Introduction to Econometrics (4th Edition)

by

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Solutions to Odd-Numbered End-of-Chapter Exercises: Chapter 14

(This version March 29, 2023)

14.1 (a.i) $X^{oos}_{RPM} = (0.52 - 0.60) / 0.28 = -0.289$ and

$X^{oos}_{TExp} = (11.1 - 13.2) / 3.8 = -0.553$

(a.ii) $\hat{Y} = 750.1 - 48.7 \times (-0.289) + 8.7 \times (-0.553) = 759.4$

(b) $Y - \hat{Y}$ = 775.3 – 759.4 = 15.9

(c) $\left(\widehat{TestScore^* - 750.1}\right) = -48.7 \times (RPM^* - 0.60) / 0.28 + 8.7 \times (TExp^* - 13.2) / 3.8$

Rearranging yields:

$\widehat{TestScore^*} = 824.2 - 173.9 \times RPM^* + 2.29 \times TExp^*$

(d) $\widehat{TestScore^*} = 824.2 - 173.9 \times 0.52 + 2.28 \times 1.11 = 759.1$, where the difference arises from rounding error.

14.3 The standard error of the regression is an estimate of $\sigma_{u^{is}}$, where u^{is} is the regression error for the in-sample observations. The MSPE is an estimate of the variance the forecast error Y^{oos} − $Y^{oos} - \hat{Y}(X^{oos}) = u^{oos} + (\beta - \tilde{\beta})X^{oos}$, where for simplicity this assumes only one predictor (X^{oos}). The MSPE is then

$$MSPE = \text{var}\left(u^{oos} + (\beta - \tilde{\beta})X^{oos}\right) = \sigma^2_{u^{oos}} + \text{var}\left((\beta - \tilde{\beta})X^{oos}\right).$$

Assuming the out-of-sample population is the same as the in-sample population then $\sigma^2_{u^{is}} = \sigma^2_{u^{oos}}$, so that the square of the standard error of the regression estimates one component of the MSPE.

14.5 (a.i) The best predictor is $E(Y) = \mu = 2$.

(a.ii) The MSPE is the variance of Y, which is $\sigma^2 = 25$.

(b.i) $Y - \overline{Y} = (Y - \mu) + (\mu - \overline{Y})$ obtains by subtracting and adding μ to $Y - \overline{Y}$.

(b.ii) $E(Y - \mu) = E(Y) - \mu = \mu - \mu = 0$

$E(Y - \overline{Y}) = E(Y) - E(\overline{Y}) = \mu - \mu = 0$.

(b.iii) $\overline{Y}$ is computed from in-sample Ys, Y is out-of-sample and is independent of the in-sample observations. Because Y and $\overline{Y}$ are independent, they are uncorrelated.

(b.iv)

$$\begin{aligned} MSPE &= E\left[(Y - \overline{Y})^2\right] \\ &= E\left[\left\{(Y - \mu) - (\overline{Y} - \mu)\right\}^2\right] \\ &= E\left[(Y - \mu)^2\right] + E\left[(\overline{Y} - \mu)^2\right] + 2E\left[(Y - \mu)(\overline{Y} - \mu)\right] \\ &= \text{var}(Y) + \text{var}(\overline{Y}) \end{aligned}$$

where the first line is the definition of the MSPE, the second follows from (b.i), the third expands the second, and the final line uses (b.iii) and the definition of variance.

(b.v) $\text{var}(Y) = 25$ and $\text{var}(\overline{Y}) = 25/n$. The result follows because $n = 10$.

14.7 (a) The bias is $E(\overline{Y} - 1) - E(Y) = -1$.

(b) The mean of the prediction error is $E[Y - (\overline{Y} - 1)] = 1$.

(c) The variance of the prediction error is $\underline{var}[Y - (\overline{Y} - 1)] = 25 + 25/10 = 27.5$

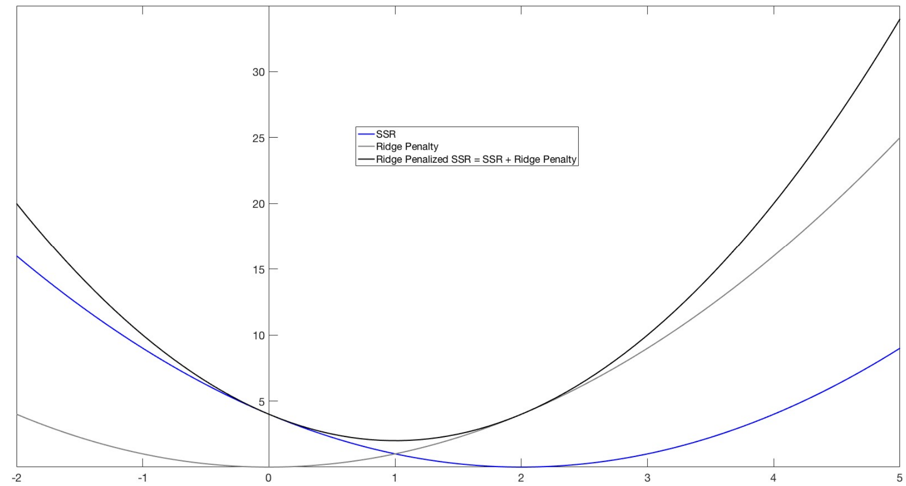
(d) This MSPE is the bias^2 + variance = 1 + 27.5 = 28.5.

(e) No, 28.5 > 27.5.

(f) No, 28.5 > 26.625.

14.9

(a)-(d)



(e)-(g) The ridge estimator is $\hat{\beta}^{Ridge} = \left(1 + \frac{\lambda^{Ridge}}{\sum_{i=1}^{n} X_i^2}\right)^{-1} \hat{\beta}$. In this problem $\sum_{i=1}^{n} X_i^2 = 1$ and $\hat{\beta}$= 2. Thus:

(e) $\hat{\beta}^{Ridge} = (1+1)^{-1}\hat{\beta} = 1$, (f) $\hat{\beta}^{Ridge} = (1+0.5)^{-1}\hat{\beta} = 4/3$, and (g) $\hat{\beta}^{Ridge} = (1+4)^{-1}\hat{\beta} = 2/5$

(h) As λ increases, the Ridge penalty becomes steeper, shifting the minimizing value of the Ridge-penalized SSR toward zero.

14.11 (a) The problem is

$$\max_{(w_1,w_2)} \text{var}(w_1x_1 + w_2x_2) \text{ subject to } w_1^2 + w_2^2 = 1.$$

Note that var($w_1x_1 + w_2x_2$) = $w_1^2 + w_2^2 + 2\rho w_1 w_2$, so the Lagrangian is

$$L = w_1^2 + w_2^2 + 2\rho w_1 w_2 + \gamma(w_1^2 + w_2^2 - 1)$$

yielding the first-order conditions:

$\partial L / \partial w_1 = 2w_1 + 2\rho w_2 + 2\gamma w_1 = 0$, $\partial L / \partial w_2 = 2w_2 + 2\rho w_1 + 2\gamma w_2 = 0$, and $w_1^2 + w_2^2 = 1$.

Inspection yields $|w_1| = |w_2|$ = sqrt(1/2). The objective function is $w_1^2 + w_2^2 + 2\rho w_1 w_2$, $\rho > 0$, so that the values of w_1 and w_2 that maximize the variance have the same sign. Thus $w_1 = w_2$ = sqrt(1/2) or $w_1 = w_2$ = −sqrt(1/2). Both solutions yield the same value of the objective function.

(b) This follows by showing that cov[($X_1 + X_2$)($X_1 - X_2$)] = 0.

(c) The result follows from var($w_1x_1 + w_2x_2$) = $w_1^2 + w_2^2 + 2\rho w_1 w_2$.