Errors in Variables and Seasonal Adjustment Procedures

JERRY A. HAUSMAN and MARK W. WATSON*

Seasonal adjustment procedures attempt to estimate the sample realizations of an unobservable economic time series in the presence of both seasonal and irregular factors. In this article, we consider a factor that has not been considered explicitly in previous treatments of seasonal adjustment: measurement error. Because of the sample design used in the Current Population Survey, measurement error will not be a white-noise process, but instead it will be characterized by serial correlation of a known form. We first consider what effect the serially correlated measurement error has on estimation of the nonseasonal component in seasonal adjustment models. We also consider the effect of measurement error on the widely used seasonal adjustment process X-11. Estimated unobserved-components models are used to estimate the precision (root mean squared error) of the official and optimal seasonally adjusted data.

KEY WORDS: Signal extraction; Measurement error; Unobserved-components model; Current Population Survey; Unemployment rate.

1. INTRODUCTION

The classic representation of economic time series decomposes the observed series into unobserved seasonal and nonseasonal components. The seasonal component represents the movements in the series caused by fairly regular seasonal phenomena such as weather, holidays, and school vacations. The nonseasonal component is the remainder, which is sometimes further decomposed into a "trend-cycle" component (which represents fairly smooth movements in the series) and an "irregular" component (representing the residual-jagged movements in the series). Within an additive framework, the decomposition can be represented as

$$x_t = n_t + s_t, \tag{1.1}$$

where x_t is the observed series and n_t and s_t are the unobserved nonseasonal and seasonal components, respectively. Seasonal adjustment procedures attempt to estimate the sample realizations of the unobserved nonseasonal component by using the observed time series. Equivalently, seasonal adjustment attempts to purge the series of its seasonal component.

Many widely used economic time series are measured with error, so a more appropriate decomposition takes the form

$$x_t^* = n_t + s_t + e_t, \tag{1.2}$$

where e_t represents the measurement error. The concept of measurement error is somewhat nonstandard, since we do not have a "real" series that we are attempting to measure. That is, n_t does not exist apart from the model specification of the components of (1.2). It still makes sense, however, to consider an underlying series, say x, that in principle could be measured perfectly. The reformulated model has $x_t^* = x_t + e_t$. With this new decomposition, the problems of "seasonal adjustment" and the problem of "estimation of n_t" are somewhat different. Seasonal adjustment purges x_t^* of the seasonal component, whereas the estimation of n_t requires that both s_t and e_t be purged from the observed series.

The usual model for measurement error in statistics and econometrics is a white-noise process. Since the irregular component of the nonseasonal is often modeled as a white-noise or low-order moving average process, it might be suggested that (1.2) corresponds closely to the classical decomposition, with e_t interpreted as the irregular component and n_t interpreted as the trend-cycle component. In this article, we show that the white-noise measurement error process may be a very poor approximation to the process generating measurement error in many economic time series. This follows because of government data collection procedures that use overlapping sample designs. These designs produce measurement error that in general will exhibit complicated forms of serial correlation.

For instance, the sample design used by the Bureau of Labor Statistics (BLS) to measure unemployment consists of eight subsamples, seven of which have been included in previous samples. This sample design is chosen presumably to induce positive correlation across months, which leads to a reduction of measurement error in estimated changes in the unemployment rates. So far as we know, no seasonal adjustment procedure takes explicit account of this measurement error.

The natural question is, what effect does this serially correlated measurement error have on estimation of the nonseasonal component n_t from Equation (1.1)? This cannot be answered apart from an examination of the goals of seasonal adjustment in the specification of the components of (1.1), since the true n_t is not observed and does not exist apart from a specific model formulation. Nerlove et al. (1979) and Pierce (1976) discussed the equation of the goal of seasonal adjustment, with Pierce setting forth a set of assumptions to guide his procedure. We choose the minimum mean squared error estimator of n_t given hypothesized autoregressive moving average models with exogenous variables (ARMAX models) for both n_t and s_t. Thus we allow for both deterministic and stochastic components in the model. After estimating the parameters of the component models, seasonal adjustment is accomplished by signal extraction techniques. When measurement error is ignored, the estimates of both n_t and s_t will be determined by the similarity of their spectra and that of the mea-

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surement error. Our empirical examples demonstrate that the increase in mean squared error can be substantial.

After comparing optimal seasonal adjustment procedures that ignore measurement error with an optimal method that incorporates measurement error, we consider the effect of measurement error on X-11. X-11 is the seasonal adjustment procedure used by the BLS and many other government agencies. It is not a model-based procedure; it does not estimate component processes for n_t and s_t and then construct minimum mean squared error estimates. Rather, X-11 consists of a set of ad hoc filters that have been found to give "reasonable looking" results for many economic time series. Using a linear approximation to the X-11 filter and the component models for n_t, s_t, and e_t, which underly the optimal filter, we can estimate the mean squared error of the X-11 estimate. Since X-11 is a moving average filter, the increase in the mean squared error brought about by measurement error is less than the variance of the measurement error. (Kaitz 1978 made this point.) Our results demonstrate the importance of this point.

An important by-product of this exercise is an estimate of the standard error of both the currently announced and the historical value of the level and the month-to-month change of the seasonally adjusted unemployment rate. The need for such standard errors has long been recognized. In its 1962 report, the President's Committee to Appraise Employment and Unemployment Statistics (the Gordon Commission) recommended "that estimates of the standard errors of seasonally adjusted data be prepared and published as soon as the technical problems have been solved" (p. 151). This recommendation was reiterated by the National Commission on Employment and Unemployment Statistics in 1978. As a preview of one of the results, our model suggests that the standard error of the currently announced level is .16% and the currently announced month-to-month change is .13%.

The plan of the article is as follows. In Section 2, we consider the signal extraction problem for n_t when n_t, s_t, and e_t have been assigned autoregressive moving average (ARMA) specifications (deterministic components are easily extracted by regression techniques). We derive expressions for the effect of omitting measurement error on otherwise optimal filters. In Section 3, we discuss the sampling procedure used by the BLS and derive the specification of measurement error for this particular sample design. We then estimate the unknown parameters of the model by maximum likelihood methods and employ a variety of diagnostic statistics as checks on our specification. Models for both the overall unemployment rate and the teenage unemployment rate are estimated. Since the latter series has considerably more variance, it offers a good comparison to the overall series. Section 4 compares the optimal procedure with X-11 and with an optimal filter that ignores measurement error. For the overall unemployment rate, the three procedures give similar results, with the efficiencies ranging from .81 to .94. For teenage unemployment, the three procedures give very dissimilar results, with the efficiency of the optimal filter that ignores measurement error as low as .33 and of X-11 as low as .15.

2. GENERAL ANALYSIS

In the last section, we defined seasonal adjustment [in the context of Eq. (1.1)] as a procedure for estimating the unobserved nonseasonal component by using data on the observed composite series. Measurement error simply adds another component to the model. The observed series is now "contaminated" not only with seasonality but with measurement error as well. The presence of this extra component reduces the precision with which the nonseasonal component can be estimated.

We will ignore any deterministic components and assume that the nonseasonal component, n_t, and the seasonal component, s_t, are generated by independent, stationary, invertible moving average processes of the form

$$n_t = \theta_n(B)\varepsilon_t \tag{2.1}$$

$$s_t = \theta_s(B)\eta_t, \tag{2.2}$$

where ε_t and η_t are white noise and $\theta_n(B)$ and $\theta_s(B)$ are polynomials in the backshift operator B. [The stationarity assumption is not crucial for what follows. An investment in additional notation would allow us to assume that $\Delta_n(B)n_t$ and $\Delta_s(B)s_t$ are stationary, where $\Delta_n(B)$ and $\Delta_s(B)$ are differencing operators with no common factors. This generalization is discussed in Cleveland and Tiao (1976), Bell (1984), and Burridge and Wallis (1983).] We will assume that the measurement error, e_t, is generated independently of n_t and s_t by another stationary, invertible moving average process,

$$e_t = \theta_e(B)\xi_t, \tag{2.3}$$

where ξ_t is white noise. Finally, let x_t be the series composed of only n_t and s_t, and let x_t^* be composed of n_t, s_t, and e_t:

$$x_t = n_t + s_t \tag{2.4}$$

$$x_t^* = n_t + s_t + e_t. \tag{2.5}$$

The problem of estimating n_t given x_t or x_t^* can be solved by using signal extraction techniques. In both cases, n_t is estimated from "noisy" measurements. When x_t is observed, the noise is s_t; when x_t^* is observed, the noise is the sum of s_t and e_t.

In general, the signal extraction procedure for n_t depends on the time subscript t. When t is not near the beginning or the end of the sample, the procedure for forming the minimum linear mean squared error estimates of n_t and n_{t+1} are very similar. Indeed, they can be made arbitrarily close by extending the sample at both ends. To avoid the notational burden of extra time subscripts, we will assume that a complete realization of the observed series is available. If the observed series is uncontaminated with measurement error, the linear minimum mean squared error estimate of n_t is given by

$$\hat{n}_t = V(B)x_t, \tag{2.6}$$

where

$$V(B) = \sum_{i=0}^{\infty} V_i(B^i + B^{-i}) \tag{2.7}$$

and the coefficients V_i can be formed from (see Whittle 1963)

$$V(z) = \frac{\sigma_\varepsilon^2\theta_n(z)\theta_n(z^{-1})}{\sigma_\varepsilon^2\theta_n(z)\theta_n(z^{-1}) + \sigma_\eta^2\theta_s(z)\theta_s(z^{-1})}. \tag{2.8}$$

The seasonal adjustment error associated with the filter $V(B)$

is given by

$$a_t = n_t - \hat{n}_t = (1 - V(B))n_t - V(B)s_t, \quad (2.9)$$

and the variance of this seasonal adjustment error is

$$\sigma_a^2 = \int_{-\pi}^{\pi} \frac{f_n(w)f_s(w)}{f_x(w)}\,dw, \quad (2.10)$$

where $f_n(w)$, $f_s(w)$, and $f_x(w)$ are the spectra of n_t, s_t, and x_t, respectively.

When the observed series is contaminated by measurement error and seasonal noise, the linear minimum mean squared error estimate of n_t is given by

$$n_t^* = V^*(B)x_t^*, \quad (2.11)$$

where

$$V^*(z) = \frac{\sigma_\varepsilon^2\theta_n(z)\theta_n(z^{-1})}{\sigma_\varepsilon^2\theta_n(z)\theta_n(z^{-1}) + \sigma_\eta^2\theta_s(z)\theta_s(z^{-1}) + \sigma_\xi^2\theta_e(z)\theta_e(z^{-1})}. \quad (2.12)$$

The signal extraction error in this case is

$$a_t^* = n_t - n_t^* \quad (2.13)$$

and

$$\sigma_{a^*}^2 = \int_{-\pi}^{\pi} \frac{f_n(w)(f_s(w) + f_e(w))}{f_{x^*}(w)}\,dw, \quad (2.14)$$

where $f_e(w)$ and $f_{x^*}(w)$ are the spectra of e_t and x_t^*. The increase in the variance of the signal extraction error caused by the measurement error is easily shown to be

$$\sigma_{a^*}^2 - \sigma_a^2 = \int_{-\pi}^{\pi} f_e(w)\left(\frac{f_n(w)}{f_x(w)}\right)\left(\frac{f_n(w)}{f_{x^*}(w)}\right) dw. \quad (2.15)$$

Since the terms in parentheses are both less than 1, the increase in mean squared error is less than the variance of e.

In the discussion of optimal seasonal adjustment procedures found in the literature, measurement error is usually (always) ignored. The literature has focused on the construction and evaluation of seasonal adjustment filters when the models for n_t and s_t are known (e.g., Grether and Nerlove 1970; Cleveland and Tiao 1976) or are estimated by using the composite series (e.g., Pierce 1976, Engle 1976, Burman 1980, and Hillmer and Tiao 1982). When measurement error is ignored, seasonal adjustment filters formed by using the actual or estimated models for n_t and s_t will be suboptimal. Consider, for example, the case in which the optimal filter $V(B)$ for seasonal adjustment of x_t is used to estimate n_t using x_t^*. This produces the series

$$\tilde{n}_t = V(B)x_t^* \quad (2.16)$$

with signal extraction error

$$\tilde{a}_t = n_t - \tilde{n}_t. \quad (2.17)$$

The increase in mean squared error that arises from the use of this suboptimal filter is given by

$$\sigma_{\tilde{a}}^2 - \sigma_{a^*}^2 = \int_{-\pi}^{\pi} f_e(w)\left(\frac{f_e(w)}{f_{x^*}(w)}\right)\left(\frac{f_n(w)}{f_x(w)}\right)^2 dw. \quad (2.18)$$

When the processes generating the measurement error and n_t are very dissimilar, so $f_e(w)$ is large when $f_n(w)$ is small and vice versa, the increase in mean squared error will be small. When the spectra of the two series have a similar shape (e.g., the series are both positively correlated), the increase in mean squared error can be as large as $\sigma_\xi^2[1 + (\sigma_\eta^2/\sigma_\varepsilon^2)]^{-1}$.

In the next section, we estimate models for two economic time series that are measured with error. We estimate models incorporating and ignoring the measurement error. These models are used to construct estimates of the nonseasonal component. The mean squared errors of these series are then calculated and compared.

3. MODELS FOR THE UNEMPLOYMENT RATE

One of the most closely watched indicators of macroeconomic performance is the civilian unemployment rate. Each month this rate is estimated by the BLS, using a rotating sample composed of approximately 56,000 households. The data exhibit clear seasonal behavior. The unadjusted rate in January and June of each year is roughly one percentage point higher than the rate in May. This empirical regularity suggests the presence of a seasonal component. Since the data published by the BLS are estimates constructed from a sample, they also contain a sampling error or measurement error component.

In this section we present a model that decomposes the series into three components—a nonseasonal component, a seasonal component, and a measurement error component. Using the observed data, we estimate the parameters of the model. The estimated model is then used to construct the optimal seasonal adjustment filter. We compare this filter to one that we estimate ignoring the measurement error. The exercise is repeated for another series, the teenage unemployment rate. This series exhibits more dramatic seasonal behavior and is subject to more severe measurement error.

We begin by describing the sample design used by the BLS. Each month the BLS surveys eight subsamples, each composed of approximately 7,000 households. Seven of these subsamples have been included in some past survey and one subsample is new. The new subsample is included in the survey for four months, left out of the survey for the next eight months, and then included for four final months. This rotation procedure produces a 75% overlap in the sample from month to month and a 50% overlap from year to year.

This sample design produces measurement error that may be serially correlated. The degree of serial correlation will depend on the "memory" in the measurement error for any subsample. Given random sampling, we can assume that errors across subsamples are uncorrelated, but we want to allow some fraction of the error in any subsample to persist from period to period. To capture this persistence, we assume that the error for the ith subsample at time t is

$$e_{it} = \gamma_i + w_{it}, \quad (3.1)$$

where γ_i is a time-invariant random effect with mean zero and variance σ_γ^2 and w_{it} is a white-noise error, uncorrelated with γ_i, that has variance σ_w^2.

The estimates reported by the BLS are the result of a composite estimation procedure that further complicates the dynamic structure of the measurement error. The current reported value is constructed as a weighted average of the current period's sample value, the previous reported value, and the monthly changes in the sample values for the continuing 75% of the sample (see Bailar 1975). We let e_t^* denote the current sample measurement error, $\tilde{e}_t$ and $\tilde{e}_{t-1}$ denote the current and lagged values of the measurement error attributable to the overlapping segment of the sample, and e_t denote the measurement error in the reported value. The composite estimation procedure will produce a reported error that follows the process

$$(1 - .5B)e_t = .5e_t^* + .5(\tilde{e}_t - \tilde{e}_{t-1}). \quad (3.2)$$

This composite estimation procedure reduces the variance of the measurement error attributable to the γ_i components, since they are "differenced out" of the second term on the right side of (3.2). To eliminate the need for two indices, i and t, let the ith subsample be surveyed for the first time in period $t = i$. This allows us to replace γ_i with γ_t and write the process generating the measurement error as

$$(1 - .5B)e_t = .5(1 + B^{12})(1 + B + B^2 + B^3)\gamma_t + \lambda_t, \quad (3.3)$$

where

$$\begin{aligned}\lambda_t = {} & .5w_{t,t} + w_{t-1,t} - .5w_{t-1,t-1} + w_{t-2,t} \\ & - .5w_{t-2,t-1} + w_{t-3,t} - .5w_{t-3,t-1} \\ & + .5w_{t-12,t} + w_{t-13,t} - .5w_{t-13,t-1} \\ & + w_{t-14,t} - .5w_{t-14,t-1} + w_{t-15,t} \\ & - .5w_{t-15,t-1} \end{aligned} \quad (3.4)$$

This implies that $E(\lambda_t^2) = 8\sigma_w^2$, $E(\lambda_t\lambda_{t-1}) = -2.5\sigma_w^2$, $E(\lambda_t\lambda_{t-\tau}) = 0$, $\tau = 2, 3, \ldots$, so λ_t has an MA(1) representation,

$$\lambda_t = \xi_t - .351\xi_{t-1}, \quad (3.5)$$

where ξ_t is white noise with variance $\sigma_\xi^2 = 7.1225\sigma_w^2$.

Equation (3.1) produces a parsimonious representation of the process generating the measurement error, e_t. The rotation incorporated in the sample design and the composite estimation procedure implies that the error follows an ARMA(1, 15) process. The characteristics of the sample design allow the BLS to estimate the standard deviation of the sampling error. (Over our sample period, it is approximately .12% for the civilian unemployment rate and .60% for the teenage unemployment rate.) Our model represents this ARMA(1, 15) process with known variance in terms of only one unknown parameter, $\sigma_\gamma^2/\sigma_w^2$.

The characteristics of the sample design have allowed us to specify fully the process generating the measurement error. The measurement error process is identified. The processes generating the nonseasonal components are not identified without additional a priori information or restrictions. There is no widespread agreement on the specific identifying restrictions that should be placed on the component models. Certainly, most would agree that any deterministic seasonal component should sum to 0 over any 12-month period, and any deterministic nonseasonal component should not be seasonally periodic. Analogously, one might insist that the spectrum of the seasonal component contain most of its power around the seasonal frequencies and the spectrum of the nonseasonal component have no extra power at the seasonal frequencies. Unfortunately, these restrictions are not strong enough to identify completely the component models. A range of different models have been proposed that satisfy these restrictions. (Indeed, models more elaborate than the univariate time series models used in this section are certainly possible. Within our framework, we can allow any of the components to depend on observed variables.)

Nerlove et al. (1979) suggested a stationary model in which the nonseasonal follows an ARMA(2, 2) process, which can be viewed as the sum of an ARMA(2, 1) trend component and a white-noise irregular component. Their seasonal component is generated by a seasonal AR process with a low-order nonseasonal moving average. The model suggested by Pierce (1976) is similar. He extracts deterministic seasonality via regression and then uses a stationary seasonal ARMA$(1, 1)_{12}$ model for the seasonal component. He places constraints on the moving average coefficient to minimize the contribution of the seasonal. Hillmer and Tiao (1982) proposed a decomposition in which the seasonal is generated by

$$(1 + B + B^2 + \cdots + B^{11})s_t = \Theta(B)\mu_t, \quad (3.6)$$

where $\Theta(B)$ is a polynomial of order 11. Harvey and Todd (1983) estimated a similar model, but they set $\Theta(B) = 1$ and (for estimation) constrained the nonseasonal to follow an ARIMA(0, 1, 1) model constructed as the sum of a random walk and white noise.

For both of our data series, we estimated two types of models. The first constrained the autoregressive polynomial of the seasonal to be of the form $(1 + B + B^2 + B^3 \cdots + B^{11})$. The second model contained a deterministic seasonal component consisting of 12 monthly seasonal constants constrained to sum to 0 together with a stochastic seasonal component with the autoregressive polynomial $(1 - \Psi B^{12})$. For both models, we included moving average terms in the seasonal process. The nonseasonal process in both cases was an ARIMA(2, 1, 0), although we have tested for the presence of moving average terms.

The parameter estimates were constructed by maximizing the likelihood function. (Details of the procedure can be found in Hausman and Watson 1984.) We formed the likelihood function by using the Kalman filter. An important by-product from the Kalman filter is the optimal current estimate (i.e., minimum mean squared error estimate conditional on the parameter values) of the components. That is, the filter calculates the expected values of n_t, s_t, and e_t conditional on the observed data up through time t. It also calculates conditional variances so that the standard error of the optimal (one-sided) estimate of the nonseasonal component can be calculated. When more data become available, the estimates of the components n_t, s_t, and e_t and their standard errors are easily updated by using smoothing formulas similar to the Kalman filter. (See, for instance, Anderson and Moore 1979.)

For both series, the models that describe the seasonal by a set of 12 seasonal dummies plus a stochastic component with an AR polynomial of the form $(1 - \Psi B^{12})$ performed much better (in terms of their likelihood values) than the models that constrained the AR polynomial of the seasonal to be $(1 + B + B^2 + \cdots + B^{11})$. For this reason, we report results for this

Table 1. Estimated Component Models

	Civilian Unemployment Rate		Teenage Unemployment Rate	
Parameter	Model 1	Model 2	Model 1	Model 2
ϕ_1	.122 (.100)	.131 (.090)	−.033 (.123)	−.223 (.148)
ϕ_2	.516 (.118)	.453 (.108)	.726 (.173)	.120 (.184)
σ_ε^2	.026 (.009)	.030 (.008)	.053 (.039)	.393 (.143)
Ψ	.482 (.278)	.555 (.216)	.687 (.101)	.678 (.103)
Θ	.570 (.495)	.632 (.526)	.449 (.270)	.343 (.273)
σ_η^2	.008 (.007)	.007 (.005)	.202 (.073)	.218 (.094)
σ_w^2	1.1×10^{-5} ($.44 \times 10^{-5}$)		.026 (.012)	
σ_γ^2	.003 (.001)		.031 (.014)	
σ_a^2	.047	.047	.774	.781
*LM*1	.46		.08	
*LM*12	.62		.43	
Q[df]	16.0 [17]	17.9 [18]	22.8 [17]	25.4 [18]

NOTE: Asymptotic standard errors are given in parentheses. Twelve seasonal constants were also estimated.

model only. (Some summary statistics comparing the models can be found in Hausman and Watson 1984.) In Table 1 under the columns labeled Model 1, we present the estimated parameters for the model:

$$x_t^* = d_t'\beta + n_t + s_t + e_t$$

$$(1 - \phi_1 B - \phi_2 B^2)(1 - B)n_t = \varepsilon_t$$

$$(1 - \Psi B^{12})s_t = (1 - \Theta B)\eta_t$$

$$(1 - .5B)e_t = .5(1 + B^{12})(1 + B + B^2 + B^3)\gamma_t + \lambda_\tau$$

$$\lambda_t = \xi_t - .351\xi_{t-1}$$

$$\text{with } \sigma_\xi^2 = 7.1225\sigma_w^2. \quad (3.7)$$

The estimates were constructed using data from 1967:1 to 1983:1. The term $d_t'\beta$ represents the zero-mean deterministic seasonal factors. The unknown parameters β were estimated by generalized least squares. Table 1 shows that the estimates of the process generating the nonseasonal components for the teenage unemployment is similar to that of the civilian unemployment rate. The teenage rate is subject to much more measurement error and to larger seasonal noise (the variance of η is .202 for the teenage rate compared with .008 for the civilian rate). The estimates seem reasonable. The variance of γ and w suggests that nearly all of the measurement error for the civilian unemployment rate is composed of the time-invariant subsample-specific effect; for the teenage rate, only 60% of the measurement error is explained by this component. The parameter σ_a^2 is the one-step-ahead forecast error variance of the measured unemployment rate. *LM*1 and *LM*12 are values of Lagrange multiplier test statistics. *LM*1 tests for an MA(1) coefficient for the nonseasonal, and *LM*12 tests for an MA(12) coefficient in the process generating s. Both of these statistics are asymptotically χ_1^2 under the null hypothesis. The last entry in the column is the Box–Pierce Q statistic followed by degrees of freedom. The *LM* and Q test statistics suggest that the models adequately explain the autocovariance properties of the data.

The results for Model 1 shown in Table 1 constrain the standard error of the measurement error to be equal to the BLS estimate of .12% for the civilian rate and .60% for the teenage rate. These standard errors are identifiable within our model, and as a final check on our specification, we have estimated the model with this constraint relaxed. The changes in the estimates are slight. The estimated standard errors were .118% and .59%, respectively. The changes in the likelihood function were also slight. The square root of the likelihood ratio statistics were .25 and .32, respectively.

Figures 1 and 2 show the estimated decomposition of the observed series based on Model 1. These estimates were formed by using the optimal signal extraction filter constructed from the parameter values in Table 1. The seasonal and measurement error variation is much more dramatic for the teenage unemployment rate. The estimated seasonal factor for the teenage unemployment rate varies from 5.5% to −2.8%, and for the civilian unemployment rate, from .86% to −.61%. Teenage unemployment rate sampling error varies from .99% to −.95%, and civilian unemployment rate sampling error varies from .13% to −.09%.

As a first step in constructing seasonal adjustment filters that ignore measurement error, we reestimated the models leaving

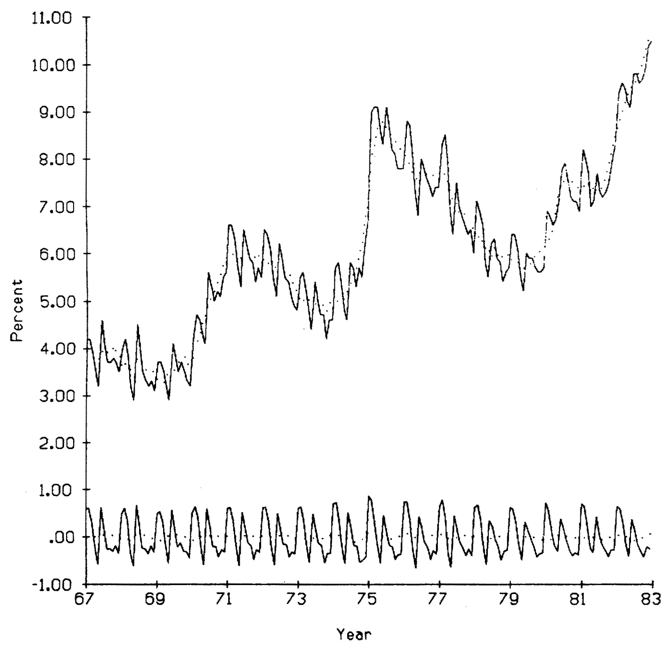

Figure 1. Components of the Civilian Unemployment Rate. Lower half: ——, seasonal component; ····, measurement error. Upper half: ——, unadjusted rate; ····, nonseasonal component.

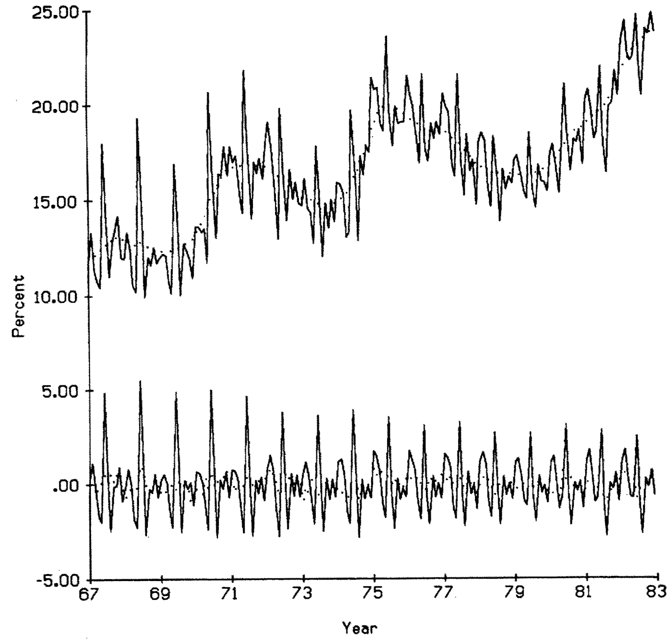

Figure 2. Components of the Teenage Unemployment Rate. Lower half: ———, seasonal component; ····, measurement error. Upper half: ———, unadjusted rate; ····, nonseasonal component.

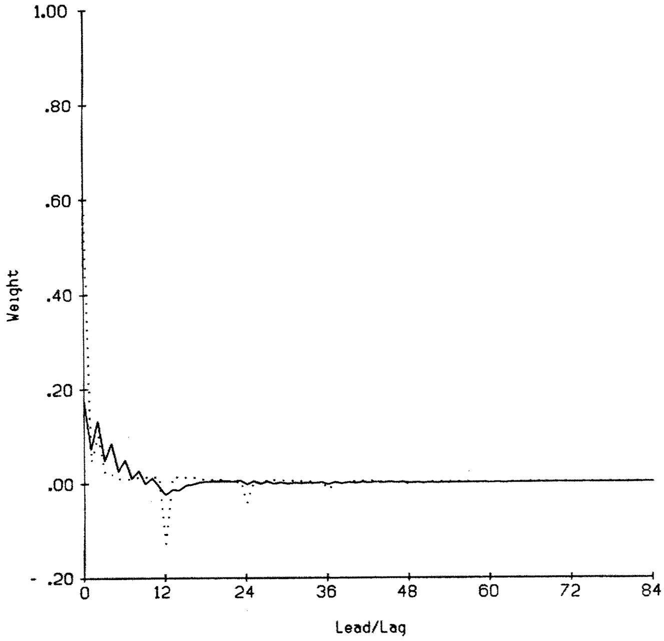

Figure 4. Optimal and Model 2 Filters for the Teenage Unemployment Rate. ———, Optimal filter; ····, model 2 filter.

out the sampling error component. The results are presented in Table 1 in the columns labeled Model 2. The measurement error in the civilian unemployment rate is small, and neglecting it has only a small effect on the model. The estimated process for the nonseasonal component of the teenage rate has changed markedly. The variance of its driving noise, ε_t, is more than seven times larger than in Model 1, and the point estimates of ϕ_1 and ϕ_2 are different.

A different "optimal" signal extraction filter is associated with both Model 1 and Model 2. The Model 1 filter constructs an optimal estimate of the true nonseasonal component, n_t, and the Model 2 filter constructs an optimal estimate of the misspecified nonseasonal component. Given a complete realization of the observed series, both of these filters will be of the form

$$\pi(B) = \pi_0 + \sum_{i=1}^{\infty} \pi_i(B^i + B^{-i}). \tag{3.8}$$

In Figure 3, we plot the filter weights implied by Model 1 and Model 2 for the civilian unemployment rate. These signal extraction filters are quite similar. Figure 4 compares the filters for the teenage unemployment rate. These filters are quite different. The misspecified filter puts far too much weight on the

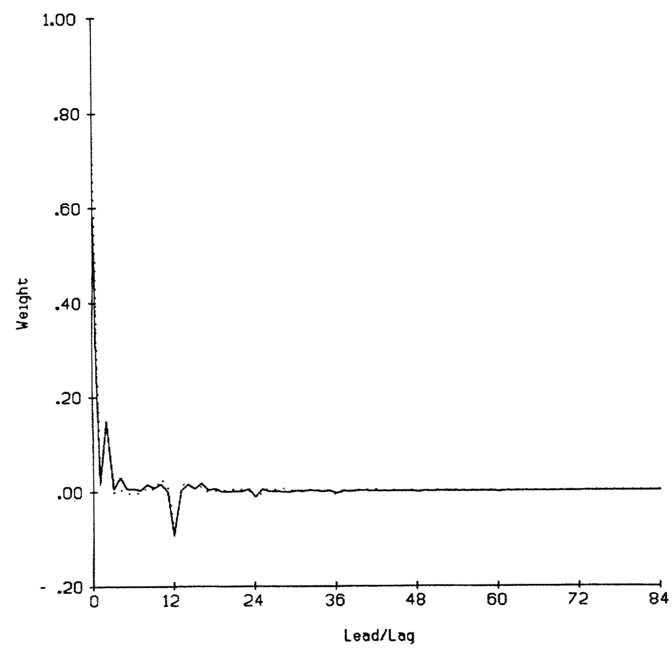

Figure 3. Optimal and Model 2 Filters for the Civilian Unemployment Rate. ———, Optimal filter; ····, model 2 filter.

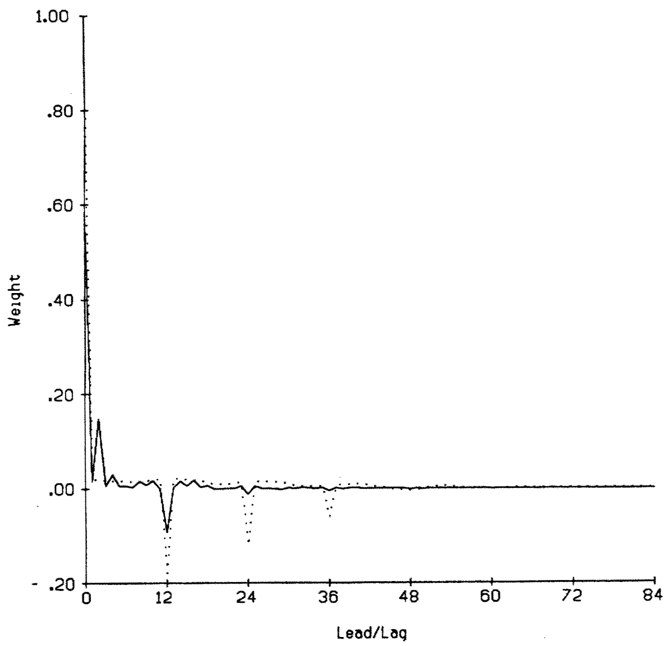

Figure 5. Optimal and X-11 Filters for the Civilian Unemployment Rate. ———, Optimal filter; ····, X-11 filter.

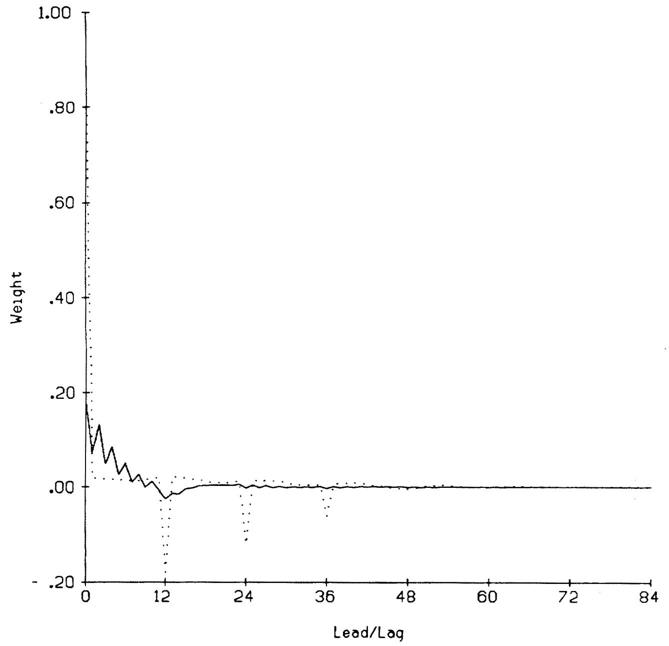

Figure 6. Optimal and X-11 Filters for the Teenage Unemployment Rate. ——, *Optimal filter;* ····, *X-11 filter.*

current observation and compensates in part for this by a large negative weight at the 12th lead/lag. The small weight placed on the current observation in Model 1 reflects the relatively small variance of its nonseasonal component. [Eq. (2.12) shows that the filter $\pi(B)$ depends on the ratio of the autocovariance-generating function of n to the autocovariance-generating function of x^*.] This component accounts for only 10% of the variance in the change of the teenage rate. In Model 2, the analogous figure is 68%, and this accounts for the large weight its filter places on the current value.

We can compare the optimal filters to other seasonal adjustment methods. The most widely used seasonal adjustment procedure is produced by the Census X-11 program. Even though the filter constructed by this program contains nonlinearities (e.g., adjustments for outliers), it can be well approximated for many series by the symmetric 84-term linear filter given in Wallis (1974). (This 84-term filter uses the 13-term Henderson moving average trend filter.) Figures 5 and 6 compare this filter with the Model 1 filter for our series. For both series, X-11 puts too much weight on the first observation and compensates for this with large negative weight at the seasonal lead/lags. A glance at these figures suggests that the increase in precision of the optimal filter from the X-11 filter will not be too large for the civilian unemployment rate, but it may be quite large for the teenage rate. In the next section, we calculate the mean squared error associated with the X-11 procedure and find this to be the case.

4. STANDARD ERRORS FOR THE SEASONALLY ADJUSTED UNEMPLOYMENT RATE

The unobserved-components model of seasonality allows us to view a seasonal adjustment procedure as an estimation method and a seasonally adjusted series as a sequence of estimates of the underlying nonseasonal component. Alternative seasonal adjustment procedures can be evaluated on the basis of how precisely they estimate the nonseasonal component. In this section, we compare various seasonal adjustment procedures for the two series analyzed in the previous section, using mean squared error (MSE) as the measure of precision. In particular, we calculate the MSE of the estimates calculated by the Census X-11 program. This comparison allows us to discuss the accuracy of the official seasonally adjusted series and the increase in accuracy that could be achieved by using an optimal filter. We also calculate the increase in MSE that arises from the measurement error component, both when it is ignored and when it is accounted for in an optimal manner.

It is useful to identify five different sources of seasonal adjustment uncertainty. First, there is a certain irreducible signal extraction uncertainty that arises when both n_t and s_t are stochastic. This uncertainty makes it impossible to deduce the value of either when only their sums are observed. Second, there is additional uncertainty that arises from measurement error. Its presence increases the "noise" in the observed series. A third increase in uncertainty may arise from the use of a suboptimal seasonal adjustment procedure. There are also two sources of uncertainty that arise from the quantity of data available. Since the components are serially correlated, future values of the observed series will contain information on the value of the current nonseasonal component. Data at time $t + 1$, and so forth, can therefore be used to reduce the uncertainty surrounding n_t. Finally, since the parameters used in constructing seasonal adjustment filters may be unknown, there is uncertainty that arises from the use of estimated parameters.

In usual calculations of MSE, it is possible at some point in time to observe the realization of the variable being estimated. In calculating one-step-ahead forecast MSE, for example, the variable being forecast is observed with a one-period lag. This setup makes it possible to calculate a sample MSE that incorporates all sources of error. In seasonal adjustment, the case is somewhat different. The variable being estimated is never observed. Sample MSE cannot be calculated. MSE can only be calculated or estimated if the processes generating the components are known or are estimated. Our estimates of MSE will be based on the models that we presented in Section 3.

If we assume that the data were generated by Model 1 with parameter values shown in Table 1, then the root mean squared error (RMSE) of various seasonally adjusted series are straightforward to calculate. All of the seasonal adjustment procedures that we will consider form a seasonally adjusted series of the form

$$\hat{n}_t = \pi(B)(x_t - d_t'B), \qquad (4.1)$$

where $\pi(B)$ is a time-invariant linear filter. Different seasonal adjustment procedures correspond to different choices of $\pi(B)$. [The X-11 filter does not subtract the term $d_t'B$ from x_t; however, it is easy to show that for X-11, $\pi(B)d_t = 0$ for all t.] This implies that the seasonal adjustment error corresponding to the filter $\pi(B)$ is

$$c_t = n_t - \hat{n}_t = [1 - \pi(B)]n_t - \pi(B)s_t - \pi(B)e_t. \qquad (4.2)$$

Recall that n_t followed an integrated process, so a necessary condition for the MSE of c_t to be finite is that $(1 - \pi(B))$ contain the factor $(1 - B)$ or, equivalently, that $\pi(1) = 1$. All seasonal adjustment filters that we consider will have this property. If we let $W(B) = \pi(B)/(1 - B)$, then from (4.2), the autocovariance-generating function of c_t, say $A_c(z)$, is

$$A_c(z) = W(z)W(z^{-1})A_n(z) + \pi(z)\pi(z^{-1})A_s(z) + \pi(z)\pi(z^{-1})A_e(z),$$

where $A_n(z)$, $A_s(z)$, and $A_e(z)$ are the autocovariance-generating functions of $(1 - B)n_t$, s_t, and e_t, respectively. Polynomial long division and multiplication can be used to calculate the coefficients in $A_c(z)$, which can then be used to form the RMSE for the level and change of the seasonally adjusted estimate. When an optimal filter is used, these calculations can be avoided, as the RMSE is easily calculated by the Kalman filter and smoother.

The first row of Table 2 shows the RMSE associated with the optimal filter applied to a complete realization of the series without measurement error. This model corresponds to the series x_t and error a_t given in Section 2. In row 2 of the table we show the RMSE associated with the optimal filter applied to a complete realization of the series measured with error. This model corresponds to the series x_t^* and error a_t^* given in Section 2. Comparing row 2 and row 1 of Table 2 shows that the increase in RMSE due to measurement error is larger for the teenage rates. Recall, however, that the measurement error variance was .36 for the teenage rate as compared with .0144 for the civilian rate. The increase in MSE for the level of the teenage rate is only 28% of its measurement error variance, whereas the increase in MSE for the level of the civilian rate is 65%. The reason for this differential increase is shown in Equation (2.15). The seasonal measurement error components are relatively unimportant in the civilian rate, so a large fraction of the spectrum of x and x^* is accounted for by the n_t. The terms $(f_n(w)/f_x(w))$ and $(f_n(w)/f_{x^*}(w))$ in (2.15) are, on average, larger for the civilian rate than the teenage rate. This emphasizes the point that the seasonal adjustment procedure significantly reduces both sources of noise—seasonal and measurement error. It also illustrates the point made by (2.14) and (2.15): Although the size of the measurement error is important, its dynamic properties and those of n_t and s_t are equally important.

The next row of Table 1 presents the results using the filter constructed from the Model 2 parameter estimates and applied to a complete realization of the series. This filter would be estimated if measurement error was present but ignored. For the civilian unemployment rate, the variance of the change in the measurement error is small relative to the variance of the change in the observed series, so the increase in MSE from the Model 1 filter is not large. The relative efficiency of this filter is quite high. In the case of the teenage rate, measurement error is much more important, and the Model 2 filter produces a much less precise estimate than the Model 1 filter. The relative efficiency for estimating the change in n_t is only .33. Our finding emphasizes the potential importance that measurement error can have on seasonal adjustment procedures.

The plots presented at the end of the last section also suggested a more dramatic reduction in efficiency from the use of Census X-11 for the teenage unemployment rate. Rows 4 and 5 of Table 2 present the RMSE for the linear approximation to the X-11 filter. The performance of the X-11 filter is close to the optimal filter for the level of civilian unemployment rate (a relative efficiency of .81), but it performs very poorly for the change in the teenage rate (a relative efficiency of .15). Indeed, for the change in the teenage rate, one is better off applying the Model 1 filter to the series measured with error than applying the X-11 filter to the series without measurement error. It is worth noting that our estimate of the RMSE of the X-11 process is approximately one-half of the estimate of Summers (1981). He failed, however, to take account of the time series properties of the unemployment rate or of the X-11 filter.

The results thus far have assumed that a complete realization of the series was available so that the two-sided filters could be applied. When adjusting current values or values from the recent past, symmetric two-sided filters cannot be used. It is possible to construct optimal one-sided filters, and indeed, the Kalman filter does just this. In row 6 of Table 2 we present the RMSE of currently adjusted values using the optimal filter. Comparing these figures with those in row 2 shows the value of future data for current seasonal adjustment. For both series, future data decreases MSE for the level of the series rather substantially (22% for the civilian rate; more than 50% for the teenage rate) but has a smaller effect on the MSE of the change in n_t.

The X-11 filter that we have presented is also two-sided and therefore cannot be used for adjusting current values. Dagum (1975) suggested applying the special nonsymmetric X-11 lag-12 filter to the observed series augmented by 12 forecasts from an ARIMA model. This method, called X-11 ARIMA, is used by the BLS for current adjustment. The application of the linear approximation to X-11 to a series augmented by 84 ARIMA forecasts is equivalent to the application of a one-sided linear filter. For both the civilian and teenage unemployment rates, we have formed their one-sided X-11 forecast filters. These can be viewed as close approximations to the X-11 ARIMA filters used by the BLS. In row 7 of Table 2, we present the RMSE associated with these filters. For both series the RMSE associated with the forecast 84 filter is nearly as small as the optimal one-sided filter for the levels of the series. The X-11 forecast 84 filter performs considerably worse than the optimal filter for the change in the teenage rate. Here its relative efficiency is .14.

As Cleveland and Tiao (1976) and Burridge and Wallis (1984) pointed out, the X-11 filter has some of the features of an optimal filter for a nonstationary time series (e.g., it is symmetric and the filter weights sum to one). They present an

Table 2. Seasonal Adjustment Root Mean Squared Error

Adjustment Method	Civilian Unemployment Rate		Teenage Unemployment Rate	
	Level	Change	Level	Change
M1, no measurement error	.091	.081	.310	.204
M1, measurement error	.137	.090	.419	.262
M2, measurement error	.141	.091	.580	.453
X-11, no measurement error	.110	.105	.436	.421
X-11, measurement error	.152	.113	.687	.666
Current adjustment using M1 with measurement error	.155	.098	.614	.279
X-11, forecast 84	.162	.128	.800	.810
M1, with parameter uncertainty*	.143	.101	.433	.300
Current adjustment with parameter uncertainty*	.157	.092	.622	.302

* Varies over the sample period. Average value is presented.

unobserved-components model for which X-11 is nearly optimal. Given the form of X-11 and the autocovariance-generating function of our observed series, we can decompose the series into a "signal" component and a "noise" component such that X-11 is the optimal signal extraction filter. The solution to this inverse optimal signal extraction problem for our series is presented in the Appendix. (This is analogous to an exercise suggested by Bell and Hillmer 1984.) There we show that the signal process for which X-11 is optimal is very nearly the model for n_t in the case of the civilian unemployment rate. For the teenage rate, the models are quite different. The fundamental underidentification in unobserved-component models makes this a very useful exercise. It yields the autocovariance of the signal and the noise that X-11 is, in a sense, trying to estimate.

The filter that we have constructed from our estimated Model 1 is only an estimate of the optimal filter. Imprecise estimates of the parameters will produce an imprecise estimate of the filter, which in turn will produce an imprecise seasonally adjusted series. By using the asymptotic distribution of the estimated parameters, we can approximate this additional source of uncertainty.

The final two rows of Table 2 present the estimated RMSE for both of our series incorporating parameter uncertainty. Details of the calculations can be found in Hausman and Watson (1984). Row 8 shows the results for the estimated two-sided filters, and this result corresponds to historical seasonal adjustment. The final row shows the results using the estimated optimal one-sided filter. These results correspond to the adjustment of current data. Comparing rows 2 and 8 and rows 6 and 9 suggests that parameter uncertainty increased MSE only slightly. This finding is true even though our point estimates of the parameters were not extremely precise, as the large standard errors for some coefficients in Table 1 indicate.

5. CONCLUSIONS

Our empirical results suggest that there may be large gains from the use of model-based rather than X-11 seasonal adjustment methods. Although this point has been made many times in the past, to the best of our knowledge the gain (in terms of MSE) has never been calculated. Our results suggest that for some series, there is a large absolute as well as relative gain.

A related issue that arises is optimal sample design. If the objective of the data collection is the estimation of the nonseasonal component, then the rotation of the sample should be designed with this in mind. If an optimal filter is used, the expressions for the MSE given in Section 2 suggest that the rotation should be chosen so that the spectrum of the measurement error is as different as possible from that of the nonseasonal component. If a nonoptimal filter (e.g., X-11) is to be used, the rotation should be chosen so that as much of it as possible is eliminated by the chosen seasonal filter. In the case of X-11, this suggests that the rotation scheme should be chosen to make the measurement error as seasonal as possible.

Finally, we have presented a procedure that allows calculation of confidence intervals for both optimal and nonoptimal seasonal adjustment techniques. The need for a measure of precision for official seasonally adjusted series has long been recognized. Similar research on other adjusted series would seem to be valuable in interpretation of other government series, such as inflation. In addition, the precision of the unemployment series is an important issue by itself, given its use in federal government aid programs at the state level, such as unemployment insurance extensions.

APPENDIX

Suppose that y_t is an observed time series and $(1 - B)y_t$ is stationary with Wold representation

$$(1 - B)y_t = \Theta_y(B)a_t, \qquad (A.1)$$

where a_t is white noise and the roots of $\Theta_y(z)$ are outside the unit circle. Let $\Theta_x(z)$ and $\Theta_\xi(z)$ be two polynomials with roots outside the

Table A.1. Autocovariances of Signal and Noise Processes Implied by Model 1 and X-11

Lag	Observed Series	Signal		Noise	
		Model 1	X-11	Model 1	X-11
		Civilian Unemployment Rate			
0	.05647	.03708	.04311	.01940	.01336
1	.00876	.00933	.01018	−.00056	−.00141
2	.01459	.02025	.01594	−.00566	−.00135
3	.00680	.00728	.00820	−.00047	−.00139
4	.00975	.01133	.01040	−.00157	−.00065
5	.00427	.00513	.00593	−.00086	−.00165
6	.00589	.00646	.00652	−.00057	−.00063
7	.00286	.00343	.00439	−.00057	−.00153
8	.00289	.00375	.00339	−.00086	−.00049
9	.00196	.00223	.00325	−.00027	−.00130
10	−.00053	.00221	.00063	−.00273	−.00116
11	.00119	.00142	.00263	−.00022	−.00143
12	.01064	.00131	−.00207	.00933	.01272
13	.00068	.00089	.00217	−.00021	−.00150
14	−.00193	.00078	−.00063	−.00271	−.00130
15	.00033	.00055	.00176	−.00022	−.00143
16	−.00029	.00047	.00037	−.00076	−.00066
17	−.00004	.00034	.00152	−.00038	−.00156
18	.00009	.00028	.00062	−.00019	−.00053
19	.00012	.00021	.00122	−.00010	−.00111
20	.00013	.00017	.00043	−.00005	−.00030
21	.00011	.00013	.00093	−.00002	−.00082
22	−.00131	.00010	−.00040	−.00141	−.00091
23	−.00038	.00008	.00052	−.00046	−.00090
24	.00378	.00006	−.00558	.00371	.00935
		Teenage Unemployment Rate			
0	1.05000	.11440	.58100	.93550	.46880
1	−.25820	−.01365	−.14420	−.24460	−.11410
2	−.09115	.08349	.00456	−17460	−.09571
3	−.02200	−.01264	−.00420	−.00936	−.01780
4	.04060	.06102	.03541	−.02042	.00519
5	−.02223	−.01117	−.00350	−.01105	−.01872
6	.03745	.04466	.02824	−.00721	.00921
7	−.01654	−.00957	−.00030	−.00697	−.01625
8	.02251	.03273	.01521	−.01022	.00730
9	−.01127	−.00802	.00474	−.00325	−.01601
10	−.09176	.02402	−.00149	−.11580	−.09027
11	−.07784	−.00661	.03243	−.07124	−.11030
12	.43190	.01765	−.01682	.41420	.44870
13	−.07649	−.00537	.03420	−.07112	−.11070
14	−.10250	.01299	−.01005	−.11550	−.09246
15	−.00697	−.00433	.01015	−.00264	−.01712
16	.00059	.00957	−.00246	−.00898	.00305
17	−.00795	−.00345	.01054	−.00449	−.01848
18	.00482	.00706	−.00274	−.00225	.00756
19	−.00386	−.00274	.00696	−.00112	−.01082
20	.00465	.00521	−.00255	−.00056	.00720
21	−.00244	−.00216	.00866	−.00028	−.01110
22	−.07721	.00386	−.00255	−.08107	−.07466
23	−.05649	−.00169	.02782	−.05479	−.08430
24	.27410	.00286	−.08270	.27130	.35580

unit circle and σ_ε^2 and σ_η^2 be two positive numbers such that

$$\sigma_a^2\Theta_y(z)\Theta_y(z^{-1}) = \sigma_\varepsilon^2\Theta_x(z)\Theta_x(z^{-1}) + (1 - z)(1 - z^{-1})\sigma_\eta^2\Theta_\xi(z)\Theta_\xi(z^{-1}); \quad (A.2)$$

then y_t can be written as

$$y_t = x_t + \xi_t, \quad (A.3)$$

where

$$(1 - B)x_t = \Theta_x(B)\varepsilon_t \quad (A.4)$$

and

$$\xi_t = \Theta_\xi(B)\eta_t \quad (A.5)$$

with $\{\varepsilon_t\}$ and $\{\eta_t\}$ independent white noise sequences. Clearly there are many choices of $\Theta_x(z)$ and $\Theta_\xi(z)$ satisfying (A.2), so there are many decompositions of the form (A.3)–(A.5). For each decomposition, there is a filter $\gamma(B)$ such that $x_t = \gamma(B)y_t$ is the minimum MSE of the signal x_t. The form of $\gamma(z)$ is

$$\gamma(z) = \sigma_\varepsilon^2\Theta_x(z)\Theta_x(z^{-1})/\sigma_a^2\Theta_y(z)\Theta_y(z^{-1}),$$

so $\gamma(z)$ is symmetric with $\gamma(z) = 1$. (See Cleveland and Tiao 1976 or Bell 1984.)

The X-11 filter, X11(z), has these features, and it is an optimal filter for the signal with autocovariance-generating function given by

$$\sigma_a^2 \text{X11}(z)\Theta_y(z)\Theta_y(z^{-1}).$$

The autocovariance-generating function of the first difference of the noise is therefore

$$\{1 - \text{X11}(z)\}\sigma_a^2\Theta_y(z)\Theta_y(z^{-1}).$$

Using the autocovariances of our observed unemployment rates, we have calculated the autocovariances of the signal/noise processes for which X-11 is optimal. These are shown in Table A.1 along with the autocovariances of n_t (under the column labeled Signal, Model 1) and the autocovariance of $(1 - B)(s_t + e_t)$ (under the column labeled Noise, Model 1). For the civilian rate, the X-11 and Model 1 columns are very similar. For the teenage rate, they are quite different. The nonseasonal component accounts for only 9% of the variance series, whereas the signal for which X-11 is optimal accounts for 56% of the variance. The X-11 filter is attributing much of the measurement error to the signal rather than the noise.

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