Articulated Robots

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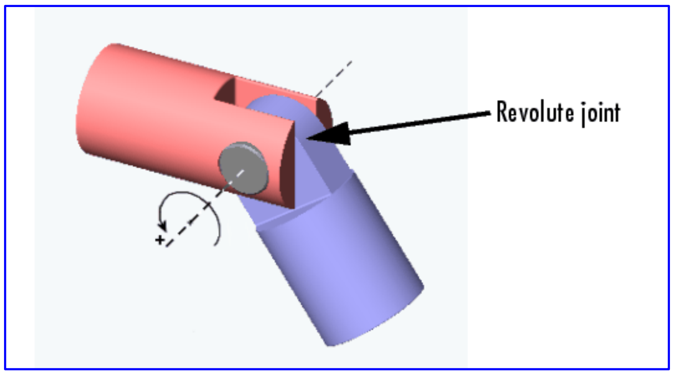
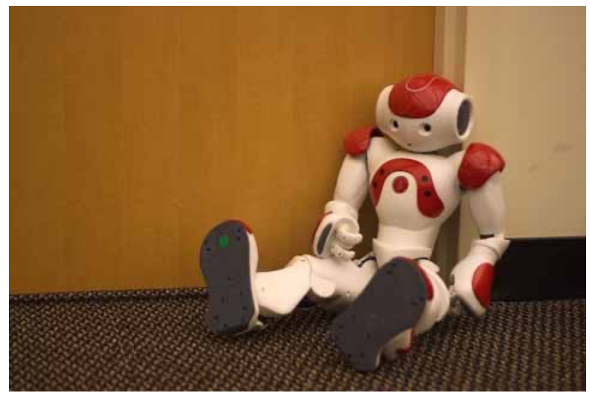
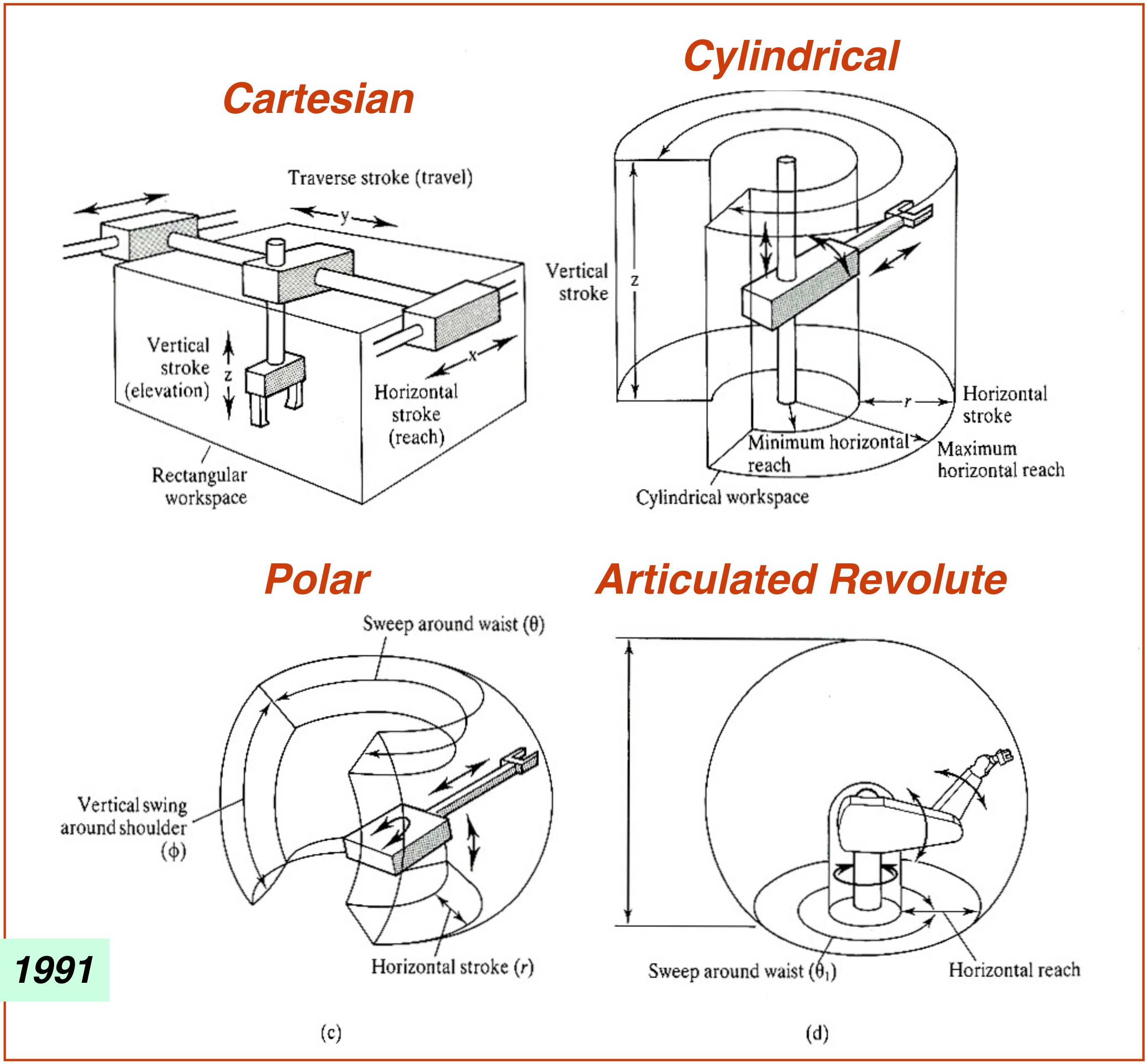
Robotics and Intelligent Systems
MAE 345, Princeton University, 2017





- **Robot configurations**
- **Joints and links**
- **Joint-link-joint transformations**
  - Denavit-Hartenberg representation

Assembly Robot Configurations

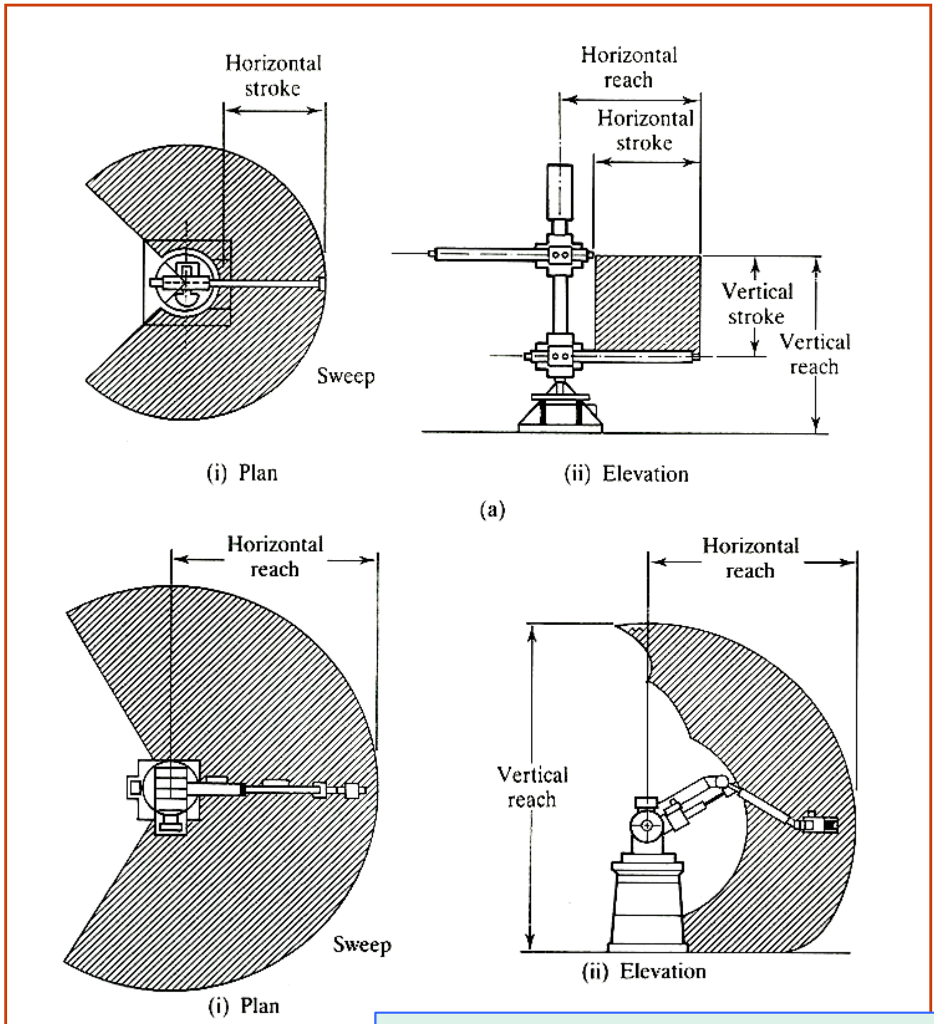


McKerrow, 1991

Assembly Robot Workspaces

Cylindrical

Articulated Revolute



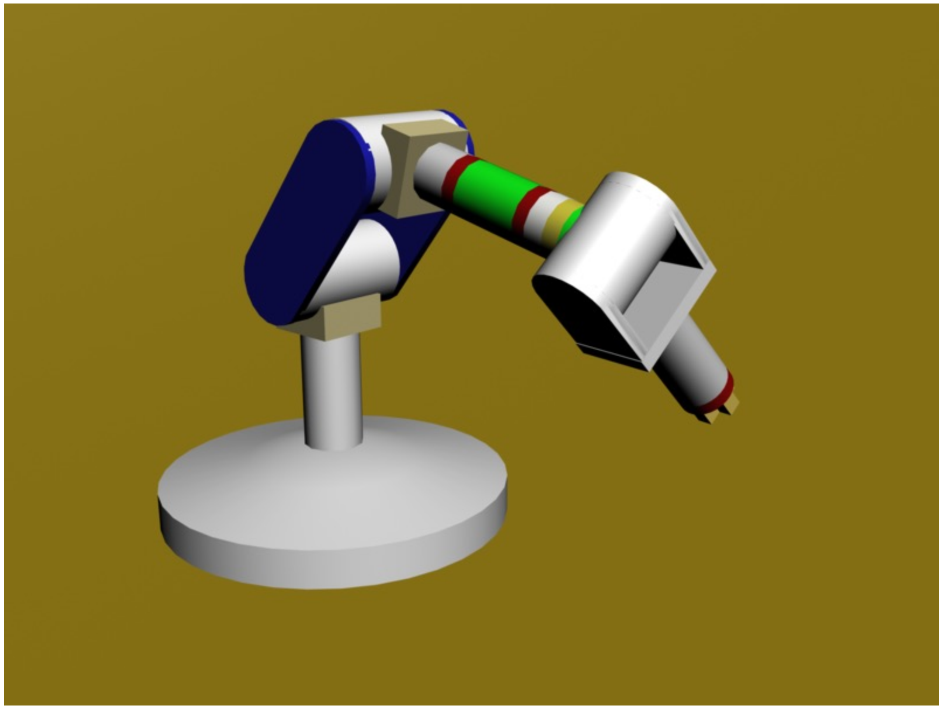
McKerrow, 1991

Tesla Model S Assembly
http://www.youtube.com/watch?v=8_lfxPI5ObM

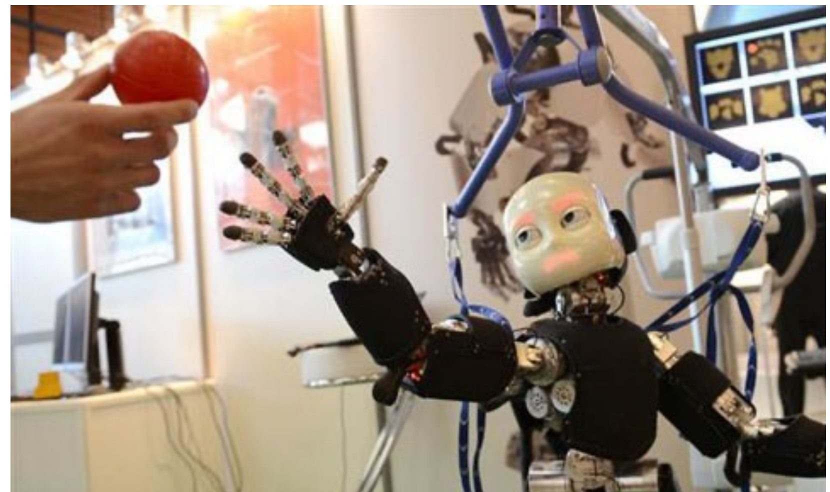
Serial Robotic Manipulators

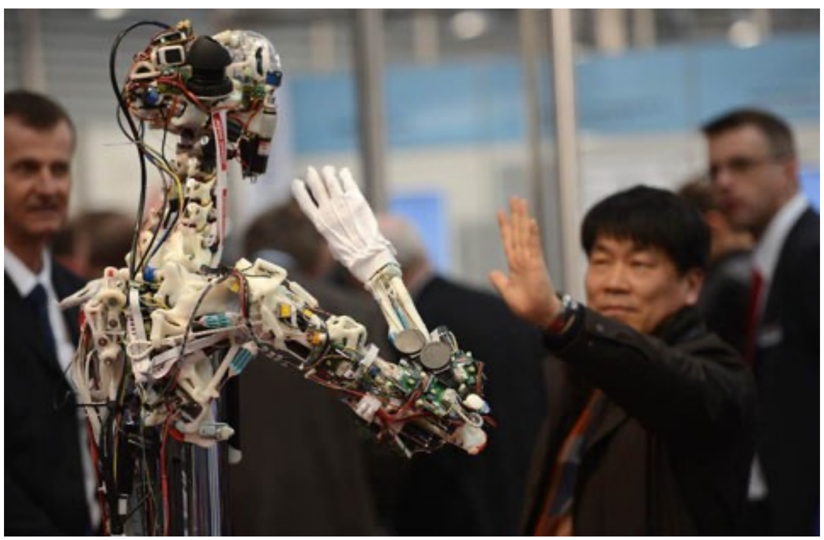
***Proximal link*: closer to the base**
***Distal link*: farther from the base**

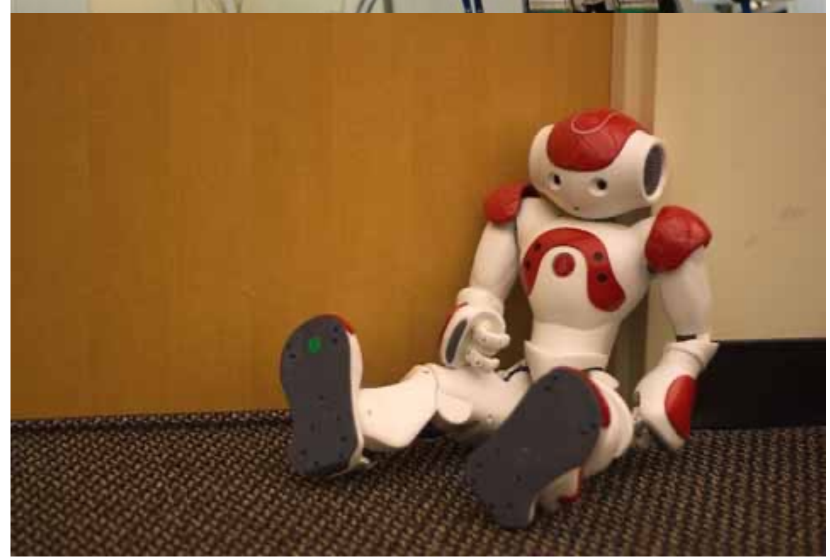
- **Serial chain of robotic links and joints**
  - **Large workspace**
  - **Low stiffness**
  - **Cumulative errors from link to link**
  - **Proximal links carry the weight and load of distal links**
  - **Actuation of proximal joints affects distal links**
  - **Limited load-carrying capability at end effecter**



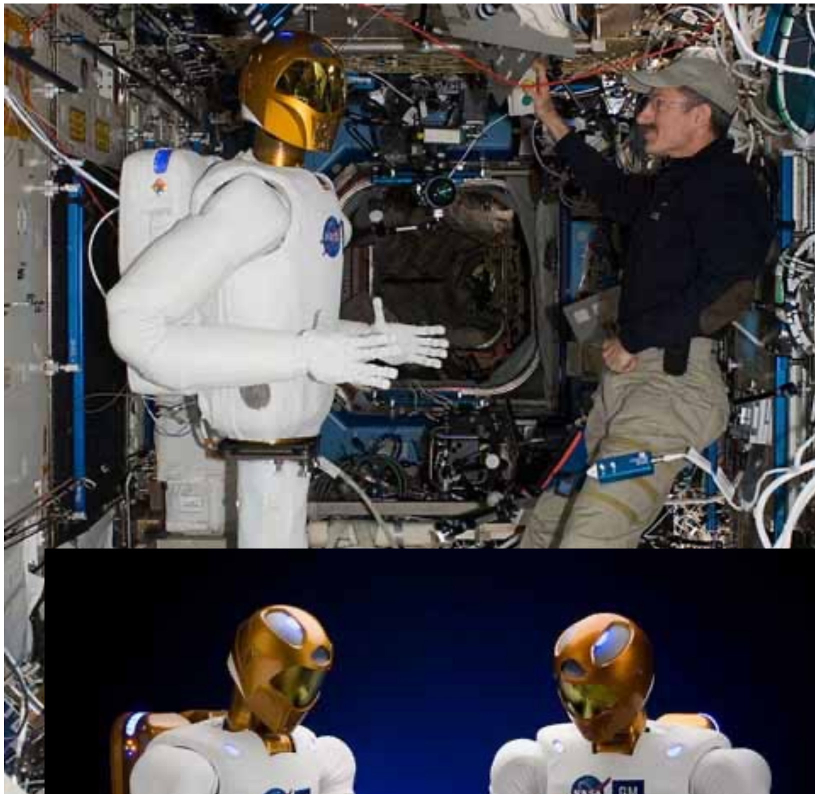
Humanoid Robots

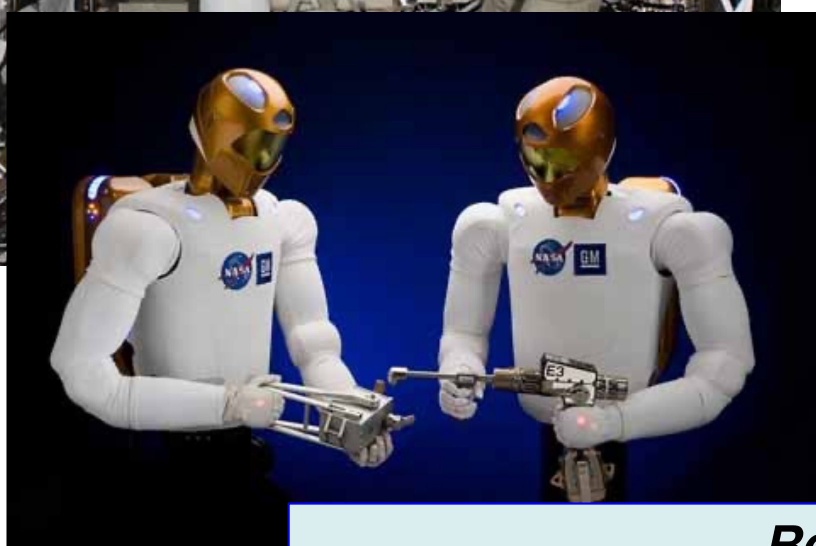


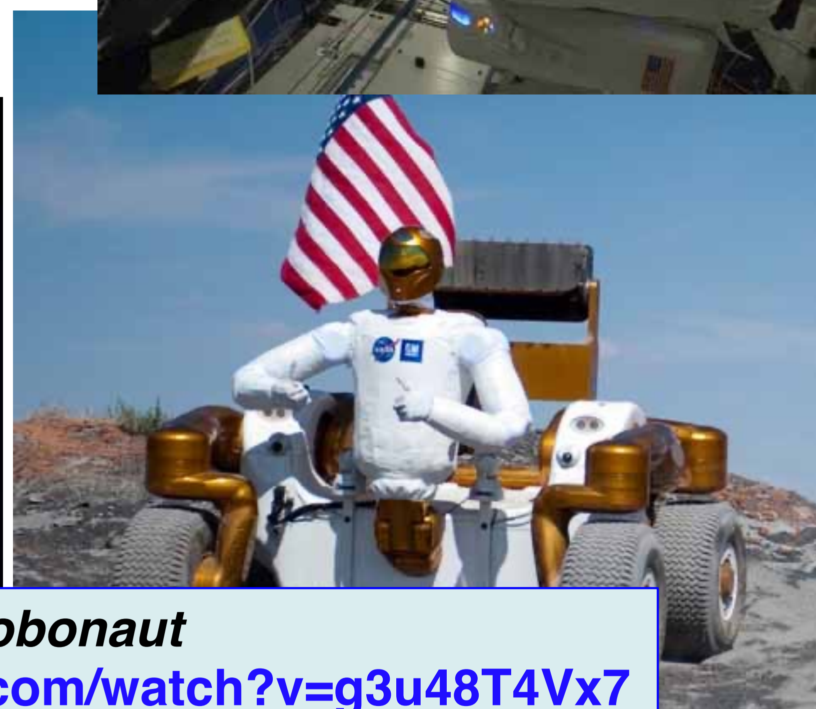




NASA/GM Robonaut



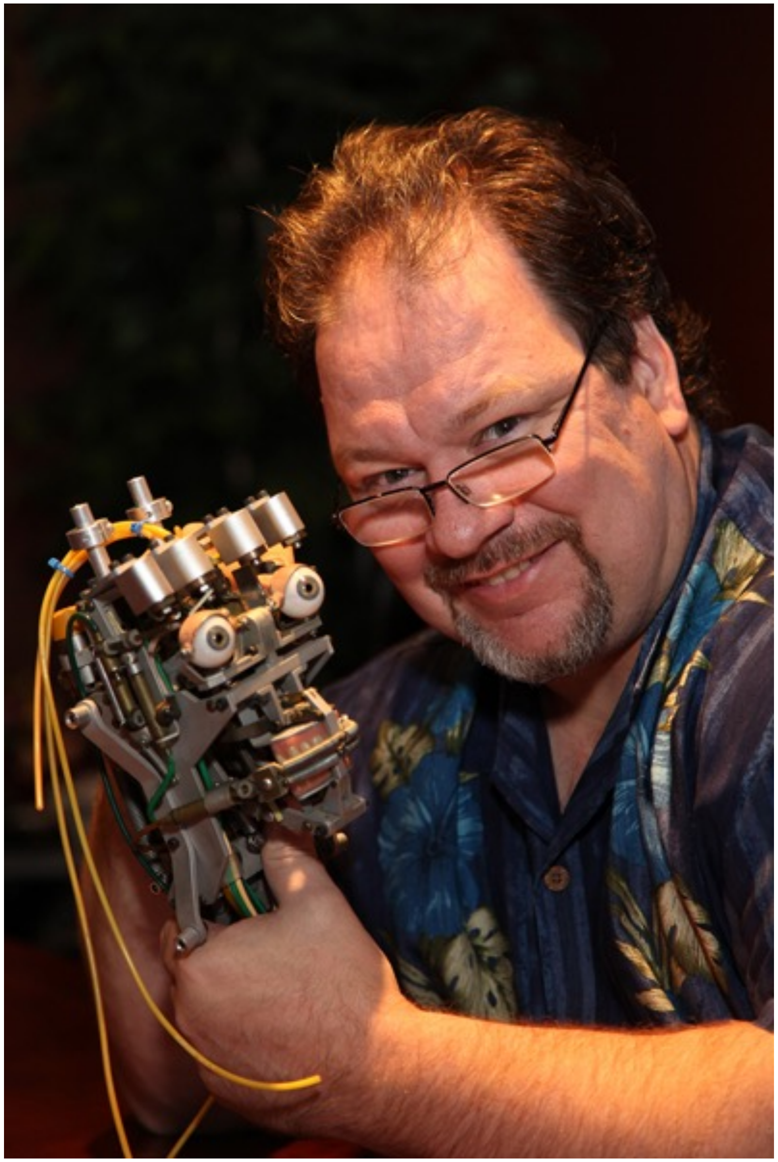




Robonaut

http://www.youtube.com/watch?v=g3u48T4Vx7k

Disney Audio-Animatronics, 1967



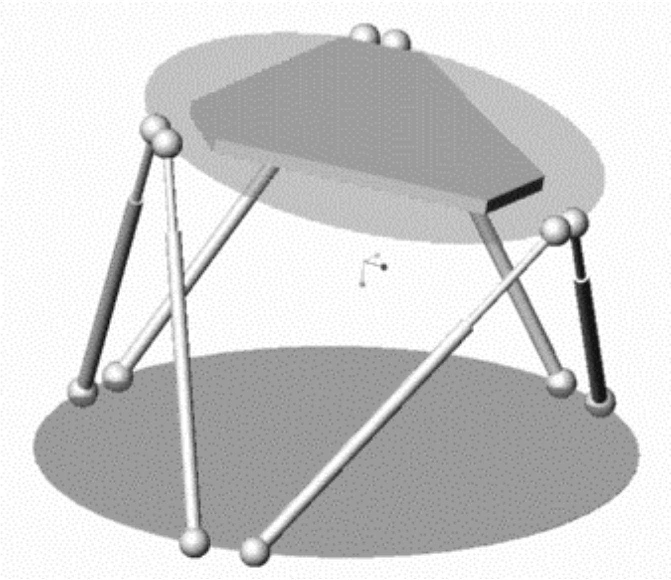
Baxter, Sawyer, and the PR2

Baxter
http://www.youtube.com/watch?v=QHAMsalhlv8

PR2
http://www.youtube.com/watch?v=HMx1xW2E4Gg

Parallel Robotic Mechanisms

- **End plate is directly actuated by multiple links and joints (*kinematic chains*)**
  - **Restricted workspace**
  - **Common link-joint configuration**
  - **Light construction**
  - **Stiffness**
  - **High load-carrying capacity**

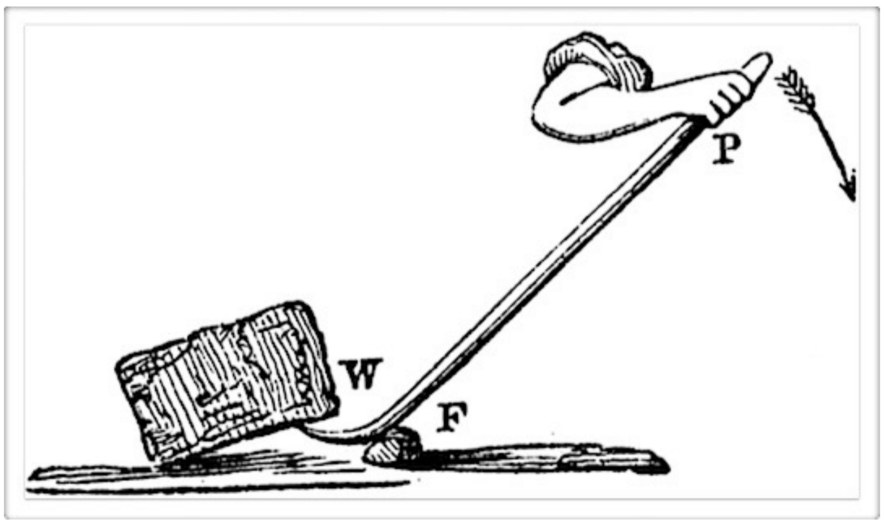


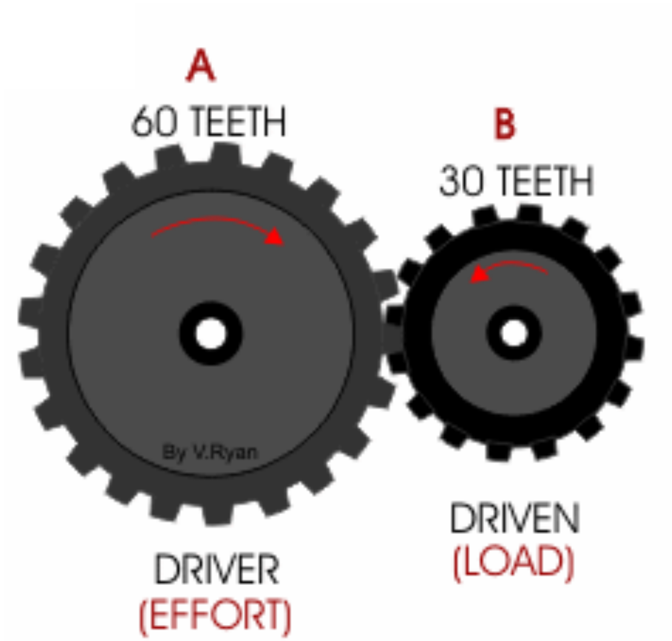
Stewart Platform
http://www.youtube.com/watch?v=QdKo9PYwGaU

Pick-and-Place Robot
http://www.youtube.com/watch?v=i4oBExl2KiQ

Gearing and Leverage

Force multiplication
Displacement ratios

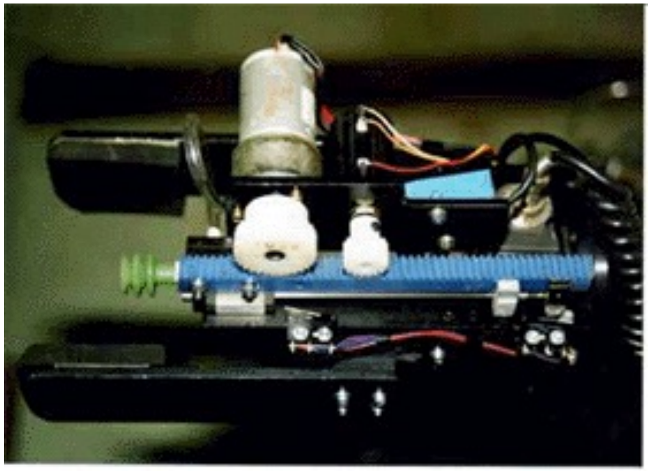


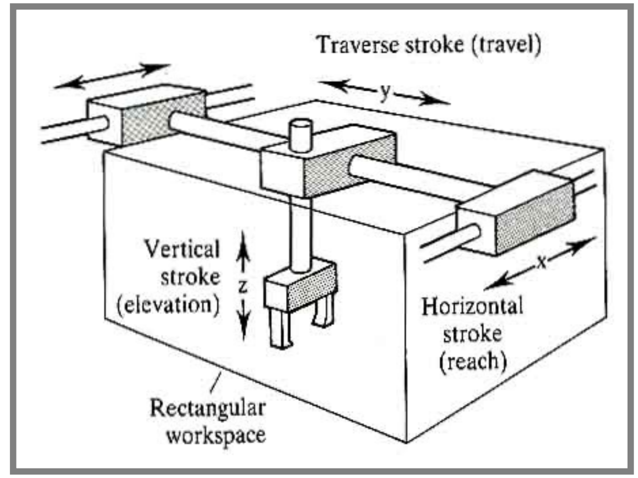


End Effecters

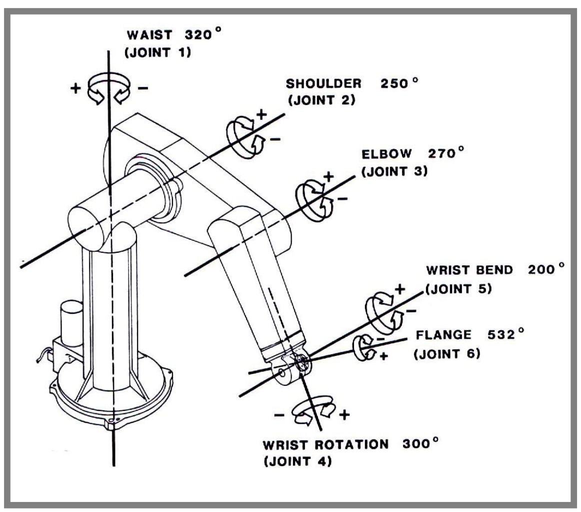
- **Machine tools**
  - Grinding, sanding
  - Inserting screws
  - Drilling
  - Hammering
- **Paint sprayer**
- **Gripper, clamp**
- **Multi-digit hand**

DARPA Prosthetic Hand
http://www.youtube.com/watch?v=QJg9igTnjlo&feature=related



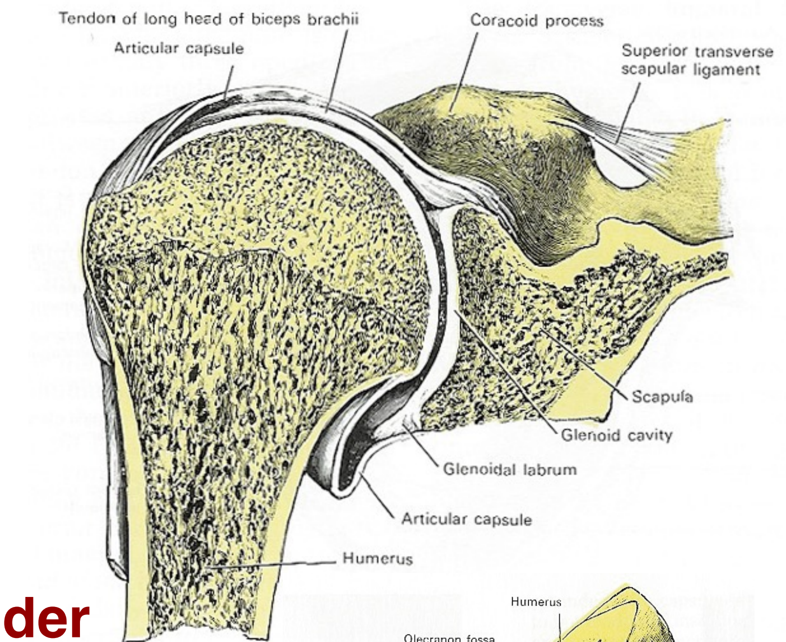


Links and Joints



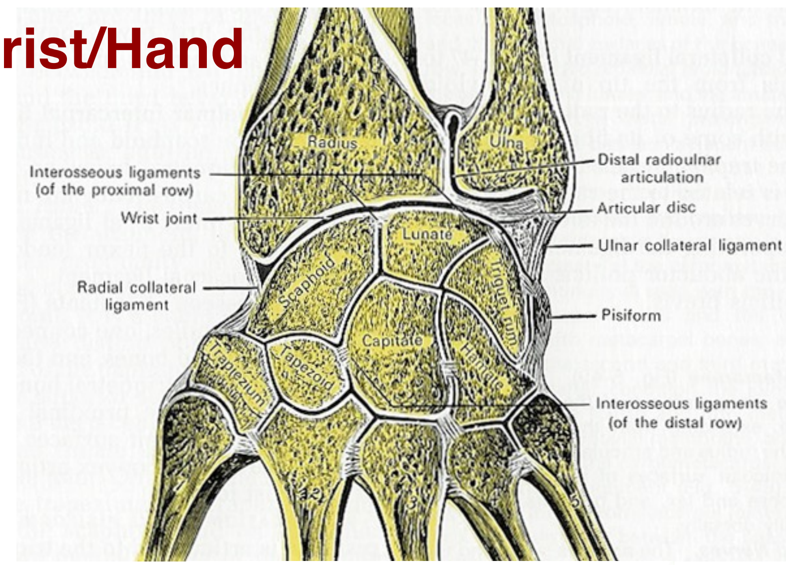
Human Joints

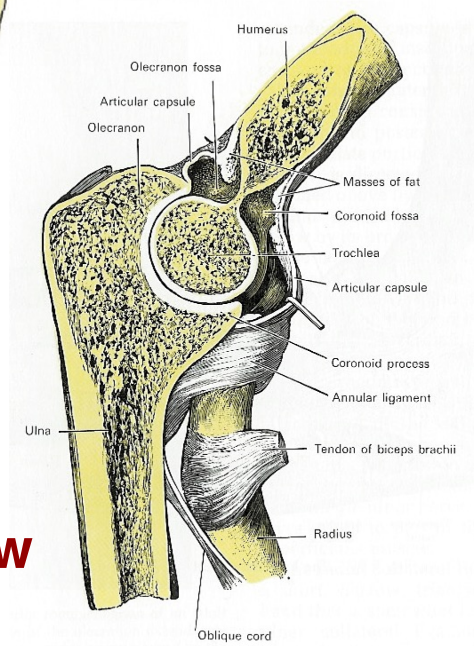
Gray's Anatomy, 1858



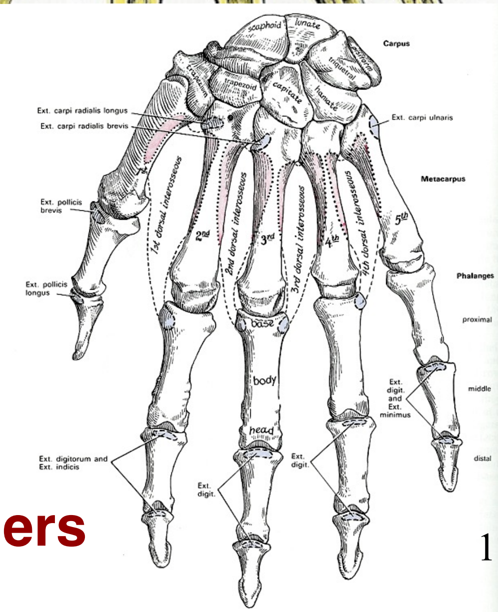
Shoulder

Wrist/Hand

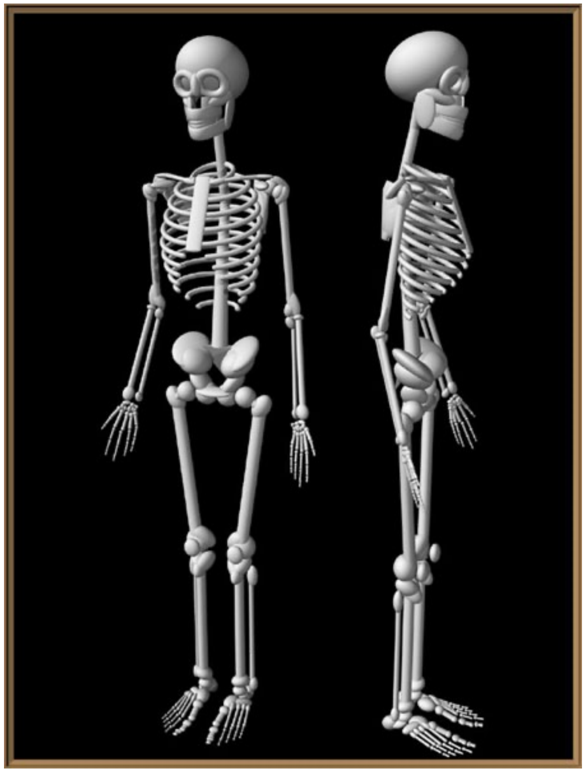




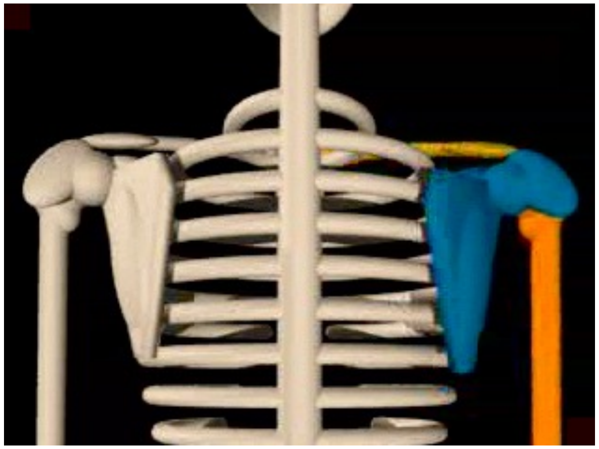
Elbow

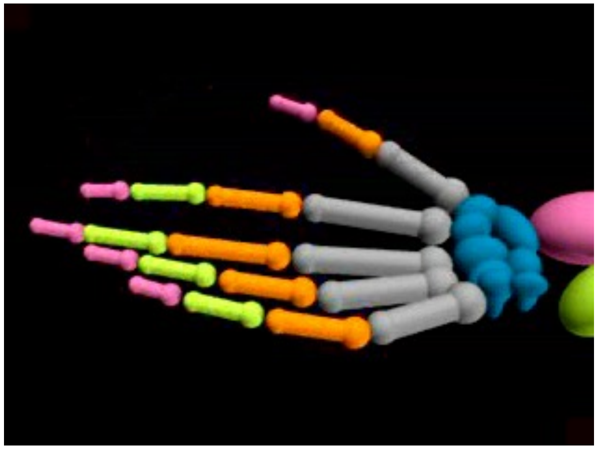


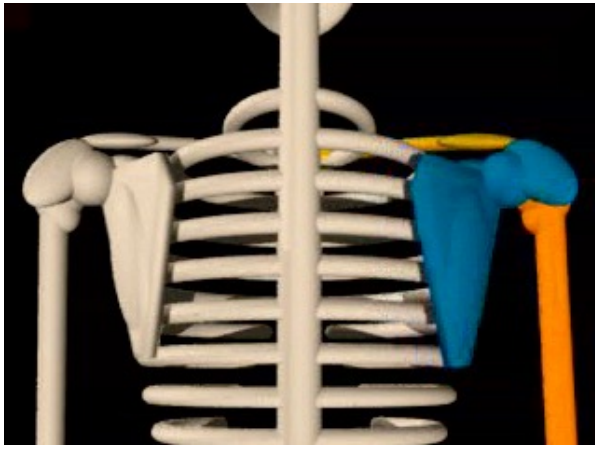
Hand/Fingers

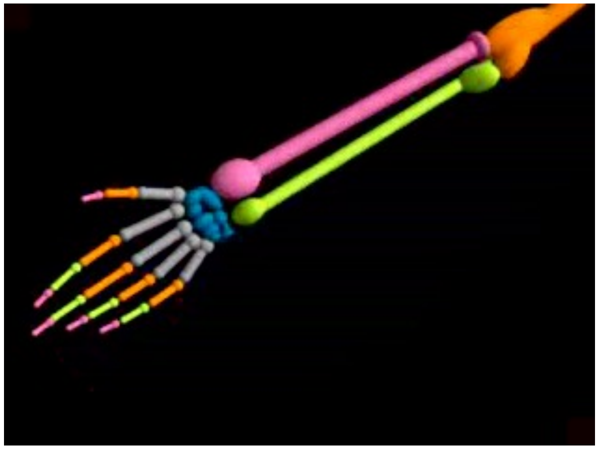


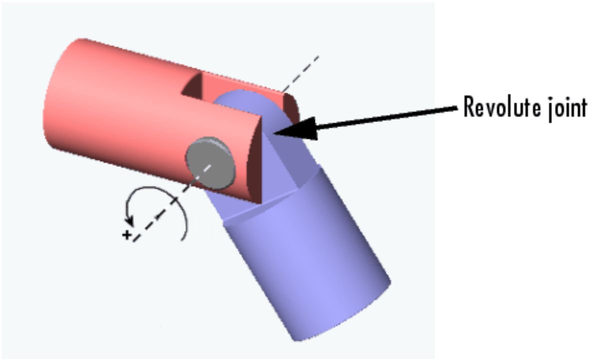
Skeleton and Muscle-Induced Motion







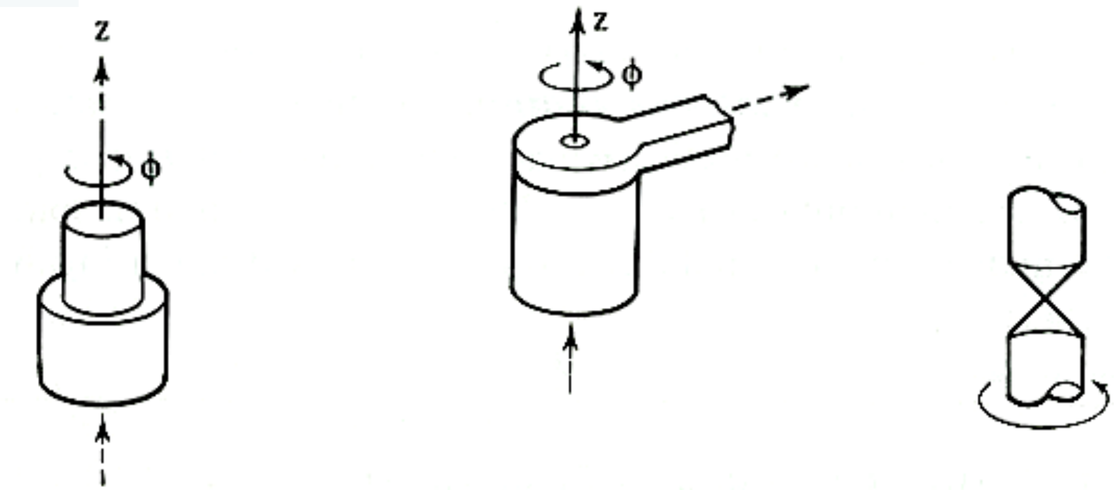




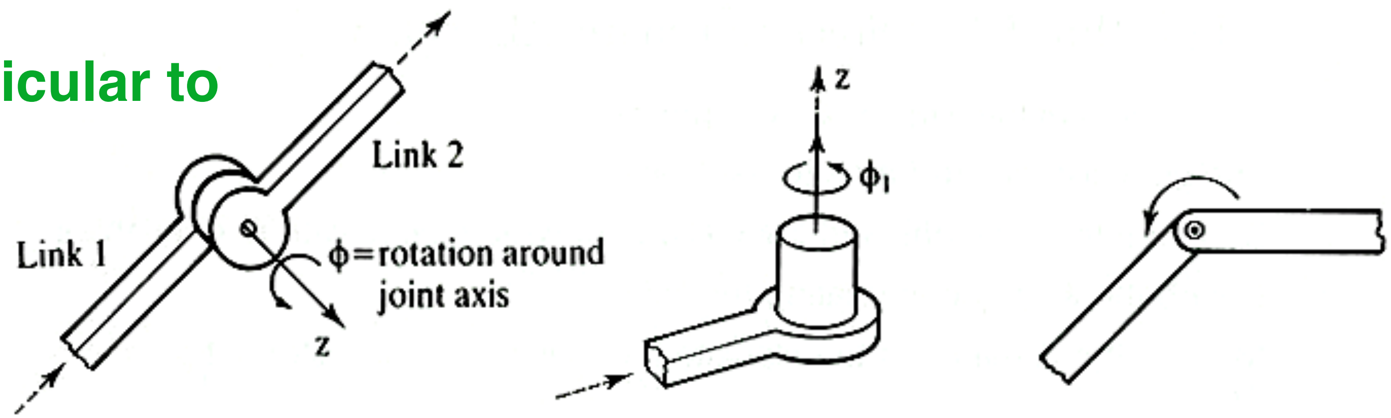
Revolute Robotic Joints

Rotation about a single axis

Parallel to Link

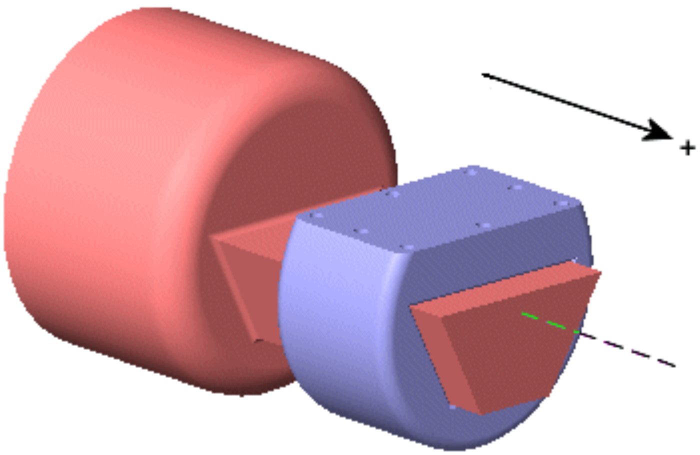


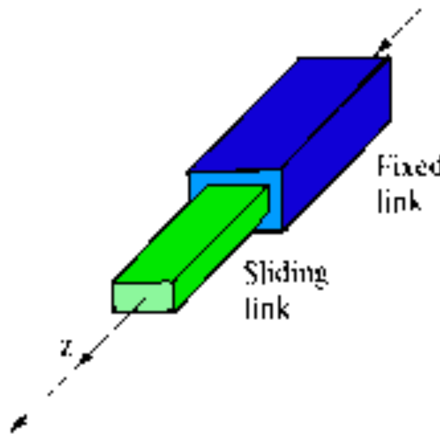
Perpendicular to Link

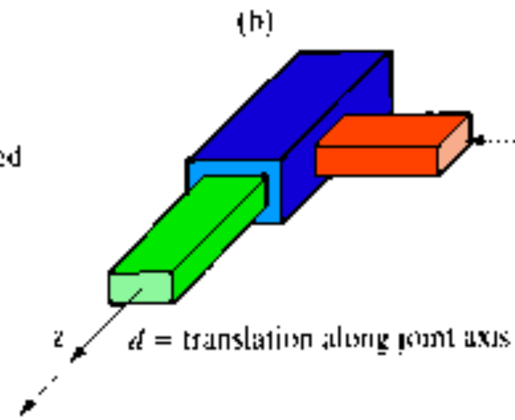


Prismatic Robotic Joints

Sliding along a single axis

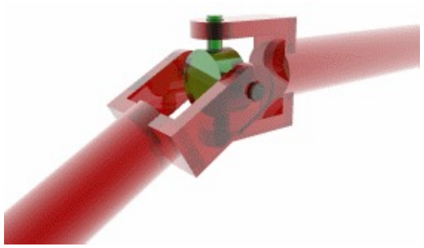




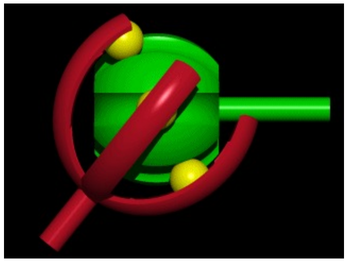


Other Robotic Joints

Universal



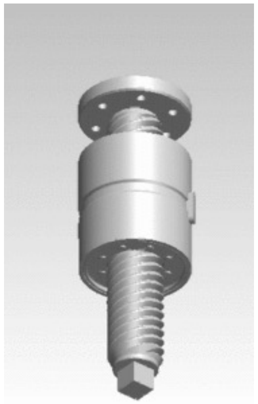
Constant-Velocity



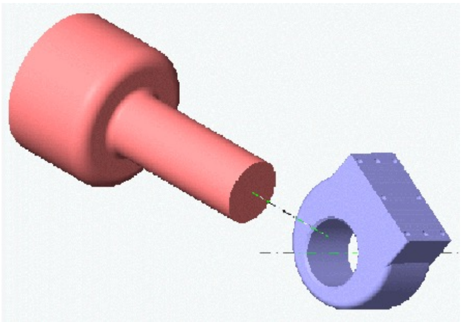
Flexible

Spherical (or ball)

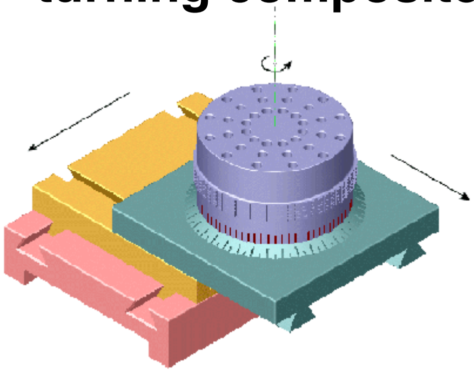
Roller Screw



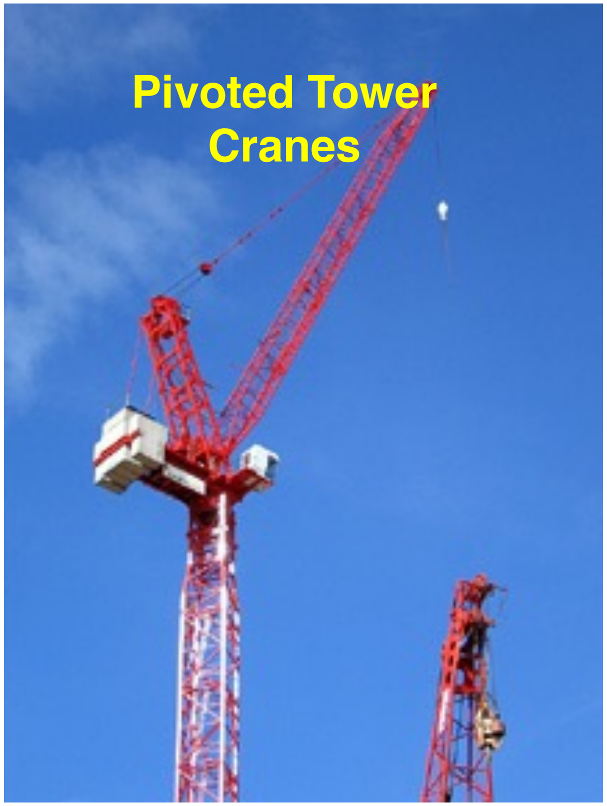
Cylindrical (sliding and turning composite)

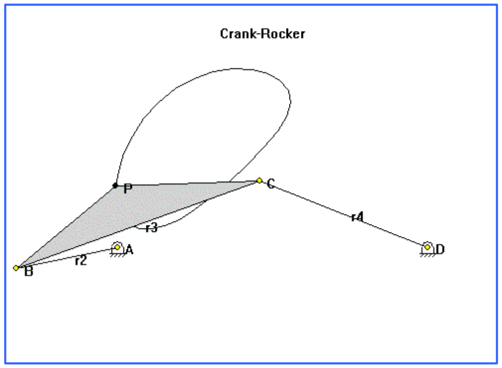


Planar (sliding and turning composite)

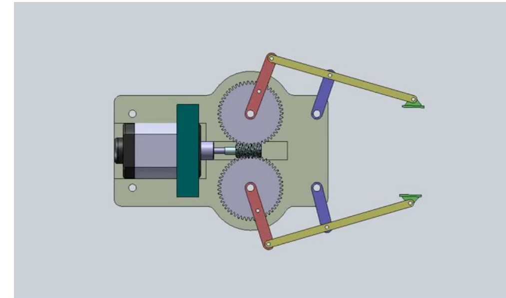


Construction Cranes

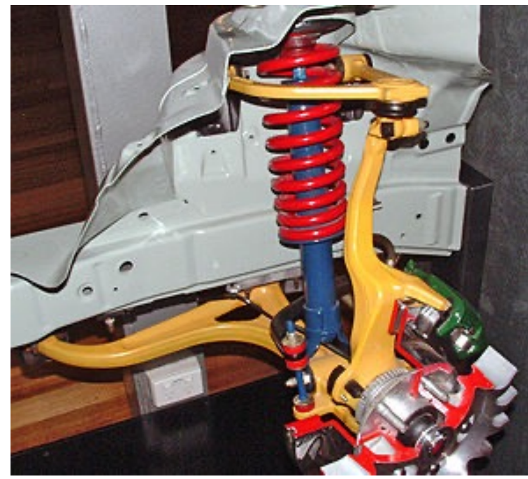


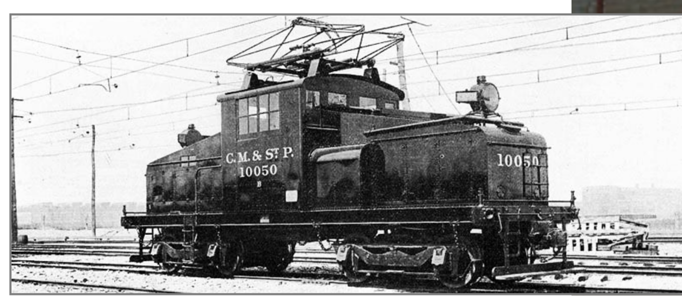


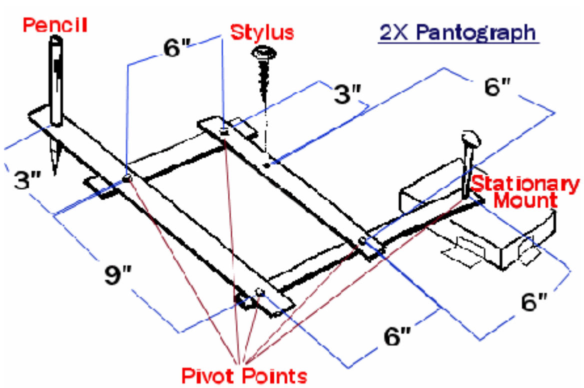
Four-Bar Linkage



- Closed-loop structure
- Rotational joints
- Planar motion
- Proportions of link lengths determine pattern of motion
- Examples
  - Double wishbone suspension
  - Pantograph
  - Scissor lift
  - Gripper



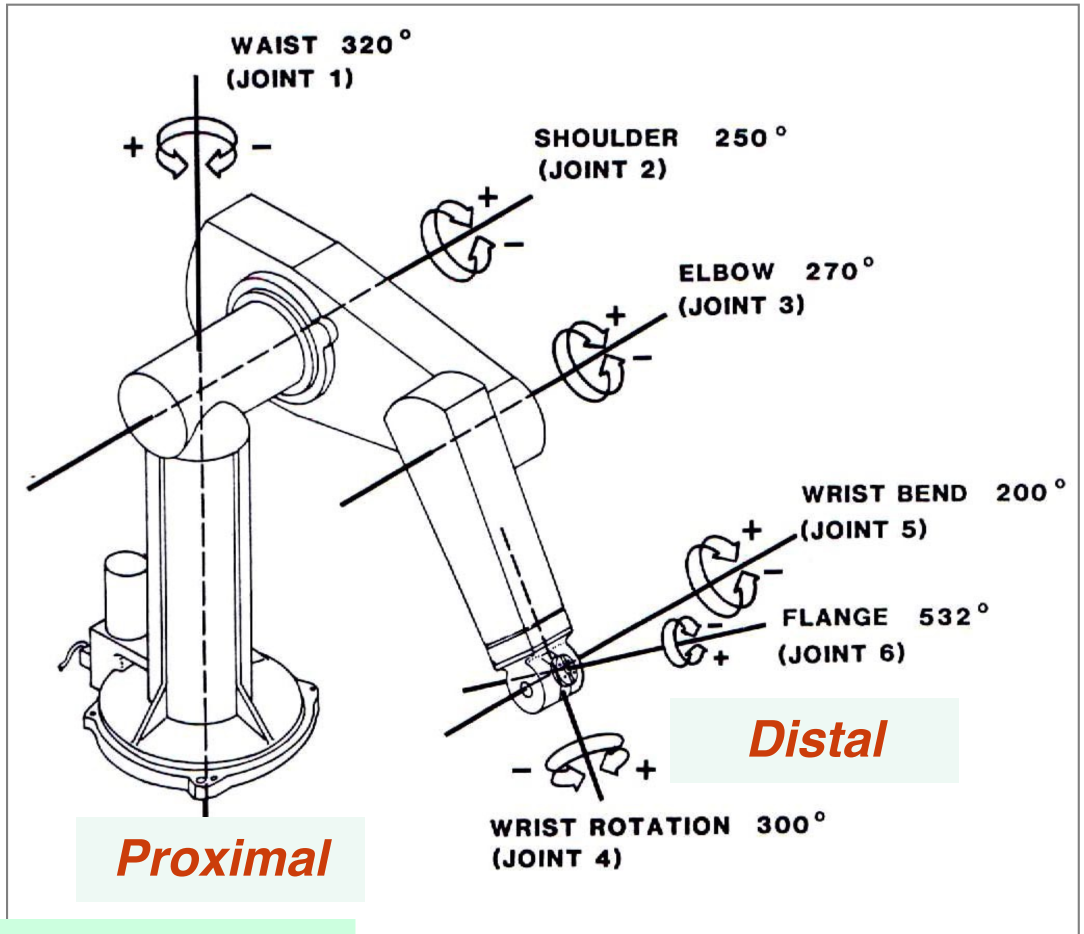




Characteristic Transformation of a Link

Link: solid structure between two joints

- **Each link type** has a **characteristic transformation matrix** relating the proximal joint to the distal joint
- **Link *n*** has
  - Proximal end: Joint n, coordinate frame $n-1$
  - *Distal end*: Joint $n+1$, coordinate frame n



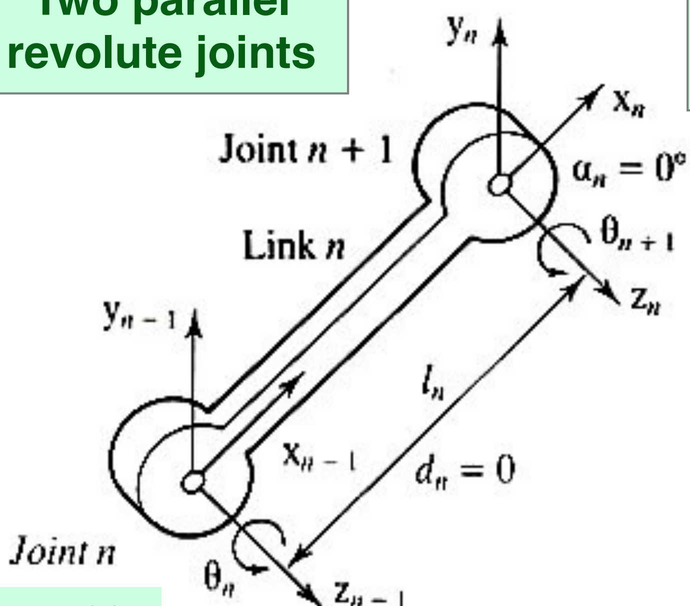
McKerrow, 1991

Links Between Revolute Joints

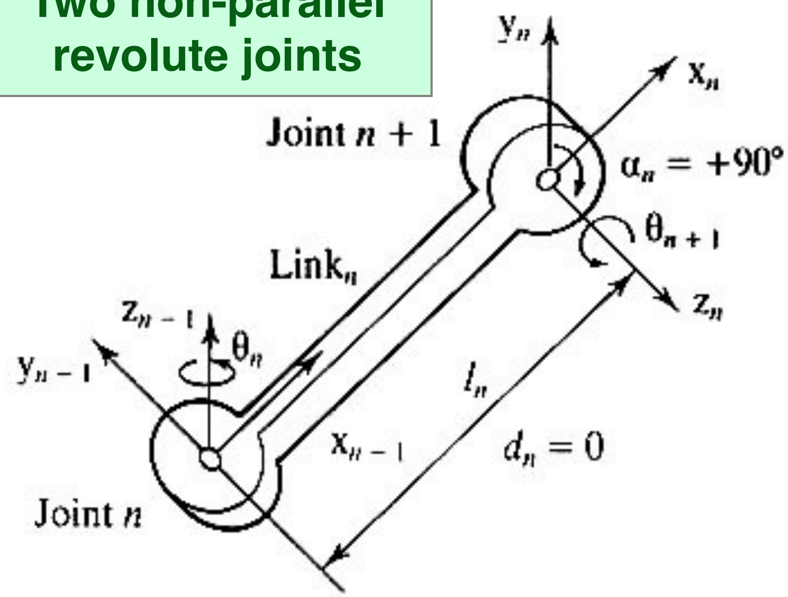
- **Link: solid structure between two joints**
  - **Proximal end: closer to the base**
  - **Distal end: farther from the base**

- **4 Link Parameters**
  - **Length of the link between rotational axes, *l*, along the common normal**
  - **Twist angle between axes, α**
  - **Angle between 2 links, θ (revolute)**
  - **Offset between links, *d* (prismatic)**
- **Joint Variable: single link parameter that is free to vary**

Type 1 Link
Two parallel revolute joints



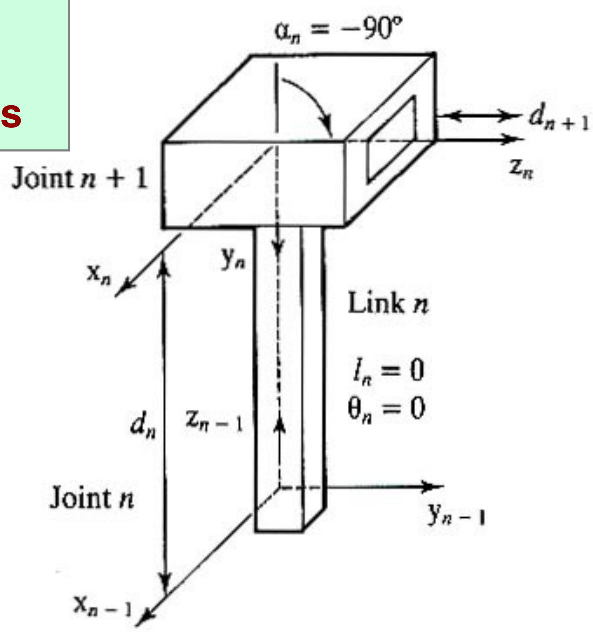
Type 2 Link
Two non-parallel revolute joints



McKerrow, 1991

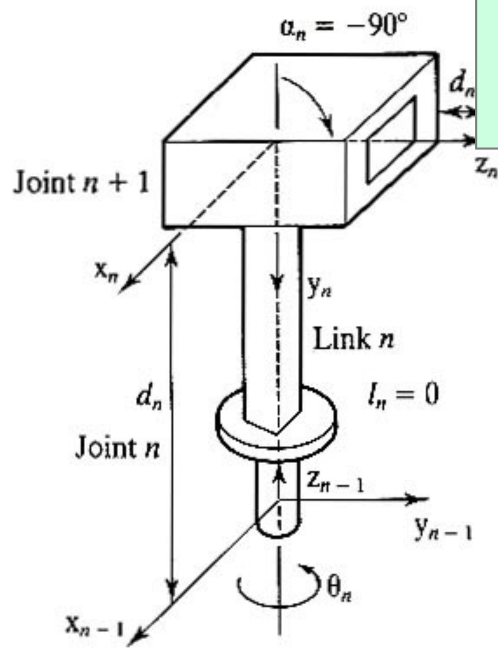
Links Involving Prismatic Joints

Type 5 Link
Intersecting prismatic joints



- **Link n extends along z_{n-1} axis**
  - **l_n = 0, along x_{n-1}**
  - **d_n = length, along z_{n-1} (variable)**
  - **θ_n = 0, about z_{n-1}**
  - **a_n = fixed orientation of $n + 1$ prismatic axis about x_{n-1}**

Type 6 Link
Intersecting revolute and prismatic joints

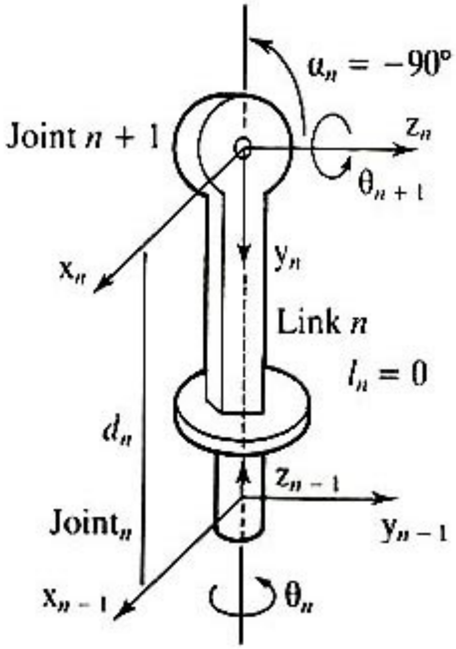


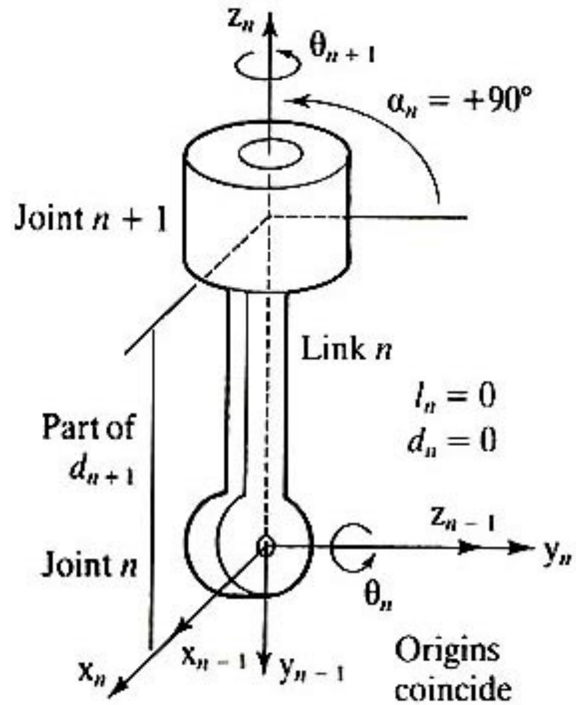
- **Link n extends along z_{n-1} axis**
  - **l_n = 0, along x_{n-1}**
  - **d_n = length, along z_{n-1} (fixed)**
  - **θ_n = variable joint angle n about z_{n-1}**
  - **a_n = fixed orientation of $n + 1$ prismatic axis about x_{n-1}**

McKerrow, 1991

Links Between Revolute Joints - 2

Type 3 Link
Two revolute joints with intersecting rotational axes (e.g., shoulder)





Type 4 Link
Two perpendicular revolute joints with common origin (e.g., elbow-wrist)

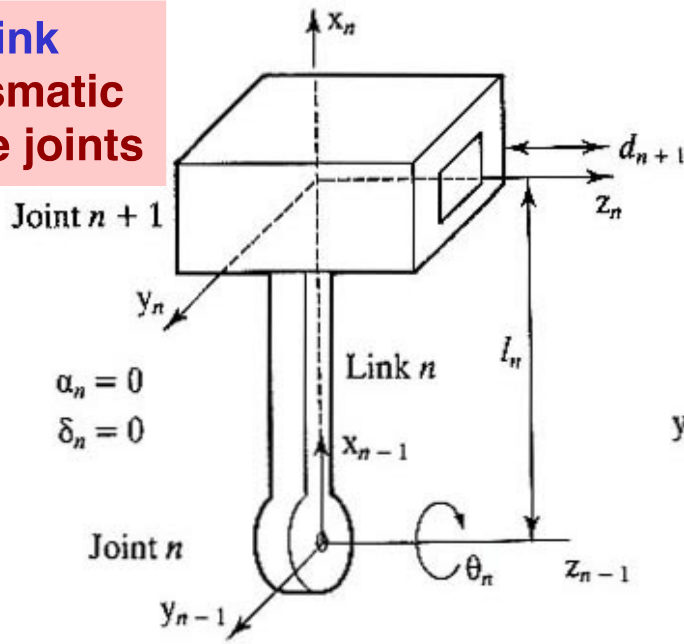
- **Link *n* extends along z_{n-1} axis**
  - l_n = 0, along x_{n-1}
  - d_n = length, along z_{n-1} (fixed)
  - θ_n = variable joint angle *n* about z_{n-1}
  - α_n = fixed orientation of *n* + 1 rotational axis about x_{n-1}

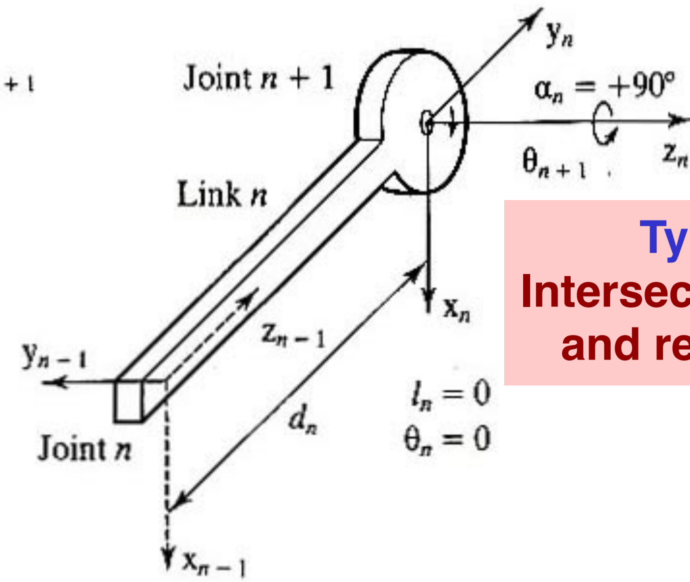
- **Link *n* extends along $-z_n$ axis**
  - l_n = 0, along x_{n-1}
  - d_n = 0, along z_{n-1}
  - θ_n = variable joint angle *n* about z_{n-1}
  - α_n = fixed orientation of *n* + 1 rotational axis about x_{n-1}

McKerrow, 1991

Links Involving Prismatic Joints - 2

Type 7 Link
Parallel prismatic and revolute joints





Type 8 Link
Intersecting prismatic and revolute joints

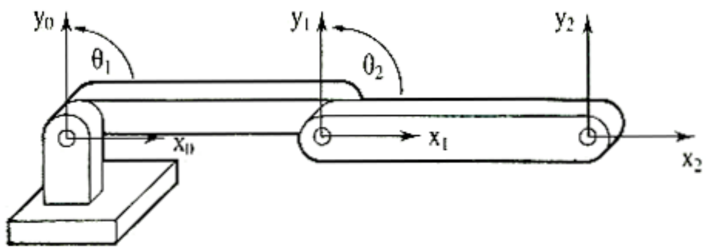
- **Link *n* extends along x_{n-1} axis**
  - l_n = length along x_{n-1}
  - d_n = 0, along z_{n-1}
  - θ_n = variable joint angle *n* about z_{n-1}
  - a_n = 0, orientation of *n* + 1 prismatic axis about x_{n-1}

- **Link *n* extends along z_{n-1} axis**
  - l_n = 0, along x_{n-1}
  - d_n = length, along z_{n-1} (variable)
  - θ_n = 0, about z_{n-1}
  - a_n = fixed orientation of *n* + 1 rotational axis about x_{n-1}

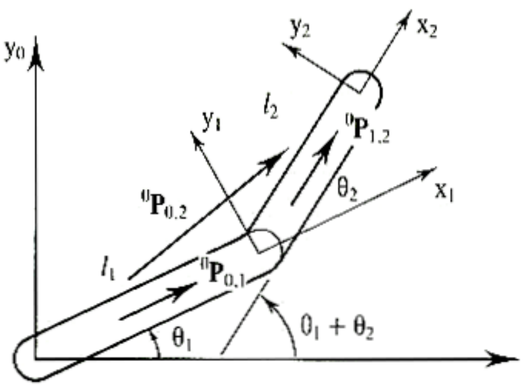
McKerrow, 1991

Two-Link/Three-Joint Manipulator

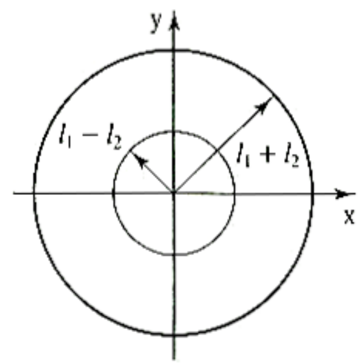
Parallel Rotation Axes



Manipulator in **zero position**



Assignment of coordinate frames



Workspace

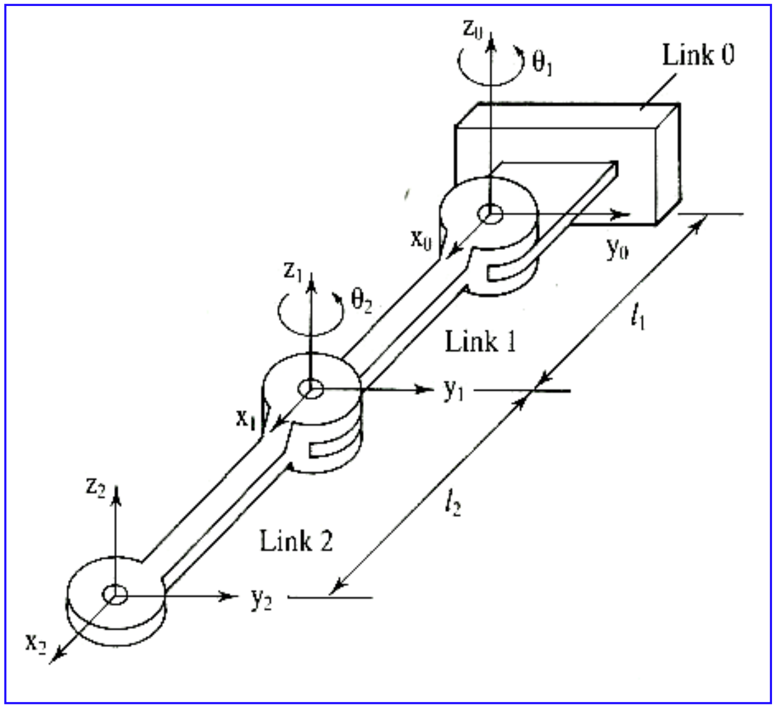
Parameters and Variables for 2-link manipulator

- Link lengths (fixed)
- Joint angles (variable)

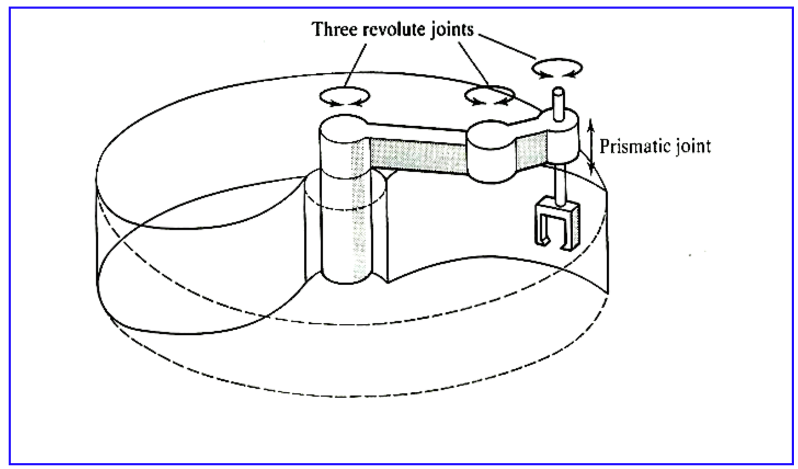
McKerrow, 1991

Four-Joint (SCARA*) Manipulator

Arm with Three Revolute Link Variables (Joint Angles)



Operation
http://www.youtube.com/watch?v=3-sbtCCyJXo



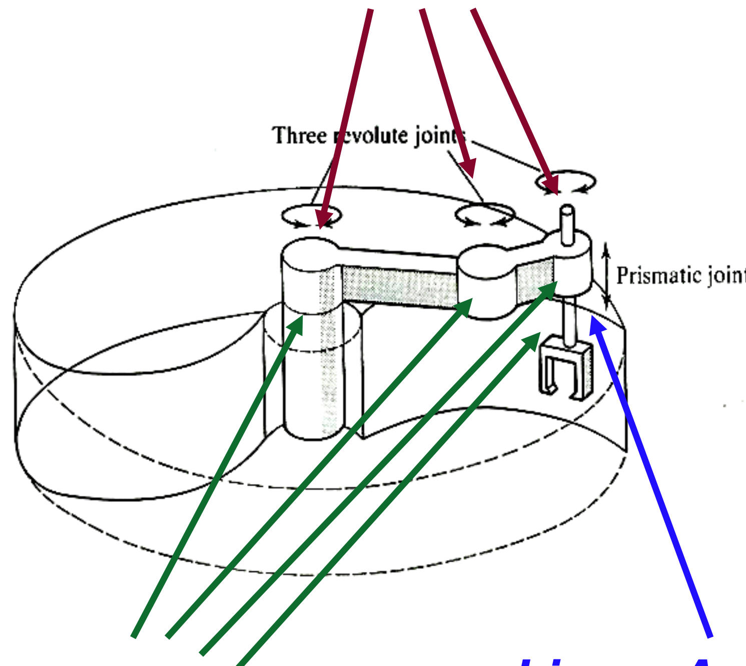
McKerrow, 1991

**Selective Compliant Articulated Robot Arm*

Joint Variables Must Be Actuated and Observed for Control

- Frames of Reference for Actuation and Control
  - World coordinates
  - Actuator coordinates
  - Joint coordinates
  - Tool coordinates

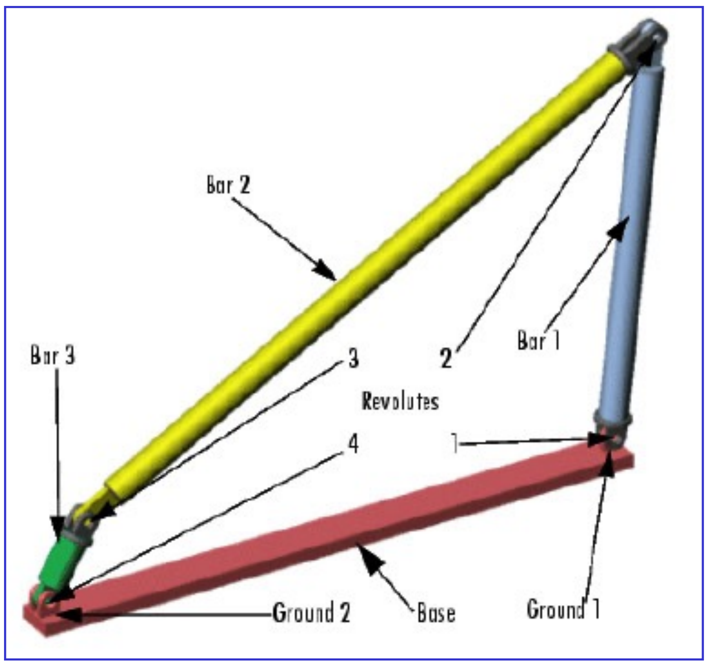
Rotary Actuators

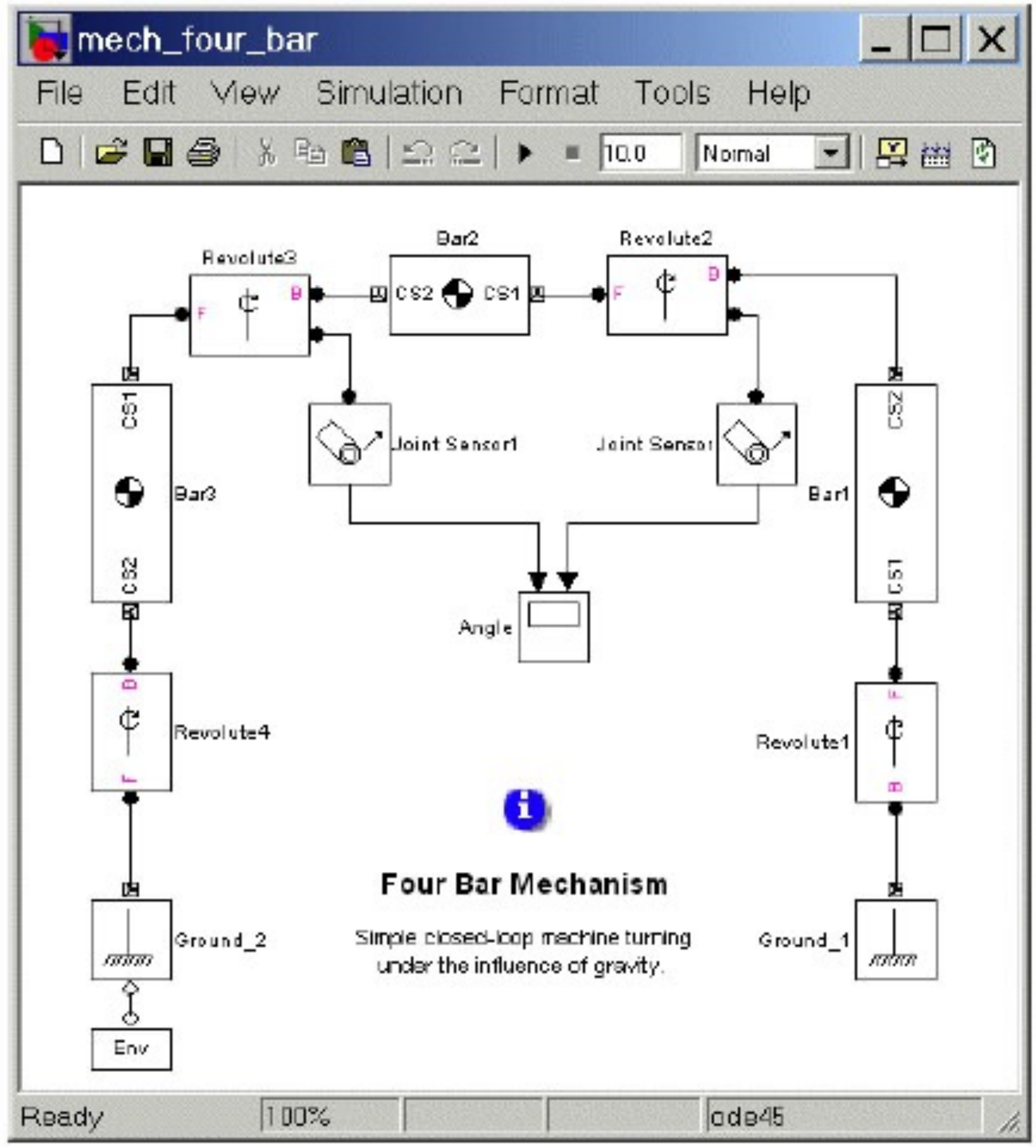


Sensors May Observe Joints Directly, Indirectly, or Not At All

Linear Actuator

Simulink/SimMechanics Representation of Four-Bar Linkage

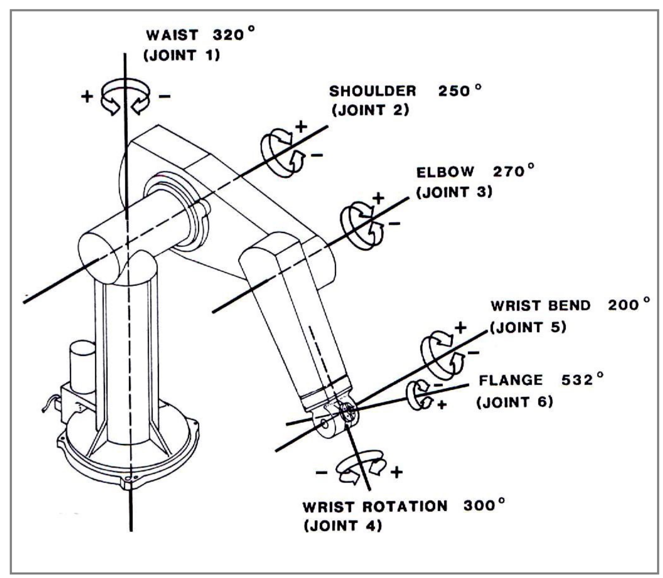




Recall: Homogeneous Transformation

$$\mathbf{s}_{new} = \left[\begin{array}{c:c} \begin{pmatrix} \text{Rotation} \\ \text{Matrix} \end{pmatrix}_{old}^{new} & \begin{pmatrix} \text{Location} \\ \text{of Old} \\ \text{Origin} \end{pmatrix}_{new} \\ \hdashline \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{array} \right] \mathbf{s}_{old} = \mathbf{A}_{old}^{new} \, \mathbf{s}_{old}$$

Transform from one joint to the next



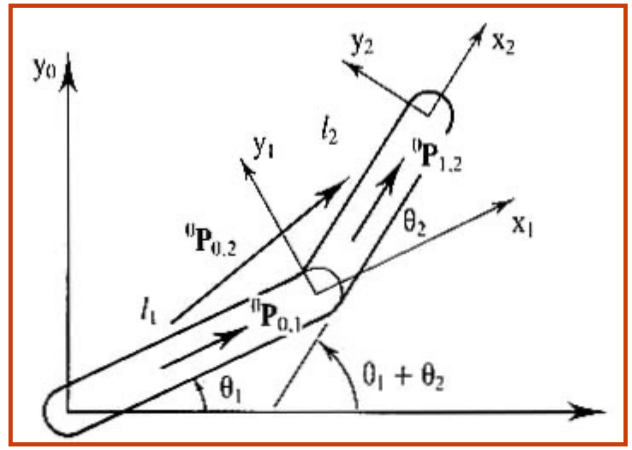
Rotation Matrix can be Derived from Euler Angles or Quaternions

$$\mathbf{A} = \left[\begin{array}{c:c} \mathbf{H}_{old}^{new} & \mathbf{r}_{old_{new}} \\ \hdashline \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{array}\right] = \left[\begin{array}{ccc:c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

Series of Homogeneous Transformations

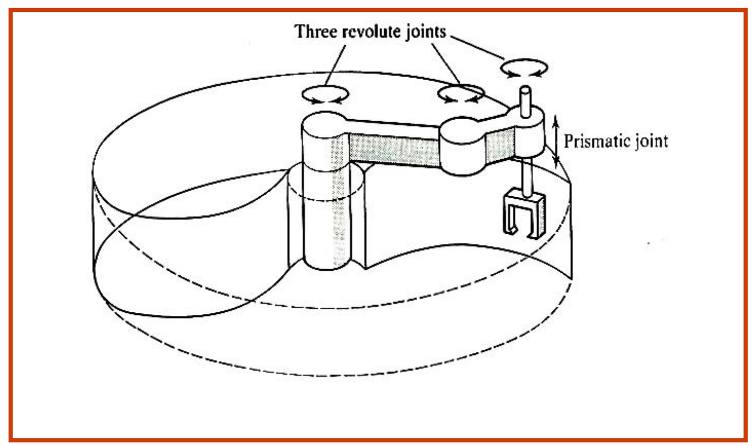
Two serial transformations can be combined in a single transformation

$$\mathbf{s}_2 = \mathbf{A}_1^2 \mathbf{A}_0^1 \ \mathbf{s}_0 = \mathbf{A}_0^2 \ \mathbf{s}_0$$



Four transformations for SCARA robot

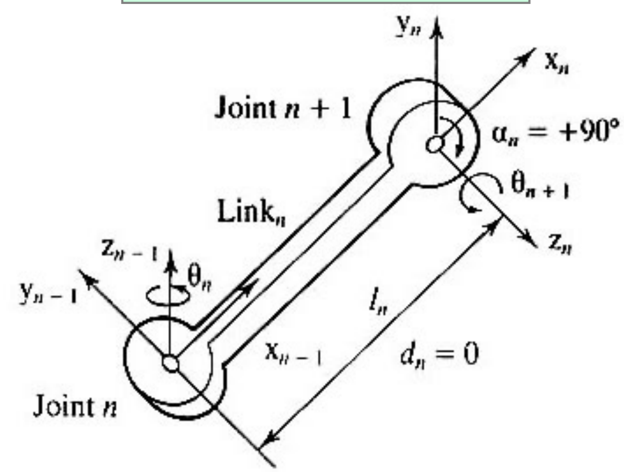
$$\mathbf{s}_4 = \mathbf{A}_3^4 \mathbf{A}_2^3 \mathbf{A}_1^2 \mathbf{A}_0^1 \ \mathbf{s}_0 = \mathbf{A}_0^4 \ \mathbf{s}_0$$



Transformation for a Single Robotic Joint-Link

- Each joint-link requires four sequential transformations:
  - Rotation about α
  - Translation along d
  - Translation along l
  - Rotation about θ

Type 2 Link



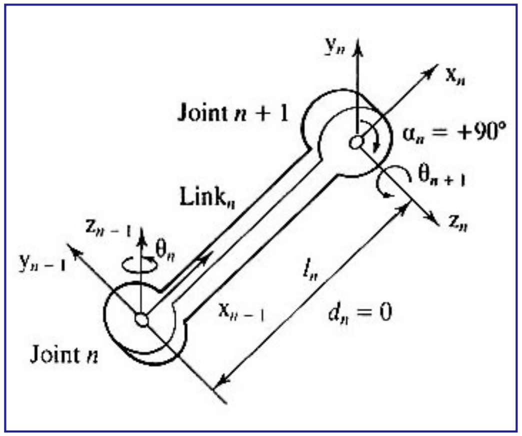
$$\mathbf{s}_{n+1} = \mathbf{A}_3^{n+1}\mathbf{A}_2^3\mathbf{A}_1^2\mathbf{A}_n^1\,\mathbf{s}_n = \mathbf{A}_n^{n+1}\,\mathbf{s}_n$$
$$= \mathbf{A}_\theta\mathbf{A}_d\mathbf{A}_l\mathbf{A}_\alpha\,\mathbf{s}_n = \mathbf{A}_n^{n+1}\,\mathbf{s}_n$$

- ... axes for each transformation (along or around) must be specified

4th | 3rd | 2nd | 1st

$$\mathbf{s}_{n+1} = \mathbf{A}(z_{n-1},\theta_n)\mathbf{A}(z_{n-1},d_n)\mathbf{A}(x_{n-1},l_n)\mathbf{A}(x_{n-1},\alpha_n)\,\mathbf{s}_n = \mathbf{A}_n^{n+1}\,\mathbf{s}_n$$

Denavit-Hartenberg Representation of Joint-Link-Joint Transformation



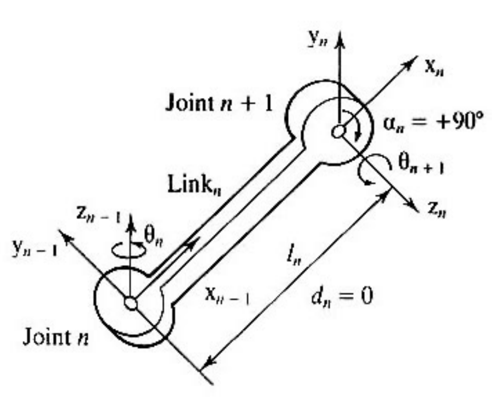
Denavit-Hartenberg Representation of Joint-Link-Joint Transformation

- **Like Euler angle rotation, transformational effects of the 4 link parameters are defined in a specific application sequence (right to left): {*θ, d, l, α*}**

- **4 link parameters**
  - **Angle** between 2 links, ***θ*** (revolute)
  - **Distance (offset)** between links, ***d*** (prismatic)
  - **Length** of the link between rotational axes, ***l***, along the common normal (prismatic)
  - **Twist angle** between axes, ***α*** (revolute)

$$
\begin{aligned}
\mathbf{A}_n &= \mathbf{A}(z_{n-1},\theta_n)\,\mathbf{A}(z_{n-1},d_n)\,\mathbf{A}(x_{n-1},l_n)\,\mathbf{A}(x_{n-1},\alpha_n) \\
&= \mathrm{Rot}(z_{n-1},\theta_n)\ \mathrm{Trans}(z_{n-1},d_n)\ \mathrm{Trans}(x_{n-1},l_n)\ \mathrm{Rot}(x_{n-1},\alpha_n) \\
&\triangleq {}^{n}\mathbf{T}_{n+1} \quad \text{in some references (e.g., McKerrow, 1991)}
\end{aligned}
$$

Denavit-Hartenberg Demo
http://www.youtube.com/watch?v=10mUtjfGmzw



Four Transformations from One Joint to the Next

(Single Link)

Rotation of θ_n about the z_{n-1} axis

$$\mathrm{Rot}(z_{n-1},\theta_n)=\begin{bmatrix}\cos\theta_n & -\sin\theta_n & 0 & 0\\ \sin\theta_n & \cos\theta_n & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Translation of d_n along the z_{n-1} axis

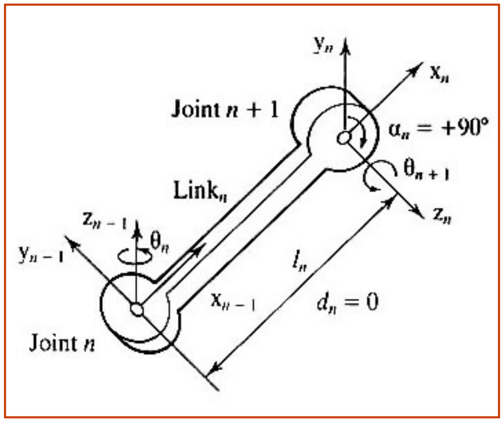
$$\mathrm{Trans}(z_{n-1},d_n)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & d_n\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Translation of l_n along the x_{n-1} axis

$$\mathrm{Trans}(x_{n-1},l_n)=\begin{bmatrix}1 & 0 & 0 & l_n\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Rotation of α_n about the x_{n-1} axis

$$\mathrm{Rot}(x_{n-1},\alpha_n)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\alpha_n & -\sin\alpha_n & 0\\ 0 & \sin\alpha_n & \cos\alpha_n & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

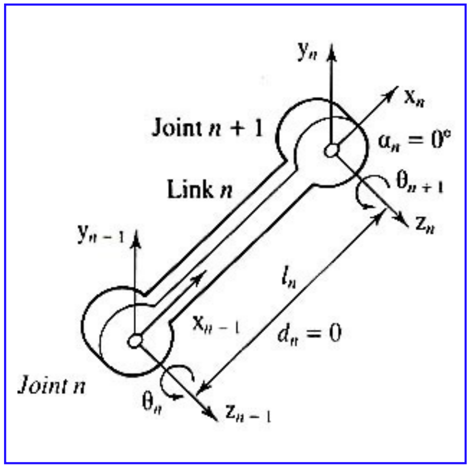


Denavit-Hartenberg Representation of Joint-Link-Joint Transformation

Then **Then** **Then** **First**

$$\mathbf{A}_n^{n-1} = \begin{bmatrix} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_n^{n-1} = \begin{bmatrix} \cos\theta_n & -\sin\theta_n \cos\alpha_n & \sin\theta_n \sin\alpha_n & l_n \cos\theta_n \\ \sin\theta_n & \cos\theta_n \cos\alpha_n & -\cos\theta_n \sin\alpha_n & l_n \sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Example: Joint-Link-Joint Transformation, Type 1 Link

Joint Variable = θ_n

$$\mathbf{A}_n^{n-1} = \begin{bmatrix} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & l_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & l_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\theta = \text{variable}$

$d = 0 \text{ m}$

$l = 0.25 \text{ m}$

$\alpha = 90 \text{ deg}$

$$\mathbf{A}_n^{n-1} = \begin{bmatrix} \cos\theta_n & 0 & \sin\theta_n & 0.25\cos\theta_n \\ \sin\theta_n & 0 & -\cos\theta_n & 0.25\sin\theta_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

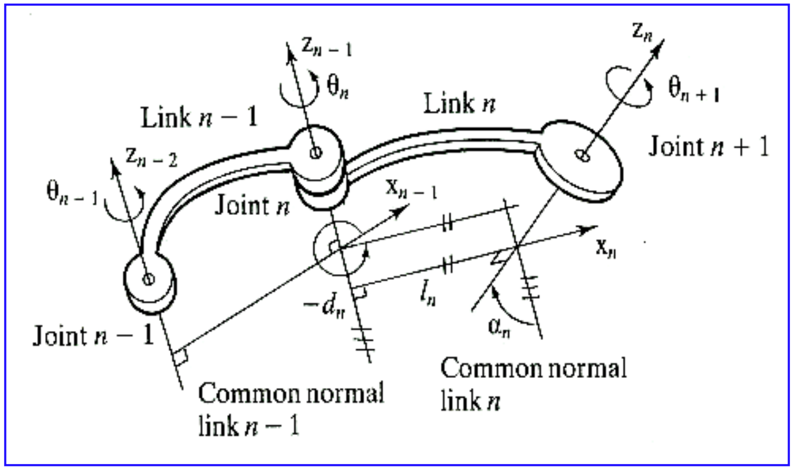
$\theta \triangleq 30 \text{ deg}$

$d = 0 \text{ m}$

$l = 0.25 \text{ m}$

$\alpha = 90 \text{ deg}$

$$\mathbf{A}_n^{n-1} = \begin{bmatrix} 0.866 & 0 & 0.5 & 0.217 \\ 0.5 & 0 & -0.866 & 0.125 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Forward and Inverse Transformations

Forward transformation: proximal to distal frame (Expression of proximal frame in distal frame)

$$\mathbf{s}_1 = \mathbf{A}_0^1 \mathbf{s}_0 \quad ; \quad s_2 = \mathbf{A}_1^2 \mathbf{s}_1 = \mathbf{A}_1^2 \mathbf{A}_0^1 \mathbf{s}_0 = \mathbf{A}_0^2 \mathbf{s}_0$$

Reverse transformation: distal to proximal frame = inverse of forward transformation

$$\mathbf{s}_0 = \left(\mathbf{A}_0^2\right)^{-1} \mathbf{s}_2 = \mathbf{A}_2^0 \mathbf{s}_2 = \mathbf{A}_1^0 \mathbf{A}_2^1 \mathbf{s}_2$$

Homogeneous Transformation Matrix is not Orthonormal

$$\mathbf{A}_2^0 = \left(\mathbf{A}_0^2\right)^{-1} \neq \left(\mathbf{A}_0^2\right)^T$$

…but a useful identity makes inversion simple

Matrix Inverse Identity

Given: a square matrix, A, and its inverse, B

$$\mathbf{A} = \left[\begin{array}{c|c} \underset{m\times m}{\mathbf{A}_1} & \underset{m\times n}{\mathbf{A}_2} \\ \hline \underset{n\times m}{\mathbf{A}_3} & \underset{n\times n}{\mathbf{A}_4} \end{array}\right] \quad ; \quad \mathbf{B} \triangleq \mathbf{A}^{-1} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix}$$

Then

$$\mathbf{A}\,\mathbf{B} = \mathbf{A}\,\mathbf{A}^{-1} = \mathbf{I}_{m+n}$$

$$= \begin{bmatrix} \mathbf{I}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_n \end{bmatrix} = \begin{bmatrix} \left(\mathbf{A}_1\mathbf{B}_1 + \mathbf{A}_2\mathbf{B}_3\right) & \left(\mathbf{A}_1\mathbf{B}_2 + \mathbf{A}_2\mathbf{B}_4\right) \\ \left(\mathbf{A}_3\mathbf{B}_1 + \mathbf{A}_4\mathbf{B}_3\right) & \left(\mathbf{A}_3\mathbf{B}_2 + \mathbf{A}_4\mathbf{B}_4\right) \end{bmatrix}$$

Equating like parts, and solving for B*i*

$$\begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} = \left[\begin{array}{c|c} \left(\mathbf{A}_1 - \mathbf{A}_2\mathbf{A}_4^{-1}\mathbf{A}_3\right)^{-1} & -\mathbf{A}_1^{-1}\mathbf{A}_2\left(\mathbf{A}_4 - \mathbf{A}_3\mathbf{A}_1^{-1}\mathbf{A}_2\right)^{-1} \\ \hline -\mathbf{A}_4^{-1}\mathbf{A}_3\left(\mathbf{A}_1 - \mathbf{A}_2\mathbf{A}_4^{-1}\mathbf{A}_3\right)^{-1} & \left(\mathbf{A}_4 - \mathbf{A}_3\mathbf{A}_1^{-1}\mathbf{A}_2\right)^{-1} \end{array}\right]$$

Apply to Homogeneous Transformation

Forward transformation (to distal frame)

$$\mathbf{A} \triangleq \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{proximal}^{distal} & \mathbf{r}_{o_{proximal}} \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{bmatrix}$$

Inverse transformation (to proximal frame)

$$\begin{bmatrix} \mathbf{H}_{proximal}^{distal} & \mathbf{r}_o \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{H}_{distal}^{proximal} & -\mathbf{H}_{distal}^{proximal}\mathbf{r}_{o_{distal}} \\ \hline \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{array} \right]$$

Apply to Homogeneous Transformation

Forward transformation

$$\mathbf{A} \triangleq \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} == \left[\begin{array}{ccc:c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

Inverse transformation

$$\left[\begin{array}{ccc:c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]^{-1} = \left[\begin{array}{ccc:c} h_{11} & h_{21} & h_{31} & -\left(h_{11}x_o + h_{21}y_o + h_{31}z_o\right) \\ h_{12} & h_{22} & h_{32} & -\left(h_{12}x_o + h_{22}y_o + h_{32}z_o\right) \\ h_{13} & h_{23} & h_{33} & -\left(h_{13}x_o + h_{23}y_o + h_{33}z_o\right) \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

Next Time: Transformations, Trajectories, and Path Planning