TESTING FOR REGRESSION COEFFICIENT STABILITY WITH A STATIONARY AR(1) ALTERNATIVE

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Abstract—We discuss the problem of testing for constant versus time varying regression coefficients. Our alternative hypothesis allows the coefficients to follow a stationary AR(1) process with unknown autoregressive parameter. Standard testing procedures are inappropriate since this parameter is identified only under the alternative. We propose a test statistic which is a function of a sequence of Score statistics, and depends only on the regressors and the OLS residuals. The distribution of the test statistic is discussed, power and size are investigated using Monte Carlo methods, and an empirical example investigating stability in the gold and silver markets is presented.

I. Introduction

The assumptions of the standard linear model are unrealistic in many economic applications. Because the assumptions may be unrealistic, it is important to subject any estimated linear model to various specification tests before the model is used for inference or forecasting purposes. In this note a test for one of the assumptions of the linear model, constant coefficients, is developed.

Many tests of the constancy of regression coefficients have been proposed. If the coefficients are suspected of discrete changes a Chow (1960) test or the test proposed by Quandt (1960) is appropriate. If the coefficients are suspected of changing smoothly through time, e.g., are generated by some economic process, another class of tests can be used. For these tests an ARIMA process is used as a proxy for the true generating process. Tests for coefficients suspected of following specific ARIMA processes have been proposed in the literature. A simple white noise process generates the random coefficients model, which can be tested using the Lagrange multiplier test of Breusch and Pagan (1979). For coefficients suspected of following a random walk, tests have been proposed by Brown, Durbin, and Evans (1975), Garbade (1977), Pagan and Tanaka (1979), LaMotte and McWhorter (1978), and a series of tests have been proposed and compared by Harvey and Phillips (1976). Cooley and Prescott (1976) introduced a model where the coefficients follow an ARIMA (0, 1, 1) process, and proposed a likelihood ratio test.

Rosenberg (1973) proposed a model where the coefficients follow a stable first-order Markov process, the so-called return to normalcy model. This is a particularly attractive specification, incorporating some of the best features of the random walk and the random coefficients models. The coefficients vary around a constant mean, a feature present in the random coefficients model, but also possess some inertia, a feature found in the random walk model.

Tests for the Rosenberg model have not been proposed, and a simple test for one varying coefficient based on the OLS residuals is developed in this note. In section II the model is presented and the testing problem is formulated. Section III discusses a solution to the testing problem based on a suggestion by Davies (1977). Section IV presents some Monte Carlo results summarized in an estimated response surface. Section V presents an empirical example, and the final section contains some concluding remarks.

II. The Rosenberg Model

For a single time varying coefficient the Rosenberg model may be written as

$$y_t = x_t'\gamma + z_t\beta_t + \epsilon_t$$
$$(\beta_t - \bar{\beta}) = \phi(\beta_{t-1} - \bar{\beta}) + u_t \qquad |\phi| < 1$$

where x_t is a $k \times 1$ vector, z_t is a scalar, γ is a $k \times 1$ vector of unknown constant coefficients, β_t is a time varying coefficient, and ϵ_t and u_t are independent gaussian white noise disturbances with variances r and q, respectively.[1] The parameters of the model can be estimated using the nonlinear maximum likelihood procedures described in Pagan (1980) and Watson and Engle (1983). When the coefficient β_t is constant, maximum likelihood estimation is greatly simplified. It is, of course, just ordinary least squares (OLS). Equation (2) implies that $\beta \sim N[\bar{\beta}, q(1-\phi^2)^{-1}]$ so that β_t is constant if and only if $q = 0$. The hypotheses of interest are therefore:

$$H_0: \quad q = 0$$
$$H_A: \quad q > 0.$$

One is tempted to use a standard large sample test, a likelihood ratio, a Wald, or a Lagrange multiplier test; however, these tests cannot be used in the usual fashion. A problem arises because the transition parameter, ϕ, is identified under H_A but not under H_0. With $q = 0$ any $\phi \in (-1, 1)$ yields the same value of the likelihood function. This implies that the information matrix will be singular under H_0, a violation of one of the standard

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[1] The results are only slightly modified if ϵ_t follows an ARMA process. See Harvey and Phillips (1979).

regularity conditions required to derive the usual asymptotic distribution (and local equivalence) of the tests listed above. To overcome this problem Davies (1977) has proposed a testing procedure which is discussed and applied to the varying coefficient problem in the next section.

III. The Davies Test

The problem with the usual Wald, likelihood ratio, and Lagrange multiplier (LM) testing procedures arises because of the unknown and unidentified parameter ϕ. Let's assume for a moment that ϕ were known so that any of the tests could be carried out in the usual way. The LM test requires estimation only under the null (i.e., OLS) so that it is the easiest of three to perform. The test statistic is a function of the exogenous variables, the OLS residuals and ϕ. Since the value of ϕ doesn't matter under the null, the LM test will have the correct size (asymptotically) regardless of the value of ϕ. With ϕ unknown, the test can be carried out using any arbitrary value of the parameter. The size of the test will not be affected by the value of ϕ chosen, but the power of the test will be affected. Davies has suggested that Roy's Union-Intersection Principle be applied in cases such as this so that the null hypothesis should be rejected if the LM test statistic is "large" when evaluated at any value of ϕ.

Specifically, let $S(\bar{\phi})$ be the normalized element of the score vector corresponding to q evaluated under the null and assuming that $\phi = \bar{\phi}$ were known. Then, under the usual regularity conditions, $S(\bar{\phi})$—the (sign corrected) square root of the LM test statistic—is asymptotically a standard normal random variable, and the Davies test statistic is

$$D \equiv \left\{ \operatorname*{Sup}_{-1<\phi<1} S(\phi) \right\}.$$

This test statistic is asymptotically locally equivalent to the Wald test statistic and the (sign corrected) square root of the likelihood ratio test statistic. Since our alternative is one-sided we reject the null when D is greater than some critical value, CV.

Unfortunately, in the model under consideration a closed form solution for $\{\text{Sup}_{-1<\phi<1} S(\phi)\}$ is difficult to derive so that the test statistic D cannot be formed. However, it can be approximated. We can, for example, carry out a simple grid search over n different values of ϕ. This approximation leads to an approximate Davies test statistic defined as

$$AD = \max\{ S(\phi_i);\ i = 1,2,\ldots,n \text{ with } -1 < \phi_i < \phi_{i+1} < 1 \}.$$

To find the critical value for the test notice that

$$\text{prob}(AD > CV) = 1 - \text{prob}[(S(\phi_1) < CV), (S(\phi_2) < CV), \ldots, (S(\phi_n) < CV)]. \quad (1)$$

Under the usual regularity conditions the asymptotic distribution of $[S(\phi_1), S(\phi_2), \ldots, S(\phi_n)]$ is multivariate normal so that this probability can, in principle, be calculated, and a value of CV equating the probability of type 1 error to the desired size of the test can be found. In practice, evaluation of the multivariate normal distribution function for $n > 5$ is very difficult (see Johnson and Kotz (1972), ch. 35). This suggests that approximations or bounds on the probabilities must be used.

A very useful and tight upper bound on the probability given in (1) which requires only the evaluation of the univariate and bivariate normal distributions is easy to derive. To motivate the bound, suppose that the event $(AD > CV)$ occurs. Then it must have been the case that one of the following events occurred:

$$(A_0)\, S(\phi_1) > CV$$

or

$$(A_1)\, S(\phi_1) < CV < S(\phi_2)$$

or

$$\vdots$$

$$(A_n)\, S(\phi_{n-1}) < CV < S(\phi_n).$$

So that the event $(AD > CV)$ is the union of the events $A_0, A_1, A_2, \ldots, A_n$. Since the probability of the union is bounded above by the sum of the probabilities we have

$$\text{prob}(AD > CV) \le \sum_{i=0}^{n} \text{prob}(A_i).$$

The probabilities of the events A_i are readily calculated (asymptotically) since they require evaluation only of the univariate and bivariate normal distributions.[2]

In an earlier expanded version of this paper (Watson (1982)), the derivation of $S(\phi)$ and regularity conditions which justified the assertions concerning asymptotic distributions made above, were presented. In that

[2] Of course, alternative bounds are available which are even easier to calculate. The most obvious is the Bonferroni bound

$$P(AD \ge CV) \le nF(-CV),$$

where F is a standard normal distribution function. While easier to calculate, the Bonferroni bound is always "looser" than the bound suggested above as it merely replaces the values

$$P[S(\phi_i) \le CV \le S(\phi_{i+1})]$$

with the larger values

$$P[CV \le S(\phi_{i+1})].$$

paper it was shown that

$$S(\phi) = \frac{s_1 + s_2(\phi)}{s_3(\phi)}$$

where

$$s_1 = \frac{1}{2}\sum_{t=1}^{T} z_t^2\left(\frac{\hat{\epsilon}_t^2}{\hat{r}} - 1\right)$$

$$s_2(\phi) = \frac{1}{\hat{r}}\sum_{t=2}^{T} \hat{\epsilon}_t z_t \sum_{i=1}^{t-1} \hat{\epsilon}_i z_i \phi^{t-i}$$

$$s_3(\phi) = \left\{\frac{1}{2}\sum_{t=1}^{T} z_t^4 + \sum_{t=2}^{T} z_t^2 \sum_{i=1}^{t-1} z_i^2 \phi^{2(t-i)} - \frac{1}{2T}\left(\sum_{t=1}^{T} z_t^2\right)^2\right\}^{1/2},$$

and a caret over a variable indicates the OLS estimate.

The interpretation of $S(\phi)$ is straightforward. If the alternative is true and the model is estimated by ordinary least squares, then we would expect the residuals to exhibit both heteroskedasticity and serial correlation. The first term in the numerator checks for heteroskedasticity, while the second term checks for serial correlation. The denominator is just a normalizing constant.

Two extreme cases are helpful in interpreting the test statistic. First assume the ϕ is 0, so that only heteroskedasticity is present. Then $S(\phi)$ is asymptotically equivalent to $\hat{\rho}\sqrt{T}$, where $\hat{\rho}$ is the sample correlation between z_t^2 and $\hat{\epsilon}_t^2$. This is a version of the Lagrange multiplier test for heteroskedasticity proposed in Breusch and Pagan (1979). Second, assume that $z_t = 1$ for all t. The errors are homoskedastic, but are generated by an ARMA (1,1) process[3] if $\phi \neq 0$. In this case the test is asymptotically equivalent to

$$T^{1/2}(1 - \phi^2)^{1/2} \sum_{i=1}^{T-1} \hat{\rho}_i \phi^i,$$

where $\hat{\rho}_i$ is the i^{th} autocorrelation coefficient of the OLS residuals.

[3] It is straightforward to show that if

$$(1 - \phi B)a_t = (1 - \alpha B)e_t$$

with e_t white noise, and

(a) $0 < |\alpha| \leq |\phi| < 1$
(b) $\phi\alpha > 0$,

then a_t can be written as

$$a_t = \xi_t + \eta_t$$
$$\eta_t = \phi\eta_{t-1} + v_t \qquad |\phi| < 1$$

where v_t and ξ_t are independent white noises. The AD test, with $z_t = 1$, can then be used to test for this class of ARMA (1, 1) disturbances. Godfrey (1978) points out that the standard Lagrange multiplier test is inappropriate for testing white noise vs. ARMA (1, 1) errors.

IV. Monte Carlo Results

The Monte Carlo experiment was designed with the estimation of response surface in mind; see for instance Mizon and Hendry (1980). For this reason the number of replications are small, since between experiment results are used to improve the efficiency of the estimated power function. The model used for the experiment included an intercept and one time varying parameter. The parameters $\bar{\beta}$ and r were constant throughout the experiment with values $\bar{\beta} = 2.0$ and $r = 1.0$. The regressor z_t was generated from an AR(1) process with variance 25 and autocorrelation coefficient α. The design variables are ϕ, α, T, and q. We chose a latin square design using $\phi = (0.0, 0.45, 0.90)$, $\alpha = (0.0, 0.4, 0.8)$, and $T = (30, 60, 100)$. The experiment was run for $q = 0.0$, 0.1, 0.4, 0.7, and 1.0. For each experiment with $q = 0$, 400 simulations were carried out. We ran 100 simulations otherwise. To maintain as much control as possible between experiments, variables were held fixed where possible. So, for instance, the same z series was used for all simulations with the same α, and the same ϵ vector was used for the i^{th} simulation in all experiments.

Some of the results are presented in table 1, for $q = 0$ (β constant), and $q = 0.1$ and 1.0. (Detailed results on all experiments are reported in Watson (1980).) The column labeled AD presents the results for the approximate Davies test searching over ϕ from 0.0 to 0.95 in steps of 0.05. The column labeled $S(0)$ presents the results treating $S(0)$ as a standard normal random variable, i.e., the Breusch and Pagan (1979) heteroskedasticity test. The next two columns show the results of tests using $S(.45)$ and $S(.95)$ as standard normal random variables. The final column shows the results using the Durbin-Watson (lower) bounds test.

The results show that the calculated size of the AD and S tests are less than the nominal size of the test. The size of the AD test is very similar to the size of the $S(0)$ and $S(.45)$ tests. The table suggests that the test has reasonable power, which increases with T, ϕ, and q. The test performs well compared to the $S(0)$ and $S(.45)$ tests even when these tests are locally asymptotically optimal. (This of course wouldn't be known in practice since ϕ_0 would be unknown.) Conversely, the $S(0)$ test worked well even when the value of ϕ was 0.9, but not as well as the AD test.[4]

[4] The poor performance of the $S(.95)$ test stands out in all of the tables. Even when the true value of ϕ is 0.90, the $S(.95)$ test performs poorly. This seems to result from the asymptotic nature of the test. The random variable $S(.95)$ is distributed standard normal only asymptotically, and clearly not for the small sample sizes considered in this study. Based on 200 simulations with a sample size of 100 and the null hypothesis true, the sample mean of $S(.95)$ was -0.62, while its standard error was 0.43. This negative mean and small standard error are responsible for the small number of rejections. Increasing the sample size to 1000, produced an $S(.95)$ which did appear standard normal.

TABLE 1.—MONTE CARLO RESULTS

Parameters			Percentage of Rejections of H_0[a]				
T	α		AD	$S(0)$	$S(.45)$	$S(.95)$	DW
			$q = 0.0$				
30	0.0		0.8	0.8	1.3	0.0	2.3
60	0.0		2.0	2.5	2.3	0.0	5.3
30	0.4		1.5	2.3	2.0	0.0	3.3
60	0.4		3.5	3.0	2.8	0.3	4.3
30	0.8		1.0	0.8	1.0	0.0	2.0
60	0.8		1.3	1.5	1.0	0.3	3.3
Parameters			Percentage of Rejections of H_0[a]				
ϕ	T	α	AD	$S(0)$	$S(.45)$	$S(.95)$	DW
			$q = 0.1$				
.00	30	0.0	50	63	51	0	4
.00	60	0.4	81	85	75	16	2
.45	30	0.8	43	41	50	0	25
.45	60	0.0	93	89	94	20	10
.90	30	14	83	64	85	37	7
.90	60	0.8	98	86	97	83	99
			$q = 1.0$				
.00	30	0.0	76	86	68	1	4
.00	60	0.4	92	92	91	22	2
.45	30	0.8	62	53	67	0	45
.45	60	0.0	99	100	99	27	12
.90	30	0.4	92	81	94	45	10
.90	60	0.8	98	90	99	87	99

[a]Using 5% asymptotic critical values for $S(0)$, $S(.45)$, $S(.95)$.

While the table shows the estimated power at specific points in the parameter space, ideally one would like to estimate a function relating the power of the test to the parameters of the model. Using the methodology employed in Mizon and Hendry (1980) and the data generated in the Monte Carlo experiment we have estimated this power function. The functional form of the power function is unknown, but we can choose a form that is suggested by the asymptotic behavior of $S(\phi_0)$, where ϕ_0 is the true value of ϕ. In particular we choose a functional form that allows the power of the test to approach 1 as $q \to \infty$ or as $T \to \infty$ (if $q > 0$), and that allows ϕ_0 to affect the power but not the size of the test.

There are many functional forms with these properties. After some experimentation we found the following form satisfactory:

$$\ln\left[\frac{\hat{P}}{1-\hat{P}}\right] = \pi_0 + \pi_1 D + \pi_2 \ln(q+1) + \pi_3 \frac{\phi D}{\ln T} + \pi_4 \frac{\alpha}{\ln T} + \pi_5 D \ln T + e$$

where $\hat{P}$ is the estimated power calculated from the Monte Carlo experiment and D is a dummy variable taking on the value 1 if $q > 0$ and zero otherwise. To correct for heteroskedasticity all variables were multiplied by $[N\hat{P}(1-\hat{P})]^{1/2}$ where N is number of replications. (After this weighting a correctly specified model should have an error term with unit variance. This can serve as a check on the specification.)

The estimated results for the test carried out at the 5% level are presented in table 2. The results appear to be quite sensible. The standard error is close to 1, and the R^2 is high. The power of the test increases with q, ϕ, and T (if $q > 0$) as expected. Letting $T \to \infty$, the estimated size of the test is 0.0339, indicating that the bound in (4) is satisfied.

V. An Empirical Example

The test presented in section III has been successively used in a variety of empirical applications. It was used and compared to other tests of parameter instability by Beck (1983) in a study of Federal Reserve reaction functions, and by Bos and Newbold (1984) in their

TABLE 2.—ESTIMATED RESPONSE SURFACE

Coefficient	Estimate	Standard Error
π_0	−3.35	.140
π_1	−6.98	.779
π_2	1.64	.269
π_3	7.93	.789
π_4	−5.76	.693
π_5	3.04	.203
S.E. = 1.021		$R^2 = 0.982$

TABLE 3.—EMPIRICAL RESULTS FOR THE MODEL[a]
$R_t = \mu + \beta r_t + \epsilon_t$

	μ	β	$\hat{\sigma}_\epsilon$	DW	AD[b]	$\hat{\phi}$[c]	P[d]
Silver	0.6	0.75	144.1	2.29	0.83	−.65	.75
	(51.5)	(9.3)					
Gold	−22.5	4.34	123.0	1.83	3.66	.20	.001
	(44.0)	(7.9)					

Maximum Likelihood Allowing
$(\beta_t - \bar{\beta}) = \phi(\beta_{t-1} - \bar{\beta}) + u_t$

	μ	$\bar{\beta}$	ϕ	σ_ϵ	σ_u
Gold	−7.4	2.91	.18	85.3	15.5
	(5.23)	(9.8)	(.16)	(23.2)	(4.6)

Note: Standard errors are in parentheses.
[a] Rates of return R_t and r_t are annual rate in percentage points.
[b] AD was calculated for $\phi \in [-.95, .95]$ in steps of 0.05.
[c] $\hat{\phi}$ is arg max $AD(\ )$.
[d] P is the upper bound on the prob value for AD.

investigation of stability of the CAPM model for 464 stocks. In this section we use the test to check for stability in an efficient market model for gold and silver prices. We'll let R_t denote the one period holding yield on these metals and ER_t denote the expectation of this yield formed at the beginning of the period. A popular specification sets $ER_t = \tilde{\mu} + \beta r_t$, where μ is a time invariant risk premium and r_t is the risk free rate of return which is assumed to be known by market participants at the beginning of the period. Efficiency in the asset market implies that the excess return, $R_t - ER_t$, is uncorrelated with information known at the beginning of the period, so that

$$R_t = \mu + \beta r_t + e_t$$

where e_t is a serially uncorrelated error term which is uncorrelated with current and lagged values of r_t, and the coefficient β is equal to one for all time periods.

We have estimated this model using weekly data on gold and silver prices over the period 1975–1979. The risk free rate of return is the return on 90-day Treasury bills with 1 week remaining until maturity.[5] All data were taken from the *Wall Street Journal*. The results are shown in table 3. The week-to-week returns on these assets are quite volatile leading to very poor fits, so that tests of the hypothesis $\beta = 1$ have very low power, and the hypothesis is not rejected for either metal. The table also presents the results of the Davies test for time varying β's. In spite of the large amount of noise in the model the hypothesis is decisively rejected for gold. We re-estimated the model for gold allowing for time variation in the parameter. The results are shown at the bottom of the table. The estimates suggest the time variation is present. Notice, however, that ϕ is not significantly different from zero, suggesting that the direction of the change in β_t is not predictable.

[5] A listing of the data is available from the author.

VI. Conclusions

The results of the last two sections indicate that the AD test is quite useful for detecting coefficient instability. The test statistic is relatively easy to construct, and conservative critical values can be quickly computed. The heteroskedasticity test of Breusch and Pagan also performed quite well in the Monte Carlo simulations. The simulations suggest that the AD test is preferred to the BP test when the sample size is moderate and the value of the transition parameter is reasonably large. We would also expect the AD test to outperform the BP test when the variance of the regressor, z_t, is small. This would reduce the heteroskedasticity in the OLS residuals and lower the power of the BP test. Serial correlation would, however, still be present, and this behavior could be indicated by the AD test.

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BIAS FROM NONSYNCHRONOUS TRADING IN TESTS OF THE LEVHARI-LEVY HYPOTHESIS

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It is well established that beta estimates are biased due to nonsynchronous trading (Dimson, 1979, 1981; Fowler and Rorke, 1981; Fowler, Rorke and Jog, 1980, 1981; Scholes and Williams, 1977, and Roll, 1981). Yet, researchers often fail to consider the effect of nonsynchronous trading on tests of hypotheses concerning the behavior of beta. This paper examines specific hypotheses about the behavior of beta developed by Levhari and Levy (1977) (described fully below). Evidence is presented which shows that results of tests based on all of the data (including data contaminated by nonsynchronous trading) may produce conclusions which differ from those which prevail when only the data not severely contaminated by nonsynchronous trading are used. This evidence clearly indicates the potential importance of considering nonsynchronous trading in tests of hypotheses concerning beta.[1]

The paper by Levhari and Levy has two parts. The first mathematically derives a systematic relationship between beta and the investment horizon. The second part examines the extent to which the empirical evidence conforms with the predictions of the mathematical model. Rejection of the mathematical model on empirical grounds may result from failure of the assumptions of the model (such as independence of returns over time) to hold or from measurement errors in the data due to factors such as nonsynchronous trading. This paper examines whether nonsynchronous trading has obfuscated previous tests of the relationship between beta and the investors' horizon postulated by Levhari and Levy.

This paper is divided into four parts. The next section provides a discussion of previous research related to this topic. The second section describes the methodology and the third presents the results. The final section provides a summary and conclusions.

Background

Levhari and Levy (1977) suggest that for "risky" securities (true $\beta > 1$), the longer the differencing interval the higher the estimated beta. On the other hand, for less risky securities (true $\beta < 1$), the longer the differencing interval the lower the estimated beta. For differencing intervals of one month and more, Levhari and Levy (1977, 1981) and Smith (1978) offer empirical

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