Long-Horizon Forecasts of Global Growth

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May 2018

preliminary

Motivation

- Module in interdisciplinary research project on social cost of carbon

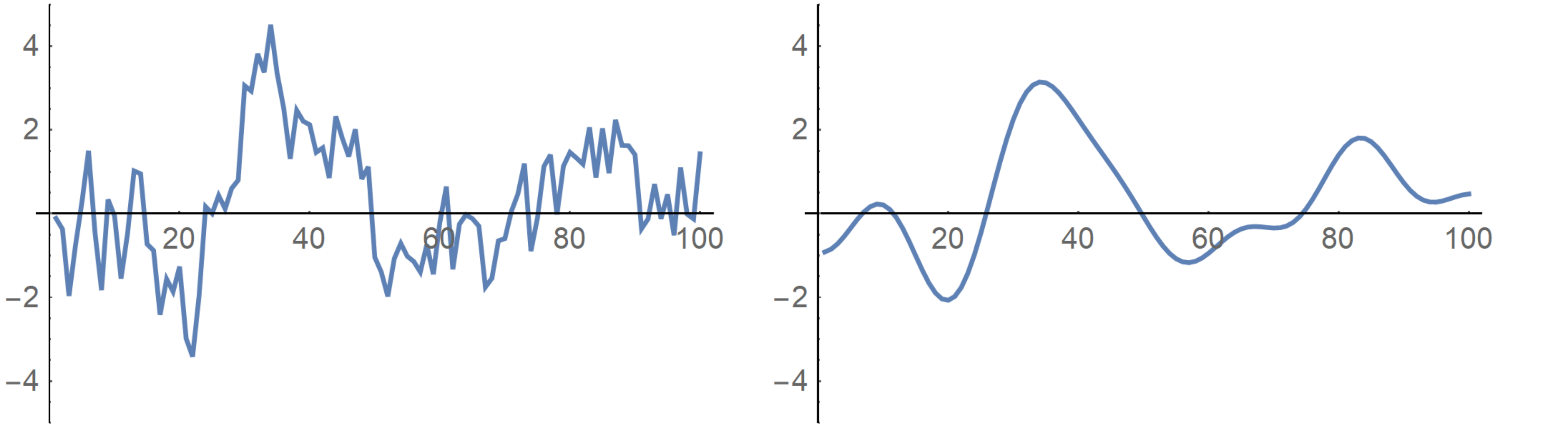
  ⇒ Probabilistic global growth forecasts over next 200 years

  ⇒ Informs baseline CO_2 emission paths and local damage function for climate change model

- Data: World Penn tables merged with Maddison data set 1915-2014

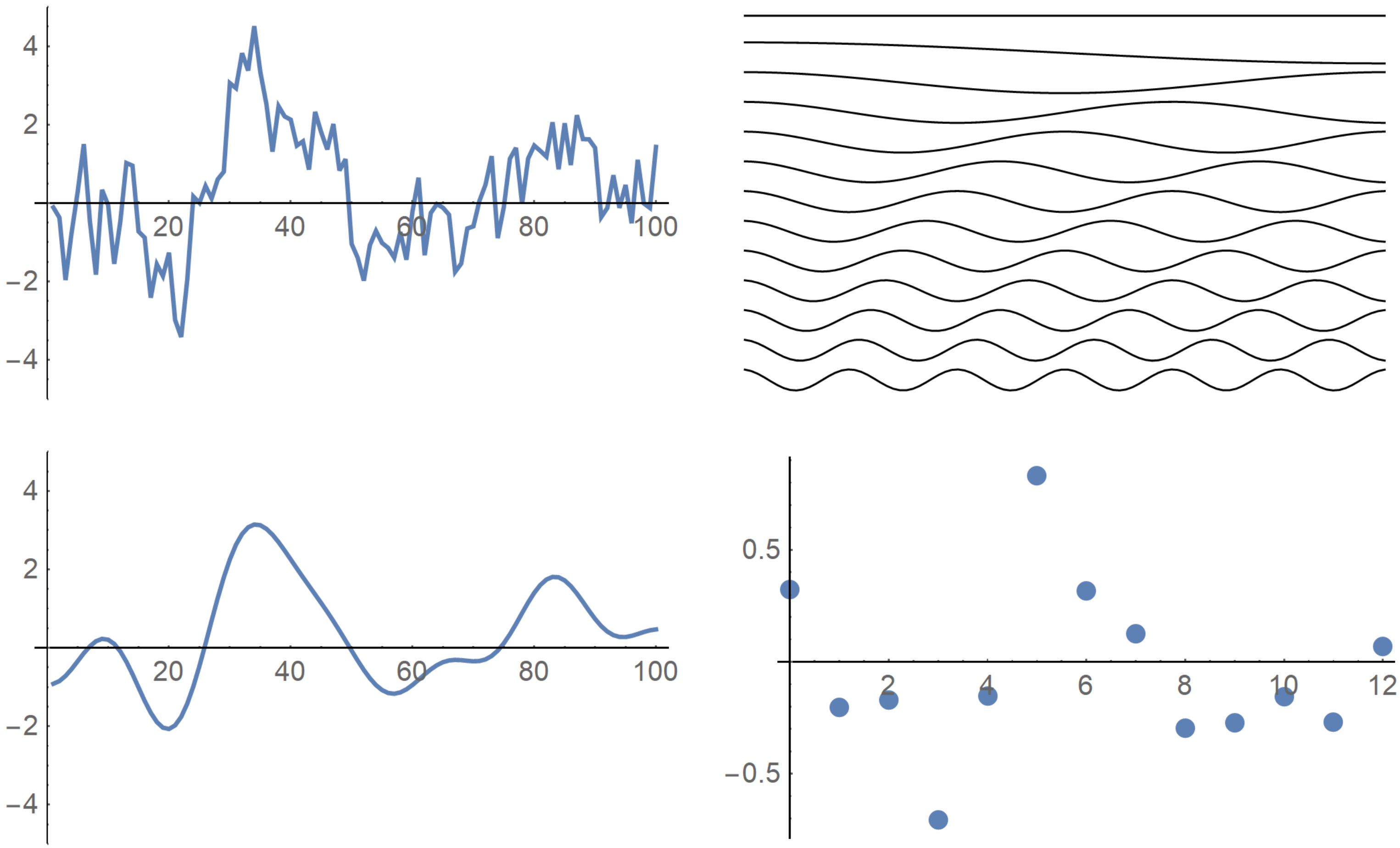
- Focus on forecasts of GDP/capita in 71 countries

Low-Frequency Approach

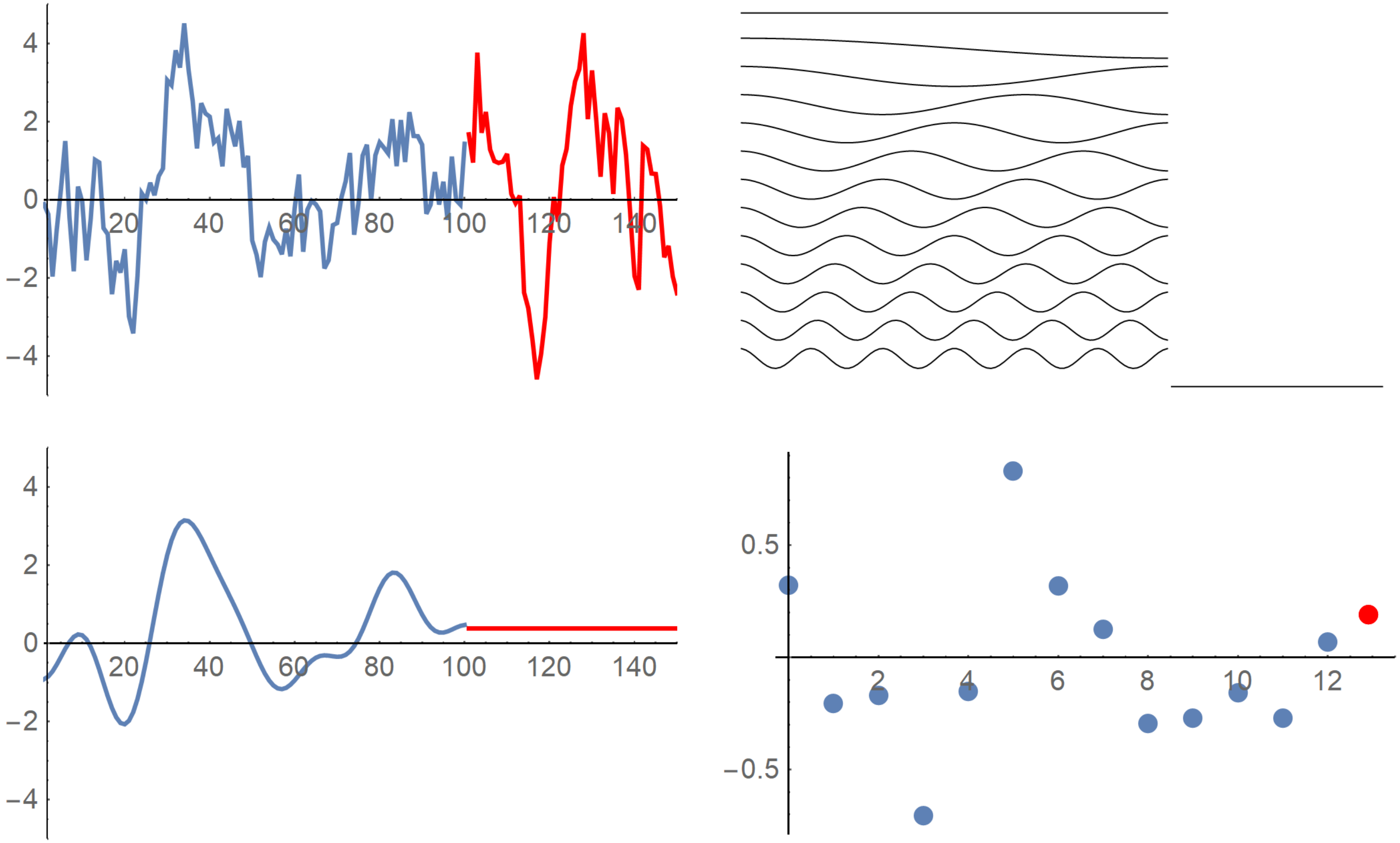


⇒ Müller and Watson (2016): For long-horizon forecasts, focus on low-frequency variation

LF Transformation: Dimension Reduction



Long-Horizon Forecasts



Approximate Gaussianity from CLT for weighted averages

$\Rightarrow$ forecast distribution from conditional multivariate normal

Forecasting of Scalar Series

- Müller and Watson (2016) consider long-horizon forecasts of average growth of scalar series

  $\Rightarrow$ Robustified Bayesian approach to account for parameter uncertainty of low-frequency dynamics

- Application to panel faces three problems
  - extrapolation of sample growth differences yields unrealistic divergence over long horizons
  - ignores cross sectional dependence
  - Unbalanced panel

Related Literature

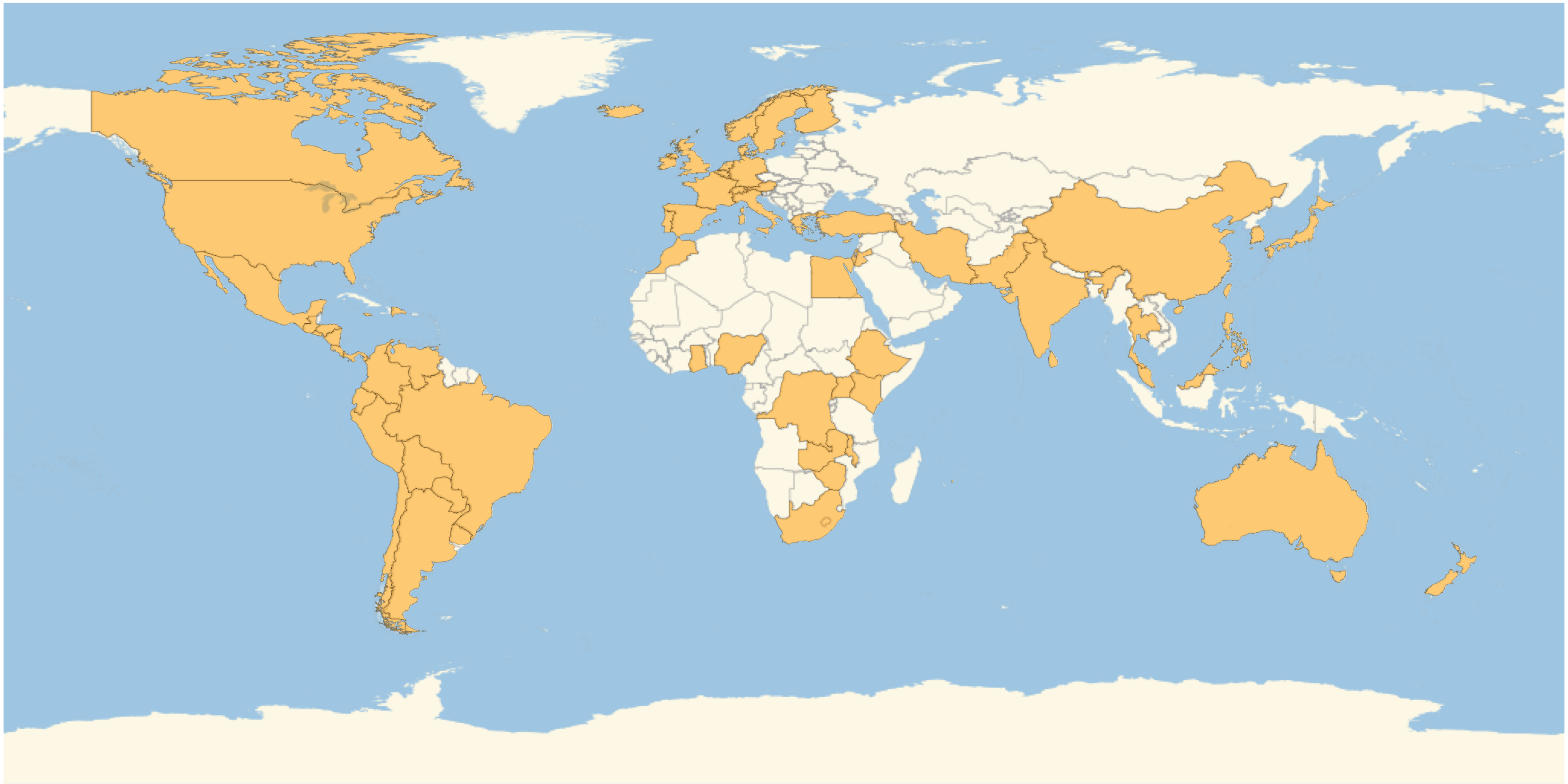
- GDP/capita: Christensen, Gilingham and Nordhaus (2017), Startz (2017)
- Long-Run Risk: Pastor and Stambaugh (2012)
- Population: Lee (2011), Raftery, Li and Sevcikova (2012), Raftery, Alkema and Gerland (2014)
- Structural model: Desmet, Nagy and Rossi-Hansberg (2017)

This Paper

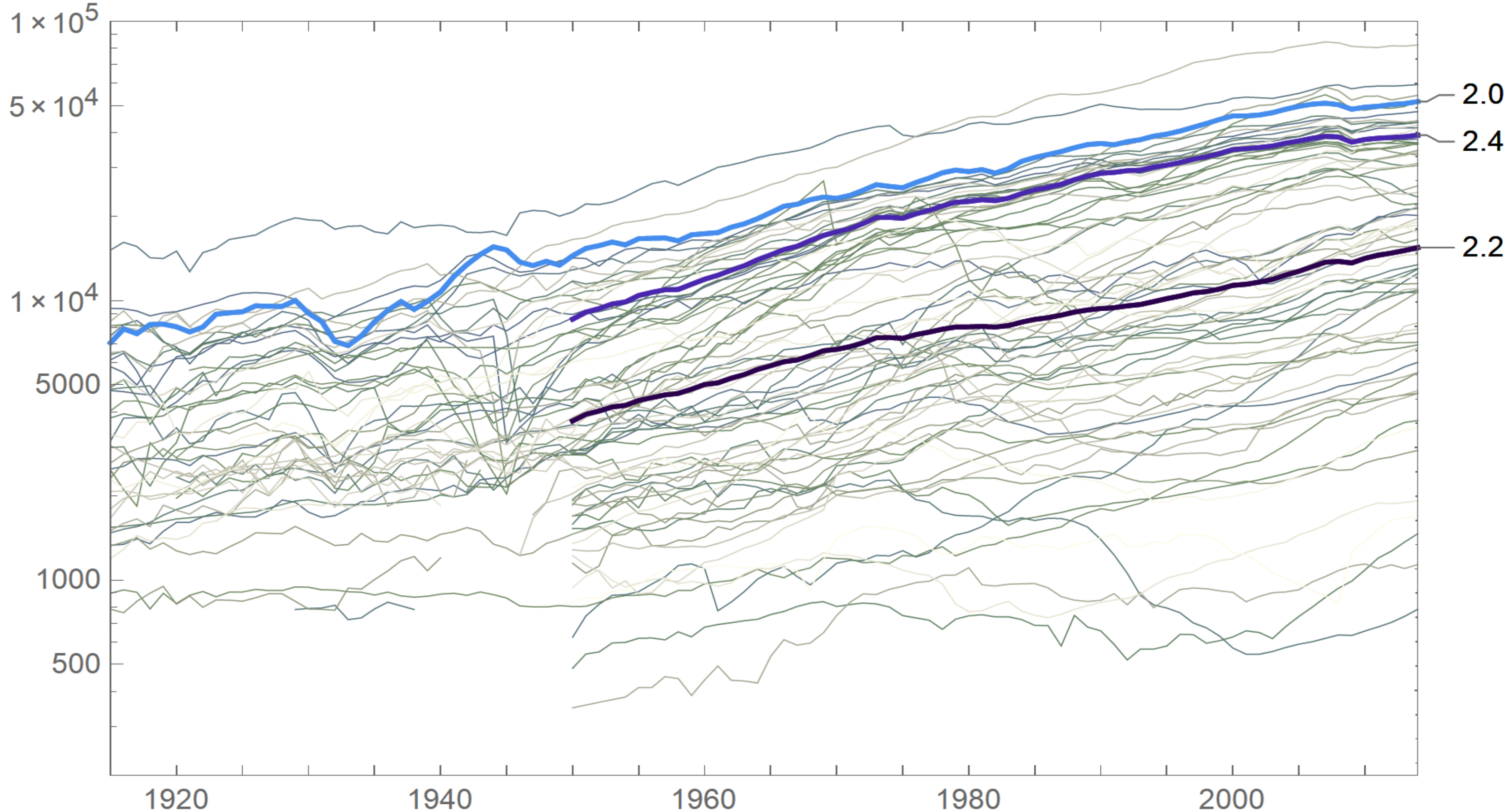
- Low-frequency linear panel model with
  - potentially unbalanced sample information
  - single nonstationary factor
  - slowly mean reverting idiosyncratic deviations with common unconditional distribution
  - cross-sectional dependence in idiosyncratic deviations

- Bayesian estimation

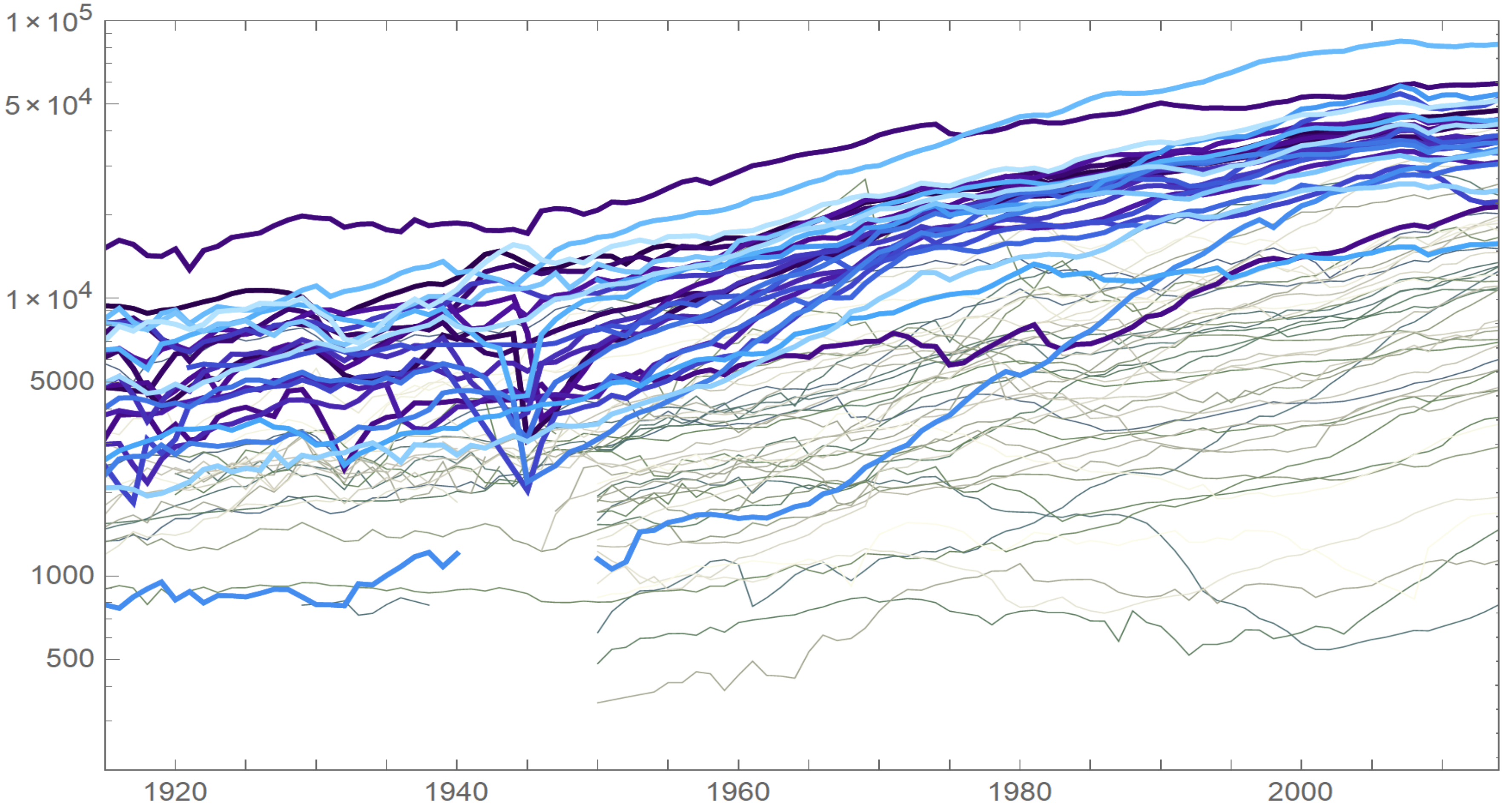
Data Coverage



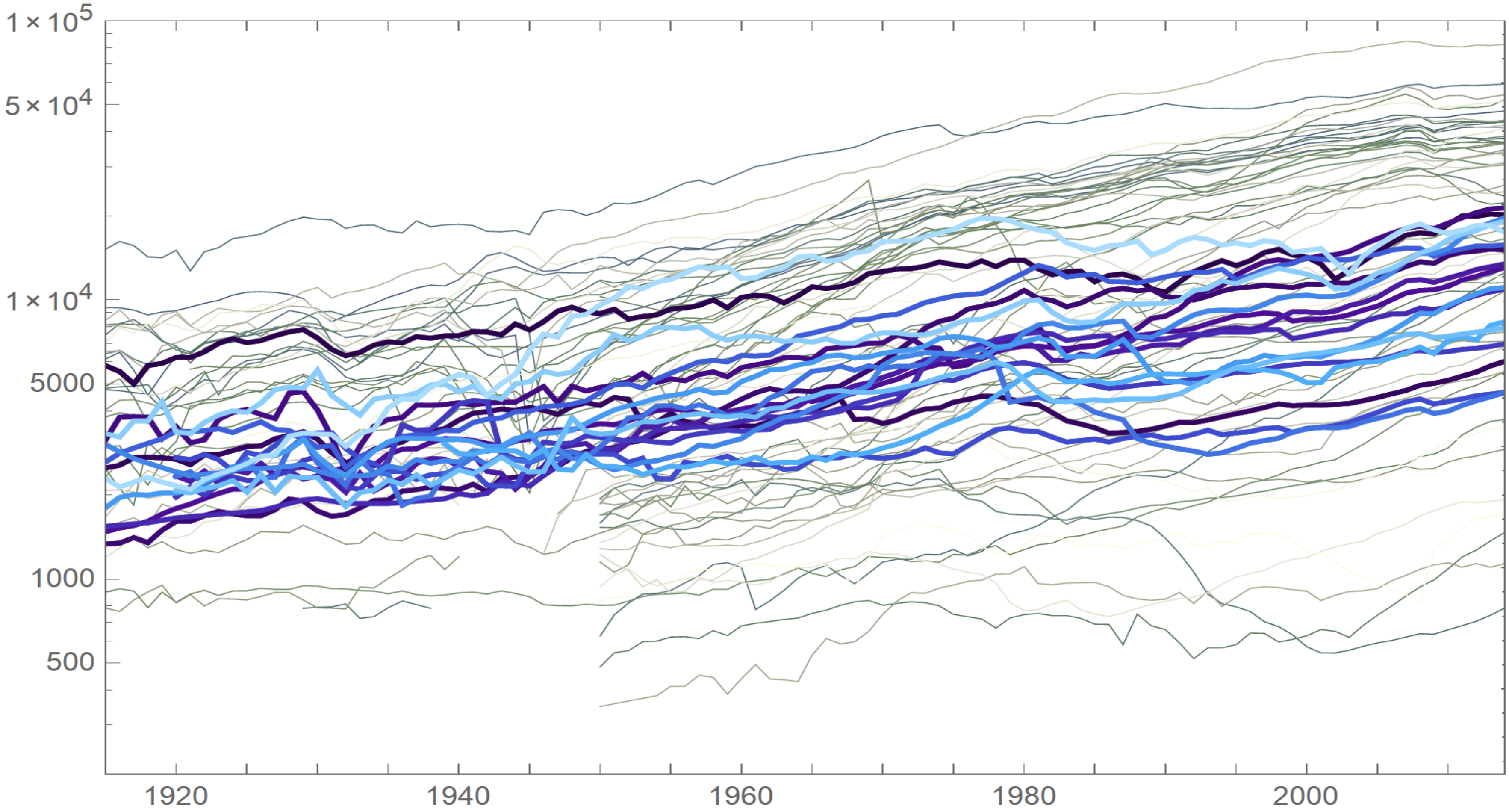
USA, World Average, OECD Average



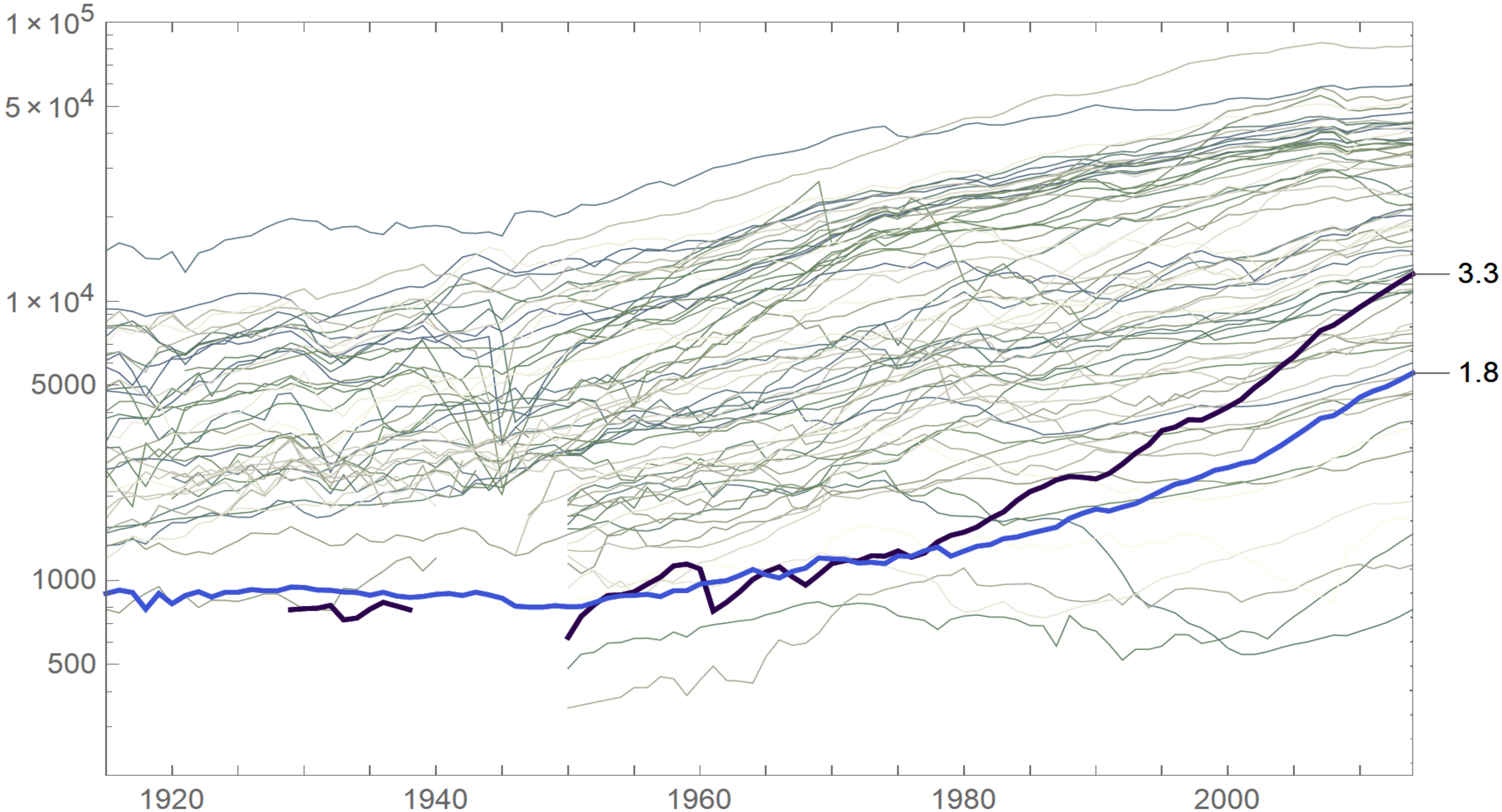
OECD Countries



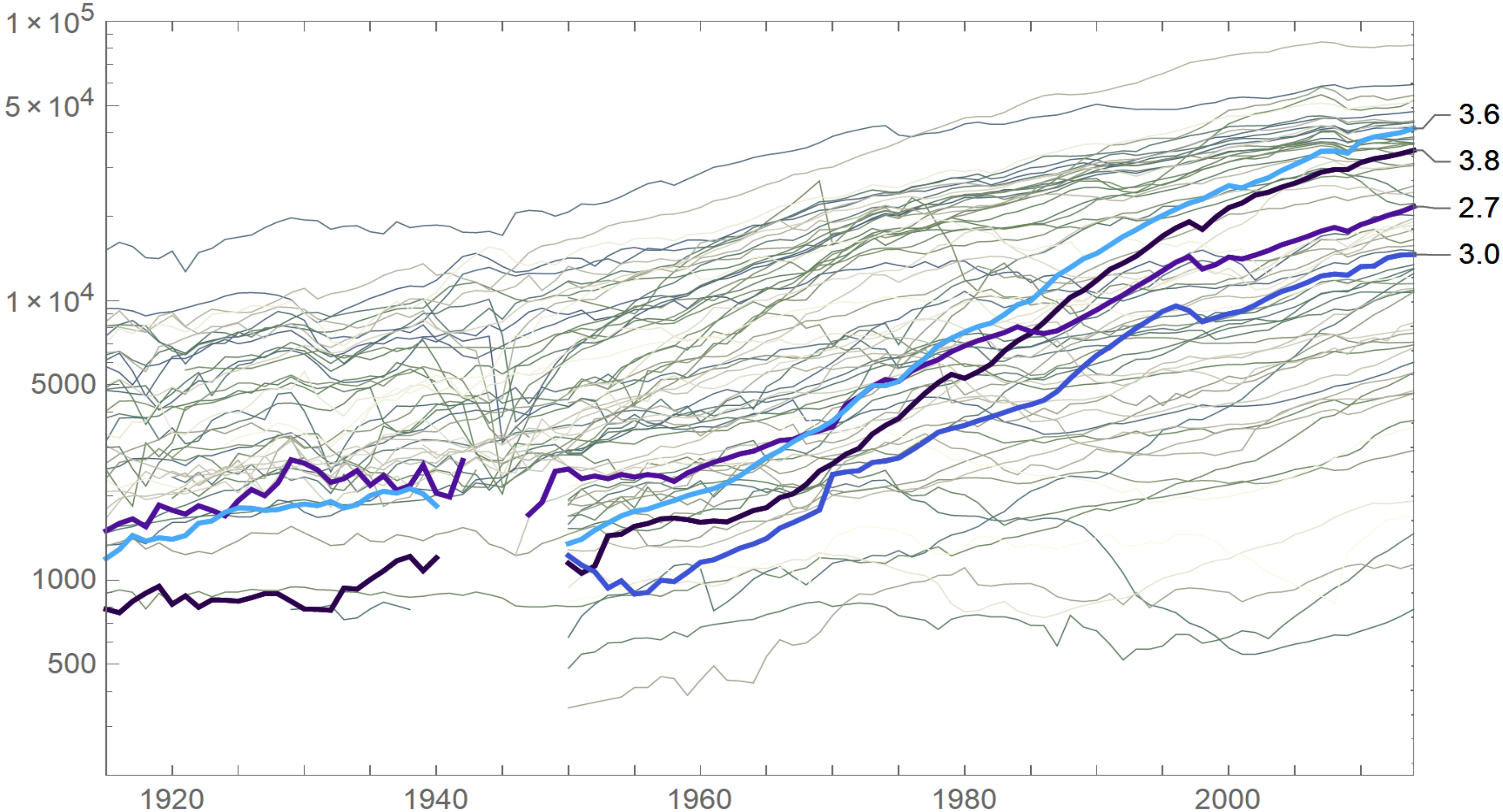
Latin America



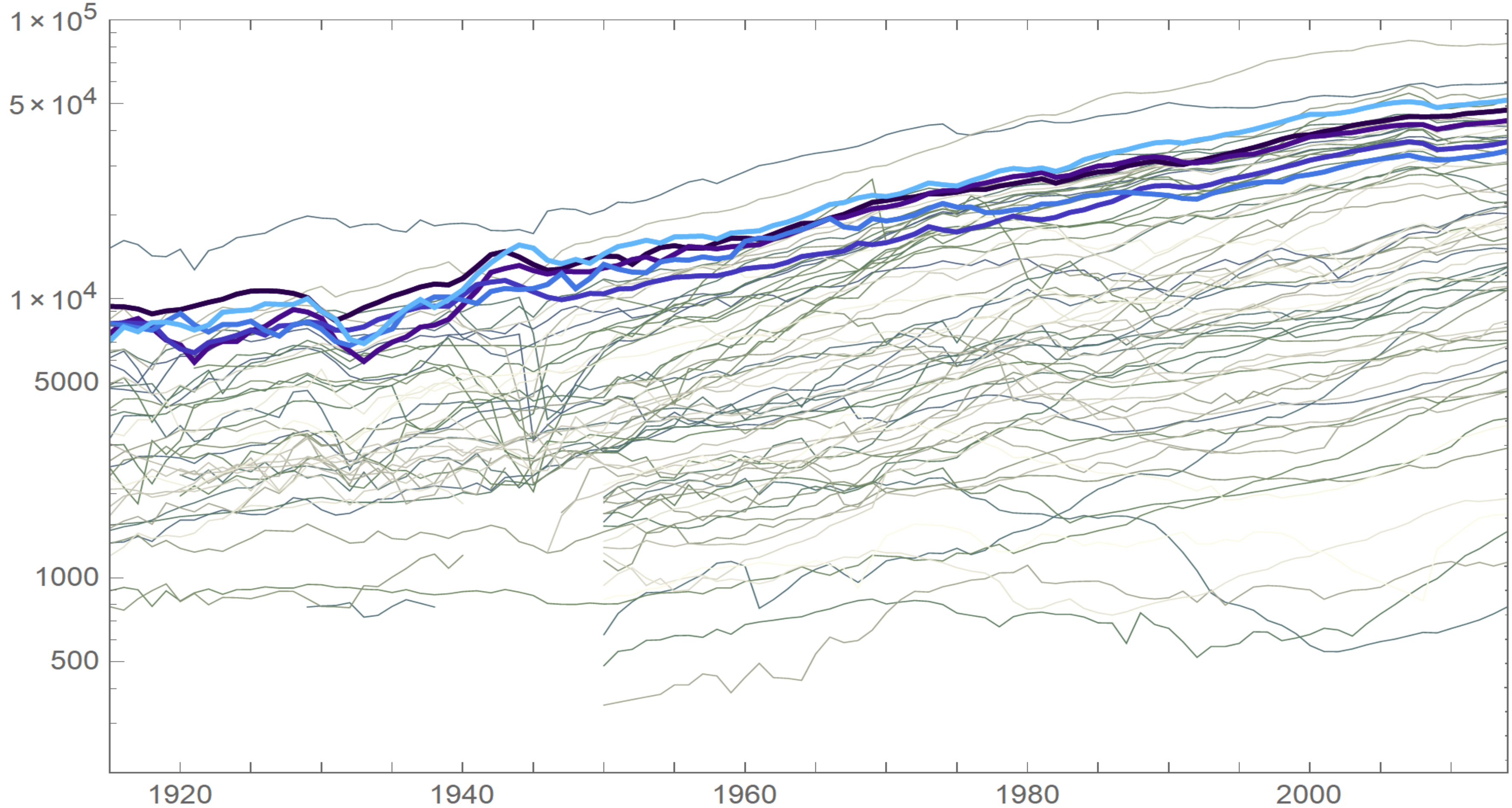
China and India



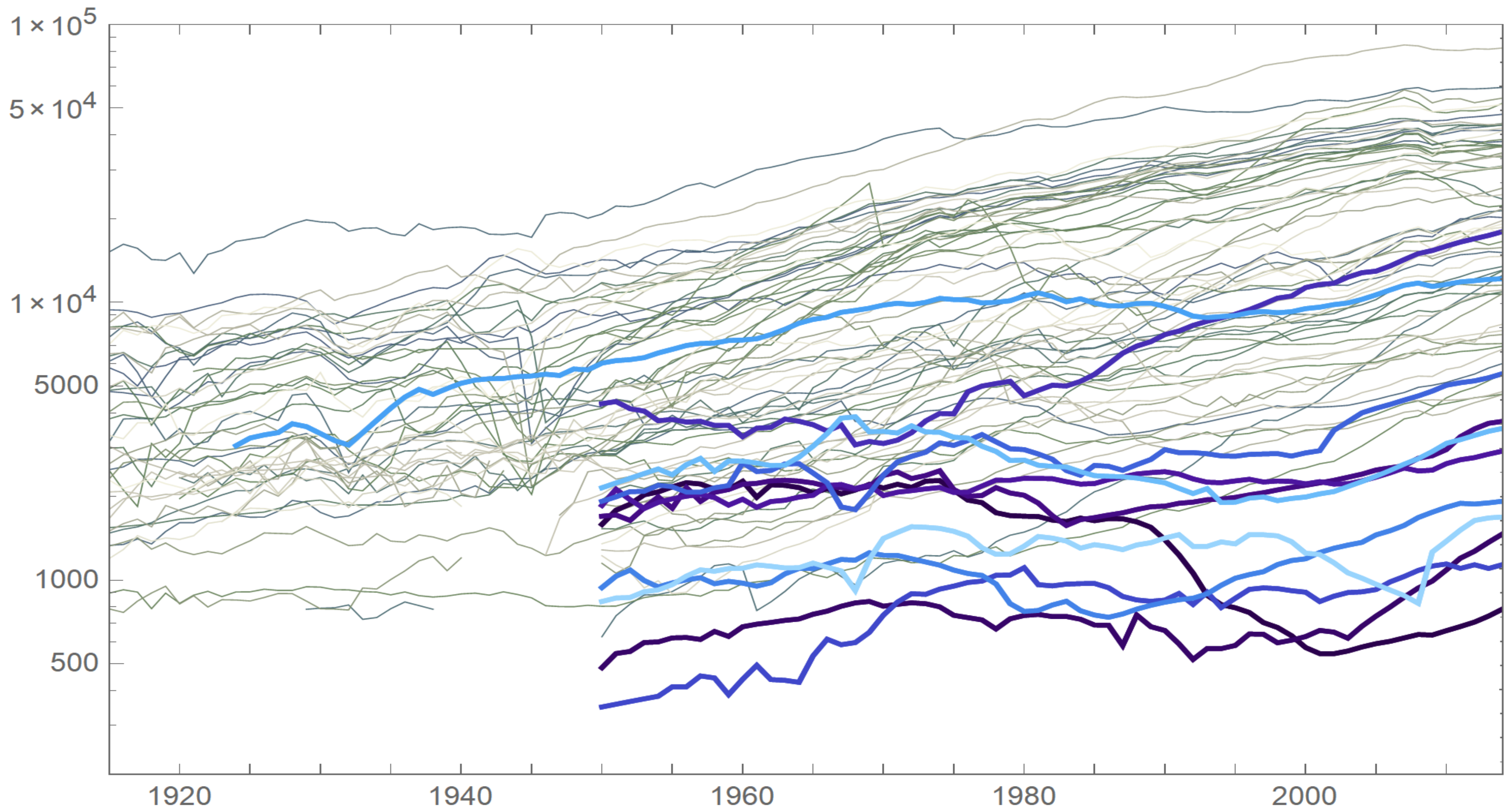
South Korea, Taiwan, Malaysia, Thailand



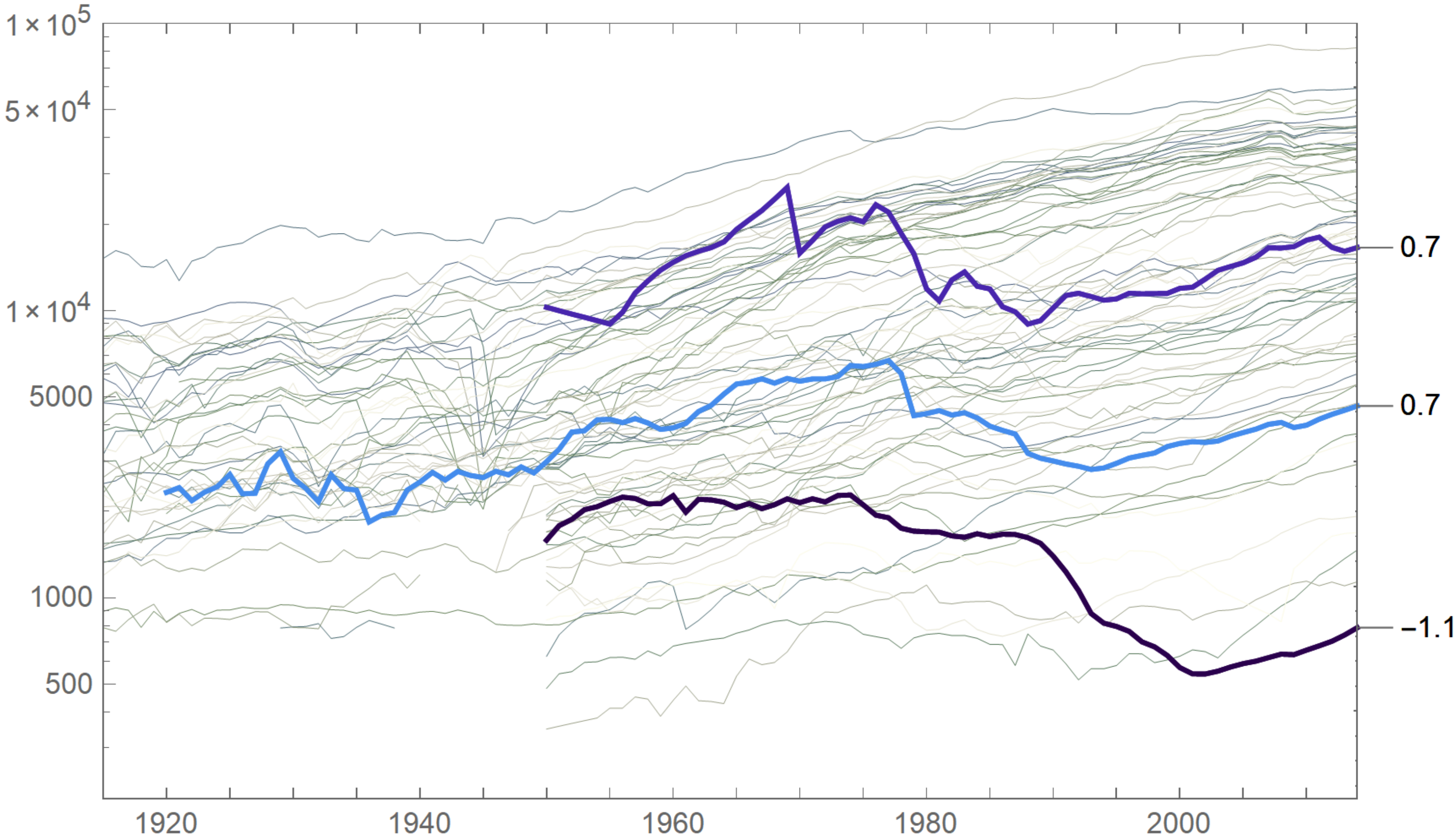
USA, Canada, GBR, New Zealand, Australia



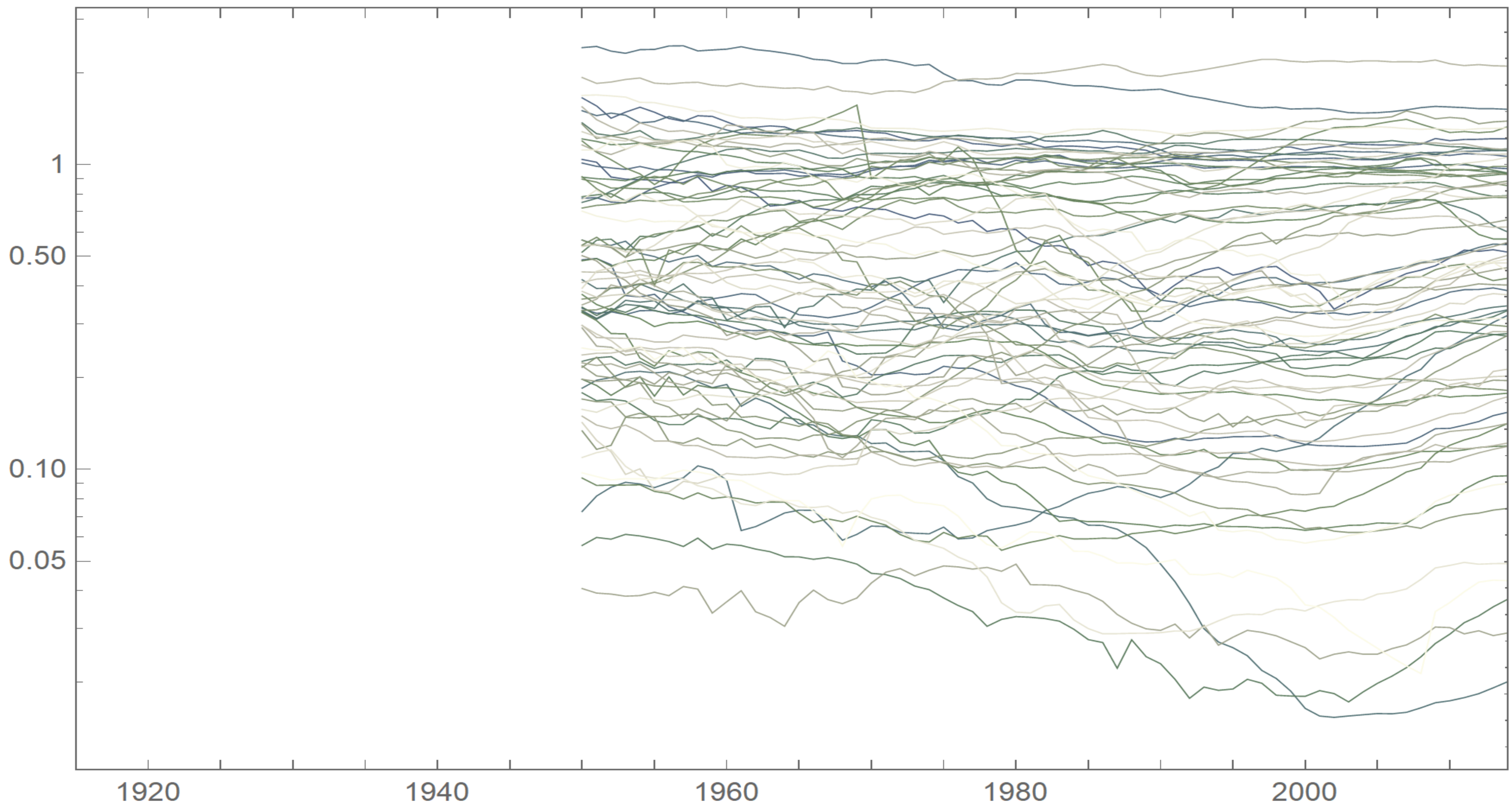
Sub-Saharan Africa



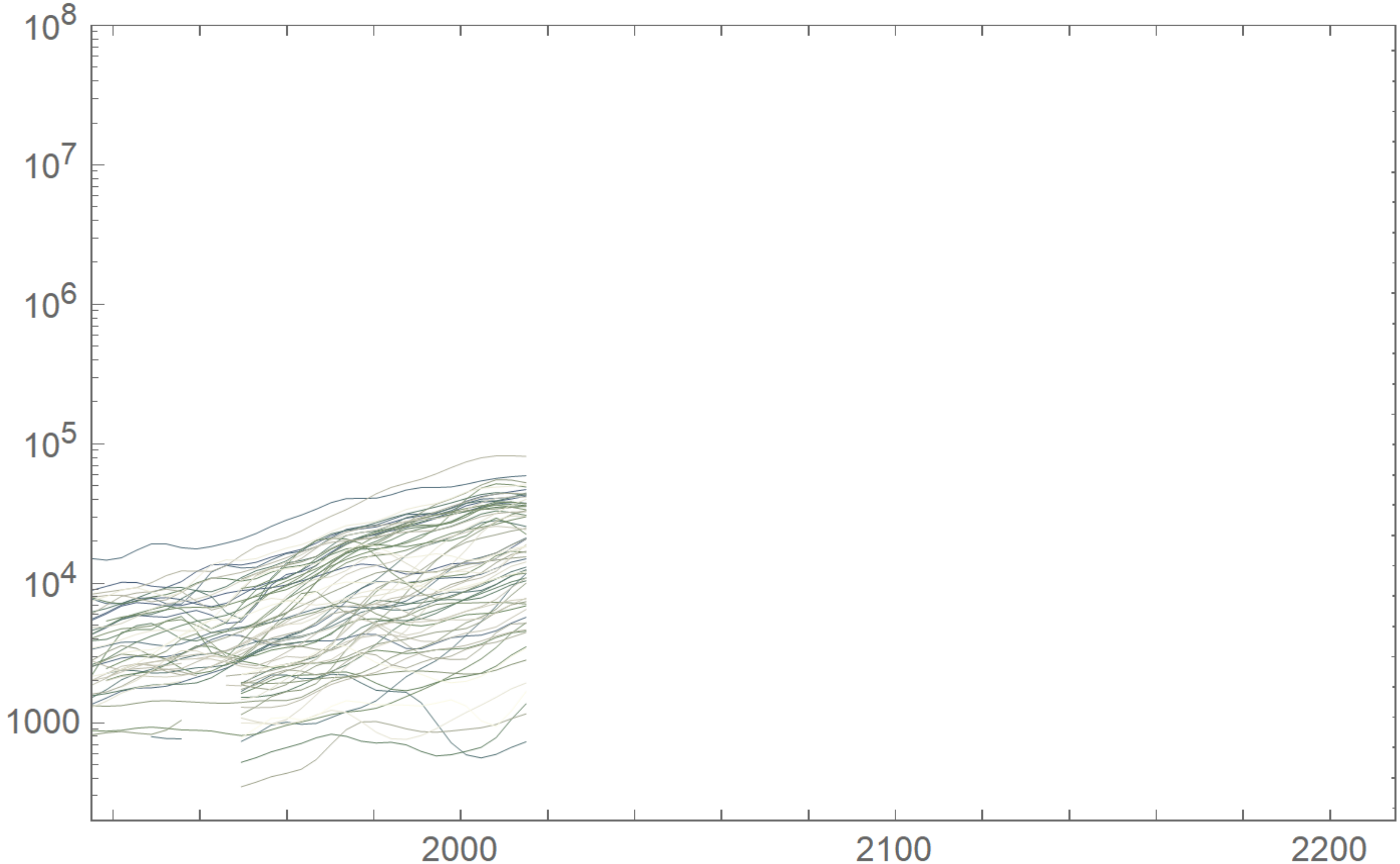
Iran, Nicaragua, Congo



Relative to OECD



Future log(GDP/capita)?



Focus on LF variability with 14 year period and longer

Univariate Model Blocks

- cd-model: stationary with

$$(1-\rho L)^d y_t = e_t, \quad e_t \sim iid\mathcal{N}(0,\sigma^2)$$

with $\rho = \rho_T = 1 - c/T$ and $c > 0$.

$\Rightarrow$ Innovation variance σ^2 normalized so that unconditional variance of y_t is equal to unity

$\Rightarrow$ flexible way of modelling low-frequency dynamics

- Local-level model for Δf_t: nonstationary with

$$\Delta f_t = m_t + e_{1t}, \quad m_t - m_0 = \frac{g}{T}\sum_{s=1}^{t} e_{2s}$$

with $(e_{1t}, e_{2t}) \sim iid\mathcal{N}(0, \sigma^2 I_2)$

$\Rightarrow$ captures slow time variation in mean growth rate

Panel Model I

- Basic model: common nonstationary factor plus stationary idiosyncratic shocks

$$
\begin{aligned}
y_{i,t} &= f_t + u_{i,t} \\
u_{i,t} &= \mu + \omega \cdot \varepsilon_{i,t}
\end{aligned}
$$

  with Δf_t local-level model and $\varepsilon_{i,t}$ independent cd-models with heterogeneous parameters

- Implies common unconditional distribution $\mathcal{N}(\mu, \omega^2)$ for $u_{i,t}$

- Priors:

  – Uninformative priors on (ω, μ, f_0, m_0)

  – Uniform prior on LLM parameter $g \in [0, 5]$

Prior on cd Parameters

- Discretize cd parameters on grid with $d \in [0.6, 2.0]$, and c such that 200 year correlation in $[0.1, 0.8]$

- Makes sense to have prior exchangeable for 71 countries, but not i.i.d.

- Dirichlet prior over distribution on grid

  $\Rightarrow$ allows data to inform distribution of cd parameters across countries

  $\Rightarrow$ Dirichlet prior shrinks towards uniform distribution on grid, but not very much (total $\alpha = 20$ vs $n = 71$)

Panel Model II

- Allow some heterogeneity for unconditional variances

$$
\begin{aligned}
y_{i,t} &= f_t + u_{i,t} \\
u_{i,t} &= \mu + \kappa_i \cdot \omega \cdot \varepsilon_{i,t}
\end{aligned}
$$

- Dirichlet prior ($\alpha = 20$) for distribution of discretized $\kappa_i \in [1/3, 3]$, shrinking towards uniform on $\log(\kappa_i)$

Panel Model III

- Add cross-sectional dependence in $u_{i,t}$ via club membership

$$\begin{aligned} y_{i,t} &= f_t + u_{i,t} \\ u_{i,t} &= \mu + \lambda_i \cdot g_{J(i),t} + \kappa_i \sqrt{1-\lambda_i^2} \cdot \omega \cdot \varepsilon_{i,t} \\ g_{j,t} &= \tau_j \cdot \omega \cdot \nu_{j,t} \end{aligned}$$

  with $\varepsilon_{i,t}$ and $\nu_{j,t}$ independent cd models

- $\sqrt{1-\lambda_i^2}$ term ensures equal unconditional variance if $\kappa_i = \tau_j = 1$

- Dirichlet prior shrinking towards uniform on $[0, 0.95]$ for λ_i, i.i.d. uniform prior on club membership $J(i) \in \{1, \ldots, 25\}$

- Label switching and lack of identification if $\lambda_i = 0$, but that's ok

Panel Model IV

- Club-of-clubs to increase flexibility of cross sectional dependence

$$
\begin{aligned}
y_{i,t} &= f_t + u_{i,t} \\
u_{i,t} &= \mu + \lambda_i \cdot g_{J(i),t} + \kappa_i \sqrt{1-\lambda_i^2} \cdot \omega \cdot \varepsilon_{i,t} \\
g_{j,t} &= \psi_j \cdot h_{K(j),t} + \tau_j \sqrt{1-\psi_j^2} \cdot \omega \cdot \nu_{j,t} \\
h_{k,t} &= \xi_k \cdot \omega \cdot \eta_{k,t}
\end{aligned}
$$

- Flat prior on club-of-club membership $K(j) \in \{1, \ldots, 10\}$

- Additional dummy observation prior: Observe $\tilde{y}_t = 0 = \epsilon_t + \sum_{i=1}^{n} s_i u_{i,t}$
  - s_i OECD population shares in 1985, $\text{Var}[\epsilon_t] = 0.1\,\text{Var}[\sum_{i=1}^{n} s_i u_{i,t}]$
  - Ensures factor is informed by large OECD countries

LF Transformation in Unbalanced Panel

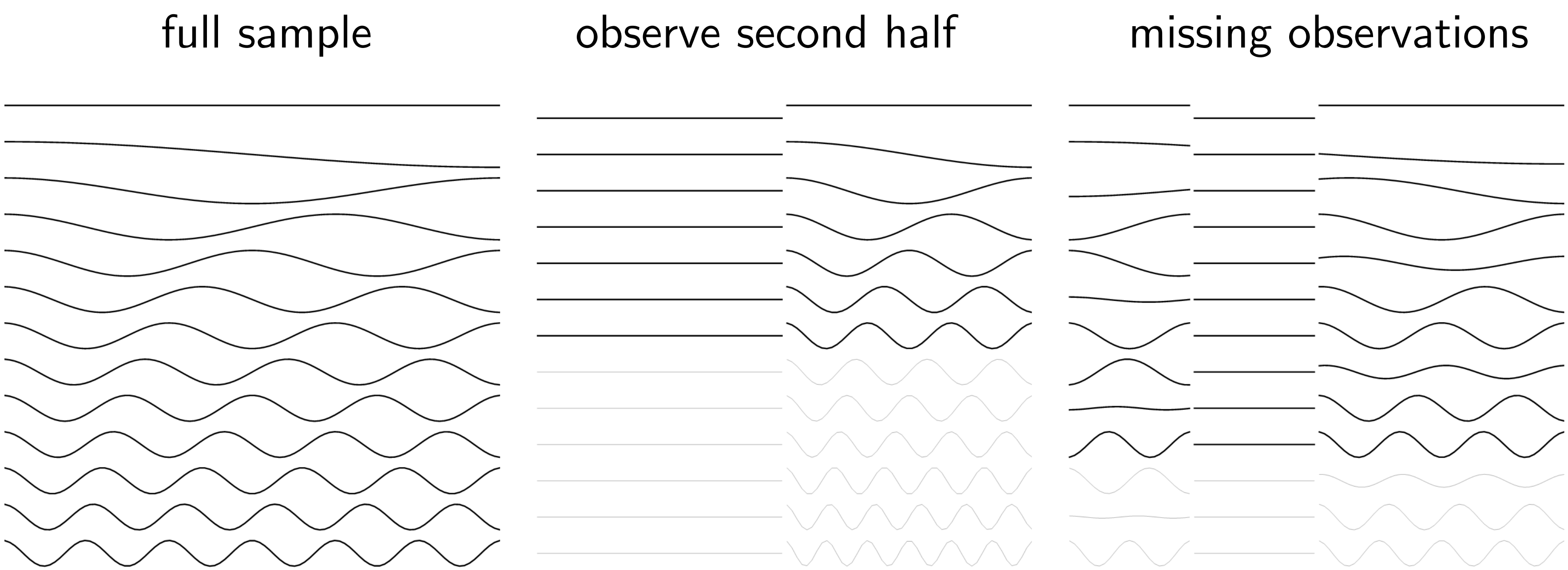
- Seek linear combinations of data that extract (only) low-frequency information

- Formalize as signal extraction problem

  $\Rightarrow$ For Random walk, low-frequency linear combinations are those with largest variance

- Compute eigenvectors of covariance matrix of (demeaned) Random Walk, keep those with eigenvalue larger than cut-off

LF Transformation in Unbalanced Panel



Bayesian Estimation

- In principle, straightfroward Gibbs sampler

- Fast due to LF sample information reduction + discretized parameters

  $\Rightarrow$ 100,000 draws take 2 minutes

Geweke Test

- Geweke (2004, JASA): If one adds "draw Y|state" in Gibbs iterations, then should recover prior predictive distribution on state as stationary distribution

- Implemented as follows: Generate 200,000 i.i.d. from prior predictive distribution of state

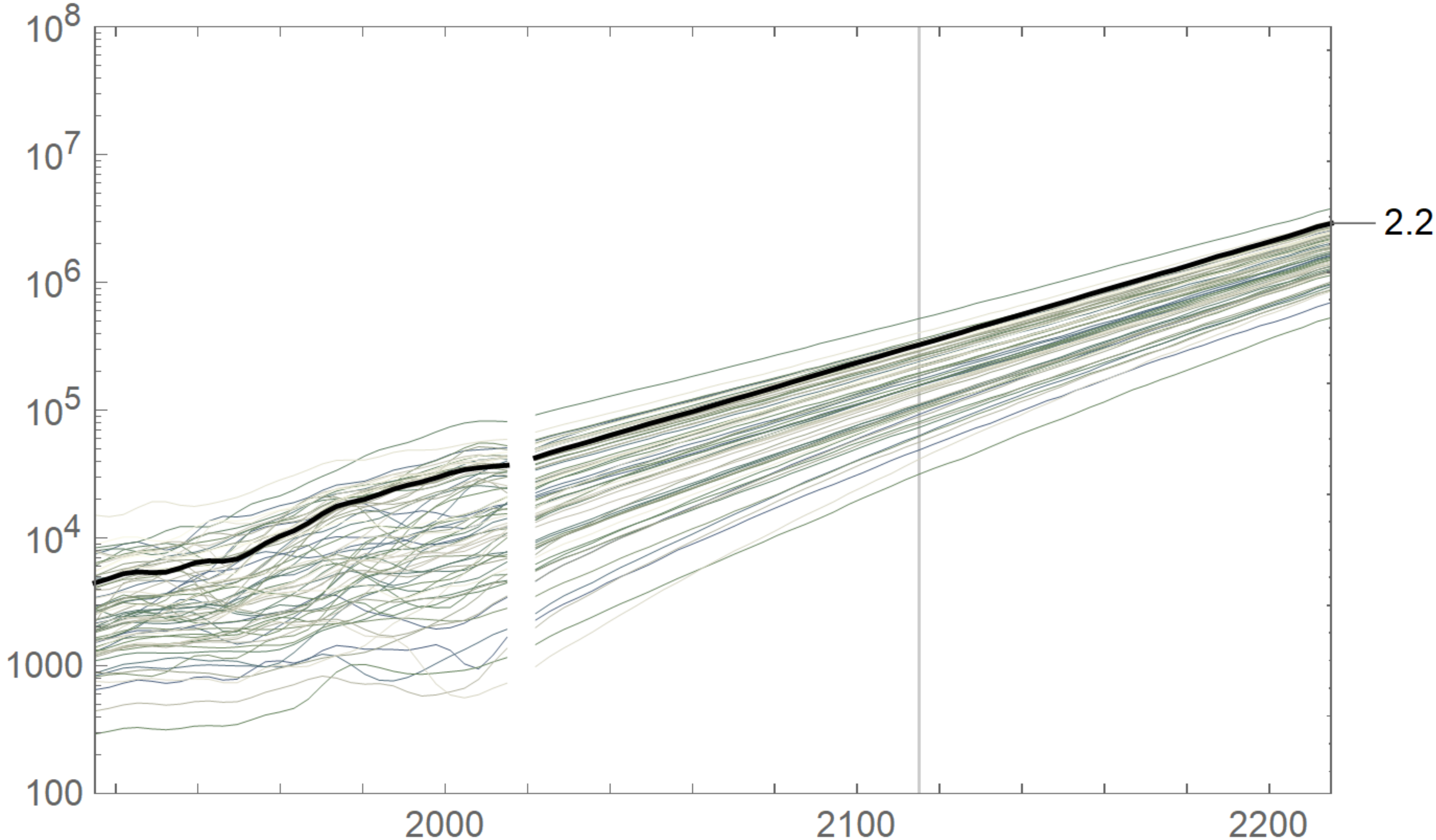
  $\Rightarrow$ Compute percentiles of (subset) of state vector and its linear combinations

- Run sampler with augmented Gibbs iterations, and compare resulting state distribution with prior percentiles via t-statistic

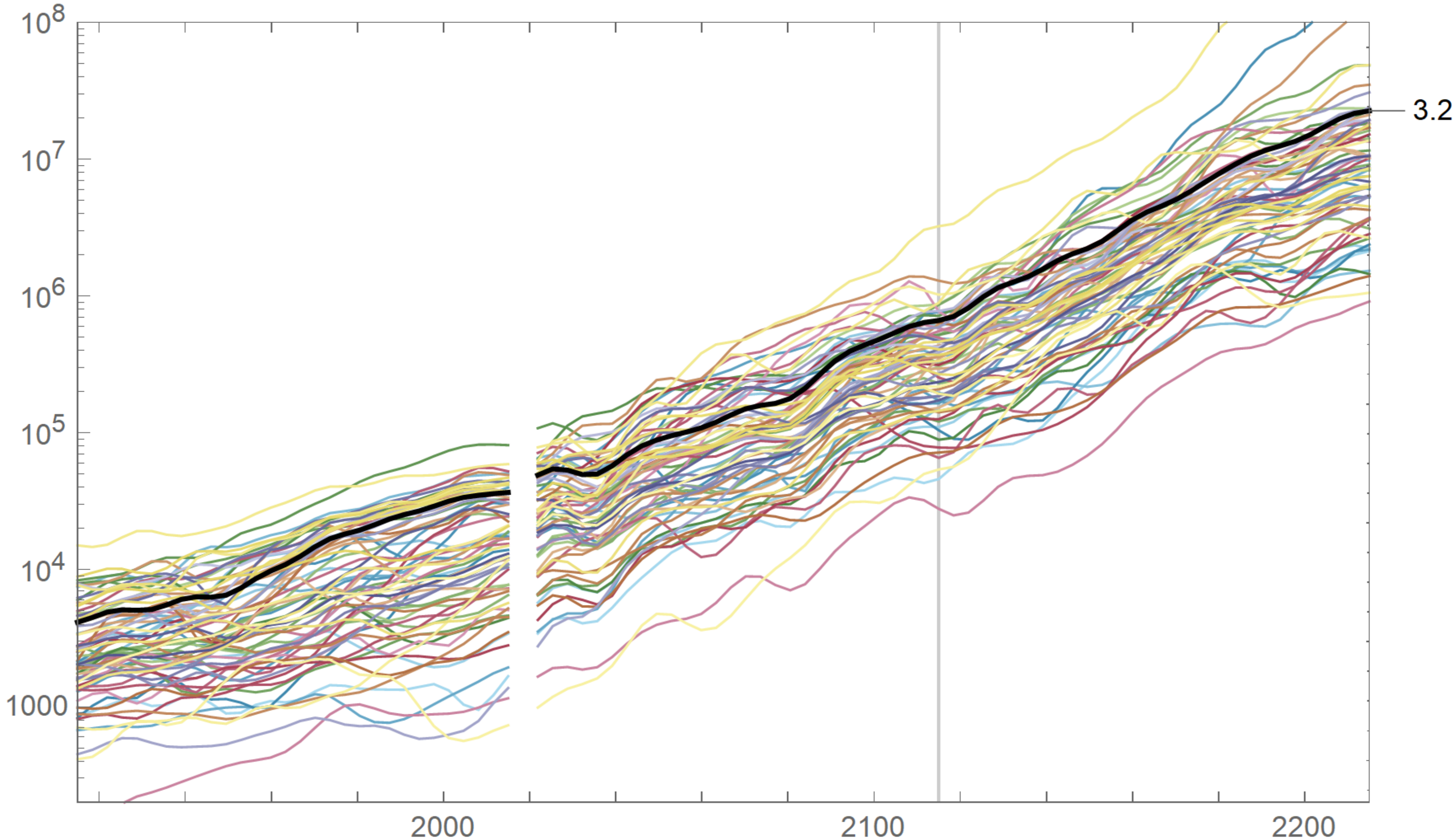
  $\Rightarrow$ Compute standard error via batch-means with 200 batches

  $\Rightarrow$ t-statistic tends to diverge whenever mistake/chain not mixing

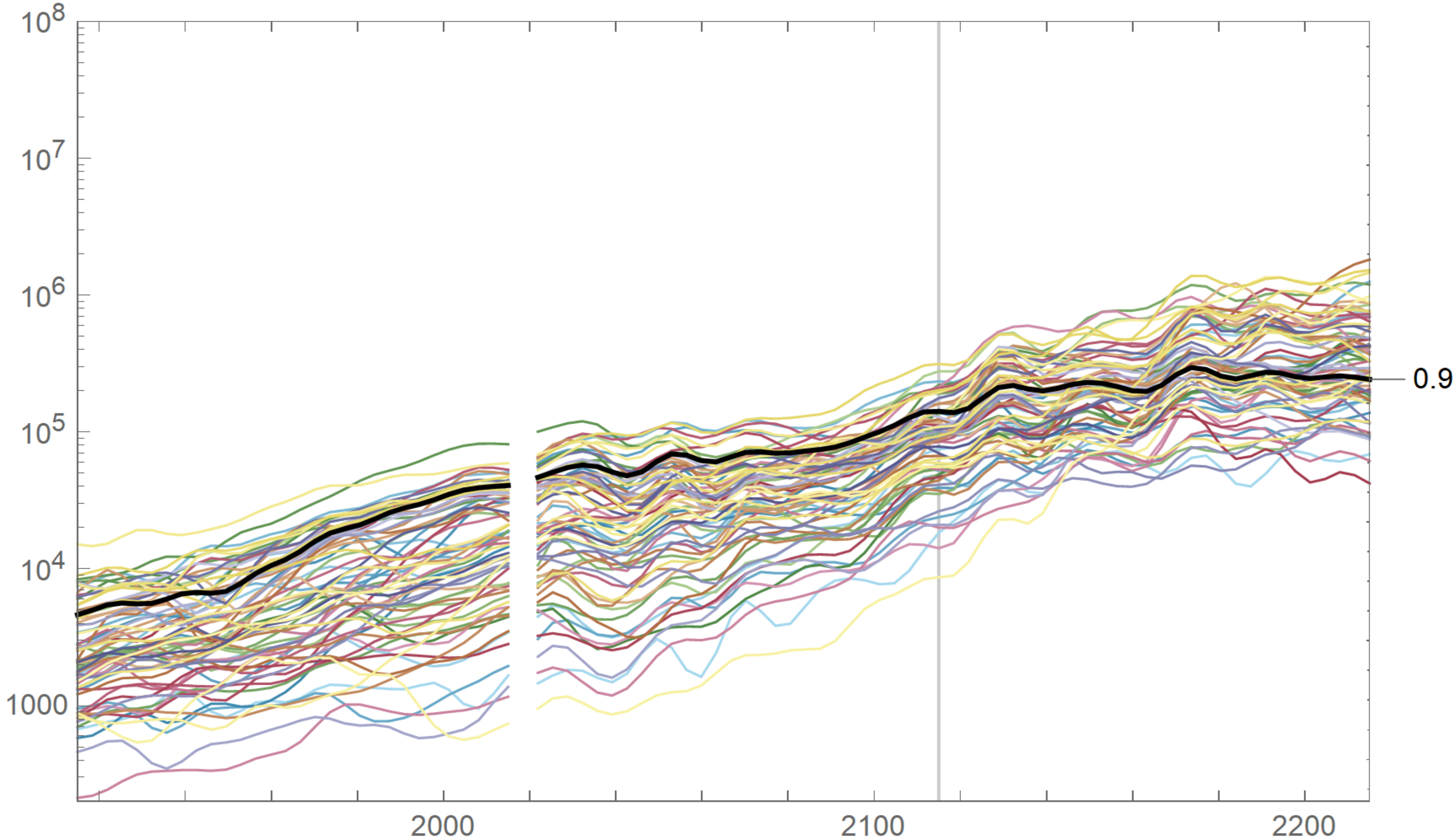
Posterior Median



A Draw from the Posterior Distribution



A Draw from the Posterior Distribution



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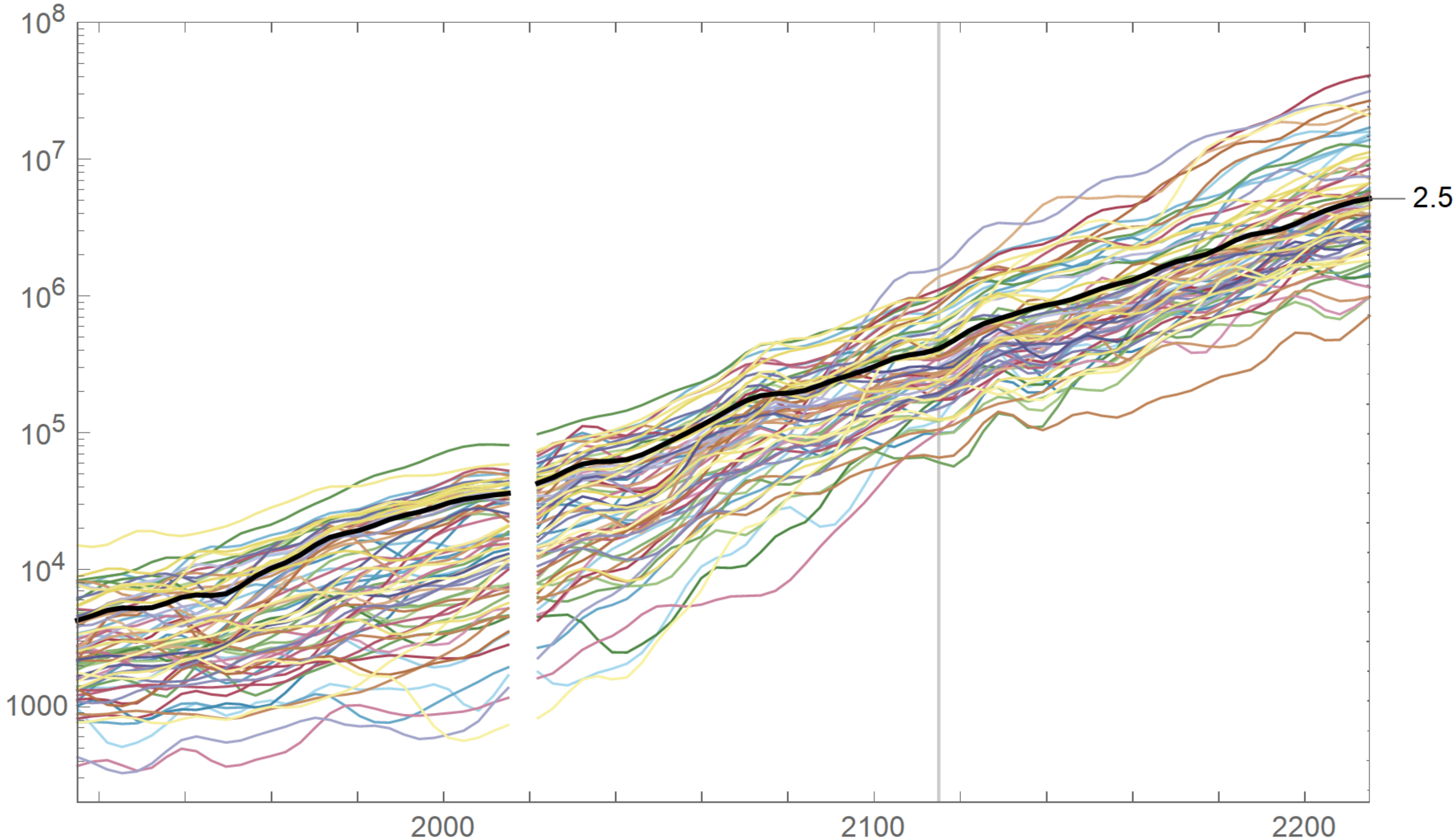

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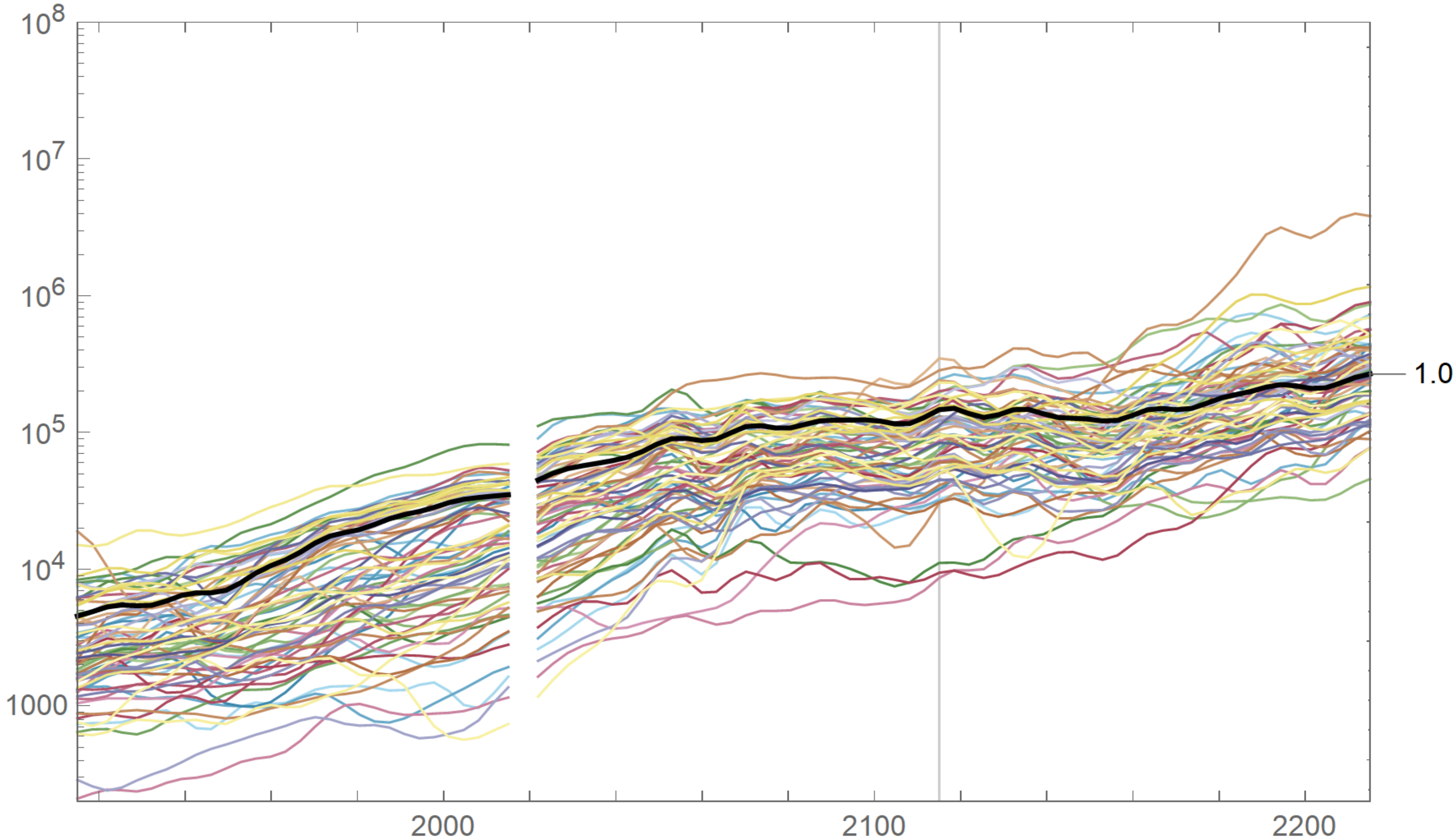

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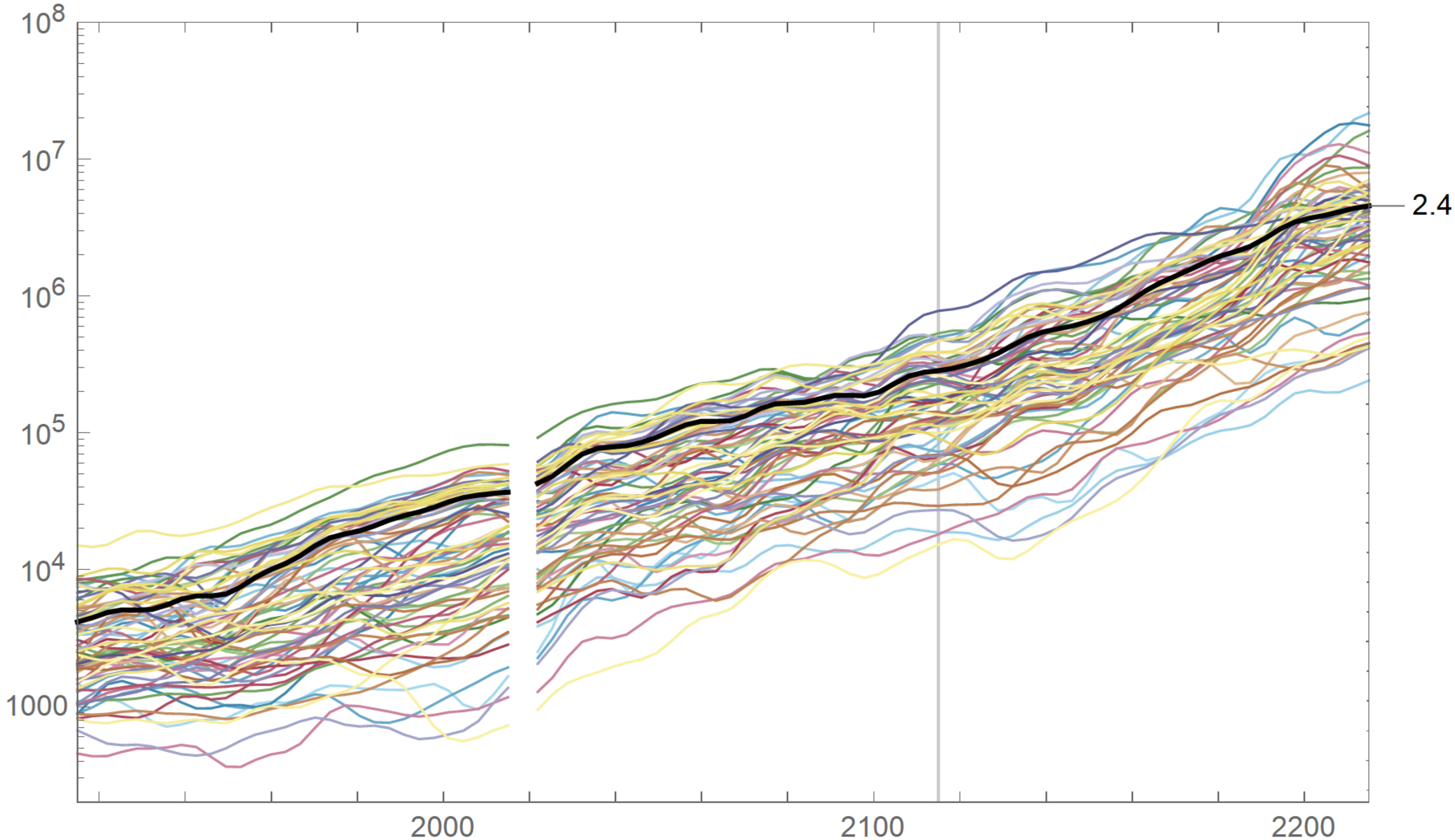
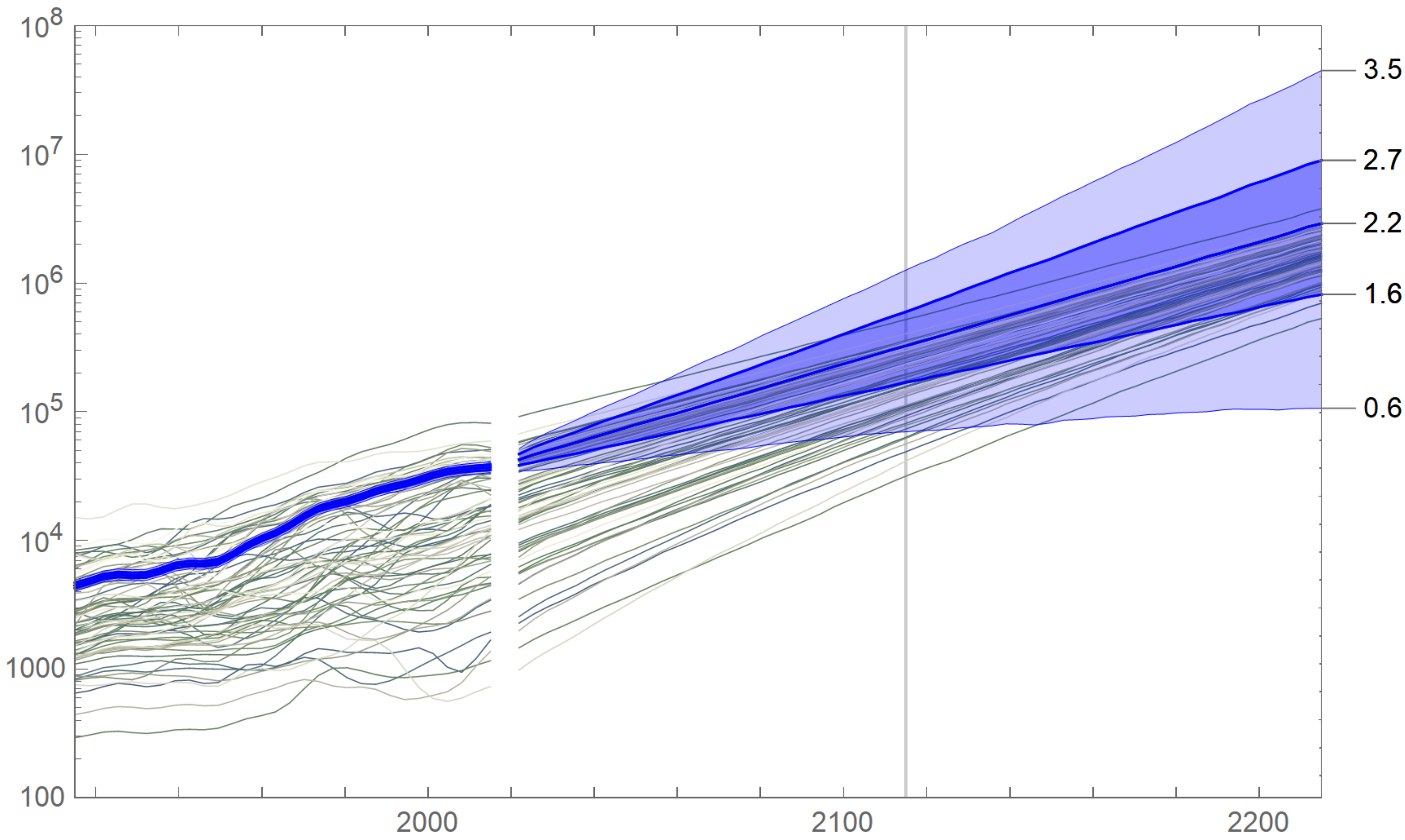
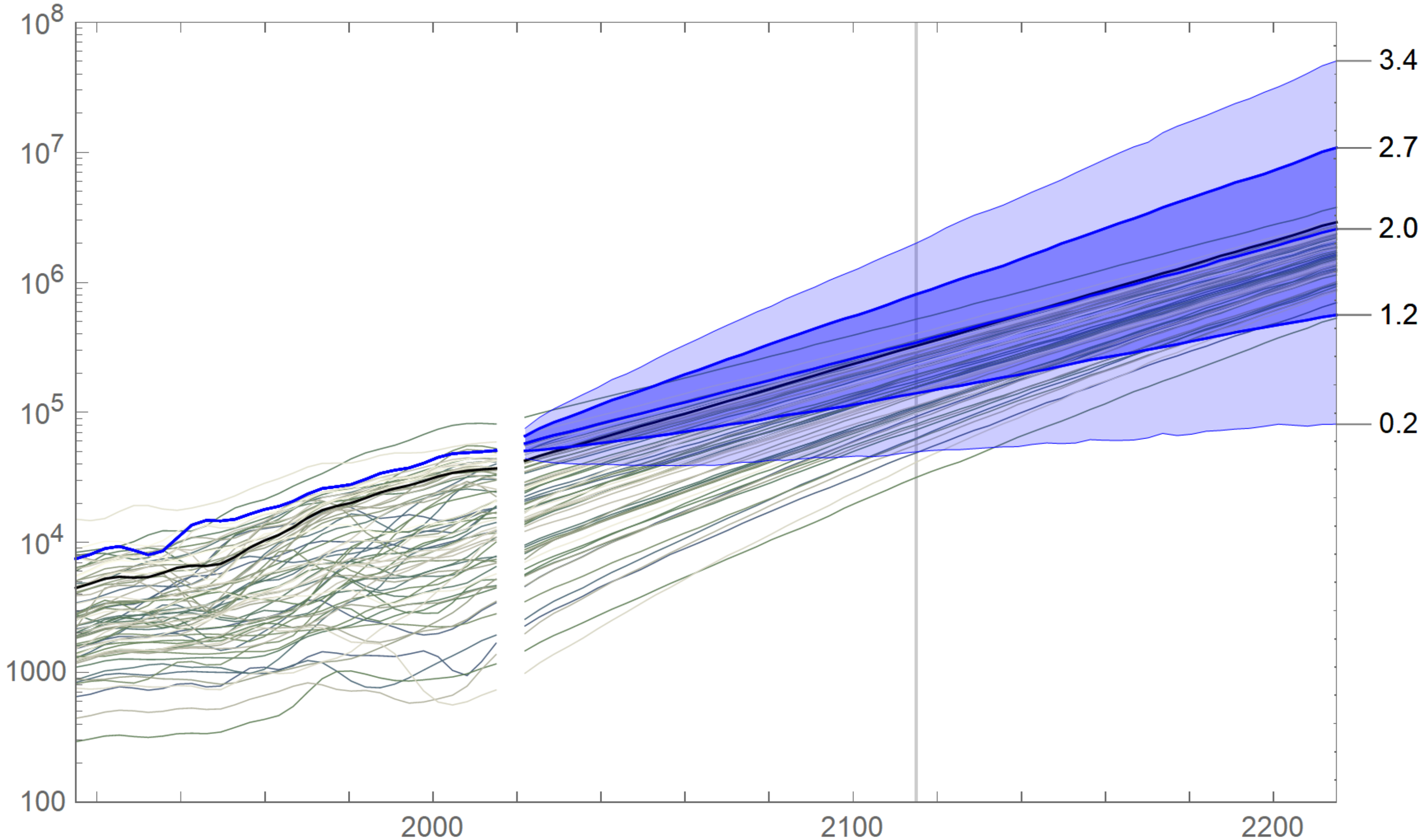


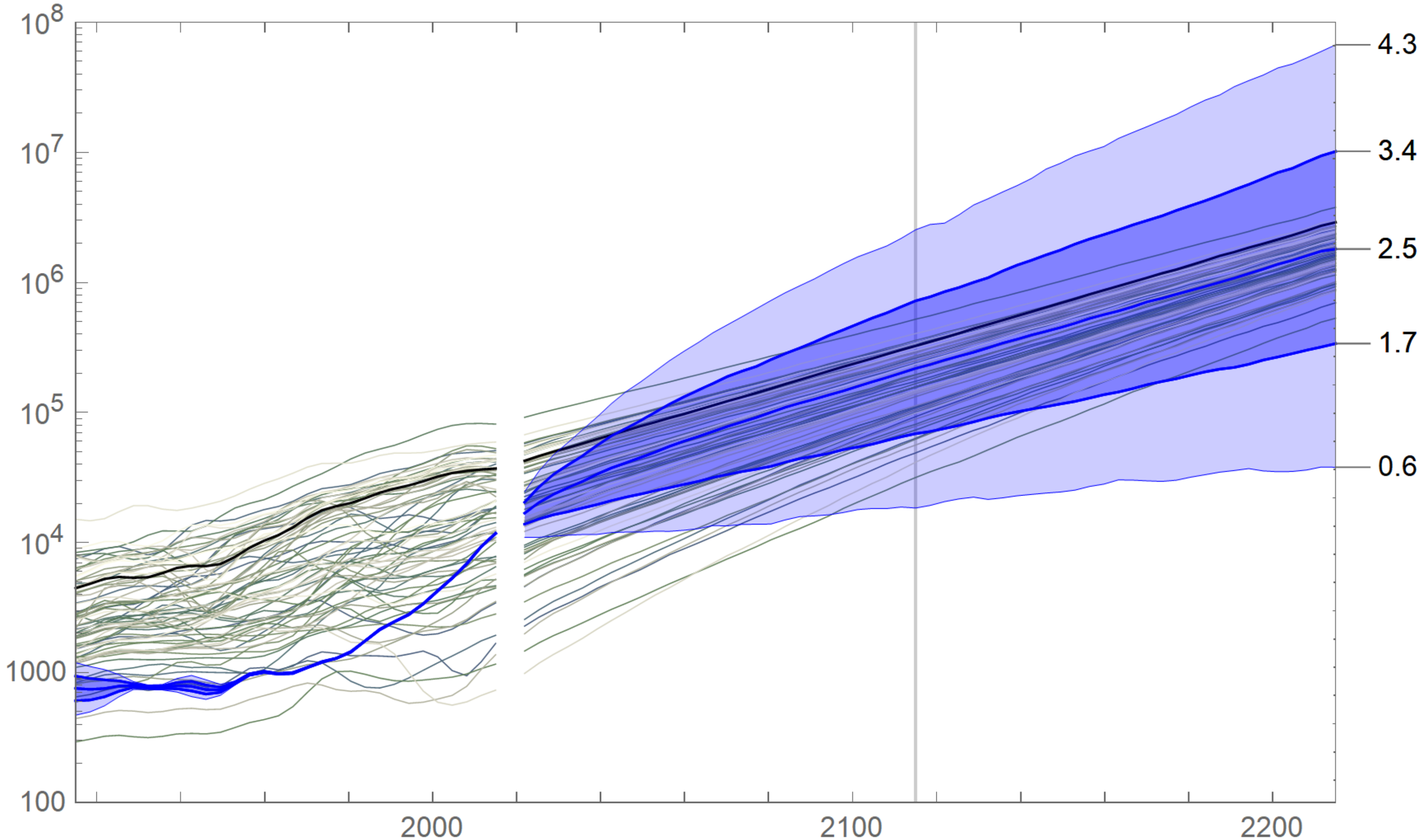
Factor Forecast Bands



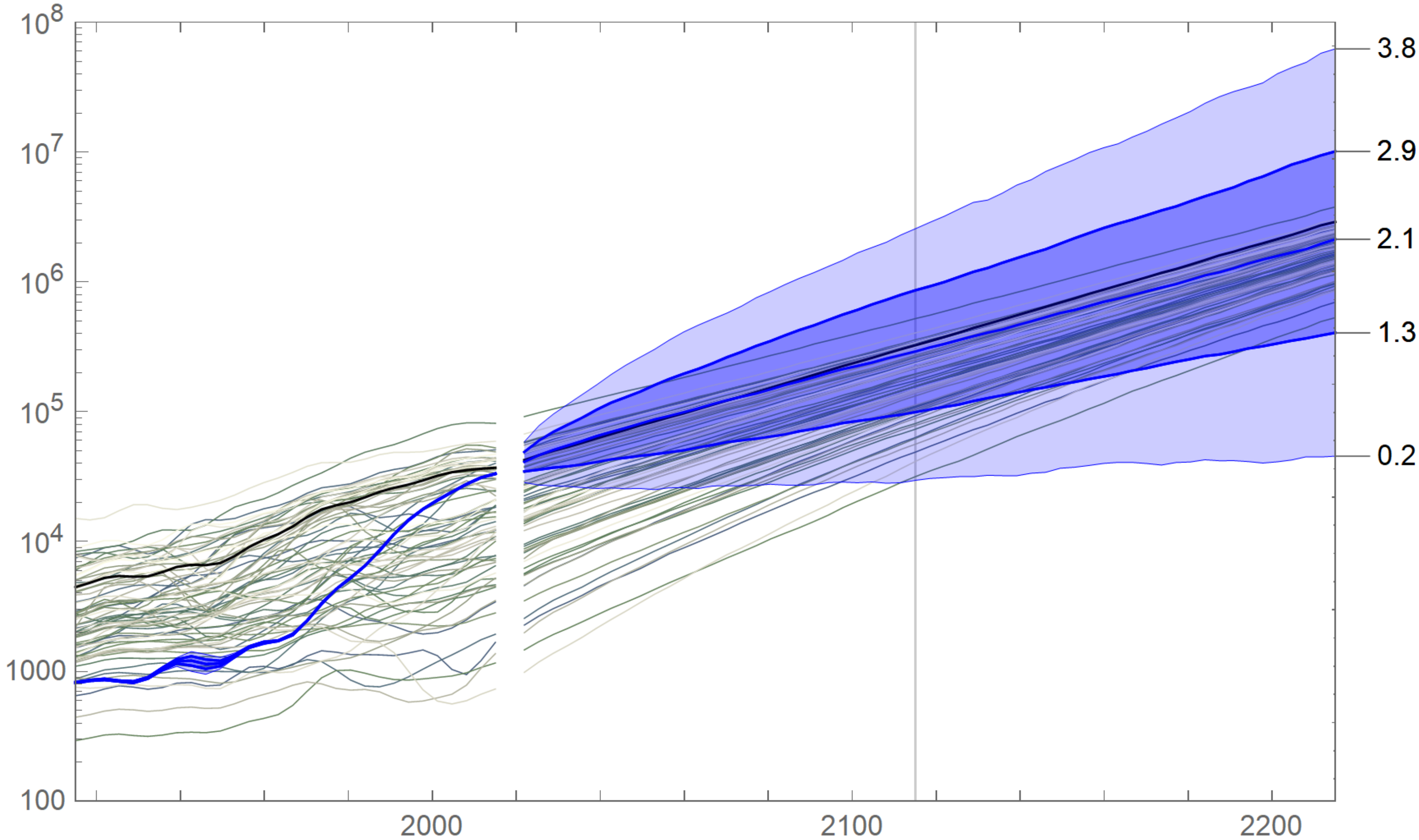
Forecast Bands USA



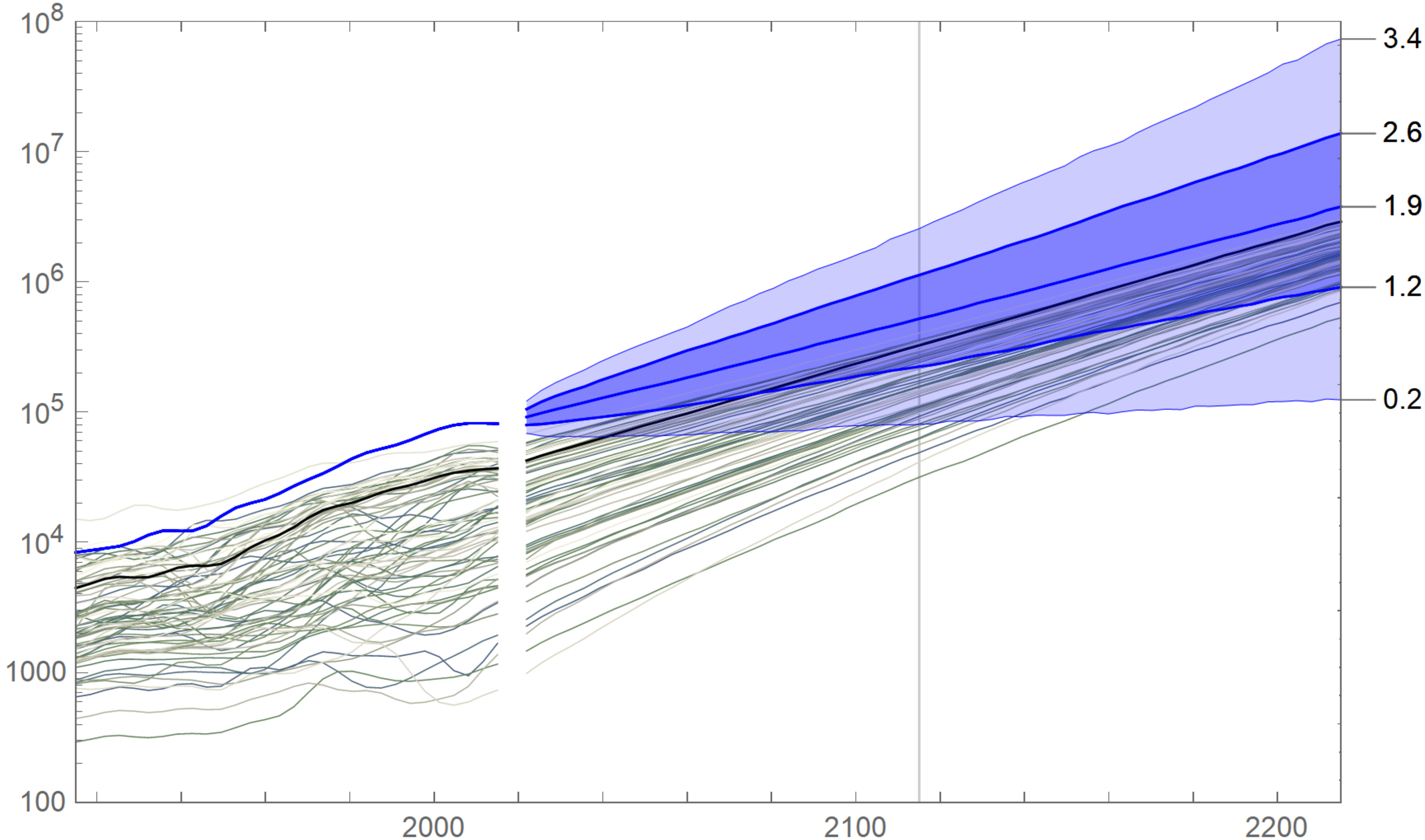
Forecast Bands China



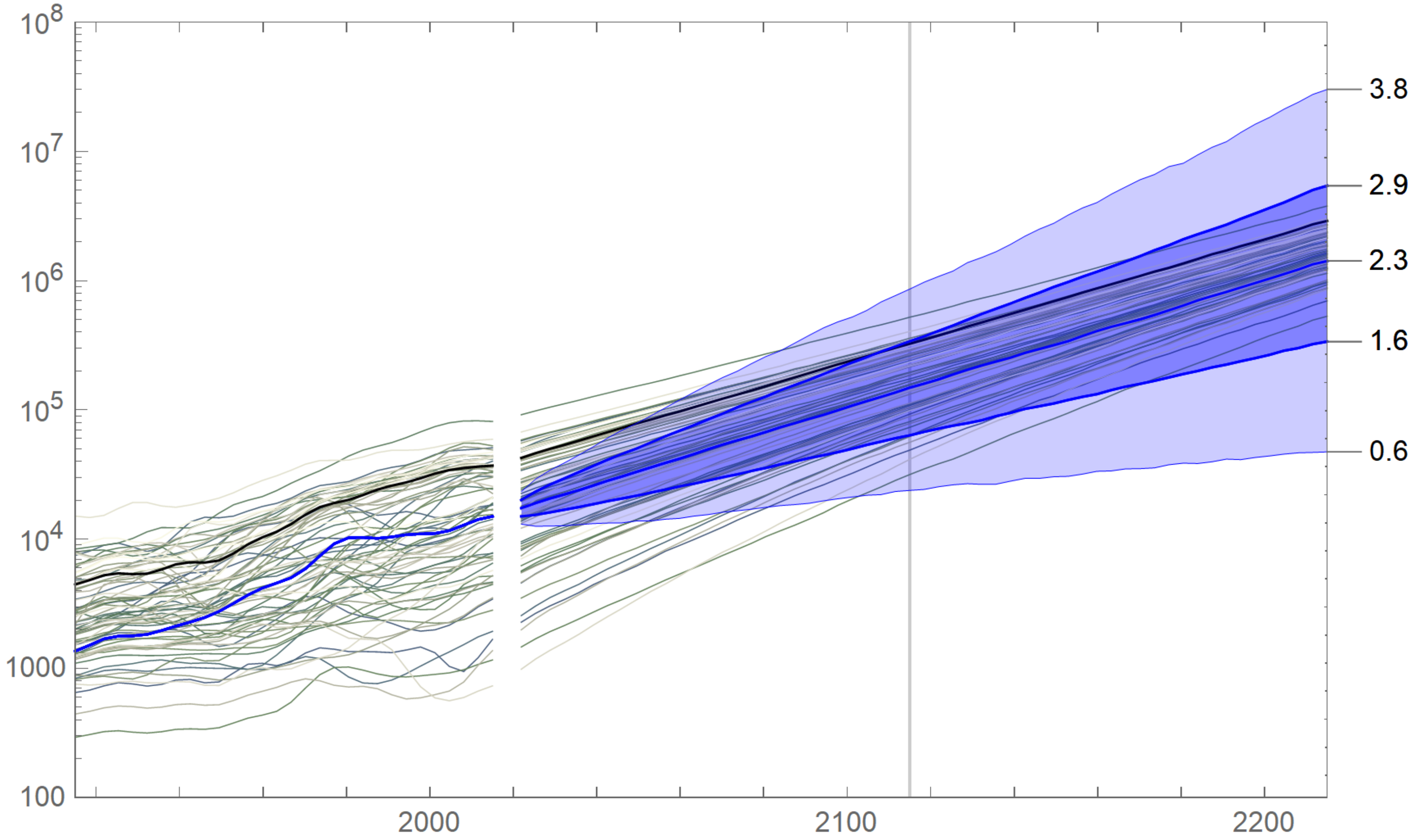
Forecast Bands South Korea



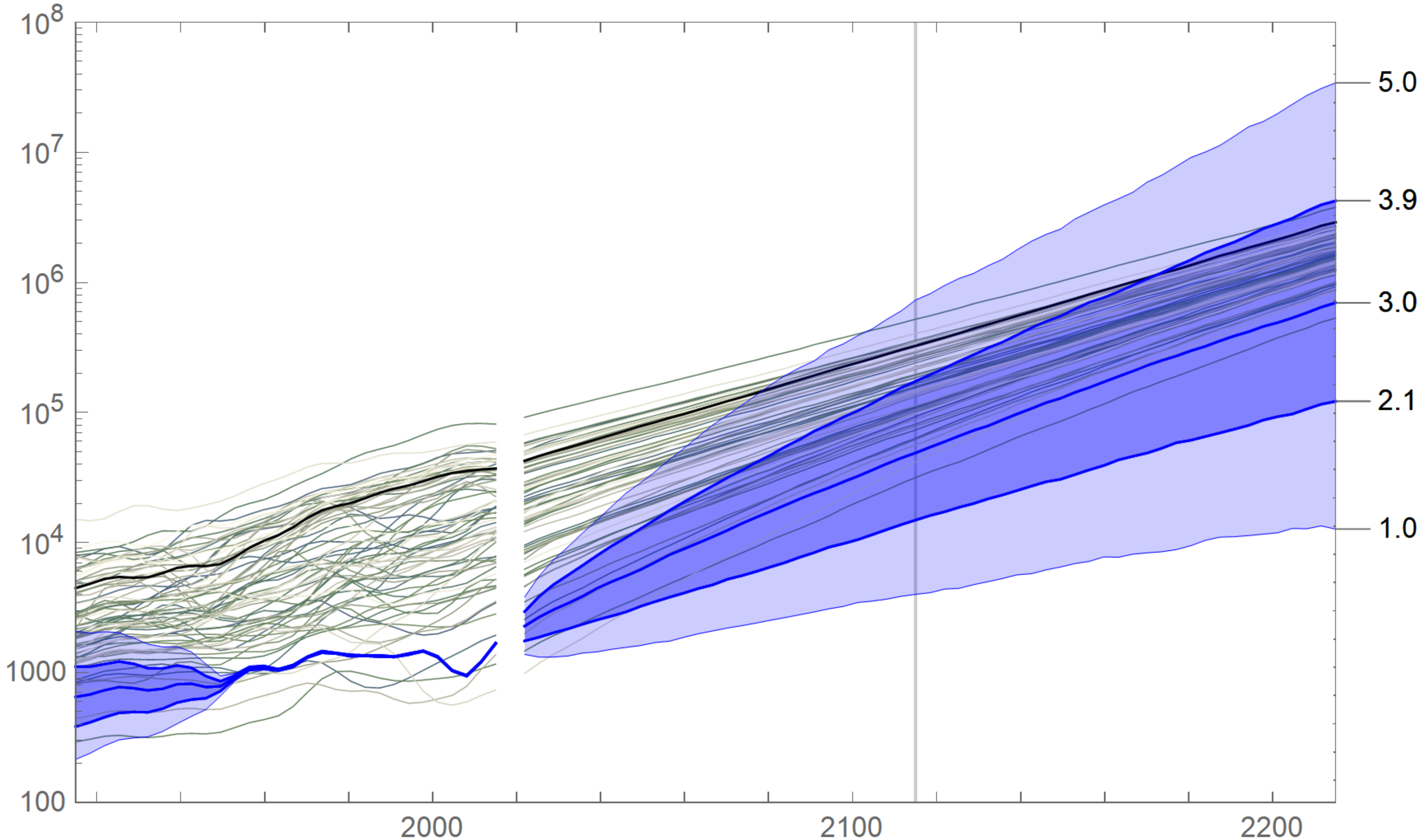
Forecast Bands Norway



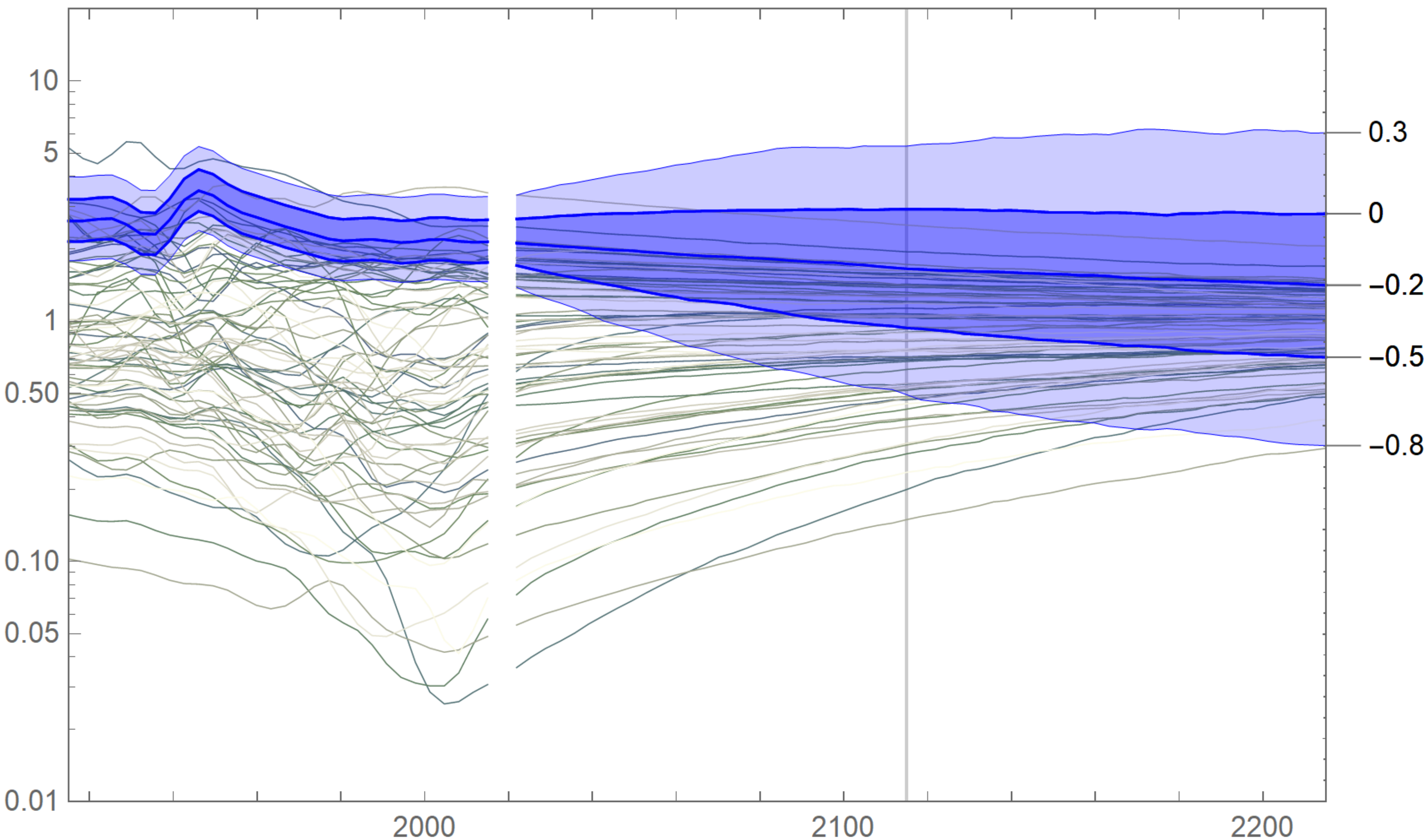
Forecast Bands Brazil



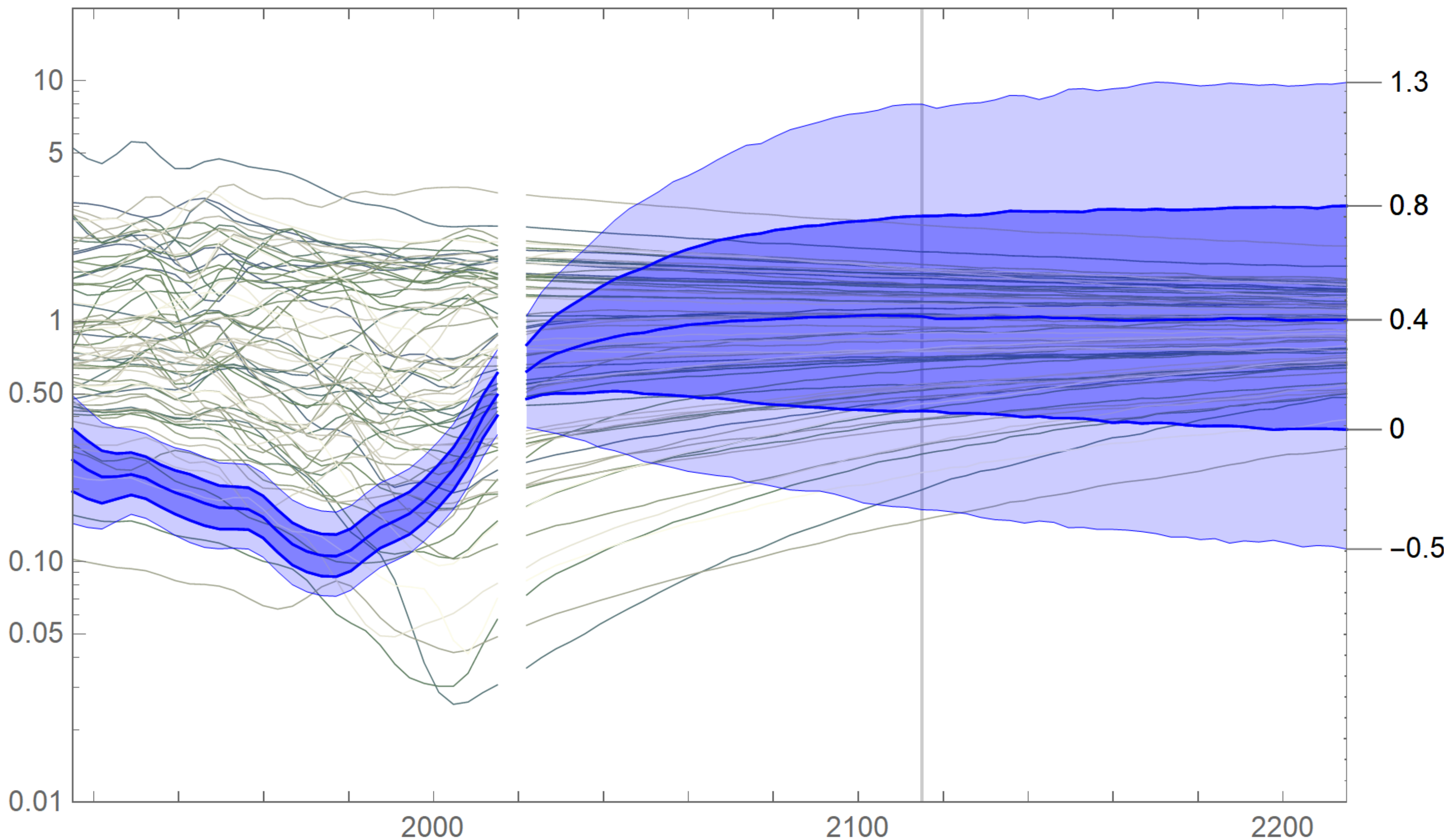
Forecast Bands Zimbabwe



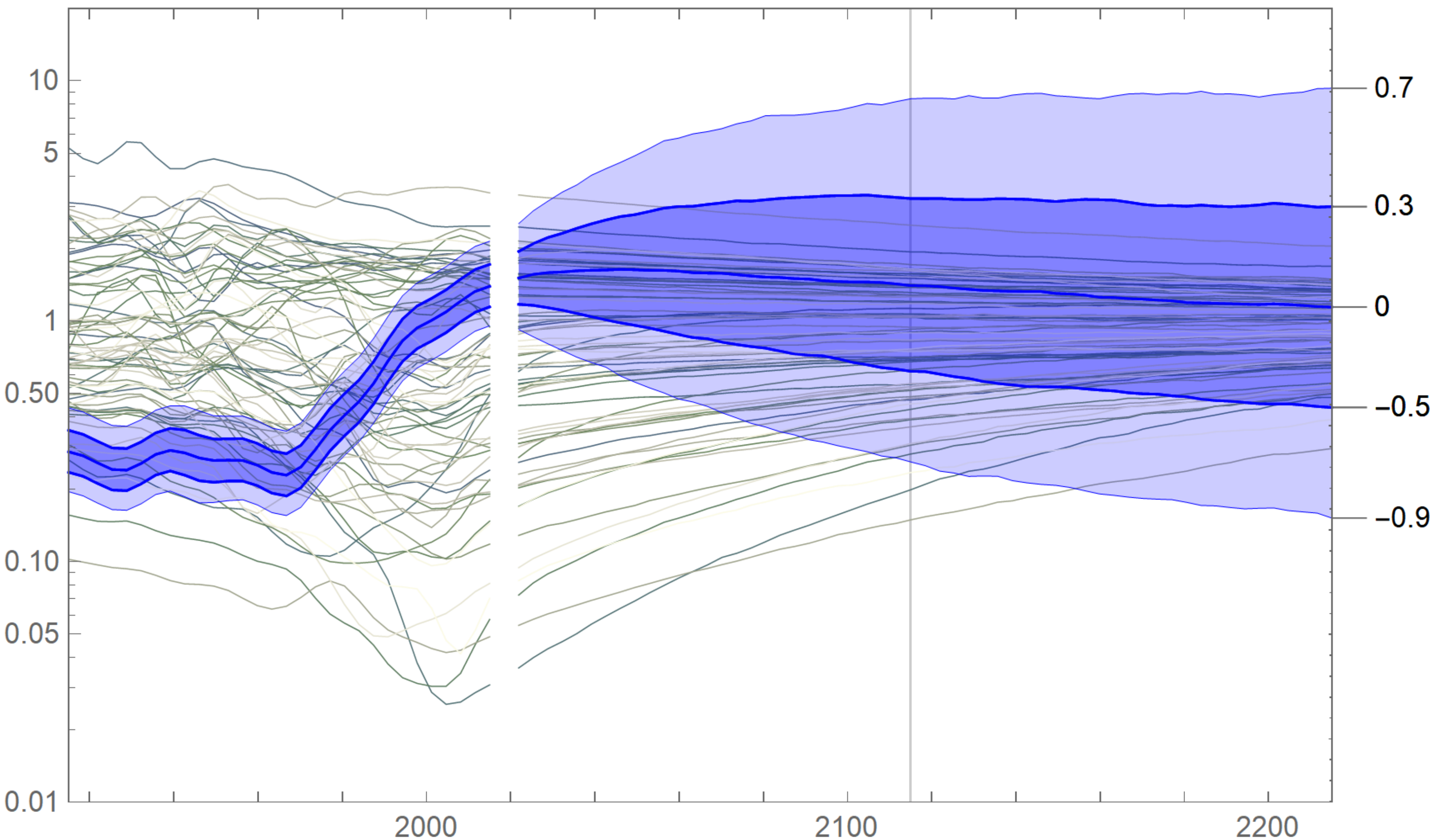
Forecast of $u_{i,t}$: USA



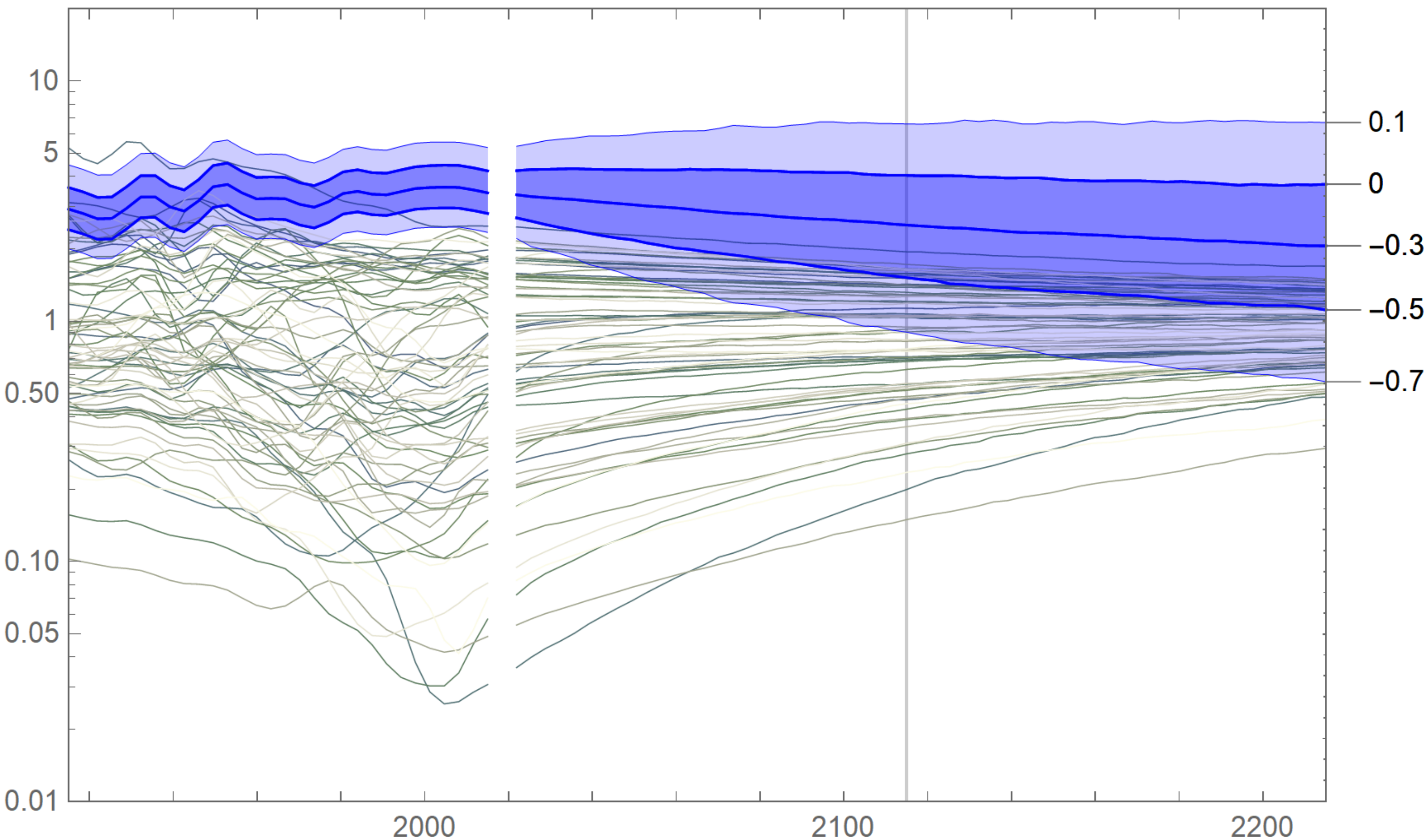
Forecast of $u_{i,t}$: China



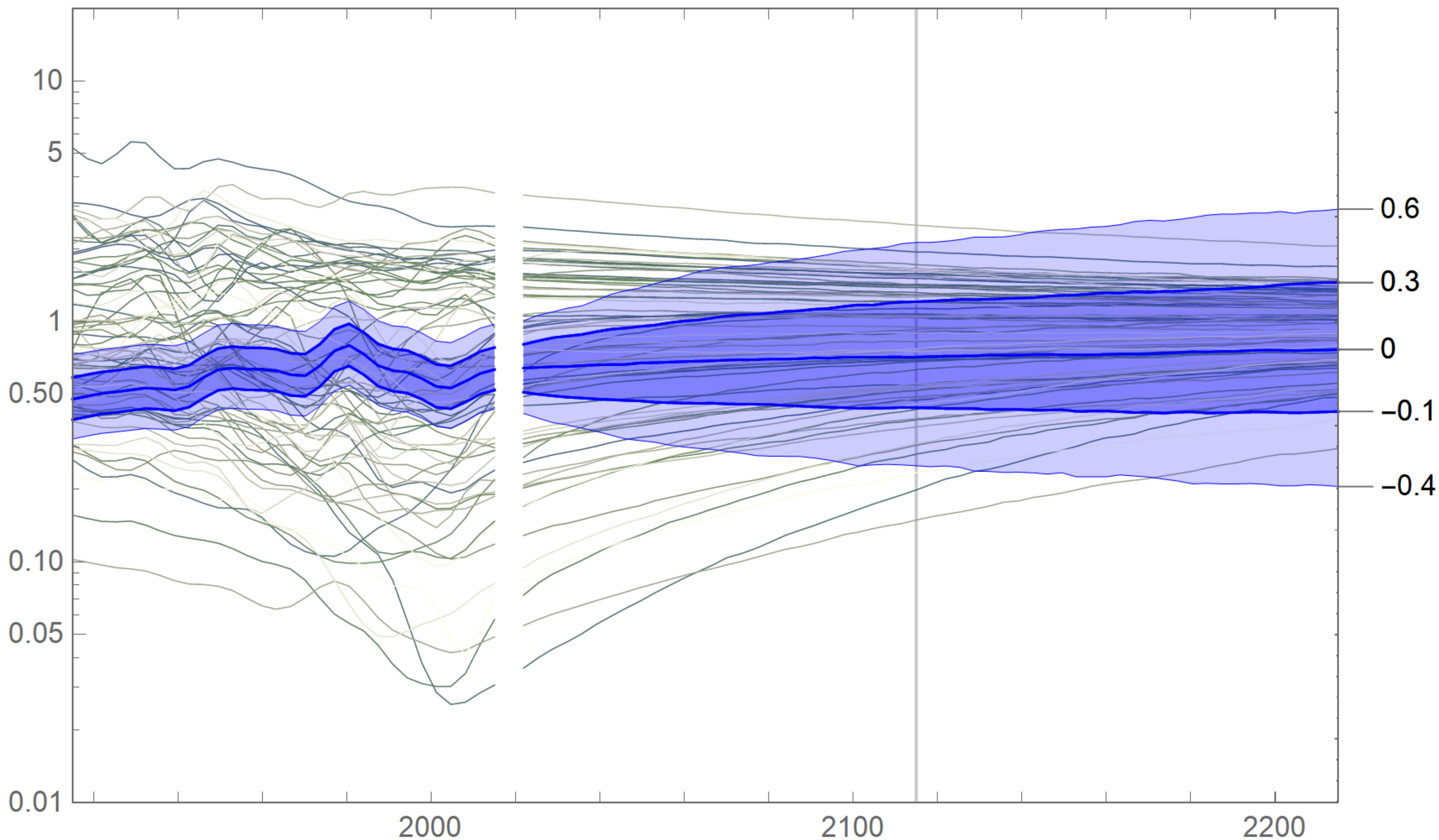
Forecast of $u_{i,t}$: South Korea



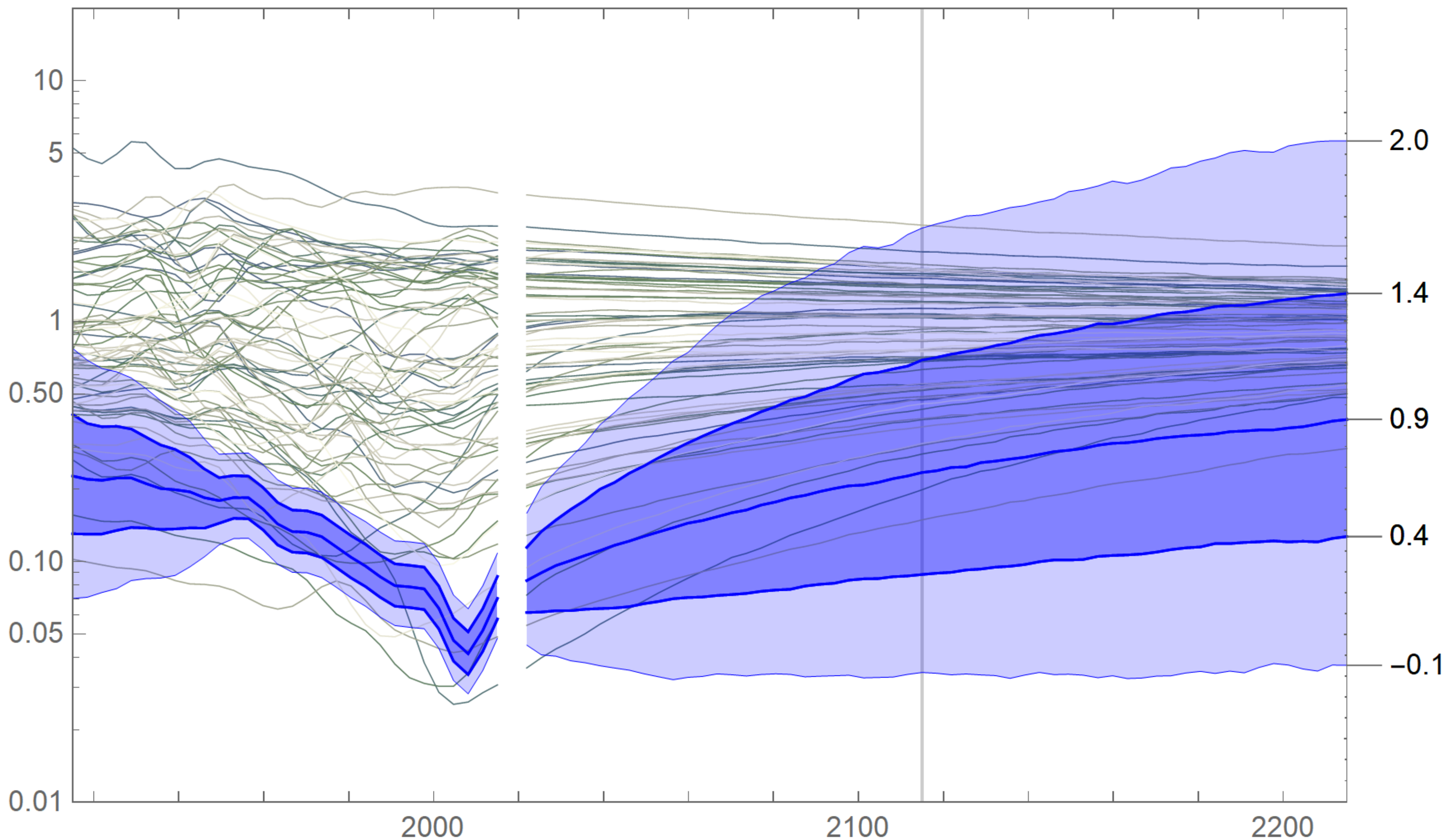
Forecast of $u_{i,t}$: Norway



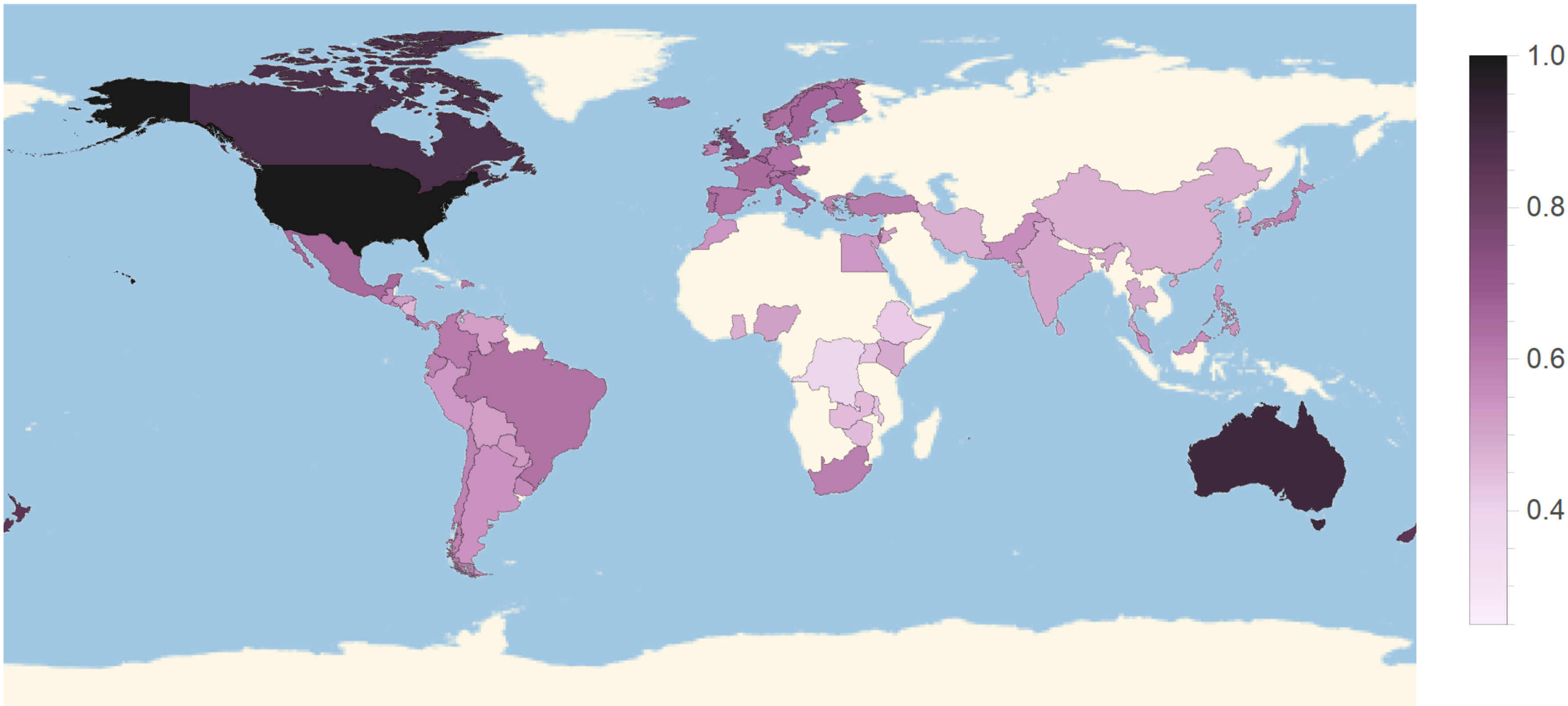
Forecast of $u_{i,t}$: Brazil



Forecast of $u_{i,t}$: Zimbabwe

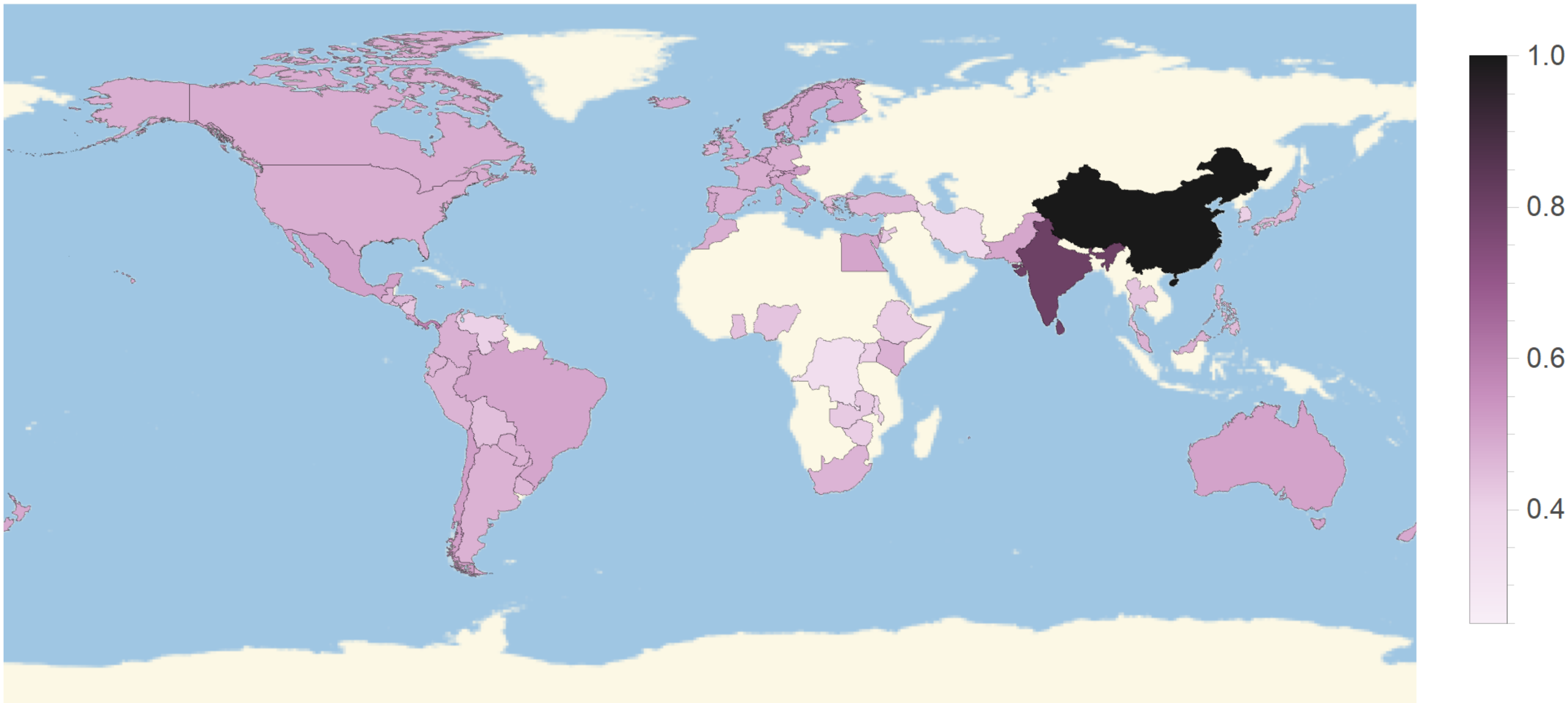


Cross-Correlations: USA

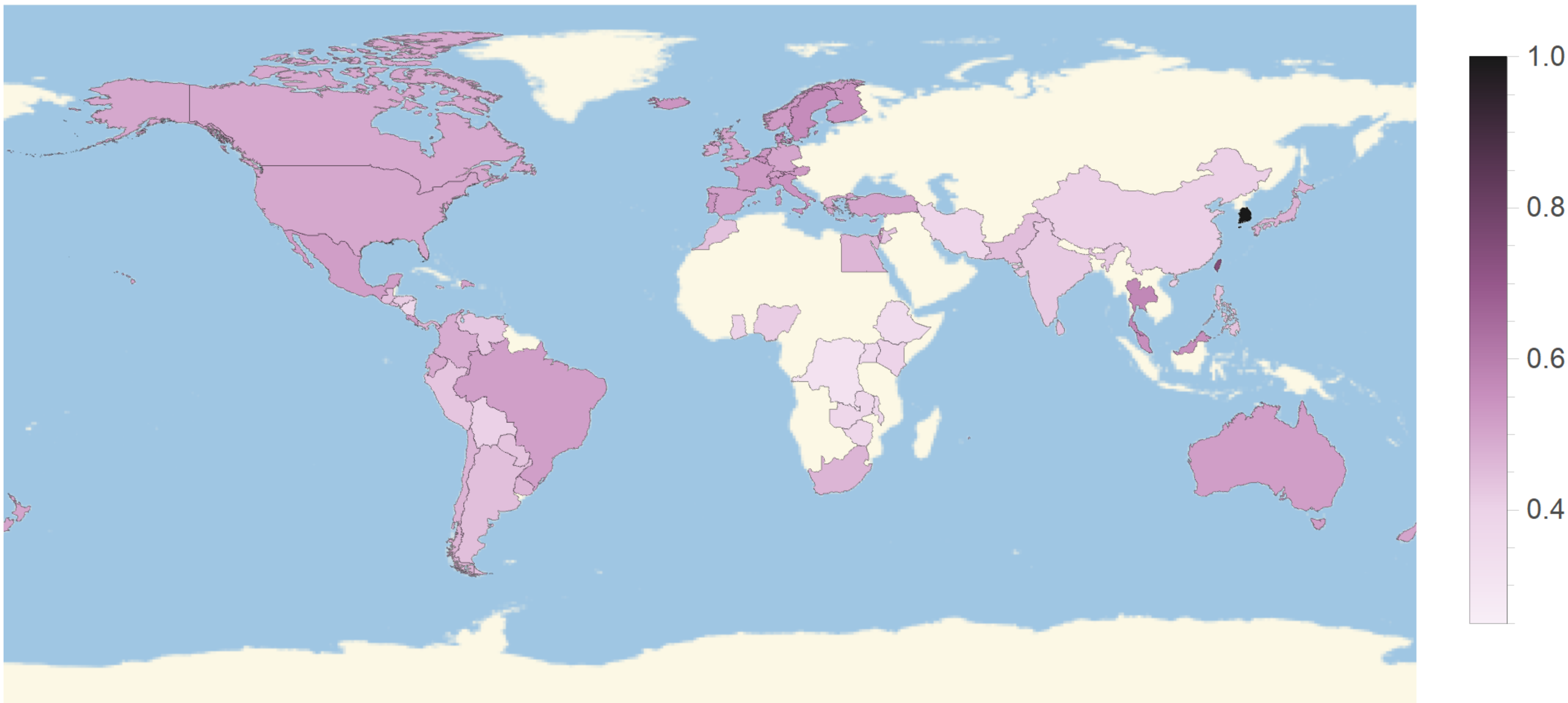


Correlation of 100 year growth rate forecasts of GDP/capita

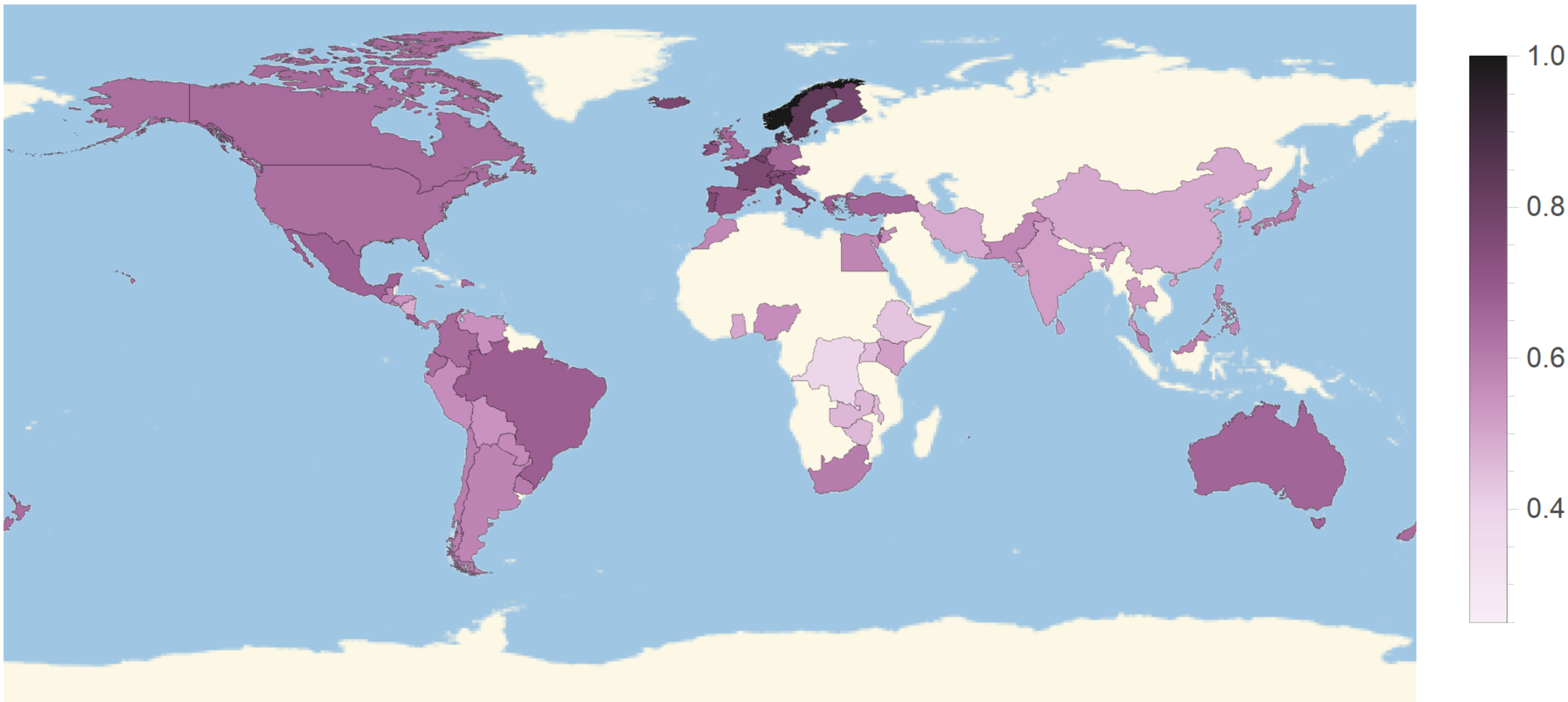
Cross-Correlations: China



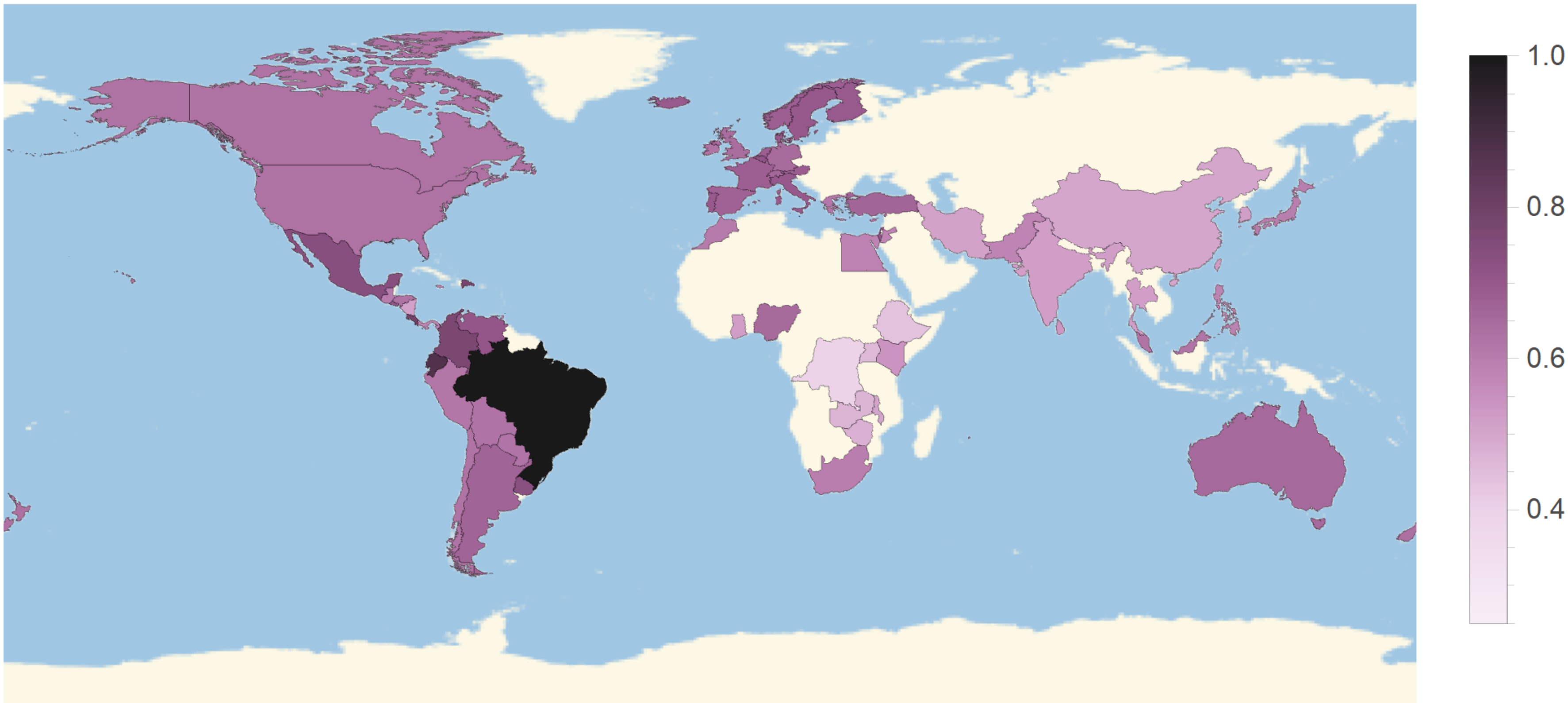
Cross-Correlations: South Korea



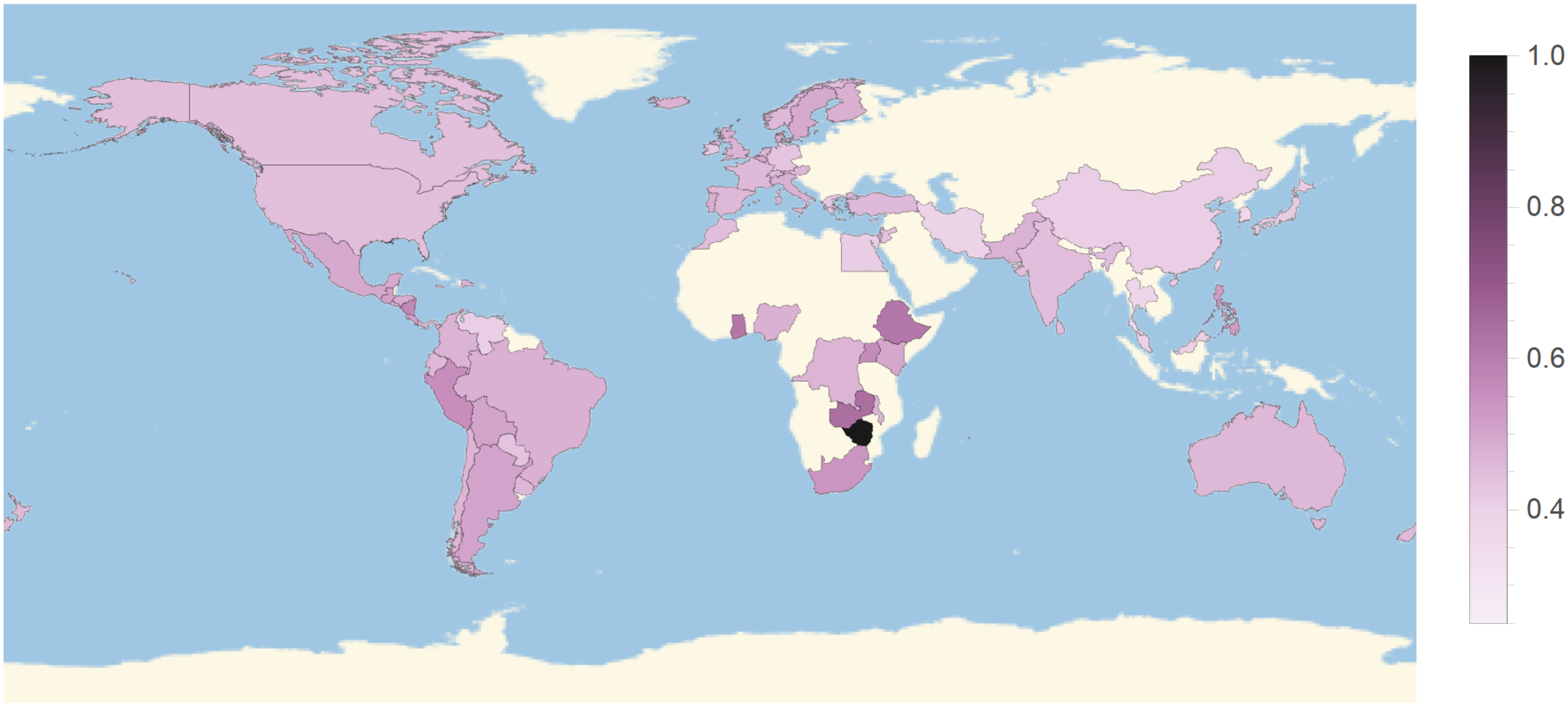
Cross-Correlations: Norway



Cross-Correlations: Brazil

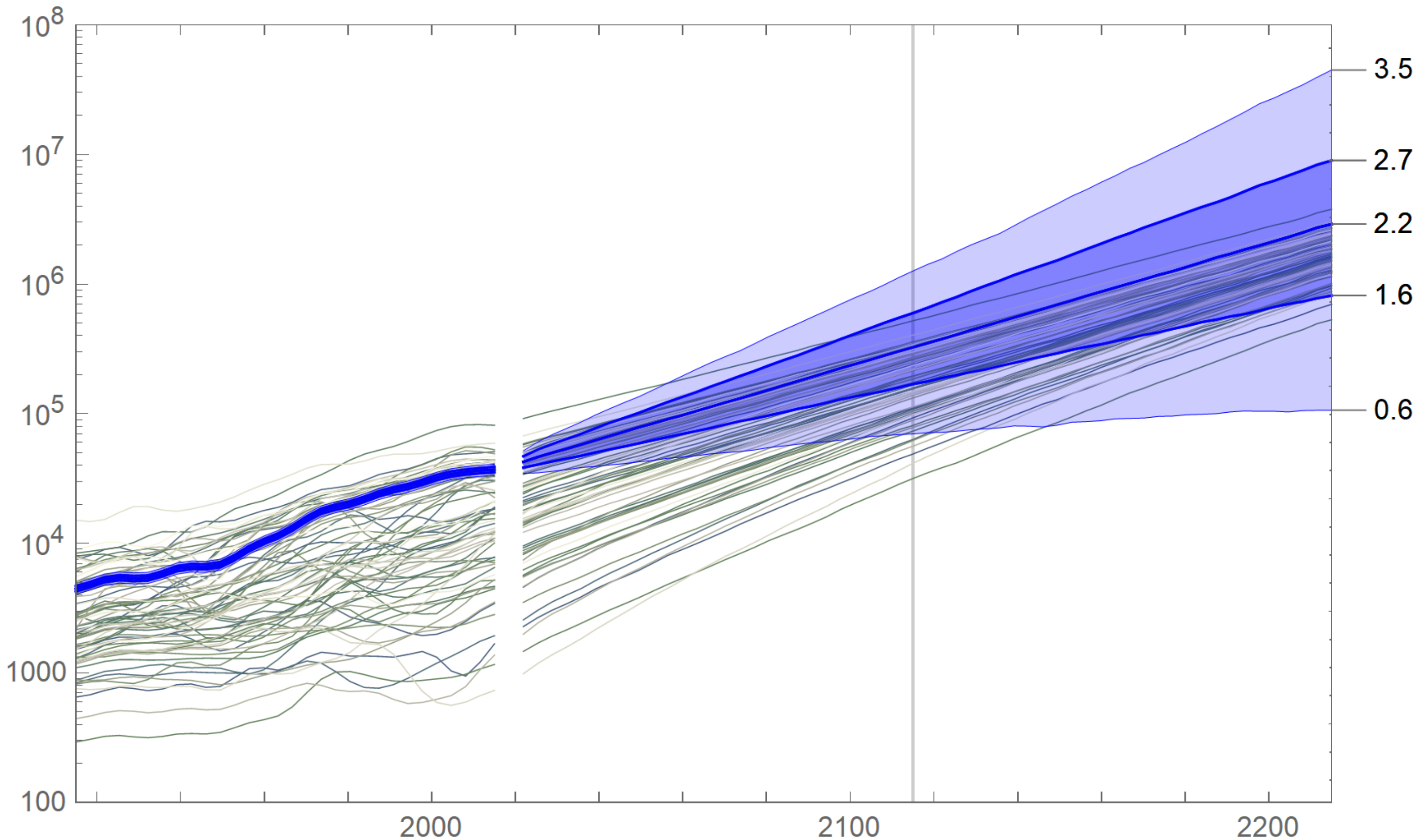


Cross-Correlations: Zimbabwe



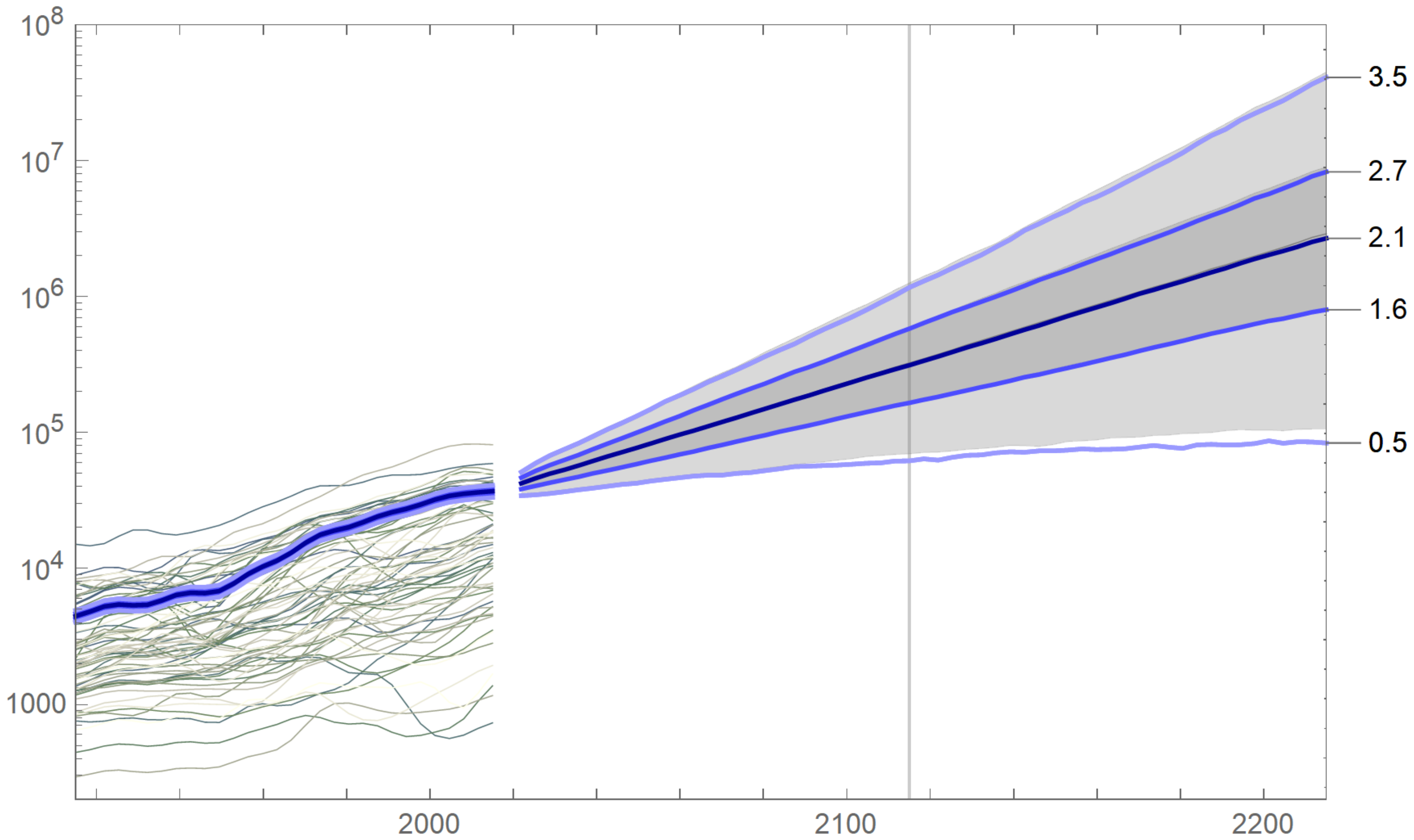
Factor Uncertainty: Baseline

Δf_t is LLM with $g \in [0, 5]$, flat prior



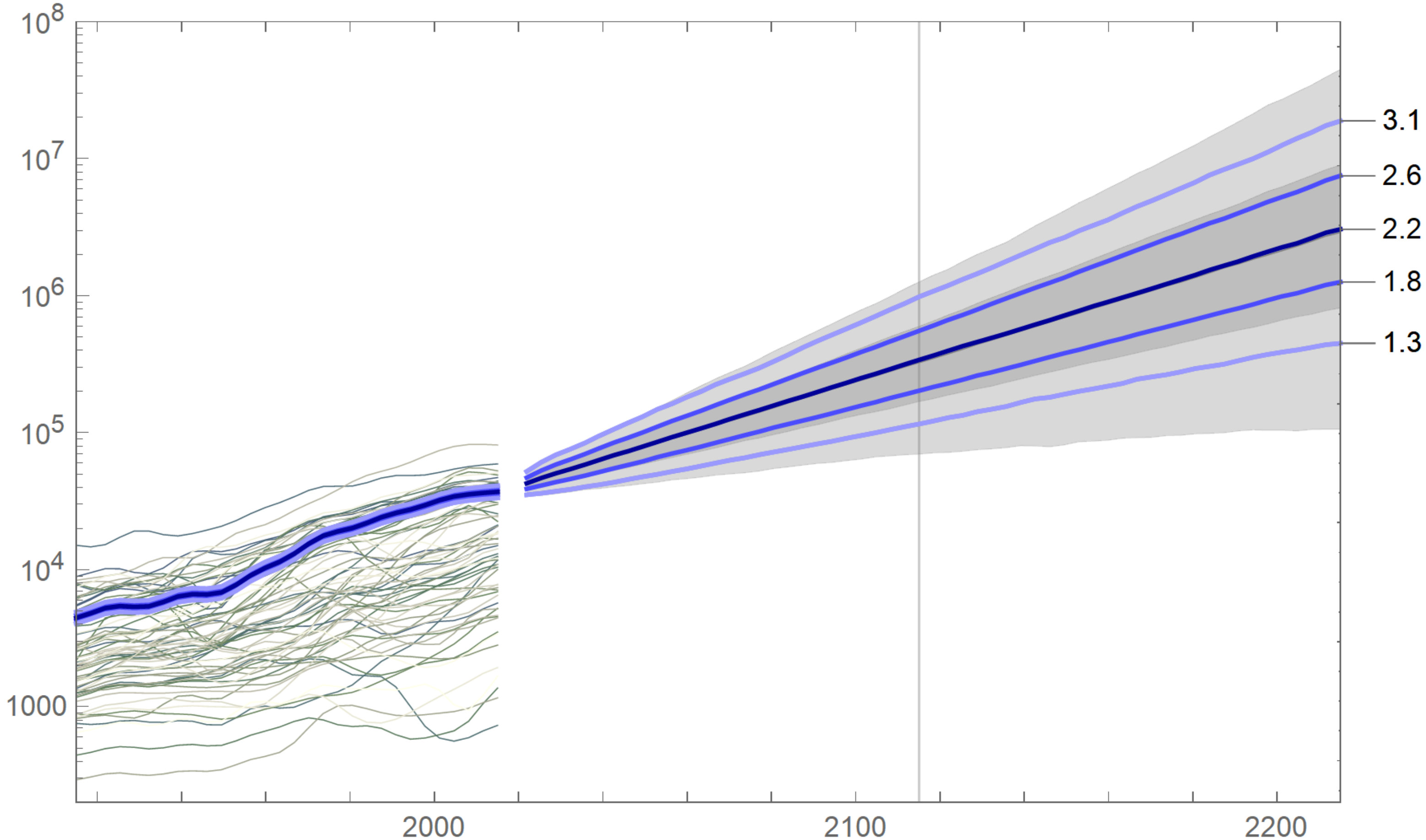
LLM with Larger g

Δf_t is LLM with $g \in [0, 10]$, flat prior



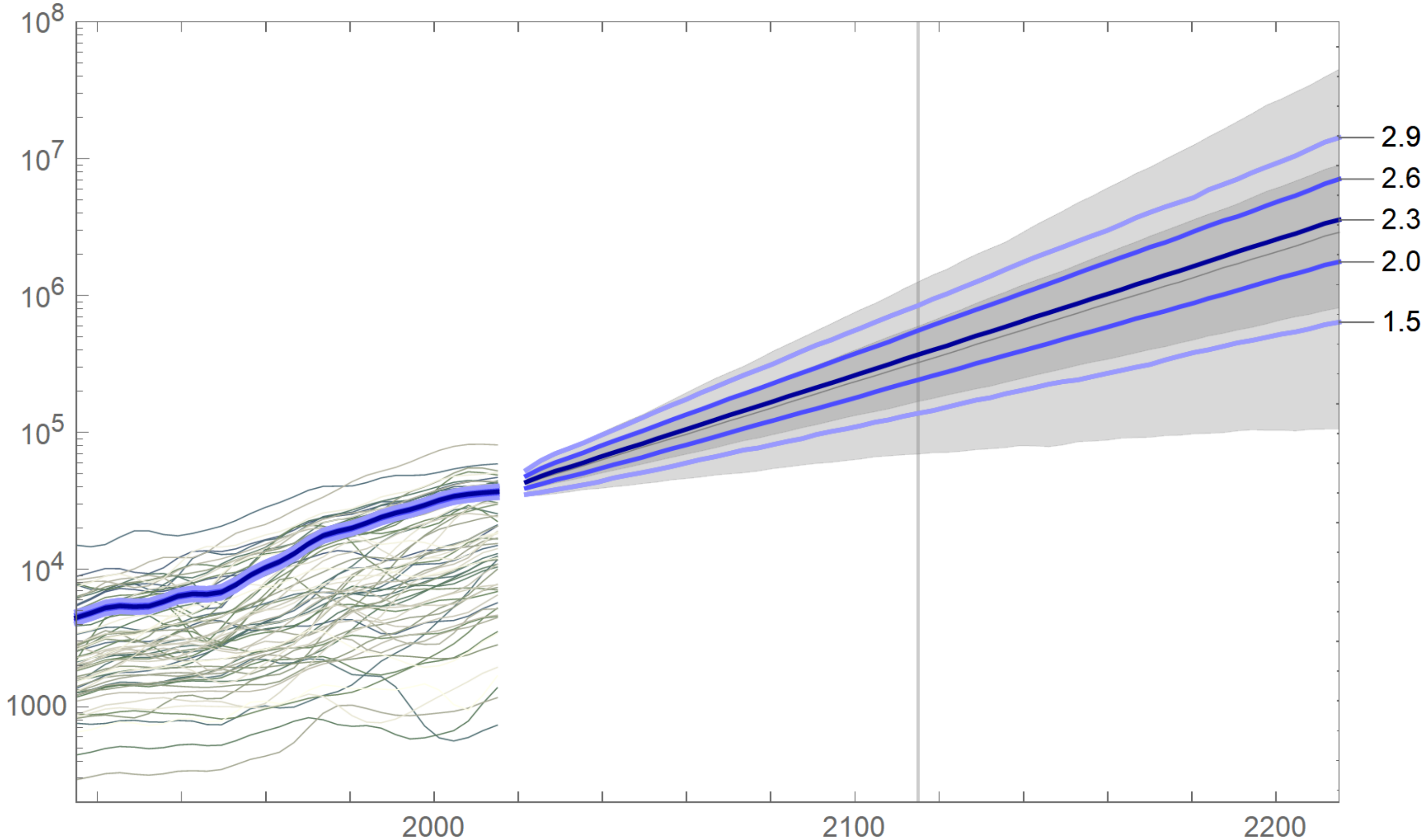
I(1) Factor

Δf_t is I(0) ($g = 0$ in LLM)



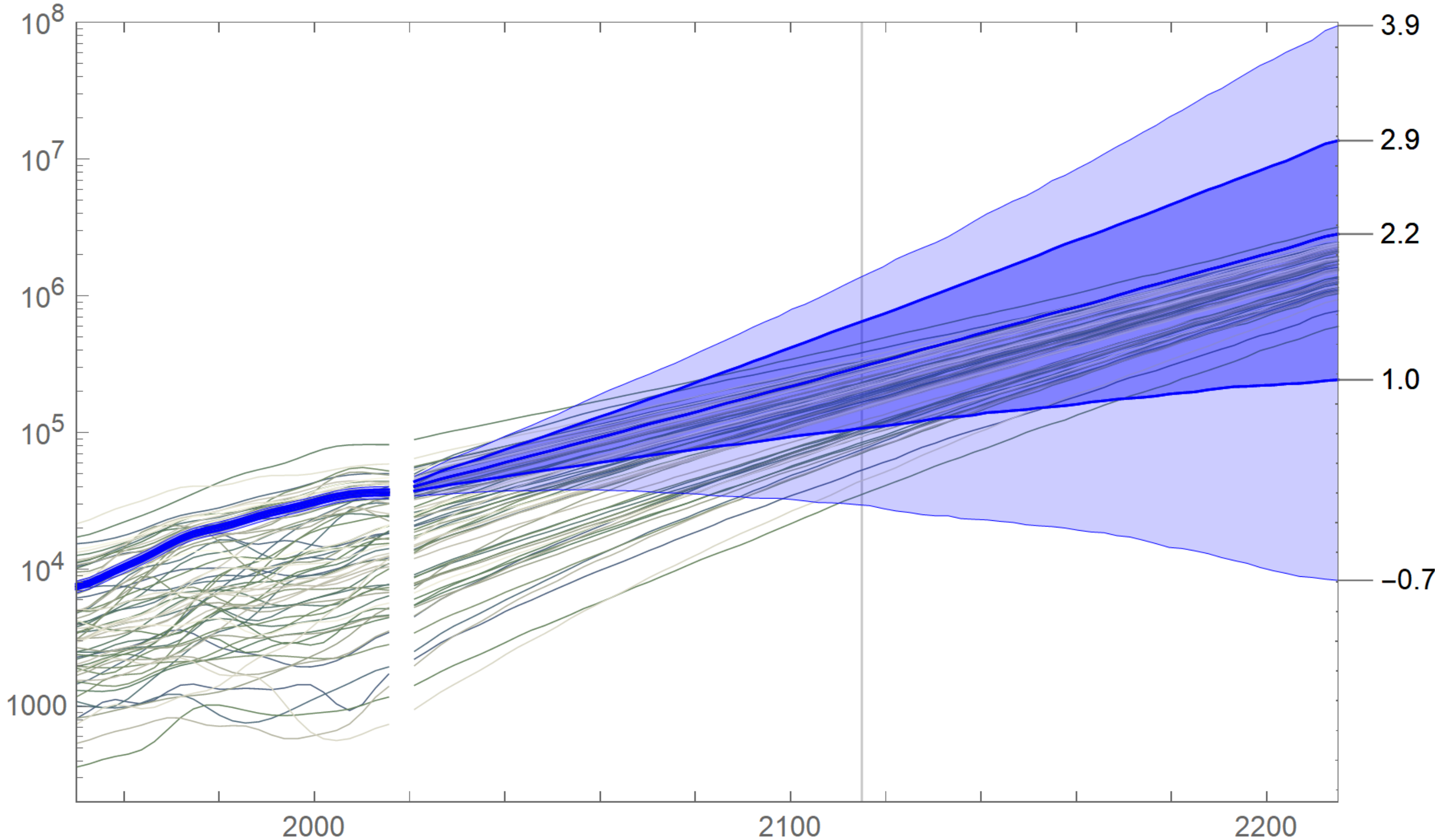
I(d) Factor

Δf_t is I(d) with $d \in [-0.4, 0.4]$, flat prior



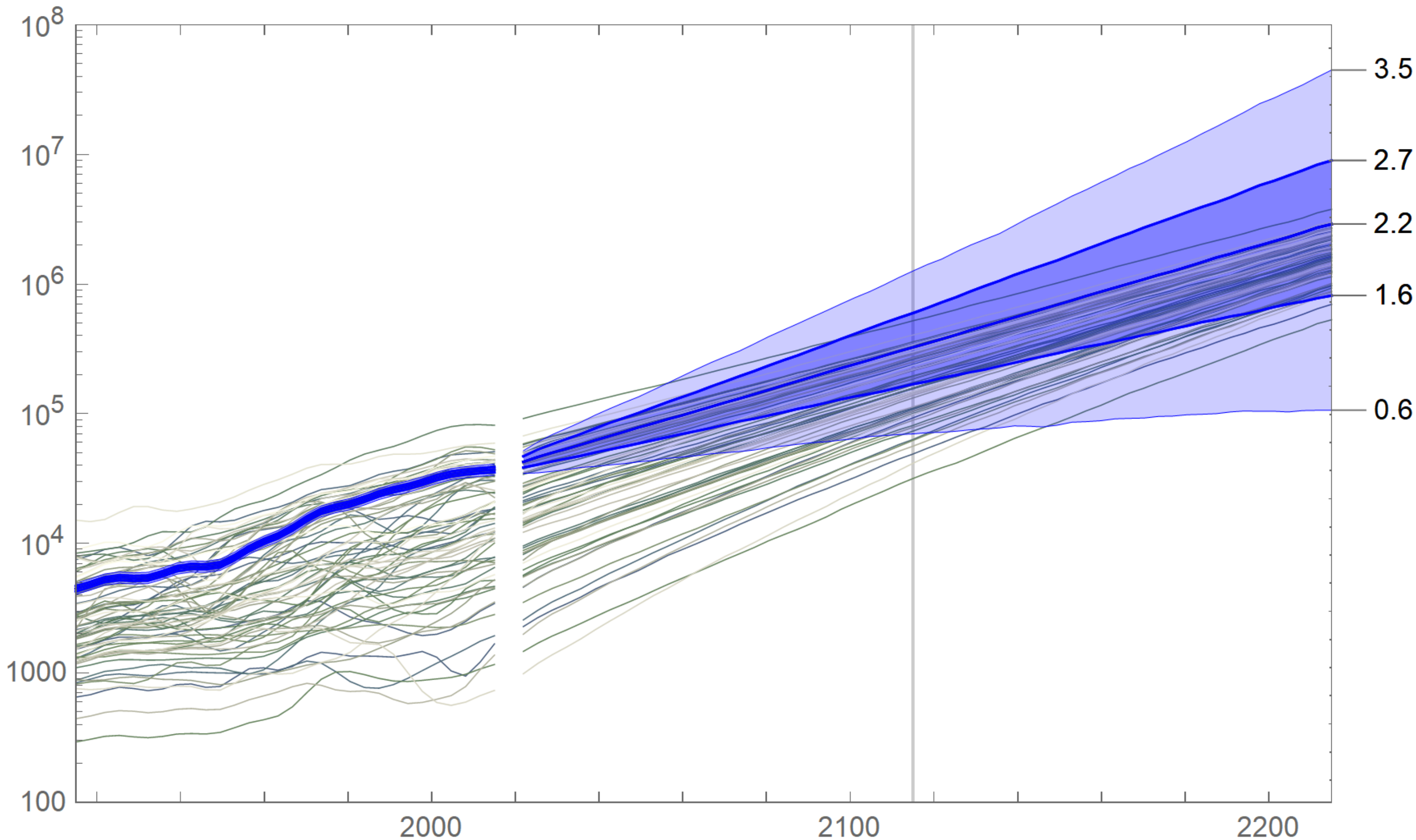
65 Year Sample: Baseline

Δf_t is LLM with $g \in [0, 5]$, flat prior



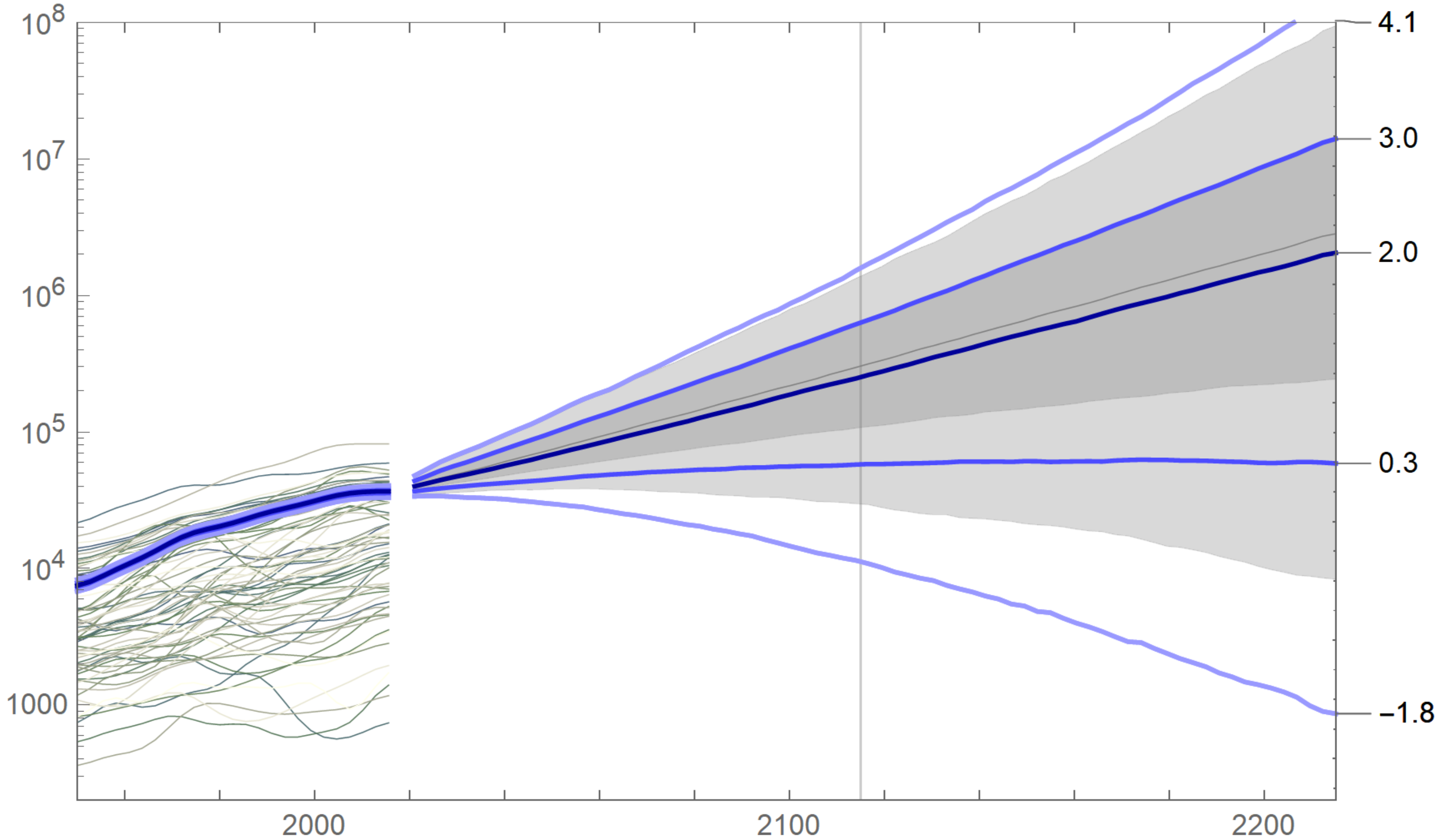
100 Year Sample: Baseline

Δf_t is LLM with $g \in [0, 5]$, flat prior



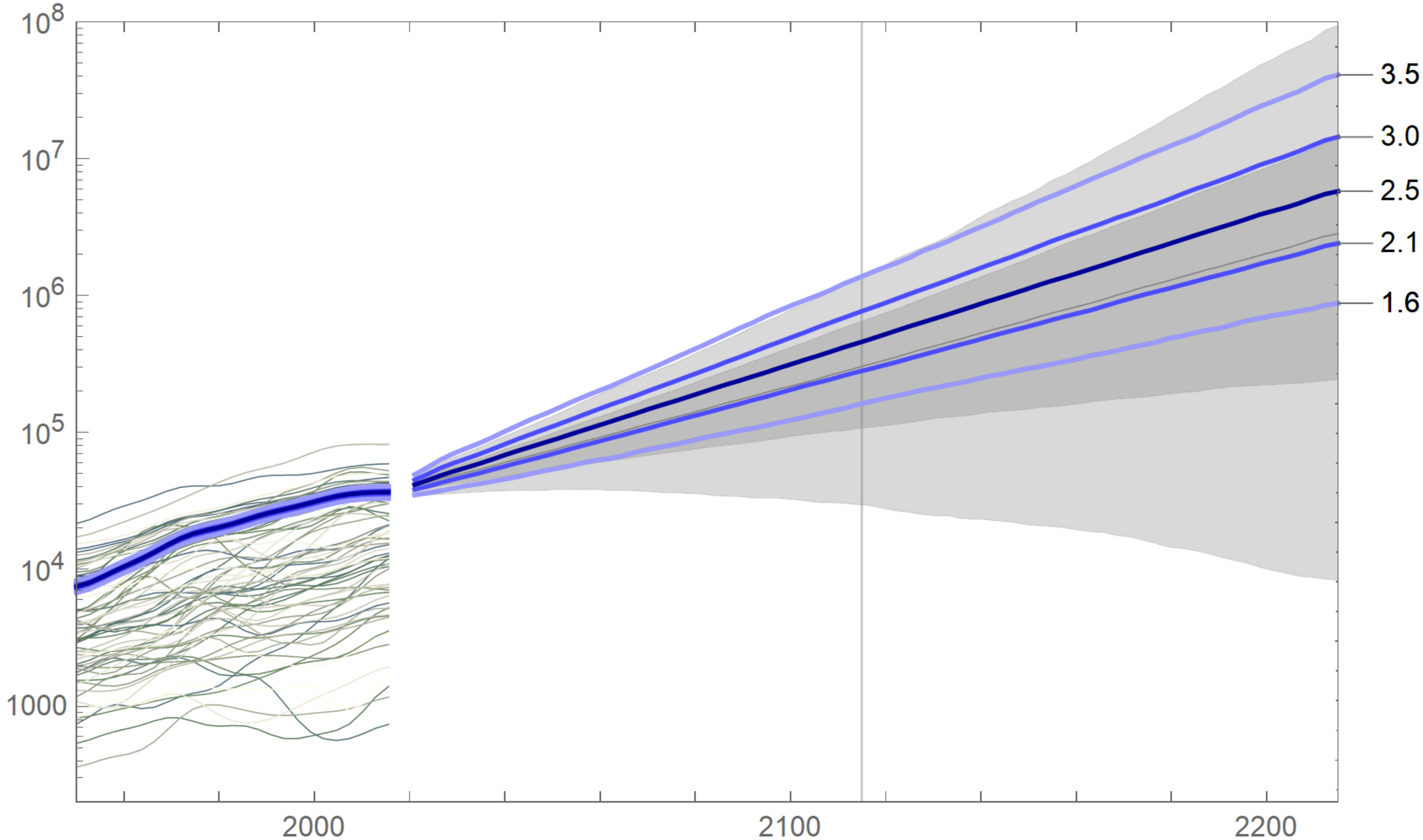
65 Year Sample: LLM with Larger g

Δf_t is LLM with $g \in [0, 10]$, flat prior



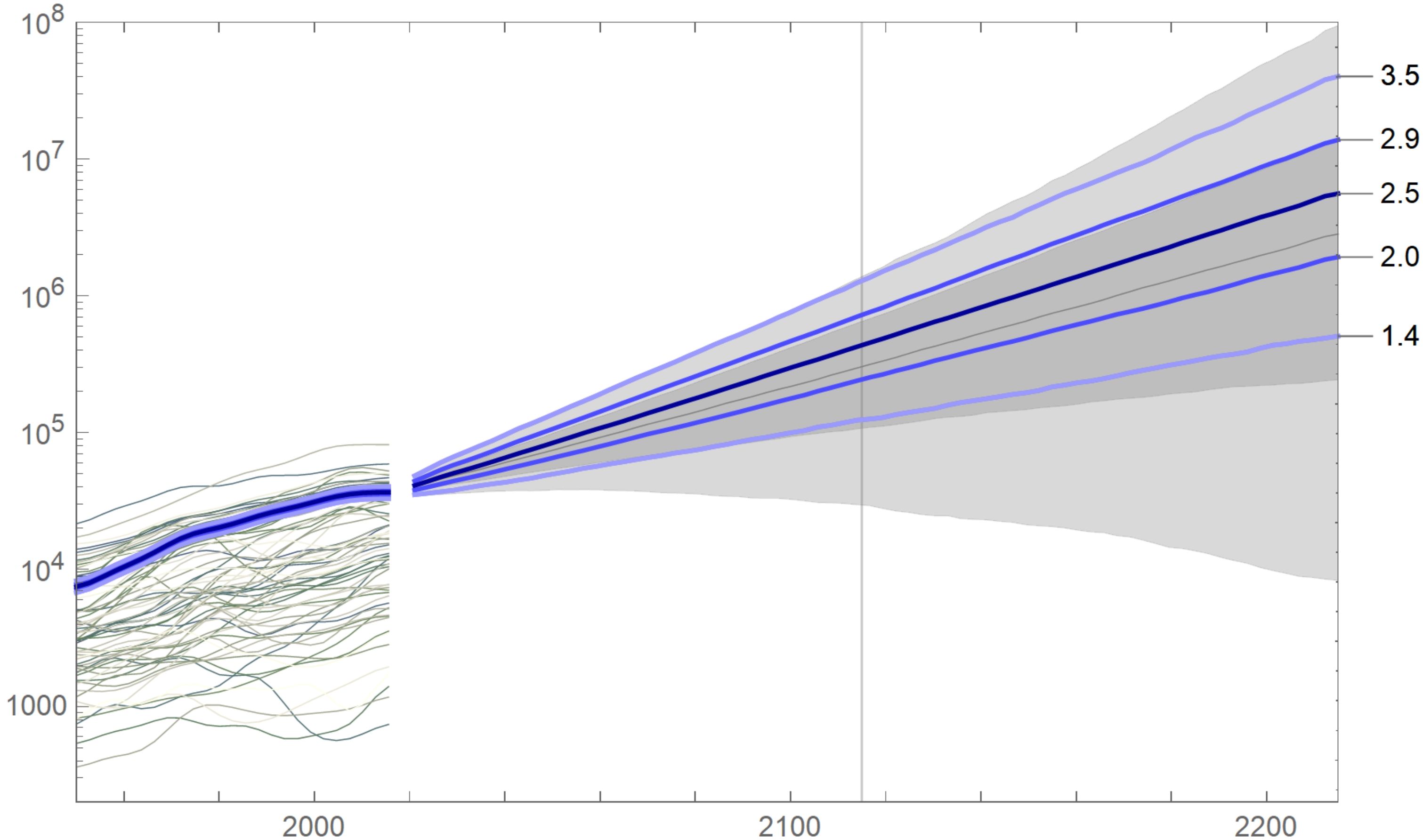
65 Year Sample: I(1) Factor

Δf_t is I(0) ($g = 0$ in LLM)



65 Year Sample: I(d) Factor

Δf_t is I(d) with $d \in [-0.4, 0.4]$, flat prior



Conclusion

- Low-frequency limited-information Bayes approach to panel forecasting

- Substantial heterogeneity across countries in

  - median growth

  - uncertainty of growth

  - cross-sectional correlations of growth

- Long-range data important to inform general uncertainty