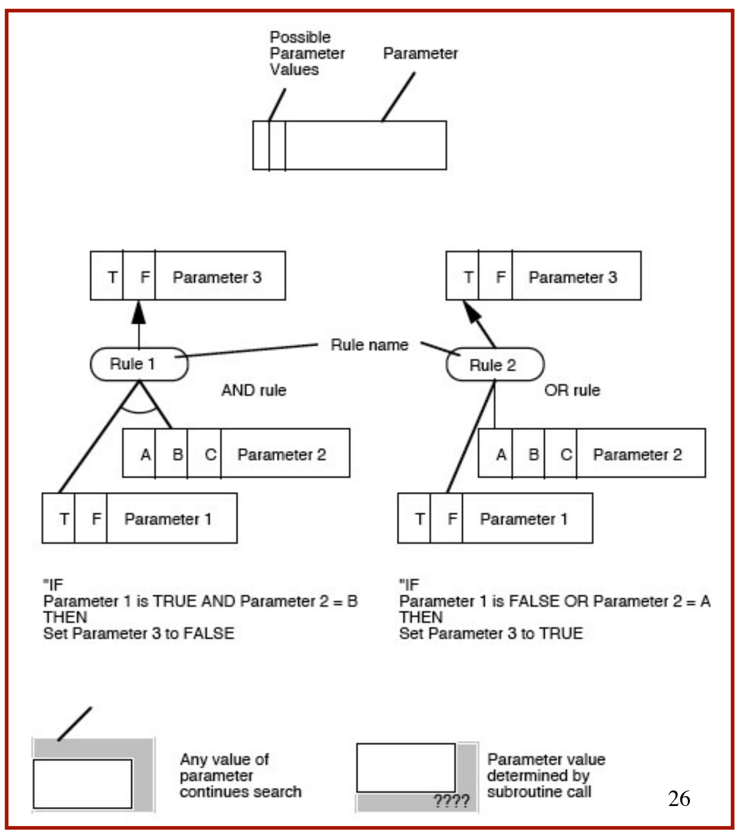
Tree Structures for Search

- Single path between root and any node
- Path between adjacent nodes = arc
- Root node
  - no precursors
- Leaf node
  - no successors
  - possible terminator



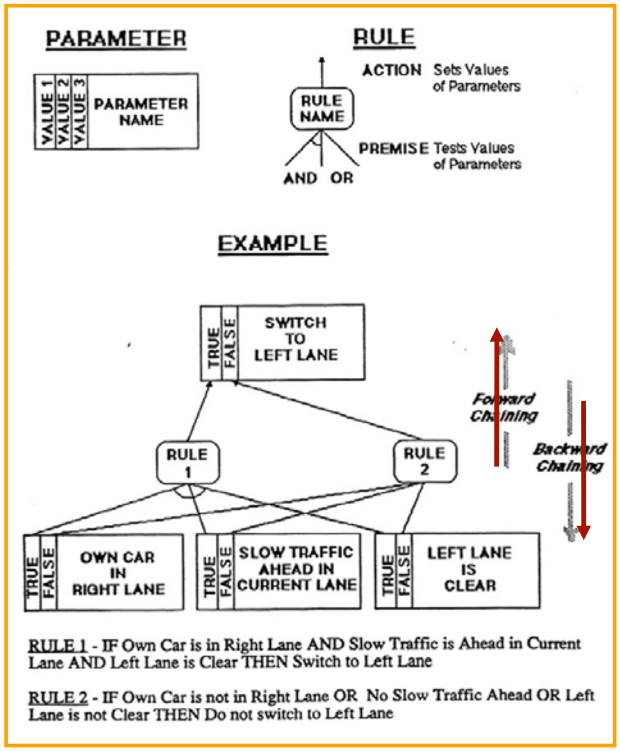
Expert System Symbology

- Parameters
  - Values
- Rules
  - Name
  - Logic
- And/Or



Structures for Search

- Multiple paths between root and some nodes
- Predicate calculus



Directions of Search

- Forward chaining
  - Reason from premises to actions
  - Data-driven: draw conclusions from facts
- Backward chaining
  - Reason from actions to premises
  - Goal-driven: find facts that support hypotheses

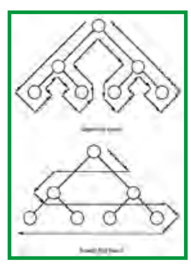
Strategies for Search

- **Realistic assessment**
  - Not necessary to consider all 10^{120} possible moves to play good chess
  - Forward and backward chaining, but not 10^{120} evaluations

- **Search categories**
  - Blind search
  - Heuristic search
  - Probabilistic search
  - Optimization

- Search forward from opening?
- Search backward from end game?
- Both?

"Blind" Tree Search

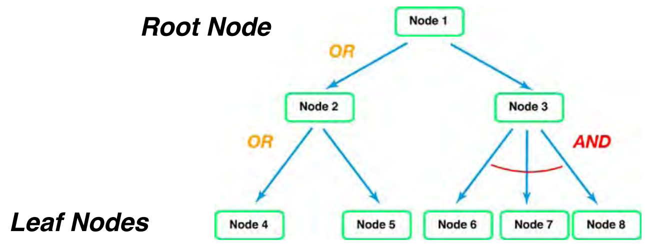


- **Node expansion**
  - Begin at root
  - Find all successors to node

- **Depth-first forward search**
  - Expand nodes descended from most recently expanded node
  - Consider other paths only after reaching node with no successors

- **Breadth-first forward search**
  - Expand nodes in order of proximity to start node
  - Consider all sequences of arc number n (from root node) before considering any of number $(n + 1)$
  - Exhaustive, but guaranteed to find the shortest path to a terminator

AND/OR Graph Search



- A node is "solved" if
  - It is a leaf node with a satisfactory goal state
  - It provides a satisfactory goal state and has "AND nodes" as successors
  - It has "OR nodes" as successors and at least one leaf provides a satisfactory goal state.
- Goal: Solve the root node

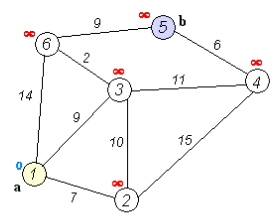
Heuristic Search

- For large problems, blind search typically leads to combinatorial explosion
- If optimal search (Lecture 12) is intractable, search for feasible (approximately optimal) solutions
- Employ heuristic knowledge about quality of possible paths
  - Decide which node to expand next
  - Discard (or *prune*) nodes that are unlikely to be fruitful
- Ordered or best-first search
  - Always expand "most promising" node

Shortest Path Routing

- Example: Double-Bucket Dijkstra algorithm
  - Forward and backward search
  - Data stored in a "heap" (value-ordered tree)
  - Length of heap update path is logarithmic in number of leaves
- Also see Lecture 5 slides

Single Dijkstra Search



Heuristic Dynamic Programming: A* and D* Search

$$\hat{J}_{k_f} = \sum_{i=1}^{k} J_i + \sum_{i=k+1}^{k_f} \hat{J}_i(arc_i)$$

- Forward search through given nodes
- Each arc bears an incremental cost
- Cost, J, estimated at k^{th} instant =
  - Cost accrued to k
  - Remaining cost to reach final point, k_f
- Goal: minimize estimated cost by choice of remaining arcs
- Choose arc_{k+1}, arc_{k+2} accordingly
- Use heuristics to estimate remaining cost

Expert Systems

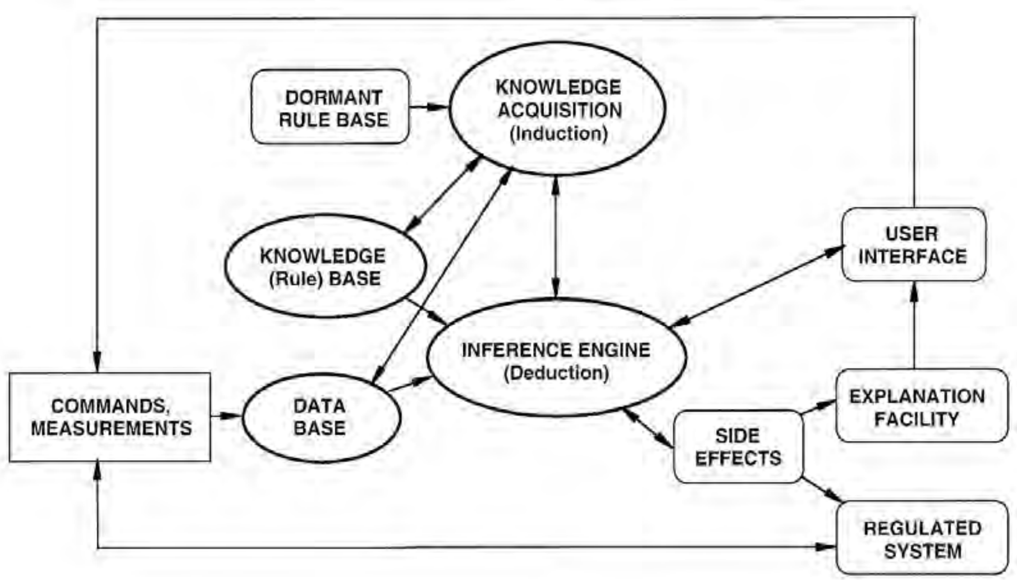
Expert Systems: Using Signals to Make Decisions

- **Program that exhibits intelligent behavior**
- **Program that uses rules to evaluate information**
- **Program meant to emulate an expert or group of experts making decisions in a specific domain of knowledge (or *universe of discourse*)**
- **Program that chains algorithms to derive conclusions from evidence**

Functions of Expert Systems

- Design
  - Conceive the form and substance of a new device, object, system, or procedure
- Diagnosis
  - Determine the nature or cause of an observed condition
- Instruction
  - Impart knowledge or skill
- Interpretation
  - Explain or analyze observations
- Monitoring
  - Observe a process, compare actual with expected observations, and indicate system status
- Negotiation
  - Propose, assess, and prioritize agreements between parties
- Planning
  - Devise actions to achieve goals
- Prediction
  - Reason about time, forecast the future
- Reconfiguration
  - Alter system structure to maintain or improve performance
- Regulation
  - Respond to commands and adjust control parameters to maintain stability and performance

Principal Elements of a Rule-Based Expert System



Critical Issues for Expert System Development

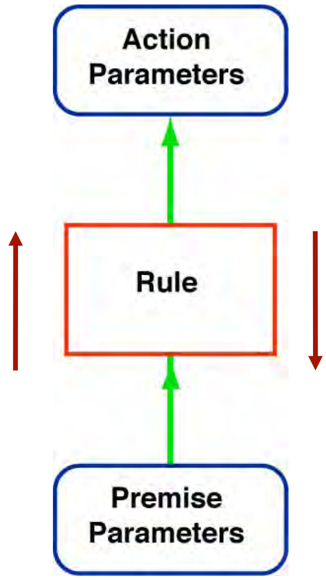
- System architecture
- Inference or reasoning method *(Deduction)*
- Knowledge acquisition *(Induction)*
- Explanation *(Abduction*)*
- User interface

* "Syllogism whose major premise is true and minor premise is probable"

Representation of Knowledge for Inference

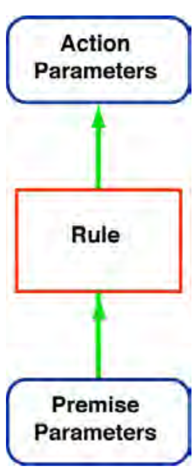
- Logic
  - Predicate calculus, 1st-order logic
  - Fuzzy logic, Bayesian belief network, …
- Search
  - Given one state, examine all possible alternative states
  - Directed acyclic graph
- Procedures
  - Function-specific routines executed within a rigid structure (e.g., flow chart)
- Semantic (propositional) networks
  - Model of associative memory
  - Tree or graph structure
  - **Nodes**: objects, concepts, and events
  - **Links**: interrelations between nodes
- Production (rule-based) systems
  - Rules
  - Data
  - Inference engine

Basic Rule Structure



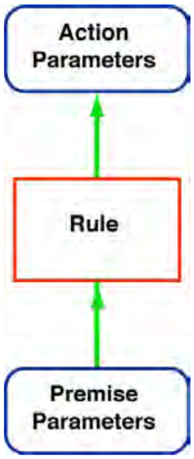
- Rule sets values of action parameters
- Rule tests values of premise parameters
- Forward chaining
  - Reasoning from premises to actions
  - Data-driven: facts to conclusions
- Backward chaining
  - Reasoning from actions to premises
  - Goal-driven: find facts that support a hypothesis
  - Analogous to numerical inversion

Elements of a Parameter



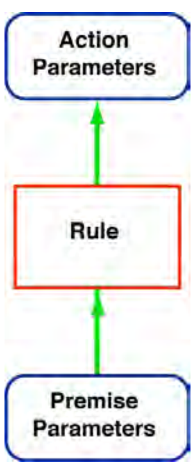
- Type
- Name
- Current value
- Rules that test the parameter
- Rules that set the parameter
- Allowable values of the parameter
- Description of parameter (for explanation)

Elements of a Rule

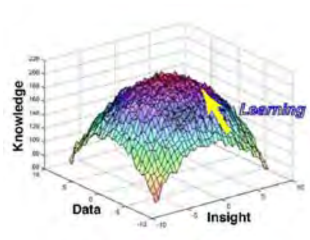


- Type
- Name
- Status
  - 0: Has not been tested
  - 1: Being tested
  - T: Premise is true
  - F: Premise is false
  - U: Premise is unknown
- Parameters tested by rule
- Parameters set by rule
- Premise: Logical statement of proposition or predicates
- Action: Logical consequence of premise being true
- Description of premise and action (for explanation)

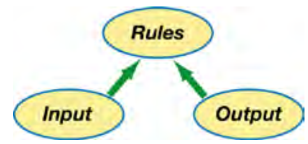
The Basic Rule: IF-THEN-ELSE



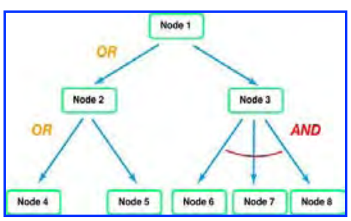
- If A = TRUE, then B, else C
- Material equivalence of propositional calculus, extended to predicate calculus and 1st-order logic, i.e., applied to logical statements
- Methods of inference lead to plans of action
- Compound rule: Logic embedded in The Basic Rule, e.g.,
  – Rule 1: If (A = B and C = D), then perform action E, else ….
  – Rule 2: If (A ≠ B or C = D), then E = F, else ….
- Nested (pre-formed compound) rule: Rule embedded in The Basic Rule, e.g.,
  – Rule 3: If (A = B), then [If (C = D), then E = F, else …], else ….



Finding Decision Rules in Data



- Identification of key attributes and outcomes
- Taxonomies developed by experts
- First principles of science and mathematics
- Trial and error
- Probability theory and fuzzy logic
- Simulation and empirical results



Example of On-Line Code Modification

- Execute a decision tree
  – Get wrong answer
- Add logic to distinguish between right and wrong cases
  – If Comfort Zone = Water,
    - then Animal = Hippo,
    - else Animal = Rhino
  – True, but Animal is Dinosaur, not Hippo
  – Ask user for right answer
  – Ask user for a rule that distinguishes between right and wrong answer: If Animal is extinct, …

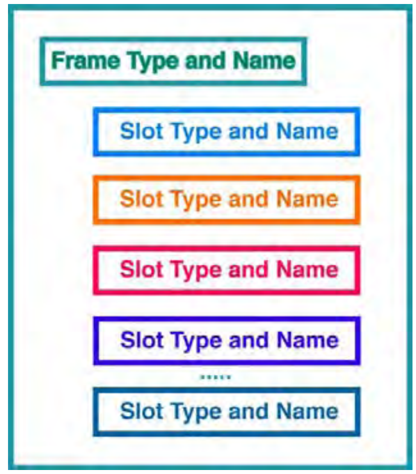
Decision Rules

Representation of Data

- **Set**
  - **Crisp sets**
  - **Fuzzy sets**
- **Schema**
  - **Diagrammatic representation**
  - **A pattern that represents elements (or objects), their attributes (or properties), and relationships between different elements**
- **Object (or Frame)**
  - **Hierarchical data structure, with inheritance**
  - **Slots: Function-specific cells for data**
  - **Scripts [usage]: frame-like structures that represent a sequence of events**
- **Database**
  - **Spreadsheets/tables/graphs**
  - **Linked spreadsheets**

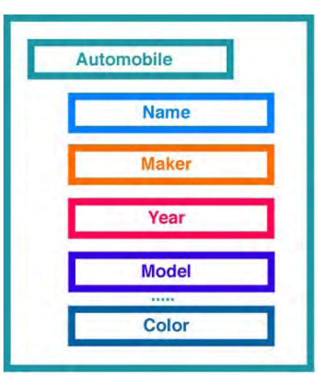
Structure of a Frame (or Object)

- Structure array in MATLAB
- Structure or property list in LISP
- Object in C++
- Ordered set of computer words that characterize a parameter or rule
- An archetype or prototype
- Object-oriented programming: Express Rules and Parameters as Frames

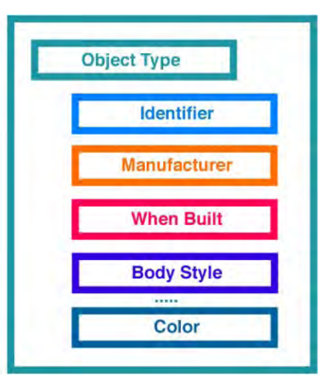


Example, Fillers, and Instance of a Frame

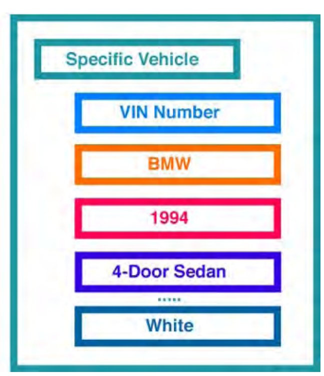
Application-Specific Frame



Generic Fillers

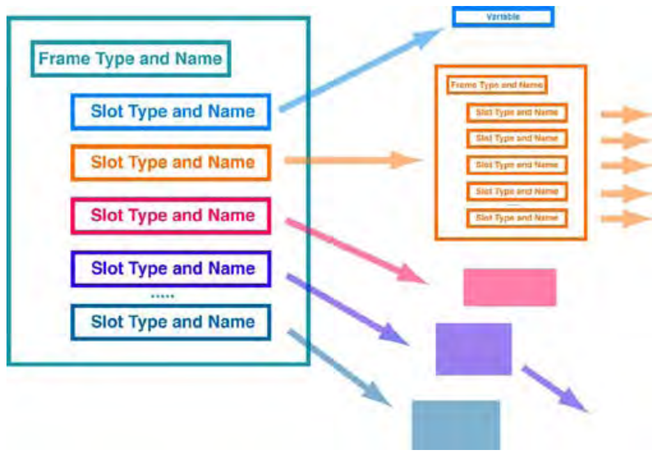


Instantiation



Inheritance and Hierarchy of Frame Attributes

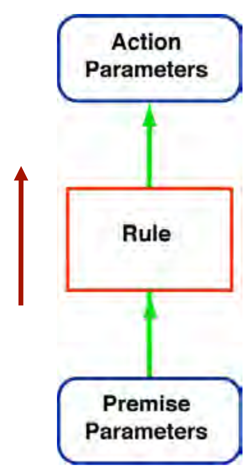
- **Legal fillers: Can be specified by**
  - **Data type**
  - **Function**
  - **Range**
- **Inheritance property**
  - **All instances of a specific frame may share certain properties or classes of properties**
- **Hierarchical property**
  - **Frames of frames may be legal**
- **Inference engine**
  - **Decodes frames**
  - **Establishes inheritance and hierarchy**
  - **Executes logical statements**



Animal Decision Tree: Forward Chaining

- **What animal is it?**

Premise Parameter:	**Size**	
Rule 1:		If 'Small', test 'Sound' Else, test 'Neck'
Action Parameter:	None	
Premise Parameter:	**Sound**	
Rule 2:		If 'Squeak', Animal = Mouse [END] Else, Animal = Squirrel [END]
Action Parameter:	Animal	
Premise Parameter:	**Neck**	
Rule 3:		If 'Long', Animal = Giraffe [END] Else, test 'Trunk'
Action Parameter:	Animal	
Premise Parameter:	**Trunk**	
Rule 4:		If 'True', Animal = Elephant [END] Else, test 'Comfort Zone'
Action Parameter:	Animal	
Premise Parameter:	**Comfort Zone**	
Rule 5:		If 'Water', Animal = Hippo [END] Else, Animal = Rhino [END]
Action Parameter:	Animal	

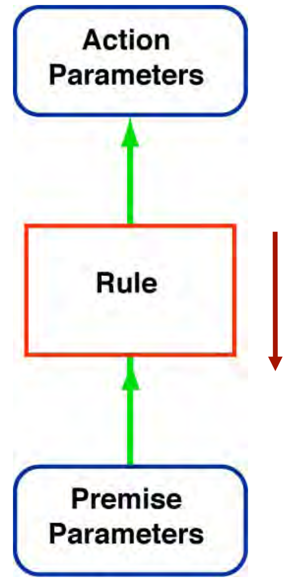


Animal Decision Tree: Backward Chaining

- **What are an animal's attributes?**

Animal = Hippo

From Rule 5,	Comfort Zone = Water
From Rule 4,	Trunk = False
From Rule 3,	Neck = Short
From Rule 1,	Size = Large



Animal Decision Tree: Parameters

Type:	**Object Attribute**
Name:	Animal
Current Value:	Variable
Rules that Test:	None
Rules that Set:	2, 3, 4, 5
Allowable Values:	Mouse, Squirrel, Giraffe, Elephant, Hippo, Rhino
Description:	Type of Animal

Type:	**Object Attribute**
Name:	Size
Current Value:	Variable
Rules that Test:	1
Rules that Set:	None
Allowable Values:	Large, Small
Description:	Size of Animal

Type:	**Object Attribute**
Name:	Sound
Current Value:	Variable
Rules that Test:	2
Rules that Set:	None
Allowable Values:	Squeak, No Squeak
Description:	Sound made by Animal

Type:	**Object Attribute**
Name:	Neck
Current Value:	Variable
Rules that Test:	3
Rules that Set:	None
Allowable Values:	Long, Short
Description:	Neck of Animal

Type:	**Object Attribute**
Name:	Trunk
Current Value:	Variable
Rules that Test:	4
Rules that Set:	None
Allowable Values:	True, False
Description:	Snout of Animal

Type:	**Object Attribute**
Name:	Comfort Zone
Current Value:	Variable
Rules that Test:	5
Rules that Set:	None
Allowable Values:	Water, Dry Land
Description:	Habitat of Animal

Animal Decision Tree: Rules

Type:	**If-Then-Else**
Name:	Rule 1
Status:	Variable (e.g., untested, being tested, tested and premise = T/F/unknown)
Parameters Tested:	Size
Parameters Set:	None
Premise:	Size = Large or Small
Action:	Test 'Sound' OR Test 'Neck'
Description:	Depending on value of 'Size', test 'Sound' or 'Neck'

Type:	**If-Then-Else**
Name:	Rule 2
Status:	Variable
Parameters Tested:	Sound
Parameters Set:	Animal
Premise:	Size = Large or Small
Action:	Set value of 'Animal' AND END
Description:	Depending on value of 'Sound', identify 'Animal' as 'Mouse' or 'Squirrel'

Type:	**If-Then-Else**
Name:	Rule 3
Status:	Variable
Parameters Tested:	Neck
Parameters Set:	Animal
Premise:	Neck = Long or Short
Action:	Set value of 'Animal AND END OR Test 'Trunk'
Description:	Depending on value of 'Neck', identify 'Animal' as 'Giraffe' or test 'Comfort Zone'

Type:	**If-Then-Else**
Name:	Rule 4
Status:	Variable
Parameters Tested:	Trunk
Parameters Set:	Animal
Premise:	Trunk = True or False
Action:	Set value of 'Animal' AND END OR Test 'Comfort Zone'
Description:	Depending on value of 'Trunk', identify 'Animal' as 'Elephant' or test 'Comfort Zone'

Type:	**If-Then-Else**
Name:	Rule 5
Status:	Variable
Parameters Tested:	Comfort Zone
Parameters Set:	Animal
Premise:	Comfort Zone = Water or Dry Land
Action:	Set value of 'Animal' AND END
Description:	Depending on value of 'Comfort Zone', identify 'Animal' as 'Hippo' or 'Rhino'

Animal Decision Tree: Programs

- **Procedural Sequence of Rules**
  - Rule1(Size, Rule2, Rule3)
  - Rule2(Sound, Animal, Animal)
  - Rule3(Neck, Animal, Rule4)
  - Rule4(Trunk, Animal, Rule5)
  - Rule5(Comfort Zone, Animal, Animal)
- **Declarative Sequence of Rules**
  - BasicRule(Size, Sound, Neck)
  - BasicRule(Sound, Animal, Animal)
  - BasicRule(Neck, Animal, Trunk)
  - BasicRule(Trunk, Animal, Comfort Zone)
  - BasicRule(Comfort Zone, Animal, Animal)

Animal Decision Tree: Procedural Logic

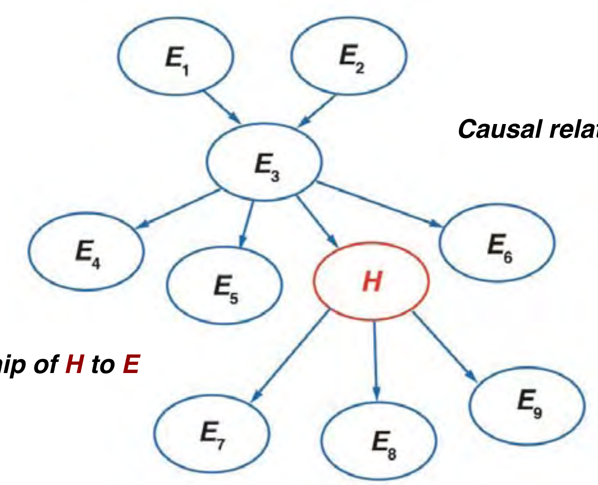
Simple exposition of decision-making
Rigid description of solution

```
If   Size = Big
     Then    If    Sound = Squeak
                   Then    Animal = Mouse
                   Else    Animal = Squirrel
             EndIf
     Else    If    Neck = Long
                   Then    Animal = Giraffe
                   Else    If    Trunk = True
                                 Then   Animal = Elephant
                                 Else   If    Comfort Zone = Water
                                              Then      Animal = Hippo
                                              Else      Animal = Rhino
                                        EndIf
                         EndIf
             EndIf
EndIf
```


Bayesian Belief Network

- **Related events, *E_i*, within a contextual domain**
- **Conditional dependence of events that may (or not) be observed**
- **Probability of unobserved event (hypothesis), *H*, to be predicted**

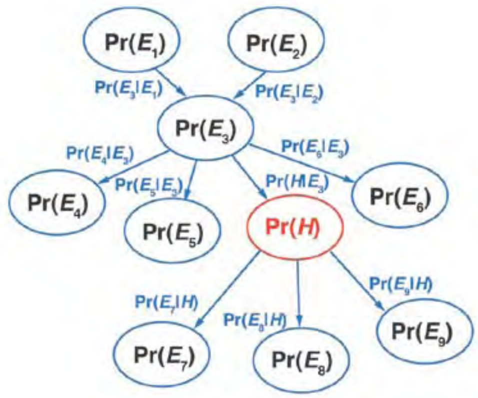
Causal relationship of E to H



Causal relationship of H to E

after *Pearl, 1991*

Network of Conditional and Unconditional Probabilities

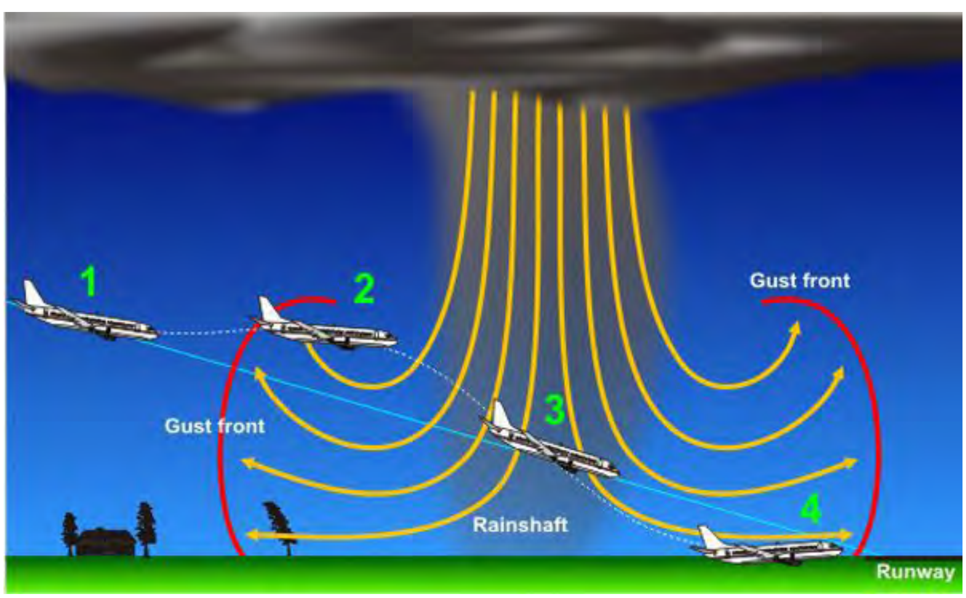


- **Conditional probabilities known**
- **Prior estimates of unconditional probabilities given**
- **When event, E_j, occurs with probability, $\Pr(E_j)$, update estimates of all unconditional probabilities, including $\Pr(H)$**

See Supplemental Material for equations

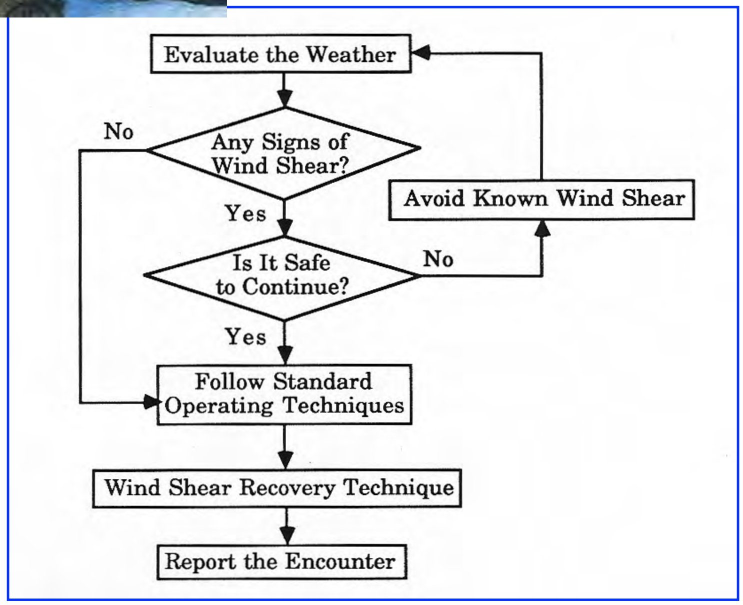
Decision Making Under Uncertainty

Aircraft Flight Through Microburst Wind Shear



http://www.princeton.edu/~stengel/StrattonJGCDProb1992.pdf

Decision Making Under Uncertainty
(FAA Guidelines)



61

Probability of Microburst Wind Shear (FAA)

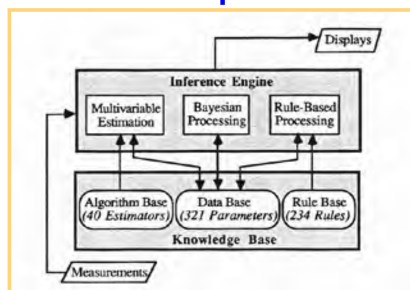
OBSERVATION	PROBABILITY OF WIND SHEAR
PRESENCE OF CONVECTIVE WEATHER NEAR FLIGHT PATH:	
– With localized strong winds (Tower reports or observed blowing dust, rings of dust, tornado-like features, etc.)	HIGH
– With heavy precipitation (Observed or radar indications of contour, red or attenuation shadow)	HIGH
– With rainshower	MEDIUM
– With lightning	MEDIUM
– With virga	MEDIUM
– With moderate or greater turbulence (Reported or radar indications)	MEDIUM
– With temperature/dew point spread between 30 and 50 degrees Fahrenheit	MEDIUM
ONBOARD WINDSHEAR DETECTION SYSTEM ALERT (Reported or observed)	HIGH
PIREP OF AIRSPEED LOSS OR GAIN:	
– 15 knots or greater	HIGH
– Less than 15 knots	MEDIUM
LLWAS ALERT/WIND VELOCITY CHANGE:	
– 20 knots or greater	HIGH
– Less than 20 knots	MEDIUM
FORECAST OF CONVECTIVE WEATHER	LOW

62

Bayesian Rules of Inference for Situation Assessment and Decision Making

(Stratton and Stengel)

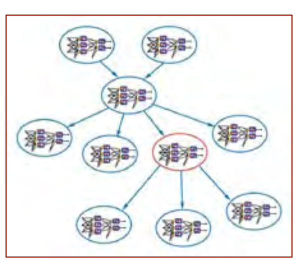
- Boxes represent *unconditional* probabilities
- Arrows represent *conditional* probabilities



http://www.youtube.com/watch?v=dKwyU1RwPto

Explanation in Machine Learning

- Expert Systems
  - *Explanation* of decisions is built-in
  - Structure relies on causal relationships
  - Best applied to problems with well-defined nodes and rules
  - Replace rules with neural networks?
- Neural Networks
  - Explanation of classification is ambiguous
  - Structure is mechanistic
  - Best applied to problems with graphical or semantic solutions
  - Restructure neural modules to reflect defined purpose?



Next Time:
State Estimation

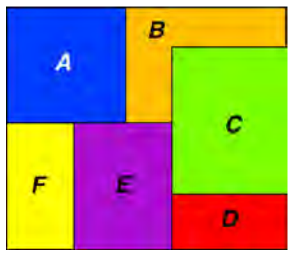
Supplementary Material

Example: Probability Spaces for Three Attributes

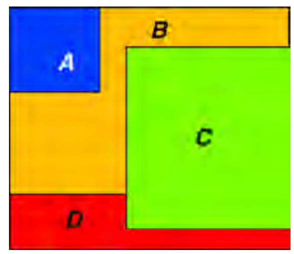
- Probability of an attribute value represented by area in diagram

Attribute #1
2 possible values

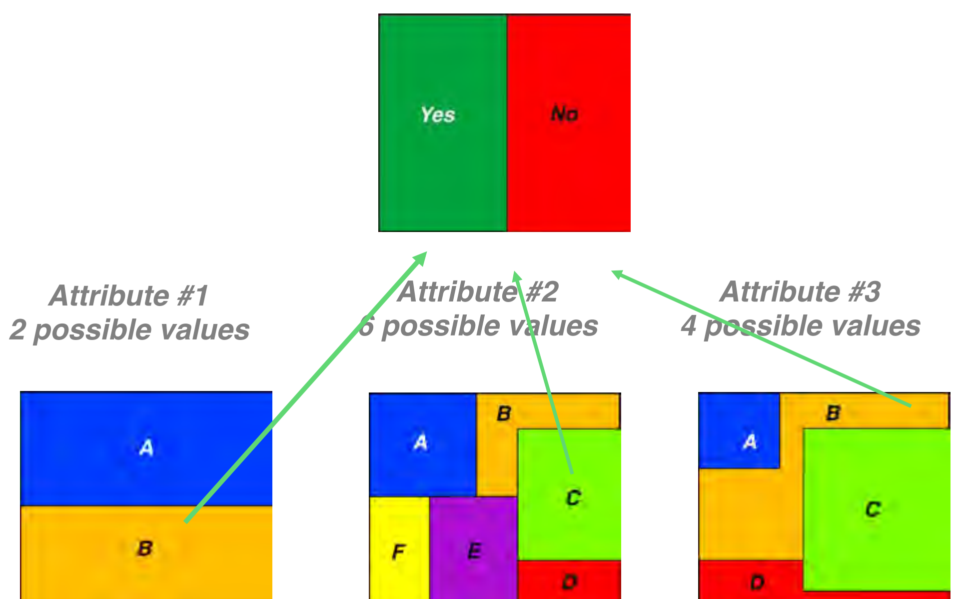
Attribute #2
6 possible values



Attribute #3
4 possible values



Example: Decision, given Values of Three Attributes



Bayesian Belief Network Relationships

H : Hypothesis
E_i : i^{th} Piece of Evidence

- **Conditional probability of hypothesis, *H***

$$\Pr(H \mid E) = \frac{\Pr(E \mid H)}{\Pr(E)} \Pr(H)$$

- **Unconditional probability of evidence, E_1**

$$\Pr(E_1) = \Pr(E_1 \mid H)\Pr(H) + \Pr(E_1 \mid \neg H)\Pr(\neg H)$$

- **Probability of hypothesis, *H*, conditioned on E_1 and E_2**

$$\Pr(H \mid E_1 \wedge E_2) = \frac{\Pr(E_1 \wedge E_2 \mid H)}{\Pr(E_1 \wedge E_2)} \Pr(H)$$

after *Stratton, Stengel, 1992*

Bayesian Belief Network Relationships

- **Probability of E_2 conditioned on E_1 and *H***

$$\Pr(E_2 \mid H \wedge E_1) = \Pr(E_2 \mid H)$$

- **Probability of E_1 and E_2 conditioned on *H***

$$\Pr(E_1 \wedge E_2 \mid H) = \Pr(E_1 \mid H)\Pr(E_2 \mid H)$$

- **Then**

$$\Pr(H \mid E_1 \wedge E_2) = \frac{\Pr(E_1 \mid H)\Pr(E_2 \mid H)}{\Pr(E_1 \wedge E_2)} \Pr(H)$$

$$= \frac{\Pr(E_2 \mid H)}{\Pr(E_1 \mid E_2)} \Pr(H \mid E_1)$$

Bayesian Belief Network Relationships

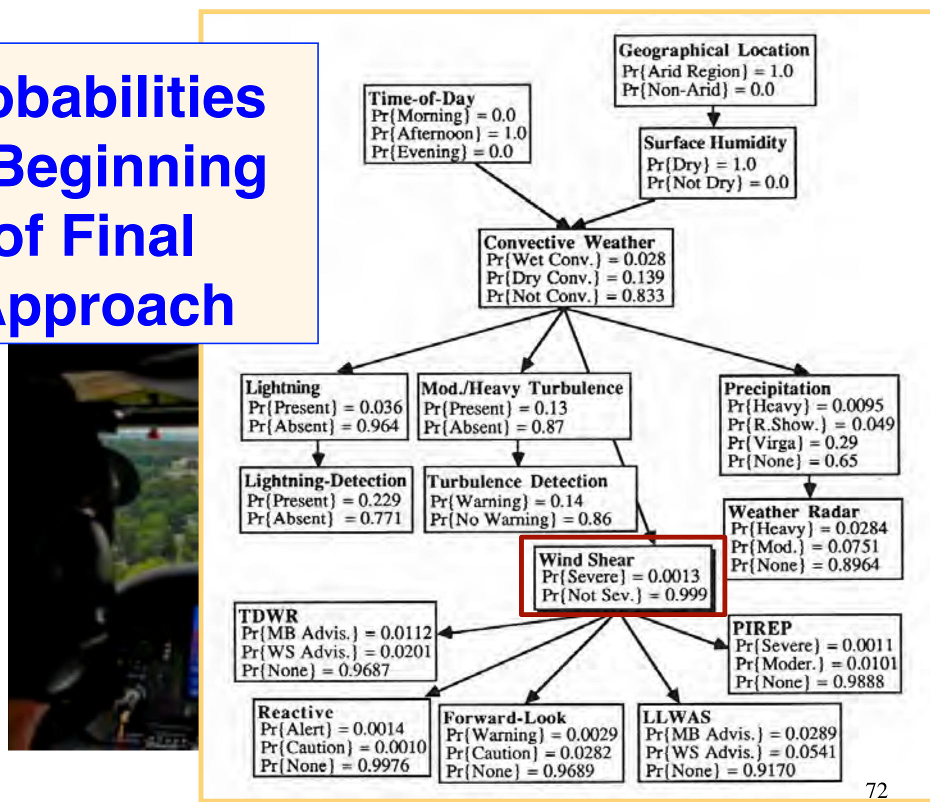
- **Pre- and post-hypothesis conditional probability**

$$\Pr(E_1 \mid E_2) = \Pr(E_2 \mid H)\Pr(H \mid E_1) + \Pr(E_2 \mid \neg H)\Pr(\neg H \mid E_1)$$

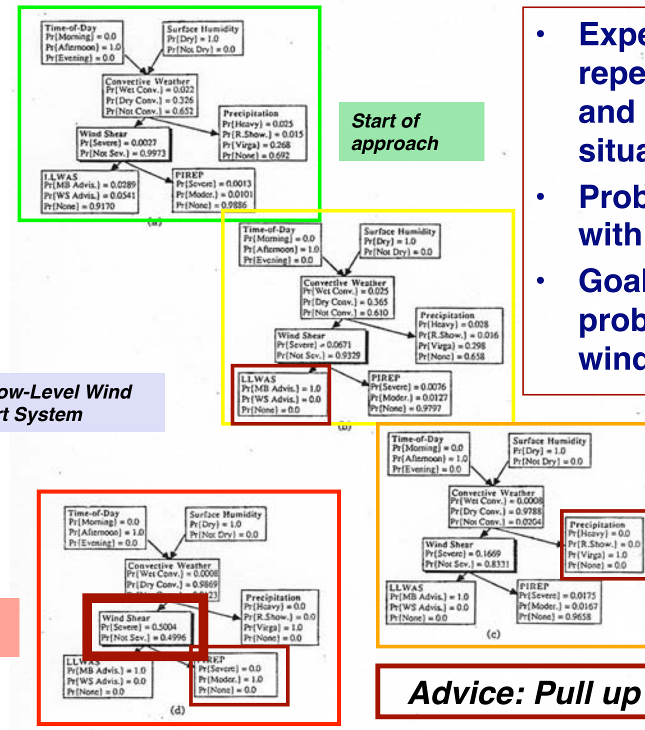
  - **Probability of hypothesis, *H*, conditioned on observation of post-hypothesis event**

$$\Pr(H \mid E_2) = \Pr(H \mid E_1)\Pr(E_1 \mid E_2) + \Pr(H \mid \neg E_1)\Pr(\neg E_1 \mid E_2)$$

Probabilities at Beginning of Final Approach



Evolution of a Wind Shear Advisory



Start of approach

- Expert system is repeatedly monitoring and assessing the situation
- Probabilities are updated with each new input
- Goal is determine probability of hazardous wind shear

__LLWAS__: Low-Level Wind Shear Alert System

__Virga__: High-altitude rain that evaporates before reaching the ground

__PIREP__: Pilot Report

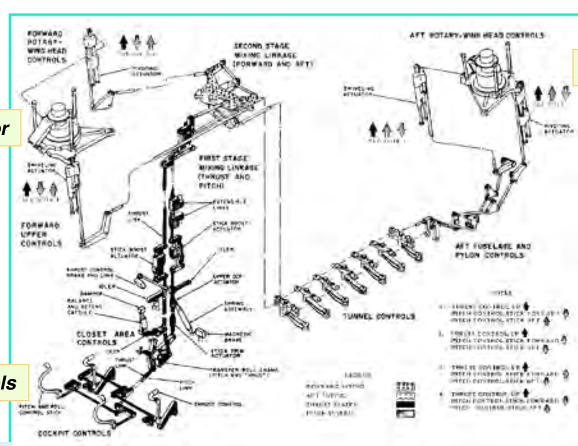
Advice: Pull up and go around

Inferential Fault Analyzer for Helicopter Control System

- Local failure analysis
  - Set of hypothetical models of specific failure
- Global failure analysis
  - Forward reasoning assesses failure impact
  - Backward reasoning deduces possible causes

Aft Rotor

Forward Rotor



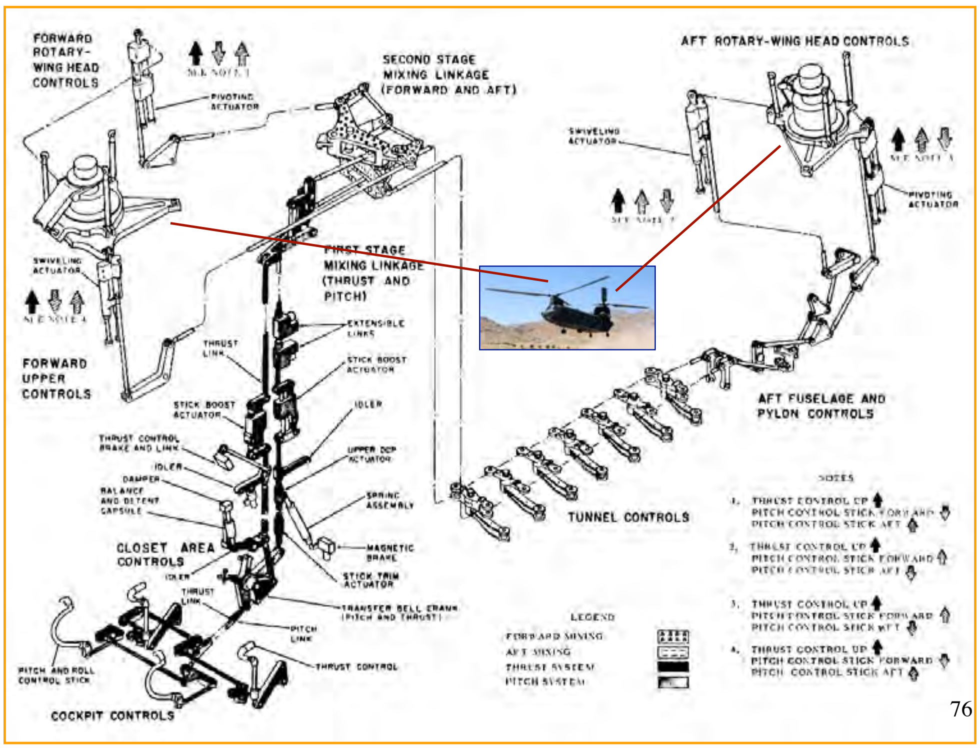
Cockpit Controls

Huang, Stengel

Heuristic Search

- **Local failure analysis**
  - **Determination based on aggregate of local models**
- **Global failure analysis**
  - **Determination based on aggregate of local failure analyses**
- **Heuristic score based on**
  - **Criticality of failure**
  - **Reliability of component**
  - **Extensiveness of failure**
  - **Implicated devices**
  - **Level of backtracking**
  - **Severity of failure**
  - **Net probability of failure model**

Mechanical Control System



Local Failure Analysis

- **Frames** store facts and facilitate search and inference
  - Components and up-/downstream linkages of control system
  - Failure model parameters
  - Rule base for failure analysis *(LISP)*

Local Failure Model #1
The cause of Nodes 9-2 (1.0) & 17-2 (1.0) being down
MAY be that Node 8-2 (1.0) is down
==

Local Failure Model #2
The cause of Nodes 9-3 (1.0) & 17-3 (1.0) being down
MAY be that Node 8-3 (1.0) is down
==

Local Failure Model #3a
The cause of Nodes 17-2 (1.0), 9-2 (1.0) & 18-2 (1.0) being down
MAY be that Node 7-2 (0.67) is down
This **IMPLICATES** Nodes 8-2, 15, 3, & 11-2
==

Local Failure Model #4
The cause of Nodes 5 (1.0) & 16 (1.0) being down
MAY be that Node 2 (1.0) is down
==

Global Failure Analysis

Global Failure Model #1
Formed from local model(s): 1,2 Score: 32.5
Flagged Devices: Torquemeter-1, Torquemeter-2, Eng-oil-temp-1, Eng-oil-temp-2, Eng-oil-press-1, Eng-oil-press-2, Flow-meter-1
Probable Cause: Engine-#1 (0,75), Engine-#2(0.75)
Implicated End-Devices: Pump-press-sensor-1, Pump-press-sensor-2, Actr-press-sensor-1,-2,-3,-4, Aft-yaw-&-roll, Aft-pitch-&-heave, Eng-chip-detector-1, Eng-chip-detector-2.
==

Global Failure Model #2
Formed from local model(s): 1,3 Score: 30.875
Flagged Devices: Torquemeter-1, Torquemeter-2, Eng-oil-temp-1, Eng-oil-temp-2, Eng-oil-press-1, Eng-oil-press-2, Flow-meter-1
Probable Cause: Engine-#2 (0,75), Fuel-System-#1(0.675)
Implicated End-Devices: Pump-press-sensor-1, Pump-press-sensor-2, Actr-press-sensor-1,-2,-3,-4, Aft-yaw-&-roll, Aft-pitch-&-heave, Eng-chip-detector-1, Eng-chip-detector-2.
==

Global Failure Model #3
Formed from local model(s): 2 Score: 19.25
Flagged Devices: Torquemeter-1, Eng-oil-temp-1, Eng-oil-press-1
Probable Cause: Engine-#1 (0,75)
Implicated End-Devices: Pump-press-sensor-1, Eng-chip-detector-1, Actr-press-sensor-1.
==