Introduction to Econometrics (4th Edition)

by

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Solutions to Odd-Numbered End-of-Chapter Exercises: Chapter 12

(This version September 18, 2018)

12.1. (a) The change in the regressor, $\ln(P_{i,1995}^{cigarettes}) - \ln(P_{i,1985}^{cigarettes})$, from a \$0.50 per pack increase in the retail price is ln(8.00) − ln(7.50) = 0.0645. The predicted percentage change in cigarette demand is $-0.94 \times 0.0645 \times 100\% = -6.07\%$. The 95% confidence interval is (−0.94 ± 1.96 × 0.21) × 0.0645 × 100% = [−8.72%, −3.41%].

(b) With a 2% reduction in income, the predicted percentage change in cigarette demand is $0.53 \times (-0.02) \times 100\% = -1.06\%$.

(c) The IV regression in column (1) will not provide a reliable answer to the question in (b) when recessions last less than 1 year. The IV regression in column (1) studies the long-run price and income elasticity. Cigarettes are addictive. The response of demand to an income decrease will be smaller in the short run than in the long run.

(d) The instrumental variable would be too weak (irrelevant) if the F-statistic in column (1) was 3.7 instead of 33.7, and standard methods for statistical inference are unreliable. Thus the IV regression would not provide a reliable answer to the question posed in (a).

12.3. (a) The estimator $\hat{\sigma}_a^2 = \frac{1}{n-2}\sum_{i=1}^{n}(Y_i - \hat{\beta}_0^{TSLS} - \hat{\beta}_1^{TSLS}\hat{X}_i)^2$ is not consistent. Write this as $\hat{\sigma}_a^2 = \frac{1}{n-2}\sum_{i=1}^{n}(\hat{u}_i - \hat{\beta}_1^{TSLS}(\hat{X}_i - X_i))^2$, where $\hat{u}_i = Y_i - \hat{\beta}_0^{TSLS} - \hat{\beta}_1^{TSLS}X_i$. Replacing $\hat{\beta}_1^{TSLS}$ with β_1, as suggested in the question, write this as

$$\begin{aligned}\hat{\sigma}_a^2 &\approx \tfrac{1}{n}\sum_{i=1}^{n}(u_i - \beta_1(\hat{X}_i - X_i))^2 \\ &= \tfrac{1}{n}\sum_{i=1}^{n}u_i^2 + \tfrac{1}{n}\sum_{i=1}^{n}[\beta_1^2(\hat{X}_i - X_i)^2 + 2u_i\beta_1(\hat{X}_i - X_i)].\end{aligned}$$

The first term on the right hand side of the equation converges to $\hat{\sigma}_u^2$, but the second term converges to something that is non-zero. Thus $\hat{\sigma}_a^2$ is not consistent.

(b) The estimator $\hat{\sigma}_b^2 = \frac{1}{n-2}\Sigma_{i=1}^{n}(Y_i - \hat{\beta}_0^{TSLS} - \hat{\beta}_1^{TSLS}X_i)^2$ is consistent. Using the same notation as in (a), we can write $\hat{\sigma}_b^2 \approx \frac{1}{n}\Sigma_{i=1}^{n}u_i^2$, and this estimator converges in probability to σ_u^2.

12.5. (a) Instrument relevance. Z_i does not enter the population regression for X_i.

(b) Z is not a valid instrument. $\hat{X}^*$ will be perfectly collinear with W. (Alternatively, the first stage regression suffers from perfect multicollinearity.)

(c) W is perfectly collinear with the constant term.

(d) Z is not a valid instrument because it is correlated with the error term.

12.7. (a) Under the null hypothesis of instrument exogeneity, the *J* statistic is distributed as a χ_1^2 random variable, with a 1% critical value of 6.63. Thus the statistic is significant, and instrument exogeneity $E(u_i|Z_{1i}, Z_{2i}) = 0$ is rejected.

(b) The *J* test suggests that $E(u_i|Z_{1i}, Z_{2i}) \neq 0$, but doesn't provide evidence about whether the problem is with Z_1 or Z_2 or both.

12.9. (a) There are other factors that could affect both the choice to serve in the military and annual earnings. One example could be education, although this could be included in the regression as a control variable. Another variable is "ability" which is difficult to measure, and thus difficult to control for in the regression.

(b) The draft was determined by a national lottery so the choice of serving in the military was random. Because it was randomly selected, the lottery number is uncorrelated with individual characteristics that may affect earning and hence the instrument is exogenous. Because it affected the probability of serving in the military, the lottery number is relevant.