Out of Sample Forecasts of Quadratic Variation: Appendix

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This appendix extends the results in Aït-Sahalia and Mancini (2007), presenting additional Monte Carlo evidence and a detailed empirical analysis for each of the thirty DJIA stocks. The notation is the same as in Aït-Sahalia and Mancini (2007).

1. Additional Monte Carlo Evidence

1.1. In-sample estimation of IV

In this section we collect in-sample estimation results for all but the first Monte Carlo experiments undertaken in Aït-Sahalia and Mancini (2007). The results for the first Monte Carlo experiment (based on the Heston volatility model) are reported in the main paper. For the detailed description of each experiment we refer the reader to the main paper. Table 1 shows the in-sample estimation of IV for the Heston jump-diffusion model; Table 2 for the log-volatility model; Table 3 for the heterogenous autoregressive realized volatility model and Table 4 for the fractional Ornstein-Uhlenbeck volatility model. In all experiments TSRV outperforms RV in terms of bias, variance, and RMSE in absolute and relative terms.

1.2. Heston Volatility Model: Forecasting $IV^{1/2}$ instead of IV

For the Heston volatility model, Table 5 shows the out-of-sample forecast of $IV^{1/2}$ rather than IV as in Aït-Sahalia and Mancini (2007). Hence in the Mincer-Zarnowitz forecast evaluating regression, the IV and the corresponding estimates are replaced by $IV^{1/2}$. The findings largely confirm the previous results in Aït-Sahalia and Mancini (2007) on the forecast of IV, meaning that the superior performance of TSRV over RV is robust to Jensen's type inequality effects.

1.3. Nearly Integrated Heston Volatility Model

In this section the volatility process is a "nearly" integrated Heston process to mimic the long memory features exhibited by volatility processes. The model parameters are the same as in Aït-Sahalia and Mancini (2007), but the mean reversion coefficient $\kappa = 1$ and the local volatility parameter $\gamma = 0.275$. A lower mean reversion coefficient κ implies an almost integrated volatility process, as $e^{-1\times\Delta} = 0.999999830$. In the present setting, γ is also lower to satisfy the Feller's condition $2\kappa\alpha \geq \gamma^2$.

Tables 6 and 7 show the in- and out-of-sample forecasts of IV and clearly TSRV outperforms RV. The non-relative performances in the first three columns of Table 6 are close to the simulation results in Aït-Sahalia and Mancini (2007). The large relative variances of RV in Table 6 are due to (relatively) large overestimation of small integrated variances. Table 7 shows that with a nearly integrated (and hence possibly smoother) instantaneous variance process all methods provide better forecasts of the integrated variances. However, at each frequency TSRV forecasts have higher R^2 than RV forecasts in the Mincer-Zarnowitz regressions.

1.4. Jump-Diffusion Model

In this section the volatility process is same the Heston jump-diffusion model as in Aït-Sahalia and Mancini (2007). The diffusion coefficients of the volatility model are the same as in the Heston model (without jumps) in Aït-Sahalia and Mancini (2007). The jump coefficients are $\lambda = \Delta \times 23{,}400/2$ which implies (on average) two volatility jumps per day, and $\xi = 0.0004$, that is the jump size is about 1% of the unconditional variance as $\alpha = 0.04$; in Aït-Sahalia and Mancini (2007), $\xi = 0.0007$ and the jump size was about 2% of the unconditional variance. Tables 8 and 9 shows the in- and out-of-sample simulation results, respectively, and also in this setting TSRV outperforms RV in estimating and forecasting integrated variances.

1.5. Jump-Return Heston Volatility Model

Empirical evidence suggests that asset returns tend to display a jump component. To investigate the impact of jumps in returns on RV and TSRV we perform the following Monte Carlo study. We simulate high frequency data using the same Heston volatility model as in Aït-Sahalia and Mancini (2007), but adding a jump component to the log-price process X,

$$\begin{aligned} dX_t &= (\mu - \sigma_t^2/2)\, dt + \sigma_t\, dW_{1,t} + J_{X,t}\, dq_{X,t} \\ d\sigma_t^2 &= \kappa(\alpha - \sigma_t^2)\, dt + \gamma\sigma_t\, dW_{2,t}, \end{aligned}$$

where $q_{X,t}$ is a Poisson process with intensity λ_X and $J_{X,t}$ is the jump size assumed to be normally distributed, $J_{X,t} \sim \mathcal{N}(\mu_X, \sigma_X)$. The processes q_X and J_X are independent of the Brownian motions. Without jumps, $\lambda_X = 0$, the log-price process X reduces to the Heston's model. Heuristically,[1] neglecting $o(dt)$

$$\begin{aligned} Var_t(dX_t) &= Var_t(\sigma_t\, dW_{1,t}) + Var_t(J_{X,t}\, dq_{X,t}) \\ &= \sigma_t^2\, dt + E_t[J_{X,t}^2\, dq_{X,t}^2] \\ &= \sigma_t^2\, dt + (\mu_X^2 + \sigma_X^2)\lambda_X\, dt, \end{aligned}$$

where Var_t and E_t are the conditional variance and expectation at time t, respectively. The total unconditional instantaneous variance is

$$E\left[\frac{Var_t(dX_t)}{dt}\right] = \alpha + (\mu_X^2 + \sigma_X^2)\lambda_X.$$

To simulate a log-return process with approximately the same unconditional variance (in finite sample) as in all the other Monte Carlo experiments we set all the diffusion coefficients as in the Heston simulation but $\alpha = 0.03$ and $\mu_X = -0.004$, $\sigma_X = 0.004$, $\lambda_X = \Delta \times 23{,}400/2$, implying (on average) two jumps a day with average size -0.4%. It is known that jumps in returns occur less frequently and have larger magnitude. Reproducing this phenomenon in simulation is conceptually straightforward but would require simulating several years of high frequency data. The computational costs would be prohibitive. As we simulate log-price trajectories of a hundred trading days, we set the parameters to induce relatively frequent, small jumps in returns. Hence all the simulated sample paths display a certain number of jumps and we can study the impact of jumps in the returns on the forecasts of IV, which is the goal of our simulation experiment. In the presence of jumps in the returns the quadratic variation of X is

$$\int_0^T \sigma_t^2\, dt + \sum_{j=1}^{q_{X,T}} J_{X,t_j}^2, \tag{1.1}$$

where $0 \le t_j \le T$ is the j-th jump time. Zhang et al. (2005) show that TSRV converges to the quadratic variation (1.1).

Table 10 shows the in-sample estimation of IV and also in this setting TSRV provides more accurate estimates of IV than RV does. Table 11 presents the Mincer-Zarnowitz regressions for the out-of-sample forecasts of IV. R^2's are lower than in the Heston simulation (without jumps in returns) because of the unpredictable jump component (with Poisson arrival times) in the quadratic variation. Of course, decreasing frequency and magnitude of jumps in returns will increase R^2's, which will converge to the R^2's in the Heston simulation. Also in this setting TSRV outperforms RV in forecasting the IV and RV forecasts have no or little additional explanatory power when compared to the TSRV forecasts.

[1] The notation $Var_t(dX_t)/dt$ is a heuristic for $\lim_{\Delta\to 0} E[(\log S_{t+\Delta} - \log S_t)^2 | \sigma_t^2, S_t]$ and the limiting operation is valid under standard regularity conditions.

1.6. *Alternative Correlation Structures for the Market Microstructure Noise*

This section complements the corresponding section in Aït-Sahalia and Mancini (2007). To summarize, we consider three possible specifications of the microstructure noise ε, that is

1. autocorrelated and independent of the latent price process

2. not autocorrelated and correlated with the latent price process

3. autocorrelated and correlated with the latent price process.

The underling driving process is the same Heston process as in Aït-Sahalia and Mancini (2007).

1.6.1 Autocorrelated Microstructure Noise

In the first specification of the microstructure noise, i.e. autocorrelated and independent of the latent price process, the detailed description of the noise process is as follows. The autocorrelation in the microstructure noise is captured using the model,

$$\varepsilon_t = U_t + V_t,$$

where U_t is a strong white noise process $U_t \sim NID(0, \sigma_U^2)$, V_t is an AR(1) process $V_t = \varrho V_{t-1} + \nu_t$ and $\nu_t \sim NID(0, \sigma_\nu^2)$. U_t and V_t are independent and

$$Var\,\varepsilon_t = \sigma_U^2 + \frac{\sigma_\nu^2}{1-\varrho^2}.$$

We set $\varrho = -0.2$, and the microstructure noise variance

$$(0.10/100)^2 = 5 \times 10^{-7} + \frac{4.8 \times 10^{-7}}{1-\varrho^2},$$

that implies $\sigma_U^2 = \sigma_\nu^2/(1-\varrho^2)$, so that the two noise components U_t and V_t equally contribute to the total variance of the microstructure noise. In-sample estimations of IV are reported in Panel A of Table 12, while out-of-sample forecasts of IV are presented in the corresponding section in Aït-Sahalia and Mancini (2007). Also in this setting TSRV outperforms RV in estimating and forecasting IV. Overall the simulation results are quite close to the ones in the iid microstructure noise case documented in Aït-Sahalia and Mancini (2007).[2] Among the considered IV estimators, the minimum variance TSRV estimator uses the highest frequency return of less than two minutes and it is the most affected by the autocorrelated noise. This confirms that the higher the sampling frequency the stronger the impact of autocorrelated microstructure noise as discussed in Aït-Sahalia et al. (2006).

1.6.2 Microstructure Noise Correlated with the Latent Price Process

In this section the microstructure noise ε is not autocorrelated, but correlated with the latent log-price process X, and all the simulation results are presented in this appendix. The correlation between the microstructure noise and the latent log-return is

$$Corr(\varepsilon_t, (X_t - X_{t-\Delta})) = \vartheta,$$

where $\Delta = 1$ second, and we consider both positive, $\vartheta = 0.2$, and negative correlations, $\vartheta = -0.2$. The correlation is imposed as follows. Under the Euler scheme

$$X_t = X_{t-\Delta} + (\mu - \sigma_t^2/2)\Delta + \sigma_t\sqrt{\Delta} z_{1,t},$$

[2]For comparative purposes, in the different simulation settings the same stochastic process (such as the Heston volatility or the iid component of the microstructure noise) is simulated using the same random numbers.

where $z_{1,t} \sim NID(0,1)$. Setting the microstructure noise $\varepsilon_t = \sigma_\varepsilon \, z_{2,t}$, where $z_{2,t} = \vartheta \, z_{1,t} + \sqrt{1-\vartheta^2} \, z_{1,t}^{\perp}$ and $z_{1,t}^{\perp} \sim NID(0,1)$ independently drawn from $z_{1,t}$, it implies the required correlation ϑ. As in the previous simulations $\sigma_\varepsilon = 0.10/100$, that is the standard deviation of the microstructure noise is 0.1% of the value of the asset price.

Tables 13 and 14 show the in- and out-of-sample results for the positive correlation, $\vartheta = 0.2$, and Tables 15 and 16 show the corresponding results for the negative correlation, $\vartheta = -0.2$. Also in these settings, TSRV outperforms RV both in- and out-of-sample forecasts of the integrated variances.

1.6.3 Autocorrelated Microstructure Noise Correlated with the Latent Price Process

In this section the microstructure noise is autocorrelated and correlated with the latent price process,

$$\varepsilon_t = U_t + V_t,$$

where $Corr(U_t, X_t - X_{t-\Delta}) = -0.2$, V_t is the same AR(1) process as in the previous section and in Aït-Sahalia and Mancini (2007) with first order coefficient $\varrho = -0.2$, $E[U^2] = E[V^2] = 5 \times 10^7$, and $U \perp V$. Panel B in Table 12 shows the in-sample estimations of IV and Table 17 reports the out-of-sample forecasts of IV. Clearly, also in this setting TSRV outperforms RV in estimating and forecasting IV. As already mentioned in Aït-Sahalia and Mancini (2007), the correlation between microstructure noise and latent return process has a minor impact on the measurement and the forecast of IV, while the autocorrelation in the microstructure noise plays a major role. Hence the simulation results in this section are quite close to ones in the autocorrelated but independent microstructure noise case documented in Section 1.6.1.

1.7. Extended Comparison between TSRV and MSRV

In this section we repeat all the Monte Carlo experiments in Aït-Sahalia and Mancini (2007) to compare the forecasting performances of TSRV and the Multi-Scale Realized Volatility (MSRV) estimator in Zhang (2006) and Aït-Sahalia et al. (2006). For an in-depth presentation of MSRV we refer the reader to Zhang (2006). We implement the MSRV estimator with slow time scales, $K_i, i = 1, \ldots, 300$, and the optimal weights a_i^* with kernel $h^*(x) = 12(x - 1/2)$. To keep the computational burden feasible for each Monte Carlo experiment we run 1,000 simulations (as opposed to the 10,000 simulations in the main paper) and we compare TSRV with slow time scale of 5mn (or 300 seconds) and the previous MSRV (based on 300 slow time scales) with all the RV estimators at different frequencies computed in Aït-Sahalia and Mancini (2007). Note that the implemented MSRV is about 300 times more computationally intensive than the TSRV estimator. The remaining part of this section shows that in all Monte Carlo experiment, both in-sample and out-of-sample, TSRV and MSRV give close forecasts of IV.

1.7.1 Heston Stochastic Volatility Model

Table 18 summarizes the in-sample estimation of IV and shows that TSRV and MSRV give close estimates of IV. Both estimates are far more accurate than RV estimates, which are reported as a "benchmark" case. Table 19 shows the Mincer-Zarnowitz regressions for the out-of-sample forecasts of TSRV and MSRV and the comparison with RV forecasts. TSRV and MSRV give close forecasts of IV and in particular the corresponding b_0's and b_1's coefficients are nearly the same. We also run Mincer-Zarnowitz regressions of the true IV on both forecasts of IV given by TSRV and MSRV (including or not RV forecasts do not change the result). The coefficient for the TSRV is quite high (≈ 4.6) while the coefficient for MSRV is quite low (≈ -3.6) and their sum is approximately one. Similar results are obtained for all the other Monte Carlo experiments and they are not reported. However, given that TSRV and MSRV forecasts are very close (and accurate) this type of regressions has to be carefully interpreted because of multicollinearity issues.

The presentation of the results for all the other Monte Carlo experiments is similarly organized and in the following we summarize those findings.

1.7.2 Jump-Diffusion Model

Tables 20 and 21 show the in-sample estimation and out-of-sample forecast of TSRV and MSRV and the comparison with RV when the same positive jump component as in Aït-Sahalia and Mancini (2007) is added to the Heston volatility process. Also in this setting, both in-sample and out-of-sample, TSRV and MSRV give close forecasts of IV and both estimators outperform all the other RV estimators.

1.7.3 Jump-Return Heston Volatility Model

Tables 22 and 23 show the in-sample estimation and out-of-sample forecast results of TSRV and MSRV and the comparison with RV when the high frequency data are generated by the jump-return Heston volatility model as in Section 1.5. Also in this setting TSRV and MSRV give close forecasts of IV and both estimators outperform all the other RV estimators.

1.7.4 Log-Volatility Model

Tables 24 and 25 show the in-sample estimation and out-of-sample forecast results of TSRV and MSRV and the comparison with RV when the high frequency data are generated by the discrete time log-volatility model used in Aït-Sahalia and Mancini (2007). Also in this setting TSRV and MSRV give close forecasts of IV and both estimators outperform all the other RV estimators. For instance in both cases, adding RV forecasts either to TSRV or MSRV forecasts, the explanatory power of Mincer-Zarnowitz regressions does not increase.

1.7.5 Heterogenous Autoregressive Realized Volatility Model

Tables 26 and 27 show the in-sample estimation and out-of-sample forecast results of TSRV and MSRV and the comparison with RV when the discrete time high frequency data are generated by the Heterogenous Autoregressive Realized Volatility model used in Aït-Sahalia and Mancini (2007). Also in this setting TSRV and MSRV give close forecasts of IV and both estimators outperform all the other RV estimators. For instance the b_0's and b_1's coefficients and the corresponding standard errors of TSRV and MSRV are nearly the same.

1.7.6 Fractional Ornstein-Uhlenbeck Model

Tables 28 and 29 show the in-sample estimation and out-of-sample forecast results of TSRV and MSRV and the comparison with RV when the high frequency data are generated by the fractional Ornstein-Uhlenbeck model in Aït-Sahalia and Mancini (2007). Also in this setting TSRV and MSRV give close forecasts of IV and both estimators outperform all the other RV estimators.

1.8. Forecasting Integrated Variance Over Different Time Spans

In Aït-Sahalia and Mancini (2007) and in the previous analysis we have considered daily IV. In this section we consider estimation and forecast of IV over two different time spans: two days and five days (i.e. one trading week). For instance the two-day IV is

$$\int_{t-2d}^{t} \sigma_s^2 \, ds,$$

where $2d$ is two trading days. Five-day IV is similarly defined. The simulation setup is the same as in the Monte Carlo experiment with the Heston volatility model. Tables 30 and 31 show in-sample estimation and out-of-sample forecast, respectively, of two-day IV. Tables 32 and 33 report the corresponding results for five-day IV. All out-of-sample forecasts are based on AR(1) models for two- and five-day IV estimated using $m = 100$ trading days of simulated high frequency data. Tables 34 and 35 show the forecasting results of two-day and five-day IV, respectively, when the corresponding AR(1) models are estimated using only $m = 60$ trading days of simulated high frequency data. Overall, all the in-sample estimations and out-of-sample forecasts of IV over different time spans and based on different lengths of simulated trajectories confirm the previous results on daily IV and TSRV largely outperforms RV.

2. Detailed Empirical Analysis of the Thirty DJIA Stocks

In this section we provide a detailed analysis for each of the individual DJIA stocks. The presentation of the empirical results parallels the empirical analysis in Aït-Sahalia and Mancini (2007), and hence the description of tables and plots will be brief. We recall that for each of the thirty DJIA stocks the sample extends from January 3, 2000 to December 31, 2004, and on each day from 9:30 until 16:00.

2.1. Unconditional Return and Integrated Volatility Distributions

Table 36 shows the summary statistics for the daily log-returns of the DJIA stocks, $r_{i,t}$, along with the corresponding names and ticker symbols. All stocks have an unconditional mean close to zero and excess kurtosis. Tables 37 and 38 show the summary statistics for the daily log-returns standardized using the integrated volatility, $r_{i,t}/\mathrm{IV}_{i,t}^{1/2}$. The standardized distributions are now closer to the Gaussian distribution. The integrated volatilities are estimated using TSRV with a slow time scale of five minutes, and two RV estimators, based on de-meaned MA(1) filtered and unfiltered five minutes log-returns.

Tables 39 and 40 present the summary statistics for the integrated volatilities, $\mathrm{IV}_{i,t}^{1/2}$, on an annal base (252 days), and Tables 41 and 42 for the logarithmic integrated volatilities, $\log(\mathrm{IV}_{i,t}^{1/2})$, on a daily base. Logarithmic integrated volatilities, $\log(\mathrm{IV}_{i,t}^{1/2})$, have unconditional distributions closer to the Gaussian distribution than the integrated volatilities, $\mathrm{IV}_{i,t}^{1/2}$. For all stocks the two RV estimators, based on MA(1) filtered and unfiltered log-returns, provide very close estimates of the integrated variances. Figures 1 and 2 show the integrated volatilities and the logarithmic integrated volatilities for the DJIA stocks given by TSRV.

2.2. Temporal Dependence and Long Memory of Integrated Volatility

Tables 43 and 44 show the Ljung-Box portmanteau test (Q_{22}), the augmented Dickey-Fuller test (ADF), and the estimate of the degree of fractional integration (d_{GPH}) using the Geweke and Porter-Hudak log-periodogram regression for the integrated volatilities, $\mathrm{IV}_{i,t}^{1/2}$. Tables 45 and 46 show the same statistics for the logarithmic integrated volatilities, $\log(\mathrm{IV}_{i,t}^{1/2})$. For nearly all stocks in terms of the Ljung-Box test and the degree of fractional integration, the integrated volatilities given by TSRV are more persistent than the integrated volatilities given by RV. The two RV estimators, based on MA(1) filtered and unfiltered log-returns, provide very close estimates of all the previous statistics.

Figure 3 shows the autocorrelations of the logarithmic integrated volatilities given by TSRV, that are almost always well above the conventional Bartlett 95% confidence band even at lag of 120 trading days. As an example Figure 4 shows the autocorrelogram of the integrated volatilities and the logarithmic integrated volatilities for INTC based on TSRV and RV estimates. The two estimates are very close, and well above the conventional Bartlett 95% confidence band even at the lag of 120 trading days.

2.3. *Forecasts of the Integrated Volatility*

In the previous sections we have shown that the two RV estimators, based on MA(1) filtered and unfiltered log-returns, provide very close estimates of the integrated volatilities. Hence in this section we present the one day ahead in- and out-of-sample forecasts given by the first RV estimator as well as the TSRV estimator. We recall that forecasts for the integrated volatility, $\mathrm{IV}_{i,t}^{1/2}$, are based on AR(1) models for the integrated variances $\mathrm{IV}_{i,t}$, and the logarithmic integrated volatilities, $\log(\mathrm{IV}_{i,t}^{1/2})$. For each stock and each time series of the integrated variances given by TSRV and RV, the AR models are estimated on a moving window of 100 trading days. To summarize, we evaluate the integrated volatility forecasts running the following three Mincer-Zarnowitz regressions,

$$
\begin{array}{llcllllll}
1) & \mathrm{IV}^{1/2} & = & b_0 & & & + & b_2\,\mathrm{RV}^{1/2} & + \ \text{error} \\
2) & \mathrm{IV}^{1/2} & = & b_0 & + & b_1\,\mathrm{TSRV}^{1/2} & & & + \ \text{error} \\
3) & \mathrm{IV}^{1/2} & = & b_0 & + & b_1\,\mathrm{TSRV}^{1/2} & + & b_2\,\mathrm{RV}^{1/2} & + \ \text{error,}
\end{array}
$$

where the benchmark integrated volatility $\mathrm{IV}^{1/2}$ is given by TSRV, $\mathrm{TSRV}^{1/2}$ and $\mathrm{RV}^{1/2}$ denote the one day ahead integrated volatility forecasts given by TSRV and RV.

For each stock, Tables 47, 48 and 49 show the in-sample forecasts based on the AR(1) model for IV, and for each of the Mincer-Zarnowitz regressions, respectively. Tables 50, 51 and 52 show the corresponding in-sample forecasts based on the AR(1) model for $\log(\mathrm{IV}_{i,t}^{1/2})$. Out-of-sample forecasts based on the AR(1) model for IV are reported in Tables 53, 54 and 55,[3] and out-of-sample forecasts based on the AR(1) model for $\log(\mathrm{IV}_{i,t}^{1/2})$ are given in Tables 56, 57 and 58. Taken as whole, all these results show that TSRV outperforms RV in forecasting integrated volatilities. As an example Figure 5 shows the in- and out-of-sample forecasts of the integrated volatility for INTC based on AR models for IV and $\log(\mathrm{IV}^{1/2})$, for about the last six months in our sample period, that is from June 24, 2004 to December 31, 2004. The TSRV forecasts tend to be closer to the benchmark integrated volatilities than the RV forecasts, while the AR models for $\log(\mathrm{IV}^{1/2})$ provide better forecasts than AR models for IV.

In the main paper, for space constraints, we present in-sample and out-of-sample forecast results in an aggregated form and based only on AR models for $\log(\mathrm{IV}^{1/2})$. For comparison purposes Tables 59 and 60 show, respectively, in-sample and out-of-sample forecast results based on AR models for IV in the same aggregated form.

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[3] In one occasion, the AR model for IV gave a negative out-of-sample forecast of $\mathrm{IV}^{1/2}$, both for TSRV and RV. That forecast was discarded.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.561	0.530	1.722	0.946	1.233	1.459
RV 5mn MA(1)	1.408	0.484	1.570	0.821	0.786	1.208
TSRV 5mn	−0.024	0.158	0.398	−0.011	0.017	0.132
RV 10mn	0.781	0.742	1.163	0.470	0.404	0.791
TSRV 10mn	−0.054	0.305	0.555	−0.022	0.034	0.185
RV 15mn	0.531	0.950	1.110	0.319	0.284	0.621
TSRV 15mn	−0.084	0.450	0.676	−0.033	0.050	0.226
RV 30mn	0.283	1.660	1.319	0.166	0.254	0.531
TSRV 30mn	−0.181	0.886	0.958	−0.072	0.096	0.318
TSRV minimum variance	0.001	0.036	0.189	−0.001	0.006	0.077

Table 1. In-sample estimates of daily IV$\times 10^4$ based on 10,000 simulated paths under the Heston jump-diffusion model, $d\sigma^2 = 5\,(0.035 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2 + J\,dq$, q is a Poisson process with intensity $\lambda = \Delta \times 23{,}400/2$ and $J \sim \text{Exp}(0.0007)$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.564	0.297	1.656	0.999	0.147	1.070
RV 5mn MA(1)	1.364	0.235	1.447	0.869	0.110	0.930
TSRV 5mn	−0.020	0.046	0.215	−0.012	0.017	0.132
RV 10mn	0.783	0.315	0.964	0.500	0.130	0.616
TSRV 10mn	−0.040	0.090	0.302	−0.025	0.034	0.185
RV 15mn	0.522	0.369	0.801	0.332	0.144	0.504
TSRV 15mn	−0.061	0.134	0.371	−0.038	0.050	0.227
RV 30mn	0.261	0.561	0.793	0.166	0.213	0.491
TSRV 30mn	−0.125	0.252	0.517	−0.077	0.094	0.316
TSRV minimum variance	−0.004	0.016	0.127	−0.003	0.006	0.080

Table 2. In-sample estimates of daily IV$\times 10^4$ based on 10,000 simulated paths under the log-volatility model, $l_t = -0.0161 + 0.35\,l_{t-d} + 0.25\,l_{t-2d} + 0.20\,l_{t-3d} + 0.10\,l_{t-4d} + 0.09\,l_{t-5d} + u_t$, where $u \sim NID(0, 0.02^2)$ and d = 1 day. The efficient log-return $r_t = X_t - X_{t-\Delta} = \sqrt{IV_t^{(d)}}\,z_t$, where $z \sim NID(0,1)$ and $\Delta = 1$ second. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.565	0.293	1.656	1.014	0.153	1.087
RV 5mn MA(1)	1.362	0.230	1.444	0.880	0.113	0.943
TSRV 5mn	−0.019	0.044	0.212	−0.012	0.017	0.132
RV 10mn	0.783	0.309	0.961	0.507	0.131	0.623
TSRV 10mn	−0.039	0.087	0.298	−0.025	0.034	0.185
RV 15mn	0.522	0.362	0.797	0.338	0.146	0.511
TSRV 15mn	−0.059	0.130	0.365	−0.038	0.050	0.227
RV 30mn	0.261	0.549	0.786	0.169	0.215	0.494
TSRV 30mn	−0.122	0.246	0.510	−0.077	0.095	0.317
TSRV minimum variance	−0.003	0.016	0.127	−0.002	0.006	0.080

Table 3. In-sample estimates of daily IV$\times 10^4$ based on 10,000 simulated paths under the HAR-RV model, $IV^{(d)} = 0.002 + 0.45\,RV^{(d)} + 0.30\,RV^{(w)} + 0.20\,RV^{(m)} + \omega^{(d)}$, where $\omega^{(d)} \sim NID(0, 0.003^2)$. The efficient log-return $r_t = X_t - X_{t-\Delta} = \sqrt{IV_t^{(d)}}z_t$, where $z \sim NID(0,1)$ and $\Delta = 1$ second. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.558	0.288	1.647	1.036	0.219	1.137
RV 5mn MA(1)	1.360	0.229	1.442	0.900	0.155	0.982
TSRV 5mn	−0.018	0.048	0.220	−0.011	0.017	0.132
RV 10mn	0.780	0.316	0.962	0.520	0.149	0.647
TSRV 10mn	−0.038	0.092	0.306	−0.023	0.033	0.183
RV 15mn	0.524	0.373	0.805	0.350	0.156	0.527
TSRV 15mn	−0.057	0.136	0.373	−0.035	0.049	0.223
RV 30mn	0.271	0.568	0.801	0.183	0.214	0.498
TSRV 30mn	−0.115	0.260	0.523	−0.070	0.093	0.313
TSRV minimum variance	−0.006	0.017	0.129	−0.004	0.006	0.080

Table 4. In-sample estimates of daily IV$\times 10^4$ based on 10,000 simulated paths under the fractional Ornstein-Uhlenbeck process, $d\sigma = 20\,(0.2 - \sigma)\,dt + 0.012\,dW_H$, where dW_H is a FBM with Hurst parameter 0.7. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.269 (0.013)		1.393 (0.007)	0.776
RV 5mn MA(1)	−0.980 (0.010)		1.277 (0.006)	0.823
RV 10mn	−0.662 (0.010)		1.217 (0.007)	0.768
RV 15mn	−0.452 (0.010)		1.146 (0.007)	0.745
RV 30mn	−0.214 (0.010)		1.049 (0.007)	0.674
TSRV 5mn	−0.019 (0.003)	1.010 (0.003)		0.936
TSRV 5mn + RV 5mn	0.151 (0.011)	1.111 (0.007)	−0.165 (0.010)	0.937
TSRV 5mn + RV 5mn MA(1)	0.176 (0.010)	1.173 (0.009)	−0.229 (0.012)	0.938
TSRV 5mn + RV 10mn	0.126 (0.007)	1.159 (0.007)	−0.212 (0.009)	0.939
TSRV 5mn + RV 15mn	0.098 (0.006)	1.160 (0.006)	−0.207 (0.008)	0.940
TSRV 5mn + RV 30mn	0.062 (0.004)	1.134 (0.005)	−0.172 (0.007)	0.940
TSRV 10mn	−0.031 (0.004)	1.022 (0.003)		0.901
TSRV 10mn + RV 5mn	−0.021 (0.014)	1.028 (0.009)	−0.010 (0.013)	0.901
TSRV 10mn + RV 5mn MA(1)	0.021 (0.014)	1.069 (0.012)	−0.064 (0.016)	0.901
TSRV 10mn + RV 10mn	0.110 (0.009)	1.181 (0.010)	−0.218 (0.013)	0.904
TSRV 10mn + RV 15mn	0.125 (0.007)	1.250 (0.009)	−0.300 (0.012)	0.907
TSRV 10mn + RV 30mn	0.086 (0.006)	1.234 (0.008)	−0.277 (0.009)	0.909
TSRV 15mn	−0.039 (0.005)	1.030 (0.004)		0.868
TSRV 15mn + RV 5mn	−0.244 (0.016)	0.896 (0.011)	0.208 (0.015)	0.870
TSRV 15mn + RV 5mn MA(1)	−0.247 (0.015)	0.834 (0.014)	0.261 (0.018)	0.870
TSRV 15mn + RV 10mn	−0.015 (0.011)	1.059 (0.012)	−0.038 (0.015)	0.868
TSRV 15mn + RV 15mn	0.081 (0.009)	1.224 (0.012)	−0.245 (0.015)	0.871
TSRV 15mn + RV 30mn	0.096 (0.007)	1.314 (0.010)	−0.354 (0.012)	0.879
TSRV 30mn	−0.029 (0.007)	1.037 (0.006)		0.780
TSRV 30mn + RV 5mn	−0.723 (0.017)	0.555 (0.012)	0.715 (0.016)	0.816
TSRV 30mn + RV 30mn	0.040 (0.009)	1.232 (0.017)	−0.224 (0.019)	0.783
TSRV minimum variance	−0.021 (0.003)	1.011 (0.002)		0.958

Table 5. Out-of-sample, one day ahead forecasts of daily $\text{IV}^{1/2}$ in percentage based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. Forecast equation is estimated using the previous 100 simulated days and regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.567	0.335	1.670	6.089	1,742.201	42.181
RV 5mn MA(1)	1.323	0.291	1.429	4.510	851.385	29.525
TSRV 5mn	−0.012	0.081	0.285	−0.008	0.173	0.416
RV 10mn	0.785	0.434	1.025	2.981	370.691	19.483
TSRV 10mn	−0.029	0.156	0.395	−0.023	0.069	0.264
RV 15mn	0.530	0.514	0.892	1.998	163.476	12.941
TSRV 15mn	−0.047	0.223	0.475	−0.033	0.062	0.252
RV 30mn	0.278	0.878	0.977	1.001	37.923	6.239
TSRV 30mn	−0.108	0.435	0.668	−0.071	0.100	0.323
TSRV minimum variance	−0.000	0.019	0.138	−0.004	0.015	0.122

Table 6. In-sample estimates of daily IV$\times 10^4$ based on 10,000 simulated paths under the "nearly integrated" Heston model, $d\sigma^2 = (0.04 - \sigma^2)\,dt + 0.275\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.479 (0.011)		0.973 (0.003)	0.904
RV 5mn MA(1)	−1.097 (0.009)		0.922 (0.003)	0.918
RV 10mn	−0.710 (0.009)		0.967 (0.003)	0.891
RV 15mn	−0.464 (0.009)		0.967 (0.004)	0.880
RV 30mn	−0.195 (0.009)		0.956 (0.004)	0.850
TSRV 5mn	0.008 (0.004)	0.997 (0.002)		0.963
TSRV 5mn + RV 5mn	0.182 (0.015)	1.101 (0.009)	−0.108 (0.009)	0.963
TSRV 5mn + RV 5mn MA(1)	0.198 (0.013)	1.152 (0.010)	−0.150 (0.010)	0.963
TSRV 5mn + RV 10mn	0.164 (0.008)	1.175 (0.008)	−0.185 (0.008)	0.964
TSRV 5mn + RV 15mn	0.115 (0.006)	1.164 (0.008)	−0.176 (0.008)	0.964
TSRV 5mn + RV 30mn	0.063 (0.005)	1.132 (0.006)	−0.145 (0.006)	0.964
TSRV 10mn	0.011 (0.005)	1.005 (0.003)		0.940
TSRV 10mn + RV 5mn	−0.135 (0.019)	0.915 (0.012)	0.091 (0.012)	0.941
TSRV 10mn + RV 5mn MA(1)	−0.103 (0.018)	0.908 (0.015)	0.091 (0.014)	0.941
TSRV 10mn + RV 10mn	0.141 (0.011)	1.161 (0.013)	−0.158 (0.012)	0.941
TSRV 10mn + RV 15mn	0.149 (0.009)	1.240 (0.012)	−0.239 (0.012)	0.943
TSRV 10mn + RV 30mn	0.091 (0.006)	1.227 (0.009)	−0.230 (0.010)	0.944
TSRV 15mn	0.014 (0.006)	1.015 (0.003)		0.922
TSRV 15mn + RV 5mn	−0.467 (0.021)	0.711 (0.013)	0.302 (0.012)	0.927
TSRV 15mn + RV 5mn MA(1)	−0.471 (0.019)	0.592 (0.016)	0.392 (0.014)	0.928
TSRV 15mn + RV 10mn	−0.024 (0.013)	0.968 (0.015)	0.047 (0.015)	0.923
TSRV 15mn + RV 15mn	0.096 (0.010)	1.164 (0.016)	−0.148 (0.015)	0.923
TSRV 15mn + RV 30mn	0.106 (0.007)	1.298 (0.013)	−0.284 (0.012)	0.926
TSRV 30mn	0.032 (0.008)	1.045 (0.004)		0.882
TSRV 30mn + RV 5mn MA(1)	−0.977 (0.016)	0.125 (0.014)	0.816 (0.012)	0.918
TSRV 30mn + RV 30mn	0.049 (0.010)	1.109 (0.021)	−0.060 (0.020)	0.882
TSRV minimum variance	−0.002 (0.003)	0.998 (0.001)		0.986

Table 7. Out-of-sample, one day ahead forecasts of daily IV in percentage based on 10,000 simulated paths under the "nearly integrated" Heston model, $d\sigma^2 = (0.04 - \sigma^2)\,dt + 0.275\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. Forecasting equation is estimated using the previous 100 simulated days and regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.561	0.619	1.748	0.778	0.672	1.131
RV 5mn MA(1)	1.422	0.573	1.611	0.687	0.448	0.959
TSRV 5mn	−0.028	0.198	0.446	−0.011	0.017	0.131
RV 10mn	0.782	0.896	1.228	0.387	0.255	0.636
TSRV 10mn	−0.062	0.384	0.622	−0.022	0.034	0.184
RV 15mn	0.532	1.163	1.203	0.263	0.201	0.519
TSRV 15mn	−0.096	0.566	0.758	−0.033	0.050	0.225
RV 30mn	0.287	2.059	1.463	0.138	0.222	0.491
TSRV 30mn	−0.207	1.113	1.075	−0.072	0.096	0.318
TSRV minimum variance	0.001	0.042	0.206	−0.001	0.005	0.072

Table 8. In-sample estimates of daily IV$\times 10^4$ based on 10,000 simulated paths under the Heston jump-diffusion model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2 + J\,dq$, q is a Poisson process with intensity $\lambda = \Delta \times 23{,}400/2$ and $J \sim \text{Exp}(0.0004)$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.266 (0.022)		0.961 (0.005)	0.804
RV 5mn MA(1)	−1.017 (0.020)		0.930 (0.004)	0.822
RV 10mn	−0.434 (0.021)		0.942 (0.005)	0.756
RV 15mn	−0.139 (0.022)		0.931 (0.006)	0.715
RV 30mn	0.252 (0.024)		0.897 (0.007)	0.617
TSRV 5mn	0.114 (0.011)	0.982 (0.003)		0.905
TSRV 5mn + RV 5mn	0.179 (0.021)	1.015 (0.010)	−0.037 (0.010)	0.905
TSRV 5mn + RV 5mn MA(1)	0.256 (0.020)	1.075 (0.011)	−0.096 (0.011)	0.906
TSRV 5mn + RV 10mn	0.256 (0.014)	1.104 (0.009)	−0.138 (0.009)	0.907
TSRV 5mn + RV 15mn	0.218 (0.013)	1.084 (0.008)	−0.121 (0.008)	0.907
TSRV 5mn + RV 30mn	0.188 (0.012)	1.053 (0.006)	−0.094 (0.007)	0.907
TSRV 10mn	0.177 (0.013)	0.982 (0.004)		0.851
TSRV 10mn + RV 5mn	−0.304 (0.024)	0.727 (0.012)	0.273 (0.012)	0.859
TSRV 10mn + RV 5mn MA(1)	−0.217 (0.024)	0.711 (0.014)	0.271 (0.014)	0.857
TSRV 10mn + RV 10mn	0.209 (0.018)	1.013 (0.013)	−0.034 (0.013)	0.851
TSRV 10mn + RV 15mn	0.286 (0.016)	1.109 (0.011)	−0.142 (0.012)	0.853
TSRV 10mn + RV 30mn	0.274 (0.015)	1.096 (0.009)	−0.140 (0.009)	0.855
TSRV 15mn	0.225 (0.015)	0.986 (0.005)		0.806
TSRV 15mn + RV 5mn	−0.647 (0.025)	0.520 (0.012)	0.492 (0.011)	0.837
TSRV 15mn + RV 5mn MA(1)	−0.600 (0.024)	0.411 (0.014)	0.566 (0.013)	0.836
TSRV 15mn + RV 10mn	0.001 (0.020)	0.763 (0.014)	0.234 (0.014)	0.811
TSRV 15mn + RV 15mn	0.227 (0.019)	0.989 (0.014)	−0.004 (0.015)	0.806
TSRV 15mn + RV 30mn	0.326 (0.017)	1.130 (0.011)	−0.166 (0.012)	0.810
TSRV 30mn	0.401 (0.019)	0.983 (0.007)		0.695
TSRV 30mn + RV 5mn MA(1)	−0.942 (0.022)	0.100 (0.011)	0.852 (0.010)	0.823
TSRV 30mn + RV 30mn	0.356 (0.021)	0.901 (0.018)	0.085 (0.017)	0.696
TSRV minimum variance	0.050 (0.007)	0.986 (0.002)		0.957

Table 9. Out-of-sample forecasts, one day ahead of daily IV in percentage based on 10,000 simulated paths under the Heston jump-diffusion model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2 + J\,dq$, q is a Poisson process with intensity $\lambda = \Delta \times 23{,}400/2$ and $J \sim \text{Exp}(0.0004)$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. The forecast equation is estimated using the previous 100 days of estimated IV, regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.559	0.349	1.667	2.114	72.619	8.780
RV 5mn MA(1)	1.359	0.301	1.466	1.757	46.733	7.058
TSRV 5mn	−0.018	0.090	0.300	−0.012	0.020	0.142
RV 10mn	0.783	0.431	1.022	1.056	19.984	4.593
TSRV 10mn	−0.031	0.169	0.413	−0.021	0.034	0.185
RV 15mn	0.543	0.550	0.919	0.709	8.870	3.062
TSRV 15mn	−0.046	0.248	0.500	−0.030	0.049	0.223
RV 30mn	0.310	0.907	1.002	0.368	2.272	1.551
TSRV 30mn	−0.092	0.481	0.700	−0.061	0.093	0.311
TSRV minimum variance	−0.008	0.033	0.181	−0.006	0.011	0.105

Table 10. In-sample estimates of daily IV$\times 10^4$ based on 10,000 simulated paths under the jump-return Heston diffusion model with efficient log-price $dX = (0.05 - \sigma^2/2)\, dt + \sigma\, dW_1 + J_X\, dq_X$, q_X is a Poisson process with intensity $\lambda_X = \Delta \times 23{,}400/2$, $J_X \sim \mathcal{N}(-0.004, 0.004)$ and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.260 (0.037)		0.917 (0.010)	0.436
RV 5mn MA(1)	−0.961 (0.033)		0.882 (0.010)	0.447
RV 10mn	−0.561 (0.030)		0.920 (0.011)	0.424
RV 15mn	−0.274 (0.028)		0.896 (0.011)	0.402
RV 30mn	0.008 (0.026)		0.861 (0.011)	0.366
TSRV 5mn	0.120 (0.020)	0.958 (0.010)		0.482
TSRV 5mn + RV 5mn	0.507 (0.068)	1.183 (0.039)	−0.234 (0.039)	0.484
TSRV 5mn + RV 5mn MA(1)	0.450 (0.062)	1.209 (0.046)	−0.246 (0.044)	0.483
TSRV 5mn + RV 10mn	0.357 (0.039)	1.197 (0.035)	−0.256 (0.036)	0.484
TSRV 5mn + RV 15mn	0.347 (0.030)	1.259 (0.031)	−0.325 (0.032)	0.487
TSRV 5mn + RV 30mn	0.250 (0.024)	1.171 (0.024)	−0.241 (0.025)	0.487
TSRV 10mn	0.132 (0.021)	0.958 (0.010)		0.463
TSRV 10mn + RV 5mn	−0.017 (0.066)	0.869 (0.039)	0.091 (0.038)	0.463
TSRV 10mn + RV 5mn MA(1)	−0.037 (0.063)	0.824 (0.048)	0.128 (0.045)	0.463
TSRV 10mn + RV 10mn	0.262 (0.042)	1.099 (0.041)	−0.146 (0.041)	0.463
TSRV 10mn + RV 15mn	0.369 (0.032)	1.306 (0.037)	−0.364 (0.037)	0.468
TSRV 10mn + RV 30mn	0.282 (0.025)	1.242 (0.028)	−0.309 (0.028)	0.469
TSRV 15mn	0.143 (0.021)	0.959 (0.011)		0.444
TSRV 15mn + RV 5mn	−0.460 (0.061)	0.592 (0.036)	0.371 (0.035)	0.450
TSRV 15mn + RV 5mn MA(1)	−0.508 (0.057)	0.430 (0.044)	0.500 (0.041)	0.453
TSRV 15mn + RV 10mn	0.002 (0.041)	0.800 (0.041)	0.161 (0.040)	0.445
TSRV 15mn + RV 15mn	0.265 (0.033)	1.153 (0.041)	−0.197 (0.040)	0.446
TSRV 15mn + RV 30mn	0.288 (0.026)	1.276 (0.033)	−0.332 (0.032)	0.450
TSRV 30mn	0.192 (0.023)	0.955 (0.012)		0.400
TSRV 30mn + RV 5mn MA(1)	−0.961 (0.045)	0.001 (0.035)	0.881 (0.030)	0.447
TSRV 30mn + RV 30mn	0.203 (0.027)	0.984 (0.041)	−0.028 (0.039)	0.400
TSRV minimum variance	0.104 (0.020)	0.962 (0.010)		0.497

Table 11. Out-of-sample forecasts, one day ahead of daily IV in percentage based on 10,000 simulated paths under the jump-return Heston diffusion model with efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma\,dW_1 + J_X\,dq_X$, q_X is a Poisson process with intensity $\lambda_X = \Delta \times 23{,}400/2$, $J_X \sim \mathcal{N}(-0.004, 0.004)$ and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. The forecast equation is estimated using the previous 100 days of estimated IV, regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
Panel A						
RV 5mn	1.559	0.319	1.658	2.492	44.998	7.156
RV 5mn MA(1)	1.347	0.274	1.445	2.021	26.737	5.552
TSRV 5mn	−0.166	0.071	0.314	−0.211	0.064	0.329
RV 10mn	0.779	0.384	0.995	1.240	12.043	3.685
TSRV 10mn	−0.108	0.135	0.383	−0.131	0.056	0.270
RV 15mn	0.523	0.458	0.855	0.843	7.097	2.794
TSRV 15mn	−0.100	0.199	0.458	−0.107	0.063	0.273
RV 30mn	0.276	0.778	0.924	0.431	1.949	1.461
TSRV 30mn	−0.134	0.395	0.642	−0.108	0.101	0.336
TSRV minimum variance	−0.692	0.157	0.797	−0.539	0.035	0.570
Panel B						
RV 5mn	1.537	0.315	1.636	2.468	42.850	6.996
RV 5mn MA(1)	1.330	0.272	1.429	1.996	24.163	5.305
TSRV 5mn	−0.167	0.071	0.315	−0.212	0.067	0.334
RV 10mn	0.770	0.387	0.990	1.216	10.272	3.428
TSRV 10mn	−0.108	0.136	0.384	−0.130	0.056	0.269
RV 15mn	0.519	0.478	0.865	0.832	6.275	2.639
TSRV 15mn	−0.100	0.202	0.461	−0.107	0.063	0.272
RV 30mn	0.267	0.771	0.918	0.409	1.330	1.224
TSRV 30mn	−0.134	0.399	0.646	−0.108	0.101	0.336
TSRV minimum variance	−0.694	0.156	0.799	−0.550	0.053	0.596

Table 12. In-sample estimates of daily IV$\times 10^4$ based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon = U + V$, U is iid in Panel A and negatively correlated, -0.2, with latent log-returns in Panel B, V is AR(1) with first order coefficient $\varrho = -0.2$, and $U \perp V$. In both panels, $E\,\varepsilon^2 = 0.001^2$. Euler discretization scheme with time step $\Delta = 1$ second.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.583	0.327	1.683	2.497	40.521	6.838
RV 5mn MA(1)	1.366	0.281	1.466	2.021	24.431	5.340
TSRV 5mn	−0.014	0.071	0.267	−0.010	0.021	0.146
RV 10mn	0.801	0.387	1.014	1.257	14.040	3.952
TSRV 10mn	−0.032	0.136	0.370	−0.022	0.035	0.188
RV 15mn	0.535	0.482	0.877	0.845	6.179	2.625
TSRV 15mn	−0.050	0.202	0.452	−0.033	0.050	0.227
RV 30mn	0.274	0.780	0.925	0.422	1.569	1.322
TSRV 30mn	−0.110	0.399	0.641	−0.072	0.097	0.319
TSRV minimum variance	−0.001	0.019	0.138	−0.003	0.010	0.099

Table 13. In-sample estimates of IV$\times 10^4$ based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, $Corr(\varepsilon_t, X_t - X_{t-\Delta}) = 0.2$, and $E\,\varepsilon^2 = 0.001^2$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.487 (0.016)		0.969 (0.005)	0.813
RV 5mn MA(1)	−1.122 (0.013)		0.915 (0.004)	0.844
RV 10mn	−0.710 (0.013)		0.959 (0.005)	0.787
RV 15mn	−0.434 (0.013)		0.946 (0.005)	0.754
RV 30mn	−0.126 (0.014)		0.912 (0.006)	0.670
TSRV 5mn	0.016 (0.006)	0.991 (0.003)		0.927
TSRV 5mn + RV 5mn	0.126 (0.016)	1.048 (0.008)	−0.063 (0.009)	0.928
TSRV 5mn + RV 5mn MA(1)	0.171 (0.015)	1.102 (0.010)	−0.112 (0.010)	0.928
TSRV 5mn + RV 10mn	0.153 (0.010)	1.116 (0.008)	−0.140 (0.008)	0.929
TSRV 5mn + RV 15mn	0.111 (0.008)	1.099 (0.007)	−0.124 (0.007)	0.929
TSRV 5mn + RV 30mn	0.084 (0.007)	1.082 (0.006)	−0.112 (0.006)	0.930
TSRV 10mn	0.021 (0.007)	0.997 (0.004)		0.888
TSRV 10mn + RV 5mn	−0.210 (0.019)	0.872 (0.010)	0.135 (0.011)	0.890
TSRV 10mn + RV 5mn MA(1)	−0.147 (0.019)	0.870 (0.014)	0.124 (0.013)	0.889
TSRV 10mn + RV 10mn	0.093 (0.013)	1.068 (0.011)	−0.077 (0.011)	0.889
TSRV 10mn + RV 15mn	0.131 (0.010)	1.139 (0.010)	−0.156 (0.011)	0.891
TSRV 10mn + RV 30mn	0.115 (0.008)	1.146 (0.008)	−0.174 (0.008)	0.893
TSRV 15mn	0.023 (0.008)	1.005 (0.004)		0.853
TSRV 15mn + RV 5mn	−0.534 (0.020)	0.695 (0.011)	0.328 (0.011)	0.864
TSRV 15mn + RV 5mn MA(1)	−0.518 (0.019)	0.577 (0.014)	0.408 (0.013)	0.866
TSRV 15mn + RV 10mn	−0.099 (0.014)	0.878 (0.013)	0.134 (0.013)	0.854
TSRV 15mn + RV 15mn	0.069 (0.012)	1.071 (0.013)	−0.069 (0.013)	0.853
TSRV 15mn + RV 30mn	0.129 (0.009)	1.203 (0.010)	−0.219 (0.011)	0.859
TSRV 30mn	0.068 (0.011)	1.013 (0.006)		0.759
TSRV 30mn + RV 5mn MA(1)	−1.004 (0.017)	0.137 (0.013)	0.806 (0.011)	0.846
TSRV 30mn + RV 30mn	0.084 (0.013)	1.053 (0.017)	−0.040 (0.017)	0.759
TSRV minimum variance	0.002 (0.004)	0.995 (0.002)		0.961

Table 14. Out-of-sample forecasts of IV in percentage based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, $Corr(\varepsilon_t, X_t - X_{t-\Delta}) = 0.2$, and $E\,\varepsilon^2 = 0.001^2$. Euler discretization scheme with time step $\Delta = 1$ second. One day ahead forecasts of IV. OLS standard errors in parenthesis. Forecasting equation is estimated using the previous 100 simulated days and regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.532	0.316	1.632	2.459	43.998	7.074
RV 5mn MA(1)	1.326	0.272	1.425	1.993	25.843	5.460
TSRV 5mn	−0.014	0.071	0.267	−0.010	0.021	0.145
RV 10mn	0.777	0.381	0.992	1.240	15.047	4.072
TSRV 10mn	−0.032	0.136	0.370	−0.022	0.035	0.188
RV 15mn	0.518	0.477	0.863	0.835	6.282	2.642
TSRV 15mn	−0.050	0.202	0.452	−0.033	0.050	0.227
RV 30mn	0.265	0.773	0.918	0.415	1.673	1.359
TSRV 30mn	−0.110	0.399	0.641	−0.071	0.097	0.319
TSRV minimum variance	−0.000	0.018	0.135	−0.003	0.010	0.098

Table 15. In-sample estimates of IV$\times 10^4$ based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, $Corr(\varepsilon_t, X_t - X_{t-\Delta}) = -0.2$, and $E\,\varepsilon^2 = 0.001^2$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.489 (0.016)		0.985 (0.005)	0.813
RV 5mn MA(1)	−1.126 (0.013)		0.929 (0.004)	0.844
RV 10mn	−0.709 (0.013)		0.969 (0.005)	0.788
RV 15mn	−0.430 (0.013)		0.951 (0.005)	0.754
RV 30mn	−0.123 (0.014)		0.914 (0.006)	0.670
TSRV 5mn	0.016 (0.006)	0.991 (0.003)		0.927
TSRV 5mn + RV 5mn	0.132 (0.016)	1.051 (0.008)	−0.067 (0.009)	0.928
TSRV 5mn + RV 5mn MA(1)	0.179 (0.015)	1.107 (0.010)	−0.119 (0.010)	0.928
TSRV 5mn + RV 10mn	0.156 (0.010)	1.118 (0.008)	−0.144 (0.008)	0.930
TSRV 5mn + RV 15mn	0.112 (0.008)	1.101 (0.007)	−0.127 (0.007)	0.930
TSRV 5mn + RV 30mn	0.084 (0.007)	1.083 (0.006)	−0.114 (0.006)	0.930
TSRV 10mn	0.021 (0.007)	0.997 (0.004)		0.888
TSRV 10mn + RV 5mn	−0.209 (0.019)	0.873 (0.010)	0.136 (0.011)	0.890
TSRV 10mn + RV 5mn MA(1)	−0.143 (0.019)	0.873 (0.014)	0.123 (0.013)	0.889
TSRV 10mn + RV 10mn	0.094 (0.013)	1.070 (0.011)	−0.080 (0.012)	0.889
TSRV 10mn + RV 15mn	0.135 (0.010)	1.146 (0.010)	−0.164 (0.011)	0.891
TSRV 10mn + RV 30mn	0.115 (0.008)	1.148 (0.008)	−0.177 (0.008)	0.893
TSRV 15mn	0.023 (0.008)	1.005 (0.004)		0.853
TSRV 15mn + RV 5mn	−0.536 (0.020)	0.695 (0.011)	0.334 (0.011)	0.865
TSRV 15mn + RV 5mn MA(1)	−0.517 (0.019)	0.580 (0.014)	0.412 (0.013)	0.865
TSRV 15mn + RV 10mn	−0.096 (0.014)	0.880 (0.013)	0.132 (0.013)	0.854
TSRV 15mn + RV 15mn	0.074 (0.012)	1.079 (0.013)	−0.078 (0.013)	0.853
TSRV 15mn + RV 30mn	0.131 (0.009)	1.208 (0.010)	−0.226 (0.011)	0.859
TSRV 30mn	0.068 (0.011)	1.013 (0.006)		0.759
TSRV 30mn + RV 5mn MA(1)	−1.004 (0.017)	0.141 (0.013)	0.815 (0.011)	0.846
TSRV 30mn + RV 30mn	0.087 (0.013)	1.061 (0.017)	−0.048 (0.017)	0.759
TSRV minimum variance	0.002 (0.004)	0.994 (0.002)		0.962

Table 16. Out-of-sample forecasts of IV in percentage based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04-\sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05-\sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, $Corr(\varepsilon_t, X_t - X_{t-\Delta}) = -0.2$, and $E\,\varepsilon^2 = 0.001^2$. Euler discretization scheme with time step $\Delta = 1$ second. One day ahead forecasts of IV. OLS standard errors in parenthesis. Forecasting equation is estimated using the previous 100 simulated days and regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\, dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\, dt$, for $i = 1, \ldots, 99$.

	b_0	b_1	b_2	R^2
RV 5mn	−1.495 (0.016)		0.986 (0.005)	0.813
RV 5mn MA(1)	−1.130 (0.013)		0.929 (0.004)	0.843
RV 10mn	−0.704 (0.013)		0.967 (0.005)	0.784
RV 15mn	−0.435 (0.013)		0.953 (0.005)	0.754
RV 30mn	−0.131 (0.014)		0.918 (0.006)	0.669
TSRV 5mn	0.166 (0.005)	0.992 (0.003)		0.927
TSRV 5mn + RV 5mn	0.290 (0.017)	1.052 (0.008)	−0.067 (0.009)	0.928
TSRV 5mn + RV 5mn MA(1)	0.343 (0.016)	1.106 (0.010)	−0.116 (0.010)	0.928
TSRV 5mn + RV 10mn	0.320 (0.010)	1.115 (0.008)	−0.139 (0.008)	0.929
TSRV 5mn + RV 15mn	0.279 (0.008)	1.103 (0.007)	−0.128 (0.007)	0.929
TSRV 5mn + RV 30mn	0.249 (0.007)	1.084 (0.006)	−0.114 (0.006)	0.930
TSRV 10mn	0.096 (0.007)	0.997 (0.004)		0.888
TSRV 10mn + RV 5mn	−0.136 (0.020)	0.877 (0.010)	0.132 (0.011)	0.890
TSRV 10mn + RV 5mn MA(1)	−0.068 (0.020)	0.880 (0.014)	0.116 (0.013)	0.889
TSRV 10mn + RV 10mn	0.178 (0.013)	1.073 (0.011)	−0.082 (0.011)	0.889
TSRV 10mn + RV 15mn	0.227 (0.011)	1.153 (0.010)	−0.171 (0.011)	0.891
TSRV 10mn + RV 30mn	0.205 (0.008)	1.150 (0.008)	−0.180 (0.008)	0.893
TSRV 15mn	0.073 (0.008)	1.006 (0.004)		0.853
TSRV 15mn + RV 5mn	−0.498 (0.021)	0.698 (0.011)	0.331 (0.011)	0.864
TSRV 15mn + RV 5mn MA(1)	−0.482 (0.020)	0.586 (0.014)	0.405 (0.013)	0.865
TSRV 15mn + RV 10mn	−0.041 (0.015)	0.893 (0.013)	0.120 (0.013)	0.854
TSRV 15mn + RV 15mn	0.135 (0.012)	1.089 (0.013)	−0.088 (0.013)	0.853
TSRV 15mn + RV 30mn	0.196 (0.010)	1.212 (0.010)	−0.231 (0.011)	0.859
TSRV 30mn	0.092 (0.010)	1.014 (0.006)		0.759
TSRV 30mn + RV 5mn MA(1)	−1.000 (0.017)	0.146 (0.013)	0.811 (0.011)	0.845
TSRV 30mn + RV 30mn	0.114 (0.013)	1.063 (0.017)	−0.051 (0.017)	0.759
TSRV minimum variance	0.334 (0.004)	1.387 (0.003)		0.942

Table 17. Out-of-sample, one day ahead forecasts of IV in percentage based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon = U + V$, $Corr(U_t, X_t - X_{t-\Delta}) = -0.2$, V is AR(1) with first order coefficient $\varrho = -0.20$, $U \perp V$, and $E\,\varepsilon^2 = 0.001$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. Forecasting equation is estimated using the previous 100 simulated days and regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.567	0.325	1.667	2.266	20.822	5.095
TSRV 5mn	−0.014	0.069	0.263	−0.014	0.017	0.133
MSRV 5mn	−0.016	0.078	0.280	−0.015	0.019	0.137

Table 18. In-sample estimates of daily IV$\times 10^4$ based on 1,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.420 (0.052)		0.957 (0.015)	0.793
RV 5mn MA(1)	−1.078 (0.041)		0.907 (0.013)	0.832
RV 10mn	−0.648 (0.043)		0.941 (0.017)	0.765
RV 15mn	−0.406 (0.041)		0.942 (0.017)	0.754
RV 30mn	−0.092 (0.043)		0.893 (0.020)	0.676
TSRV 5mn	0.046 (0.017)	0.975 (0.009)		0.924
TSRV 5mn + RV 5mn	0.215 (0.050)	1.061 (0.025)	−0.097 (0.027)	0.925
TSRV 5mn + RV 5mn MA(1)	0.216 (0.046)	1.094 (0.031)	−0.122 (0.030)	0.926
TSRV 5mn + RV 10mn	0.172 (0.030)	1.086 (0.023)	−0.128 (0.025)	0.926
TSRV 5mn + RV 15mn	0.128 (0.025)	1.068 (0.022)	−0.109 (0.024)	0.926
TSRV 5mn + RV 30mn	0.141 (0.020)	1.115 (0.019)	−0.169 (0.020)	0.929
MSRV 5mn	0.047 (0.018)	0.974 (0.009)		0.918
MSRV 5mn + RV 5mn	0.165 (0.052)	1.035 (0.026)	−0.068 (0.028)	0.919
MSRV 5mn + RV 5mn MA(1)	0.149 (0.047)	1.046 (0.032)	−0.073 (0.031)	0.919
MSRV 5mn + RV 10mn	0.144 (0.031)	1.061 (0.024)	−0.098 (0.026)	0.919
MSRV 5mn + RV 15mn	0.106 (0.026)	1.041 (0.023)	−0.078 (0.025)	0.919
MSRV 5mn + RV 30mn	0.133 (0.021)	1.102 (0.020)	−0.153 (0.021)	0.922

Table 19. Out-of-sample, one day ahead forecasts of daily IV in percentage based on 1,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04-\sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05-\sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. Forecast equation is estimated using the previous 100 simulated days and regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.566	0.531	1.727	0.963	2.155	1.756
TSRV 5mn	−0.027	0.152	0.391	−0.014	0.016	0.129
MSRV 5mn	−0.029	0.172	0.416	−0.015	0.018	0.136

Table 20. In-sample estimates of daily IV$\times 10^4$ based on 1,000 simulated paths under the Heston jump-diffusion model, $d\sigma^2 = 5\,(0.035 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2 + J\,dq$, q is a Poisson process with intensity $\lambda = \Delta \times 23{,}400/2$ and $J \sim \text{Exp}(0.0007)$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.246 (0.066)		0.950 (0.015)	0.791
RV 5mn MA(1)	−0.995 (0.057)		0.919 (0.014)	0.819
RV 10mn	−0.433 (0.061)		0.930 (0.017)	0.748
RV 15mn	−0.167 (0.060)		0.926 (0.018)	0.726
RV 30mn	0.233 (0.064)		0.875 (0.021)	0.637
TSRV 5mn	0.133 (0.029)	0.968 (0.010)		0.907
TSRV 5mn + RV 5mn	0.253 (0.061)	1.030 (0.029)	−0.068 (0.030)	0.908
TSRV 5mn + RV 5mn MA(1)	0.278 (0.058)	1.063 (0.034)	−0.099 (0.034)	0.908
TSRV 5mn + RV 10mn	0.254 (0.040)	1.071 (0.025)	−0.117 (0.027)	0.909
TSRV 5mn + RV 15mn	0.209 (0.035)	1.047 (0.023)	−0.093 (0.025)	0.909
TSRV 5mn + RV 30mn	0.227 (0.031)	1.086 (0.020)	−0.146 (0.021)	0.912
MSRV 5mn	0.142 (0.030)	0.967 (0.010)		0.899
MSRV 5mn + RV 5mn	0.187 (0.063)	0.990 (0.030)	−0.025 (0.032)	0.900
MSRV 5mn + RV 5mn MA(1)	0.189 (0.060)	0.998 (0.035)	−0.032 (0.035)	0.900
MSRV 5mn + RV 10mn	0.220 (0.042)	1.033 (0.026)	−0.076 (0.028)	0.900
MSRV 5mn + RV 15mn	0.185 (0.037)	1.012 (0.024)	−0.053 (0.026)	0.900
MSRV 5mn + RV 30mn	0.221 (0.033)	1.066 (0.020)	−0.122 (0.022)	0.902

Table 21. Out-of-sample forecasts, one day ahead of daily IV in percentage based on 1,000 simulated paths under the Heston jump-diffusion model, $d\sigma^2 = 5\,(0.035 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2 + J\,dq$, q is a Poisson process with intensity $\lambda = \Delta \times 23{,}400/2$ and $J \sim \text{Exp}(0.0007)$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. The corresponding forecast equation is estimated using the previous 100 days of estimated IV, regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.586	0.361	1.696	1.867	39.358	6.546
TSRV 5mn	−0.010	0.084	0.290	−0.010	0.016	0.128
MSRV 5mn	−0.012	0.094	0.306	−0.011	0.018	0.134

Table 22. In-sample estimates of daily IV$\times 10^4$ based on 1,000 simulated paths under the jump-return Heston diffusion model with efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma\,dW_1 + J_X\,dq_X$, q_X is a Poisson process with intensity $\lambda_X = \Delta \times 23{,}400/2$, $J_X \sim \mathcal{N}(-0.004, 0.004)$ and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.070 (0.115)		0.856 (0.033)	0.404
RV 5mn MA(1)	−0.809 (0.103)		0.828 (0.031)	0.419
RV 10mn	−0.490 (0.094)		0.888 (0.034)	0.405
RV 15mn	−0.222 (0.086)		0.869 (0.034)	0.392
RV 30mn	−0.008 (0.082)		0.863 (0.035)	0.372
TSRV 5mn	0.170 (0.063)	0.920 (0.031)		0.466
TSRV 5mn + RV 5mn	1.143 (0.216)	1.499 (0.127)	−0.596 (0.127)	0.477
TSRV 5mn + RV 5mn MA(1)	1.001 (0.199)	1.566 (0.150)	−0.627 (0.143)	0.476
TSRV 5mn + RV 10mn	0.425 (0.122)	1.170 (0.107)	−0.271 (0.111)	0.469
TSRV 5mn + RV 15mn	0.358 (0.094)	1.167 (0.096)	−0.269 (0.099)	0.470
TSRV 5mn + RV 30mn	0.271 (0.078)	1.084 (0.081)	−0.187 (0.085)	0.468
MSRV 5mn	0.168 (0.063)	0.921 (0.031)		0.464
MSRV 5mn + RV 5mn	1.081 (0.216)	1.469 (0.128)	−0.561 (0.127)	0.474
MSRV 5mn + RV 5mn MA(1)	0.919 (0.198)	1.510 (0.150)	−0.570 (0.142)	0.472
MSRV 5mn + RV 10mn	0.395 (0.121)	1.147 (0.107)	−0.243 (0.111)	0.466
MSRV 5mn + RV 15mn	0.334 (0.093)	1.141 (0.096)	−0.239 (0.099)	0.467
MSRV 5mn + RV 30mn	0.260 (0.078)	1.074 (0.081)	−0.173 (0.085)	0.466

Table 23. Out-of-sample forecasts, one day ahead of daily IV in percentage based on 1,000 simulated paths under the jump-return Heston diffusion model with efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma\,dW_1 + J_X\,dq_X$, q_X is a Poisson process with intensity $\lambda_X = \Delta \times 23{,}400/2$, $J_X \sim \mathcal{N}(-0.004, 0.004)$ and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. The forecast equation is estimated using the previous 100 days of estimated IV, regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 99$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.583	0.281	1.669	1.002	0.135	1.068
TSRV 5mn	−0.020	0.045	0.213	−0.013	0.017	0.129
MSRV 5mn	−0.020	0.050	0.224	−0.013	0.018	0.136

Table 24. In-sample estimates of daily IV$\times 10^4$ based on 1,000 simulated paths under the log-volatility model, $l_t = -0.0161 + 0.35\, l_{t-d} + 0.25\, l_{t-2d} + 0.20\, l_{t-3d} + 0.10\, l_{t-4d} + 0.09\, l_{t-5d} + u_t$, where $u \sim NID(0, 0.02^2)$ and d = 1 day. The efficient log-return $r_t = X_t - X_{t-\Delta} = \sqrt{IV_t^{(d)}} z_t$, where $z \sim NID(0,1)$ and $\Delta = 1$ second. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$.

	b_0	b_1	b_2	R^2
RV 5mn	−1.394 (0.083)		0.966 (0.027)	0.568
RV 5mn MA(1)	−1.159 (0.068)		0.949 (0.023)	0.627
RV 10mn	−0.528 (0.065)		0.926 (0.028)	0.528
RV 15mn	−0.237 (0.058)		0.912 (0.028)	0.507
RV 30mn	0.225 (0.055)		0.813 (0.032)	0.397
TSRV 5mn	−0.052 (0.025)	1.061 (0.016)		0.819
TSRV 5mn + RV 5mn	−0.097 (0.064)	1.043 (0.028)	0.023 (0.031)	0.819
TSRV 5mn + RV 5mn MA(1)	−0.061 (0.058)	1.056 (0.033)	0.006 (0.033)	0.819
TSRV 5mn + RV 10mn	−0.015 (0.042)	1.085 (0.027)	−0.033 (0.029)	0.819
TSRV 5mn + RV 15mn	−0.014 (0.036)	1.093 (0.026)	−0.043 (0.029)	0.819
TSRV 5mn + RV 30mn	−0.025 (0.031)	1.086 (0.022)	−0.039 (0.025)	0.819
MSRV 5mn	−0.052 (0.026)	1.062 (0.016)		0.808
MSRV 5mn + RV 5mn	−0.120 (0.066)	1.035 (0.029)	0.036 (0.032)	0.809
MSRV 5mn + RV 5mn MA(1)	−0.094 (0.060)	1.038 (0.034)	0.027 (0.034)	0.809
MSRV 5mn + RV 10mn	−0.033 (0.043)	1.074 (0.028)	−0.017 (0.030)	0.808
MSRV 5mn + RV 15mn	−0.028 (0.037)	1.082 (0.027)	−0.027 (0.030)	0.809
MSRV 5mn + RV 30mn	−0.033 (0.031)	1.079 (0.023)	−0.027 (0.025)	0.809

Table 25. Out-of-sample, one day ahead forecasts of daily IV in percentage based on 1,000 simulated paths under the log-volatility model, $l_t = -0.0161 + 0.35\, l_{t-d} + 0.25\, l_{t-2d} + 0.20\, l_{t-3d} + 0.10\, l_{t-4d} + 0.09\, l_{t-5d} + u_t$, where $u \sim NID(0, 0.02^2)$ and $d = 1$ day. The efficient log-return $r_t = X_t - X_{t-\Delta} = \sqrt{IV_t^{(d)}} z_t$, where $z \sim NID(0,1)$ and $\Delta = 1$ second. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. OLS standard errors in parenthesis. Forecast equation is estimated using the previous 200 days of estimated daily IV.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.579	0.274	1.664	1.032	0.148	1.101
TSRV 5mn	−0.020	0.042	0.206	−0.013	0.017	0.129
MSRV 5mn	−0.020	0.047	0.217	−0.013	0.018	0.136

Table 26. In-sample estimates of daily IV$\times 10^4$ based on 1,000 simulated paths under the HAR-RV model, $IV^{(d)} = 0.002 + 0.45\,RV^{(d)} + 0.30\,RV^{(w)} + 0.20\,RV^{(m)} + \omega^{(d)}$, where $\omega^{(d)} \sim NID(0, 0.003^2)$. The efficient log-return $r_t = X_t - X_{t-\Delta} = \sqrt{IV_t^{(d)}} z_t$, where $z \sim NID(0,1)$ and $\Delta = 1$ second. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$.

	b_0	b_1	b_2	R^2
RV 5mn	−1.589 (0.054)		1.007 (0.017)	0.777
RV 5mn MA(1)	−1.218 (0.042)		0.951 (0.014)	0.818
RV 10mn	−0.773 (0.041)		0.995 (0.017)	0.768
RV 15mn	−0.514 (0.039)		0.997 (0.018)	0.746
RV 30mn	−0.090 (0.038)		0.906 (0.021)	0.657
TSRV 5mn	−0.015 (0.018)	1.021 (0.011)		0.889
TSRV 5mn + RV 5mn	−0.170 (0.058)	0.944 (0.029)	0.087 (0.031)	0.890
TSRV 5mn + RV 5mn MA(1)	−0.145 (0.053)	0.930 (0.037)	0.092 (0.035)	0.890
TSRV 5mn + RV 10mn	−0.084 (0.035)	0.959 (0.029)	0.070 (0.030)	0.890
TSRV 5mn + RV 15mn	−0.055 (0.029)	0.976 (0.027)	0.053 (0.029)	0.890
TSRV 5mn + RV 30mn	−0.033 (0.022)	0.994 (0.022)	0.032 (0.022)	0.890
MSRV 5mn	−0.015 (0.018)	1.020 (0.012)		0.886
MSRV 5mn + RV 5mn	−0.194 (0.059)	0.932 (0.030)	0.101 (0.031)	0.887
MSRV 5mn + RV 5mn MA(1)	−0.177 (0.054)	0.908 (0.037)	0.115 (0.036)	0.887
MSRV 5mn + RV 10mn	−0.098 (0.035)	0.947 (0.029)	0.084 (0.031)	0.887
MSRV 5mn + RV 15mn	−0.067 (0.029)	0.962 (0.027)	0.068 (0.029)	0.886
MSRV 5mn + RV 30mn	−0.036 (0.022)	0.989 (0.022)	0.038 (0.023)	0.886

Table 27. Out-of-sample, one day ahead forecasts of daily IV in percentage based on 1,000 simulated paths under the HAR-RV model, $IV^{(d)} = 0.002 + 0.45\,RV^{(d)} + 0.30\,RV^{(w)} + 0.20\,RV^{(m)} + \omega^{(d)}$, where $\omega^{(d)} \sim NID(0, 0.003^2)$. The efficient log-return $r_t = X_t - X_{t-\Delta} = \sqrt{IV_t^{(d)}} z_t$, where $z \sim NID(0,1)$ and $\Delta = 1$ second. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. OLS standard errors in parenthesis. Forecast equation is estimated using the previous 200 days of estimated daily IV, and then aggregating such estimates to obtain weekly, and monthly IV.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	1.566	0.304	1.660	1.038	0.210	1.135
TSRV 5mn	−0.016	0.047	0.217	−0.010	0.017	0.130
MSRV 5mn	−0.015	0.052	0.228	−0.009	0.019	0.137

Table 28. In-sample estimates of daily IV$\times 10^4$ based on 1,000 simulated paths under the fractional Ornstein-Uhlenbeck process, $d\sigma = 20\,(0.2 - \sigma)\,dt + 0.012\,dW_H$, where dW_H is a FBM with Hurst parameter 0.7. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−1.106 (0.093)		0.856 (0.029)	0.464
RV 5mn MA(1)	−0.906 (0.076)		0.846 (0.025)	0.526
RV 10mn	−0.405 (0.072)		0.841 (0.029)	0.450
RV 15mn	−0.123 (0.065)		0.811 (0.030)	0.423
RV 30mn	0.164 (0.062)		0.770 (0.032)	0.362
TSRV 5mn	0.012 (0.028)	1.001 (0.017)		0.779
TSRV 5mn + RV 5mn	0.300 (0.070)	1.107 (0.029)	−0.144 (0.032)	0.783
TSRV 5mn + RV 5mn MA(1)	0.305 (0.062)	1.152 (0.033)	−0.180 (0.034)	0.785
TSRV 5mn + RV 10mn	0.254 (0.048)	1.148 (0.029)	−0.199 (0.032)	0.787
TSRV 5mn + RV 15mn	0.182 (0.040)	1.125 (0.027)	−0.172 (0.030)	0.786
TSRV 5mn + RV 30mn	0.151 (0.036)	1.113 (0.025)	−0.168 (0.028)	0.786
MSRV 5mn	0.012 (0.029)	1.000 (0.018)		0.765
MSRV 5mn + RV 5mn	0.286 (0.072)	1.104 (0.030)	−0.138 (0.033)	0.769
MSRV 5mn + RV 5mn MA(1)	0.274 (0.065)	1.139 (0.035)	−0.162 (0.036)	0.770
MSRV 5mn + RV 10mn	0.232 (0.049)	1.137 (0.030)	−0.183 (0.033)	0.772
MSRV 5mn + RV 15mn	0.170 (0.042)	1.119 (0.029)	−0.162 (0.031)	0.771
MSRV 5mn + RV 30mn	0.145 (0.037)	1.111 (0.026)	−0.164 (0.029)	0.772

Table 29. Out-of-sample, one day ahead forecasts of daily IV in percentage based on 1,000 simulated paths under the fractional OU model, $d\sigma = 20\,(0.2 - \sigma)\,dt + 0.012\,dW_H$, where dW_H is a FBM with Hurst parameter 0.7. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. Forecast equation is estimated using the previous 86 days of estimated IV, regressing the corresponding estimates of $\int_{T_i}^{T_{i+1}} \sigma_t^2\,dt$ on a constant and $\int_{T_{i-1}}^{T_i} \sigma_t^2\,dt$, for $i = 1, \ldots, 85$.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	3.931	1.045	4.062	2.361	21.730	5.226
RV 5mn MA(1)	3.384	0.902	3.514	1.915	12.958	4.077
TSRV 5mn	−0.051	0.228	0.480	−0.012	0.010	0.100
RV 10mn	1.968	1.203	2.253	1.187	6.813	2.867
TSRV 10mn	−0.105	0.437	0.669	−0.026	0.017	0.134
RV 15mn	1.298	1.476	1.778	0.786	2.582	1.789
TSRV 15mn	−0.160	0.653	0.824	−0.039	0.025	0.162
RV 30mn	0.663	2.597	1.743	0.399	0.797	0.978
TSRV 30mn	−0.316	1.314	1.189	−0.076	0.047	0.230

Table 30. In-sample estimates of annualized two-day IV$\times 10^2$ based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−3.607 (0.034)		0.956 (0.004)	0.855
RV 5mn MA(1)	−2.704 (0.028)		0.903 (0.003)	0.874
RV 10mn	−1.725 (0.028)		0.952 (0.004)	0.838
RV 15mn	−1.077 (0.028)		0.947 (0.004)	0.816
RV 30mn	−0.304 (0.030)		0.908 (0.005)	0.745
TSRV 5mn	0.110 (0.014)	0.974 (0.003)		0.924
TSRV 5mn + RV 5mn	0.312 (0.048)	1.020 (0.011)	−0.048 (0.011)	0.924
TSRV 5mn + RV 5mn MA(1)	0.451 (0.044)	1.079 (0.013)	−0.102 (0.013)	0.924
TSRV 5mn + RV 10mn	0.380 (0.028)	1.085 (0.010)	−0.119 (0.010)	0.925
TSRV 5mn + RV 15mn	0.320 (0.021)	1.088 (0.009)	−0.124 (0.009)	0.925
TSRV 5mn + RV 30mn	0.229 (0.017)	1.059 (0.007)	−0.097 (0.007)	0.925
TSRV 10mn	0.109 (0.016)	0.985 (0.003)		0.900
TSRV 10mn + RV 5mn	−0.585 (0.051)	0.823 (0.012)	0.167 (0.012)	0.902
TSRV 10mn + RV 5mn MA(1)	−0.388 (0.050)	0.825 (0.016)	0.152 (0.015)	0.901
TSRV 10mn + RV 10mn	0.206 (0.033)	1.028 (0.013)	−0.044 (0.013)	0.900
TSRV 10mn + RV 15mn	0.361 (0.025)	1.136 (0.012)	−0.158 (0.012)	0.902
TSRV 10mn + RV 30mn	0.294 (0.019)	1.138 (0.009)	−0.166 (0.009)	0.904
TSRV 15mn	0.117 (0.018)	0.995 (0.004)		0.876
TSRV 15mn + RV 5mn	−1.374 (0.051)	0.641 (0.012)	0.361 (0.012)	0.887
TSRV 15mn + RV 5mn MA(1)	−1.293 (0.049)	0.530 (0.015)	0.434 (0.014)	0.887
TSRV 15mn + RV 10mn	−0.271 (0.035)	0.818 (0.014)	0.179 (0.014)	0.878
TSRV 15mn + RV 15mn	0.186 (0.029)	1.039 (0.015)	−0.045 (0.015)	0.876
TSRV 15mn + RV 30mn	0.337 (0.021)	1.201 (0.011)	−0.216 (0.011)	0.881
TSRV 30mn	0.185 (0.023)	1.014 (0.005)		0.809
TSRV 30mn + RV 5mn MA(1)	−2.400 (0.040)	0.134 (0.013)	0.794 (0.011)	0.875
TSRV 30mn + RV 30mn	0.208 (0.027)	1.042 (0.018)	−0.027 (0.017)	0.809

Table 31. Out-of-sample forecasts of annualized two-day IV$\times 10^2$ based on 10,000 simulated paths, 100 trading days each, under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. AR(1) model for non-overlapping two-day IV$\times 10^2$ used for the out-of-sample forecasts.

	Bias	Var	RMSE	Rel. Bias	Rel. Var	Rel. RMSE
RV 5mn	3.930	0.415	3.982	2.190	11.898	4.086
RV 5mn MA(1)	3.380	0.382	3.436	1.778	6.872	3.167
TSRV 5mn	−0.051	0.093	0.308	−0.013	0.004	0.064
RV 10mn	1.963	0.483	2.082	1.099	3.115	2.079
TSRV 10mn	−0.101	0.179	0.435	−0.025	0.007	0.088
RV 15mn	1.309	0.605	1.523	0.729	1.374	1.380
TSRV 15mn	−0.152	0.268	0.540	−0.038	0.010	0.108
RV 30mn	0.657	0.991	1.193	0.364	0.382	0.717
TSRV 30mn	−0.301	0.535	0.791	−0.076	0.019	0.158

Table 32. In-sample estimates of annualized five-day IV$\times 10^2$ based on 10,000 simulated paths under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second.

	b_0	b_1	b_2	R^2
RV 5mn	−3.170 (0.037)		0.895 (0.004)	0.814
RV 5mn MA(1)	−2.328 (0.032)		0.846 (0.004)	0.823
RV 10mn	−1.420 (0.030)		0.894 (0.004)	0.804
RV 15mn	−0.787 (0.029)		0.884 (0.005)	0.790
RV 30mn	−0.129 (0.029)		0.863 (0.005)	0.749
TSRV 5mn	0.297 (0.020)	0.915 (0.004)		0.851
TSRV 5mn + RV 5mn	0.854 (0.086)	1.051 (0.021)	−0.138 (0.021)	0.852
TSRV 5mn + RV 5mn MA(1)	0.993 (0.080)	1.142 (0.026)	−0.216 (0.024)	0.853
TSRV 5mn + RV 10mn	0.761 (0.046)	1.129 (0.019)	−0.219 (0.020)	0.853
TSRV 5mn + RV 15mn	0.589 (0.032)	1.106 (0.017)	−0.197 (0.017)	0.853
TSRV 5mn + RV 30mn	0.434 (0.023)	1.052 (0.012)	−0.144 (0.013)	0.853
TSRV 10mn	0.309 (0.021)	0.923 (0.004)		0.839
TSRV 10mn + RV 5mn	−0.108 (0.085)	0.820 (0.021)	0.104 (0.021)	0.839
TSRV 10mn + RV 5mn MA(1)	0.106 (0.084)	0.855 (0.027)	0.063 (0.025)	0.839
TSRV 10mn + RV 10mn	0.593 (0.051)	1.058 (0.023)	−0.136 (0.022)	0.839
TSRV 10mn + RV 15mn	0.623 (0.035)	1.139 (0.020)	−0.218 (0.020)	0.841
TSRV 10mn + RV 30mn	0.481 (0.024)	1.108 (0.015)	−0.191 (0.014)	0.842
TSRV 15mn	0.330 (0.021)	0.928 (0.004)		0.825
TSRV 15mn + RV 5mn	−1.001 (0.080)	0.596 (0.020)	0.330 (0.019)	0.830
TSRV 15mn + RV 5mn MA(1)	−0.936 (0.077)	0.501 (0.025)	0.395 (0.023)	0.830
TSRV 15mn + RV 10mn	0.074 (0.051)	0.804 (0.023)	0.123 (0.022)	0.825
TSRV 15mn + RV 15mn	0.439 (0.038)	1.006 (0.022)	−0.077 (0.022)	0.825
TSRV 15mn + RV 30mn	0.509 (0.026)	1.134 (0.017)	−0.208 (0.016)	0.828
TSRV 30mn	0.411 (0.024)	0.941 (0.005)		0.788
TSRV 30mn + RV 5mn MA(1)	−2.044 (0.058)	0.110 (0.019)	0.752 (0.017)	0.824
TSRV 30mn + RV 30mn	0.403 (0.029)	0.930 (0.022)	0.010 (0.020)	0.788

Table 33. Out-of-sample forecasts of annualized five-day IV$\times 10^2$ based on 10,000 simulated paths, 100 trading days each, under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. AR(1) model for non-overlapping five-day IV$\times 10^2$ used for out-of-sample forecasts.

	b_0	b_1	b_2	R^2
RV 5mn	−3.484 (0.034)		0.939 (0.004)	0.852
RV 5mn MA(1)	−2.613 (0.028)		0.891 (0.003)	0.870
RV 10mn	−1.650 (0.028)		0.938 (0.004)	0.836
RV 15mn	−0.957 (0.027)		0.921 (0.004)	0.816
RV 30mn	−0.246 (0.029)		0.894 (0.005)	0.759
TSRV 5mn	0.143 (0.015)	0.965 (0.003)		0.920
TSRV 5mn + RV 5mn	0.440 (0.049)	1.033 (0.011)	−0.072 (0.011)	0.920
TSRV 5mn + RV 5mn MA(1)	0.564 (0.046)	1.096 (0.014)	−0.127 (0.013)	0.920
TSRV 5mn + RV 10mn	0.481 (0.028)	1.107 (0.011)	−0.150 (0.011)	0.921
TSRV 5mn + RV 15mn	0.410 (0.021)	1.121 (0.010)	−0.166 (0.010)	0.922
TSRV 5mn + RV 30mn	0.292 (0.017)	1.086 (0.008)	−0.133 (0.008)	0.922
TSRV 10mn	0.160 (0.017)	0.969 (0.003)		0.896
TSRV 10mn + RV 5mn	−0.454 (0.054)	0.825 (0.013)	0.149 (0.012)	0.897
TSRV 10mn + RV 5mn MA(1)	−0.309 (0.052)	0.819 (0.016)	0.144 (0.015)	0.897
TSRV 10mn + RV 10mn	0.303 (0.034)	1.033 (0.014)	−0.066 (0.014)	0.896
TSRV 10mn + RV 15mn	0.422 (0.026)	1.135 (0.013)	−0.171 (0.013)	0.897
TSRV 10mn + RV 30mn	0.331 (0.019)	1.126 (0.010)	−0.167 (0.010)	0.899
TSRV 15mn	0.182 (0.018)	0.974 (0.004)		0.873
TSRV 15mn + RV 5mn	−1.234 (0.054)	0.636 (0.013)	0.344 (0.012)	0.882
TSRV 15mn + RV 5mn MA(1)	−1.191 (0.051)	0.525 (0.016)	0.422 (0.015)	0.883
TSRV 15mn + RV 10mn	−0.157 (0.037)	0.820 (0.015)	0.156 (0.015)	0.875
TSRV 15mn + RV 15mn	0.260 (0.029)	1.026 (0.015)	−0.052 (0.015)	0.873
TSRV 15mn + RV 30mn	0.358 (0.021)	1.151 (0.012)	−0.183 (0.012)	0.876
TSRV 30mn	0.233 (0.022)	0.998 (0.005)		0.817
TSRV 30mn + RV 5mn MA(1)	−2.222 (0.042)	0.166 (0.013)	0.754 (0.012)	0.872
TSRV 30mn + RV 30mn	0.227 (0.026)	0.991 (0.018)	0.007 (0.016)	0.817

Table 34. Out-of-sample forecasts of annualized two-day IV$\times 10^2$ based on 10,000 simulated paths, 60 trading days each, under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. AR(1) model for non-overlapping two-day IV$\times 10^2$ used for the out-of-sample forecasts.

	b_0	b_1	b_2	R^2
RV 5mn	−2.799 (0.037)		0.845 (0.004)	0.798
RV 5mn MA(1)	−1.992 (0.033)		0.799 (0.004)	0.806
RV 10mn	−1.078 (0.031)		0.832 (0.004)	0.784
RV 15mn	−0.389 (0.029)		0.804 (0.005)	0.761
RV 30mn	0.224 (0.029)		0.782 (0.005)	0.723
TSRV 5mn	0.465 (0.021)	0.867 (0.004)		0.832
TSRV 5mn + RV 5mn	0.419 (0.078)	0.856 (0.019)	0.011 (0.019)	0.832
TSRV 5mn + RV 5mn MA(1)	0.616 (0.072)	0.917 (0.023)	−0.047 (0.022)	0.832
TSRV 5mn + RV 10mn	0.608 (0.041)	0.935 (0.017)	−0.069 (0.017)	0.833
TSRV 5mn + RV 15mn	0.560 (0.028)	0.934 (0.014)	−0.067 (0.014)	0.833
TSRV 5mn + RV 30mn	0.514 (0.023)	0.922 (0.011)	−0.056 (0.011)	0.833
TSRV 10mn	0.496 (0.021)	0.869 (0.004)		0.817
TSRV 10mn + RV 5mn	−0.391 (0.077)	0.651 (0.019)	0.219 (0.018)	0.819
TSRV 10mn + RV 5mn MA(1)	−0.190 (0.075)	0.641 (0.024)	0.214 (0.022)	0.819
TSRV 10mn + RV 10mn	0.440 (0.045)	0.841 (0.020)	0.027 (0.019)	0.817
TSRV 10mn + RV 15mn	0.608 (0.031)	0.951 (0.017)	−0.081 (0.016)	0.817
TSRV 10mn + RV 30mn	0.566 (0.024)	0.953 (0.013)	−0.085 (0.013)	0.818
TSRV 15mn	0.527 (0.022)	0.869 (0.004)		0.803
TSRV 15mn + RV 5mn	−1.002 (0.074)	0.495 (0.018)	0.376 (0.017)	0.812
TSRV 15mn + RV 5mn MA(1)	−0.883 (0.070)	0.401 (0.022)	0.438 (0.021)	0.812
TSRV 15mn + RV 10mn	0.095 (0.045)	0.660 (0.020)	0.208 (0.019)	0.806
TSRV 15mn + RV 15mn	0.529 (0.033)	0.871 (0.019)	−0.001 (0.018)	0.803
TSRV 15mn + RV 30mn	0.617 (0.025)	0.986 (0.015)	−0.115 (0.014)	0.804
TSRV 30mn	0.643 (0.024)	0.870 (0.005)		0.764
TSRV 30mn + RV 5mn MA(1)	−1.651 (0.053)	0.131 (0.016)	0.686 (0.015)	0.807
TSRV 30mn + RV 30mn	0.608 (0.028)	0.823 (0.020)	0.044 (0.018)	0.764

Table 35. Out-of-sample forecasts of annualized five-day IV$\times 10^2$ based on 10,000 simulated paths, 60 trading days each, under the Heston model, $d\sigma^2 = 5\,(0.04 - \sigma^2)\,dt + 0.5\,\sigma\,dW_2$. The efficient log-price $dX = (0.05 - \sigma^2/2)\,dt + \sigma dW_1$, and correlation $\rho = -0.5$ between Brownian motions. The observed log-price $Y = X + \varepsilon$, where $\varepsilon \sim NID(0, 0.001^2)$. Euler discretization scheme with time step $\Delta = 1$ second. OLS standard errors in parenthesis. AR(1) model for non-overlapping five-day IV$\times 10^2$ used for out-of-sample forecasts.

		$r_{i,t}$			
Symbol	Name	Mean	Std.	Skew.	Kurt.
AA	Alcoa Inc	−0.090	2.159	0.304	4.361
AXP	American Express	−0.022	2.064	−0.142	5.022
BA	Boeing Company	0.062	1.934	0.049	4.945
C	Citigroup	−0.043	1.877	0.322	8.436
CAT	Caterpillar Inc	0.058	1.853	0.176	4.270
DD	Du Pont De Nemours	−0.005	1.737	0.392	6.652
DIS	Walt Disney	0.043	1.997	0.105	4.105
EK	Eastman Kodak	−0.058	1.866	−0.612	9.170
GE	General Electric	−0.033	1.852	0.276	5.217
GM	General Motors	−0.132	1.890	0.215	4.448
HD	Home Depot Inc	−0.100	2.099	0.346	5.447
HON	Honeywell	−0.129	2.288	−0.471	9.667
HPQ	Hewlett-Packard	0.019	2.646	0.299	6.015
IBM	Int. Business Machines	0.020	1.834	0.237	6.294
INTC	Intel Corp	−0.103	2.787	0.079	4.450
IP	International Paper	−0.076	1.894	0.397	5.315
JNJ	Johnson & Johnson	0.056	1.363	0.057	5.390
JPM	J. P. Morgan	0.008	2.279	0.759	12.277
KO	Coca-Cola	0.118	1.473	0.217	5.437
MCD	McDonalds	0.060	1.757	−0.254	6.621
MMM	Minnesota Mng Mfg	0.043	1.534	0.424	6.349
MO	Philip Morris	0.091	1.844	−0.438	8.437
MRK	Merck	0.059	1.657	0.076	5.675
MSFT	Microsoft	−0.049	2.052	0.276	4.672
PG	Procter & Gamble	0.080	1.473	−0.503	10.794
SBC	Sbc Communications	−0.016	2.010	0.331	4.741
T	AT & T Corp	−0.049	2.299	0.196	4.862
UTX	United Technologies	−0.038	1.894	−0.436	8.386
WMT	Wal-Mart Stores	−0.034	1.853	0.149	5.508
XOM	Exxon Mobil	0.024	1.400	0.572	8.855
Median		−0.011	1.883	0.205	5.477
Mean		−0.008	1.922	0.113	6.394
Min.		−0.132	1.363	−0.612	4.105
Max.		0.118	2.787	0.759	12.277

Table 36. Summary statistics for the daily log-return distributions in percentage for each of the thirty DJIA stocks, $r_{i,t}$. The sample period extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations.

	TSRV $r_{i,t}/\text{IV}_{i,t}^{1/2}$				RV $r_{i,t}/\text{IV}_{i,t}^{1/2}$			
Stock	Mean	Std.	Skew.	Kurt.	Mean	Std.	Skew.	Kurt.
AA	−0.042	1.044	0.200	2.633	−0.045	0.981	0.176	2.612
AXP	0.015	1.006	0.006	2.647	0.016	0.966	0.032	2.667
BA	0.048	0.967	−0.007	2.803	0.048	0.914	0.041	2.775
C	−0.014	0.976	0.154	2.931	−0.015	0.906	0.147	2.703
CAT	0.032	1.049	0.027	2.736	0.032	0.990	0.043	2.758
DD	−0.002	0.943	0.266	3.063	0.003	0.894	0.287	3.107
DIS	0.027	0.956	0.043	2.694	0.024	0.881	0.022	2.648
EK	−0.028	0.937	0.117	3.009	−0.026	0.882	0.103	3.039
GE	−0.009	1.026	0.181	2.774	−0.014	0.962	0.135	2.695
GM	−0.098	1.092	0.170	2.770	−0.091	1.019	0.161	2.777
HD	−0.031	1.012	0.158	2.780	−0.029	0.944	0.140	2.702
HON	−0.031	0.966	0.111	2.922	−0.028	0.915	0.099	2.947
HPQ	0.013	0.998	−0.005	2.763	0.010	0.943	0.003	2.612
IBM	0.023	1.042	0.215	3.061	0.014	0.975	0.091	2.649
INTC	−0.040	1.058	0.119	2.703	−0.041	1.011	0.106	2.729
IP	−0.033	0.986	0.137	2.764	−0.027	0.935	0.162	2.835
JNJ	0.039	0.936	−0.014	2.771	0.036	0.870	−0.002	2.684
JPM	0.013	1.007	0.145	2.835	0.011	0.950	0.136	2.832
KO	0.086	0.947	0.007	3.173	0.083	0.887	0.047	3.031
MCD	0.054	0.931	0.043	2.865	0.049	0.864	0.015	2.911
MMM	0.019	0.983	0.109	2.964	0.021	0.939	0.145	2.899
MO	0.081	1.029	0.018	2.897	0.074	0.921	0.038	2.929
MRK	0.023	0.997	0.078	2.737	0.024	0.929	0.105	2.615
MSFT	−0.031	1.007	0.116	2.564	−0.031	0.958	0.106	2.582
PG	0.103	0.927	−0.080	3.475	0.105	0.875	0.080	2.836
SBC	−0.005	0.978	0.195	2.857	−0.005	0.917	0.147	2.781
T	−0.059	1.077	−0.019	2.929	−0.045	0.973	0.058	2.793
UTX	−0.013	0.990	0.063	2.642	−0.014	0.950	0.069	2.626
WMT	−0.018	1.023	0.020	2.965	−0.018	0.945	0.056	2.842
XOM	0.046	0.946	0.104	2.763	0.043	0.895	0.106	2.773
Median	0.005	0.993	0.106	2.791	0.006	0.937	0.101	2.774
Mean	0.006	0.995	0.089	2.850	0.005	0.933	0.095	2.780
Min.	−0.098	0.927	−0.080	2.564	−0.091	0.864	−0.002	2.582
Max.	0.103	1.092	0.266	3.475	0.105	1.019	0.287	3.107

Table 37. Summary statistics for the standardized log-return distributions for each of the thirty DJIA stocks, $r_{i,t}/\text{IV}_{i,t}^{1/2}$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. TSRV is computed using a slow time scale of 5mn. RV is computed using de-meaned MA(1) filtered 5mn log-returns, as detailed in the text.

	RV $r_{i,t}/\text{IV}_{i,t}^{1/2}$			
Stock	Mean	Std.	Skew.	Kurt.
AA	−0.044	0.981	0.179	2.613
AXP	0.016	0.966	0.033	2.667
BA	0.048	0.914	0.038	2.774
C	−0.015	0.906	0.149	2.705
CAT	0.032	0.990	0.040	2.759
DD	0.003	0.894	0.288	3.108
DIS	0.024	0.881	0.021	2.648
EK	−0.025	0.882	0.105	3.040
GE	−0.014	0.962	0.136	2.696
GM	−0.090	1.019	0.167	2.780
HD	−0.029	0.944	0.143	2.704
HON	−0.027	0.915	0.104	2.948
HPQ	0.010	0.942	0.003	2.611
IBM	0.013	0.975	0.089	2.648
INTC	−0.040	1.011	0.109	2.723
IP	−0.026	0.936	0.165	2.835
JNJ	0.036	0.870	−0.004	2.683
JPM	0.011	0.950	0.137	2.831
KO	0.082	0.887	0.041	3.033
MCD	0.048	0.864	0.012	2.912
MMM	0.020	0.939	0.143	2.898
MO	0.073	0.921	0.034	2.930
MRK	0.024	0.929	0.103	2.615
MSFT	−0.030	0.958	0.110	2.580
PG	0.104	0.875	0.076	2.835
SBC	−0.005	0.917	0.147	2.781
T	−0.045	0.973	0.060	2.793
UTX	−0.013	0.950	0.071	2.627
WMT	−0.018	0.945	0.058	2.842
XOM	0.043	0.895	0.105	2.771
Median	0.006	0.937	0.103	2.773
Mean	0.006	0.933	0.095	2.780
Min.	−0.090	0.864	−0.004	2.580
Max.	0.104	1.019	0.288	3.108

Table 38. Summary statistics for the standardized log-return distributions for each of the thirty DJIA stocks, $r_{i,t}/\text{IV}_{i,t}^{1/2}$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. RV is computed using unfiltered 5mn log-returns, as detailed in the text.

	TSRV $IV_{i,t}^{1/2}$				RV $IV_{i,t}^{1/2}$			
Stock	Mean	Std.	Skew.	Kurt.	Mean	Std.	Skew.	Kurt.
AA	0.311	0.114	1.254	4.981	0.331	0.127	1.296	5.229
AXP	0.288	0.153	1.245	6.040	0.302	0.162	1.258	6.002
BA	0.289	0.117	1.486	7.348	0.308	0.127	1.354	6.437
C	0.283	0.145	1.926	11.512	0.301	0.159	2.110	13.762
CAT	0.265	0.102	1.465	6.695	0.285	0.120	1.437	6.202
DD	0.263	0.115	1.237	5.588	0.279	0.125	1.094	4.790
DIS	0.310	0.136	2.147	14.714	0.336	0.150	2.322	18.874
EK	0.281	0.117	2.149	10.923	0.299	0.124	1.996	9.801
GE	0.263	0.121	1.433	7.072	0.281	0.131	1.441	6.964
GM	0.250	0.104	1.605	7.089	0.269	0.120	1.886	10.224
HD	0.296	0.132	1.424	6.150	0.319	0.158	2.549	18.126
HON	0.327	0.148	1.866	9.153	0.349	0.171	3.072	28.451
HPQ	0.378	0.180	1.580	8.018	0.399	0.195	1.709	9.202
IBM	0.248	0.124	1.525	6.943	0.262	0.134	1.678	8.803
INTC	0.394	0.172	1.069	4.376	0.415	0.185	0.980	3.928
IP	0.281	0.128	1.303	5.112	0.299	0.141	1.257	4.866
JNJ	0.210	0.092	1.903	10.534	0.226	0.100	1.765	9.119
JPM	0.310	0.160	2.161	15.295	0.329	0.175	2.216	14.268
KO	0.225	0.094	1.485	6.755	0.241	0.107	1.549	6.621
MCD	0.270	0.109	1.656	8.037	0.296	0.132	2.511	19.025
MMM	0.226	0.100	1.450	5.831	0.237	0.108	1.477	5.837
MO	0.239	0.125	2.981	20.713	0.269	0.147	2.343	14.758
MRK	0.237	0.103	2.010	10.680	0.252	0.112	1.840	8.892
MSFT	0.293	0.136	1.191	5.323	0.312	0.153	1.375	6.626
PG	0.210	0.108	2.721	20.666	0.223	0.121	2.968	22.302
SBC	0.297	0.136	1.329	5.430	0.316	0.144	1.330	5.610
T	0.309	0.131	1.465	7.124	0.344	0.154	1.586	7.955
UTX	0.265	0.123	1.735	9.280	0.276	0.132	1.827	10.322
WMT	0.261	0.126	1.394	6.042	0.281	0.139	1.373	5.846
XOM	0.212	0.096	1.634	7.708	0.224	0.102	1.642	7.558
Median	0.275	0.124	1.506	7.106	0.297	0.133	1.660	8.379
Mean	0.276	0.125	1.661	8.704	0.295	0.139	1.775	10.213
Min.	0.210	0.092	1.069	4.376	0.223	0.100	0.980	3.928
Max.	0.394	0.180	2.981	20.713	0.415	0.195	3.072	28.451

Table 39. Summary statistics for the volatility distributions for each of the thirty DJIA stocks on annual base (252 days), $IV_{i,t}^{1/2}$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. TSRV is computed using a slow time scale of 5mn. RV is compute using de-meaned MA(1) filtered 5mn log-returns, as detailed in the text.

	RV $IV_{i,t}^{1/2}$			
Stock	Mean	Std.	Skew.	Kurt.
AA	0.331	0.127	1.296	5.229
AXP	0.302	0.162	1.258	6.002
BA	0.308	0.127	1.354	6.437
C	0.301	0.159	2.110	13.774
CAT	0.285	0.120	1.437	6.202
DD	0.279	0.125	1.095	4.792
DIS	0.336	0.150	2.324	18.896
EK	0.299	0.124	1.995	9.791
GE	0.281	0.131	1.441	6.959
GM	0.269	0.120	1.885	10.224
HD	0.319	0.158	2.548	18.118
HON	0.349	0.171	3.070	28.429
HPQ	0.399	0.195	1.709	9.201
IBM	0.262	0.134	1.678	8.802
INTC	0.415	0.185	0.980	3.925
IP	0.299	0.141	1.257	4.865
JNJ	0.226	0.100	1.766	9.132
JPM	0.329	0.175	2.219	14.294
KO	0.241	0.107	1.549	6.619
MCD	0.296	0.132	2.512	19.027
MMM	0.237	0.108	1.477	5.837
MO	0.269	0.147	2.343	14.760
MRK	0.252	0.112	1.840	8.890
MSFT	0.312	0.153	1.376	6.635
PG	0.223	0.121	2.968	22.304
SBC	0.316	0.144	1.330	5.610
T	0.344	0.154	1.586	7.957
UTX	0.276	0.132	1.831	10.356
WMT	0.281	0.139	1.373	5.845
XOM	0.224	0.102	1.643	7.558
Median	0.297	0.133	1.660	8.379
Mean	0.295	0.139	1.775	10.216
Min.	0.223	0.100	0.980	3.925
Max.	0.415	0.195	3.070	28.429

Table 40. Summary statistics for the volatility distributions for each of the thirty DJIA stocks on annual base (252 days), $IV_{i,t}^{1/2}$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. RV is compute using unfiltered 5mn log-returns, as detailed in the text.

	TSRV $\log(\text{IV}_{i,t}^{1/2})$				RV $\log(\text{IV}_{i,t}^{1/2})$			
Stock	Mean	Std.	Skew.	Kurt.	Mean	Std.	Skew.	Kurt.
AA	0.611	0.342	0.251	2.974	0.670	0.358	0.260	2.865
AXP	0.454	0.551	−0.300	2.566	0.498	0.552	−0.265	2.520
BA	0.527	0.379	0.118	2.980	0.586	0.394	0.048	2.872
C	0.463	0.482	−0.010	2.851	0.520	0.493	−0.016	2.872
CAT	0.446	0.358	0.214	3.070	0.507	0.391	0.213	2.866
DD	0.418	0.419	0.062	2.591	0.470	0.438	−0.016	2.505
DIS	0.588	0.403	0.073	3.236	0.663	0.417	−0.034	3.221
EK	0.500	0.363	0.460	3.600	0.561	0.367	0.413	3.447
GE	0.406	0.447	−0.063	2.825	0.471	0.449	−0.031	2.800
GM	0.380	0.380	0.296	3.035	0.446	0.398	0.321	3.073
HD	0.535	0.418	0.156	2.782	0.600	0.433	0.290	3.154
HON	0.638	0.405	0.299	3.117	0.692	0.423	0.335	3.241
HPQ	0.765	0.456	−0.045	2.911	0.816	0.460	0.042	2.788
IBM	0.335	0.469	0.128	2.692	0.388	0.476	0.101	2.698
INTC	0.819	0.424	0.040	2.523	0.865	0.439	0.006	2.424
IP	0.478	0.426	0.176	2.681	0.533	0.445	0.164	2.553
JNJ	0.200	0.401	0.153	3.137	0.265	0.412	0.121	3.064
JPM	0.552	0.491	−0.135	3.010	0.605	0.497	−0.072	3.046
KO	0.271	0.388	0.213	2.915	0.331	0.406	0.266	2.928
MCD	0.458	0.375	0.164	3.228	0.539	0.400	0.177	3.404
MMM	0.269	0.405	0.324	2.761	0.311	0.416	0.314	2.802
MO	0.305	0.440	0.340	3.725	0.405	0.485	0.220	3.024
MRK	0.321	0.389	0.272	3.411	0.381	0.399	0.272	3.207
MSFT	0.512	0.457	−0.102	2.715	0.562	0.476	−0.026	2.632
PG	0.176	0.437	0.428	3.181	0.229	0.454	0.431	3.265
SBC	0.530	0.435	0.090	2.774	0.595	0.434	0.084	2.717
T	0.586	0.402	0.064	2.868	0.686	0.419	0.104	2.819
UTX	0.420	0.430	0.120	2.925	0.456	0.442	0.098	2.904
WMT	0.393	0.452	0.217	2.490	0.461	0.464	0.207	2.437
XOM	0.200	0.413	0.255	2.862	0.254	0.415	0.258	2.875
Median	0.456	0.418	0.154	2.913	0.514	0.433	0.142	2.872
Mean	0.452	0.421	0.142	2.948	0.512	0.435	0.143	2.901
Min.	0.176	0.342	−0.300	2.490	0.229	0.358	−0.265	2.424
Max.	0.819	0.551	0.460	3.725	0.865	0.552	0.431	3.447

Table 41. Summary statistics for the log-volatility distributions for each of the thirty DJIA stocks in percentage on daily base, $\log(\text{IV}_{i,t}^{1/2})$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. TSRV is computed using a slow time scale of 5mn. RV is compute using de-meaned MA(1) filtered 5mn log-returns, as detailed in the text.

	RV $\log(\text{IV}_{i,t}^{1/2})$			
Stock	Mean	Std.	Skew.	Kurt.
AA	0.670	0.358	0.260	2.866
AXP	0.498	0.552	−0.265	2.520
BA	0.586	0.394	0.048	2.873
C	0.520	0.493	−0.016	2.872
CAT	0.507	0.391	0.213	2.866
DD	0.470	0.438	−0.016	2.506
DIS	0.663	0.417	−0.033	3.223
EK	0.561	0.367	0.413	3.443
GE	0.471	0.449	−0.031	2.800
GM	0.447	0.398	0.321	3.073
HD	0.600	0.433	0.289	3.154
HON	0.692	0.423	0.334	3.241
HPQ	0.816	0.460	0.043	2.788
IBM	0.388	0.476	0.101	2.698
INTC	0.865	0.439	0.007	2.424
IP	0.533	0.445	0.164	2.552
JNJ	0.265	0.412	0.121	3.065
JPM	0.605	0.497	−0.071	3.047
KO	0.331	0.406	0.266	2.927
MCD	0.540	0.400	0.177	3.405
MMM	0.311	0.416	0.315	2.802
MO	0.405	0.485	0.220	3.025
MRK	0.381	0.399	0.271	3.207
MSFT	0.562	0.476	−0.027	2.635
PG	0.229	0.454	0.431	3.265
SBC	0.595	0.434	0.084	2.717
T	0.686	0.419	0.104	2.819
UTX	0.456	0.442	0.099	2.907
WMT	0.461	0.464	0.207	2.437
XOM	0.254	0.415	0.259	2.876
Median	0.514	0.433	0.143	2.872
Mean	0.512	0.435	0.143	2.901
Min.	0.229	0.358	−0.265	2.424
Max.	0.865	0.552	0.431	3.443

Table 42. Summary statistics for the log-volatility distributions for each of the thirty DJIA stocks in percentage on daily base, $\log(\text{IV}_{i,t}^{1/2})$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. RV is compute using unfiltered 5mn log-returns, as detailed in the text.

	TSRV $IV_{i,t}^{1/2}$			RV $IV_{i,t}^{1/2}$		
Stock	Q_{22}	ADF	d_{GPH}	Q_{22}	ADF	d_{GPH}
AA	7,223.271	−5.244	0.434	6,778.727	−5.229	0.426
AXP	10,684.396	−4.656	0.471	9,952.638	−4.593	0.411
BA	7,040.730	−4.061	0.482	6,310.282	−4.305	0.435
C	9,840.163	−4.145	0.472	9,522.654	−3.995	0.443
CAT	7,479.587	−4.711	0.431	7,042.493	−4.863	0.376
DD	11,817.960	−4.906	0.502	11,709.776	−4.921	0.476
DIS	7,213.061	−3.780	0.448	6,674.643	−4.005	0.427
EK	2,998.953	−4.559	0.428	2,881.977	−4.633	0.404
GE	8,726.849	−4.591	0.485	7,606.358	−4.745	0.408
GM	7,335.697	−4.240	0.434	5,927.294	−4.151	0.396
HD	9,346.318	−4.762	0.487	6,527.008	−4.707	0.425
HON	4,669.980	−6.191	0.435	3,778.045	−6.132	0.352
HPQ	8,317.969	−4.363	0.441	7,064.500	−4.161	0.351
IBM	11,232.868	−4.875	0.481	9,777.032	−5.001	0.435
INTC	11,747.853	−4.268	0.516	11,456.245	−4.265	0.489
IP	12,206.724	−4.504	0.439	11,689.641	−4.593	0.379
JNJ	8,598.779	−4.209	0.433	7,346.210	−4.394	0.391
JPM	9,048.517	−4.060	0.507	7,780.415	−4.039	0.469
KO	9,801.712	−4.699	0.502	8,460.611	−4.649	0.404
MCD	4,837.682	−3.916	0.387	4,032.255	−4.031	0.301
MMM	8,464.822	−5.520	0.425	6,978.335	−5.485	0.383
MO	3,612.947	−4.755	0.269	6,010.163	−4.205	0.290
MRK	6,642.232	−4.054	0.430	5,955.876	−4.282	0.387
MSFT	10,547.717	−5.177	0.502	9,824.078	−5.334	0.412
PG	8,683.131	−4.433	0.490	6,841.794	−4.301	0.381
SBC	11,788.790	−2.662	0.427	9,885.972	−2.686	0.387
T	5,677.289	−4.570	0.438	5,233.710	−4.202	0.367
UTX	8,960.507	−4.518	0.480	8,416.134	−4.696	0.473
WMT	12,567.612	−4.201	0.480	11,061.534	−4.224	0.415
XOM	11,139.192	−3.615	0.488	10,033.151	−3.634	0.452
Median	8,704.990	−4.511	0.460	7,205.355	−4.350	0.406
Mean	8,608.444	−4.475	0.455	7,751.985	−4.482	0.405
Min.	2,998.953	−6.191	0.269	2,881.977	−6.132	0.290
Max.	12,567.612	−2.662	0.516	11,709.776	−2.686	0.489

Table 43. Summary statistics of the time series dependence for the volatility for each of the thirty DJIA stocks, $IV_{i,t}^{1/2}$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. TSRV is computed using a slow time scale of 5mn. RV is compute using de-meaned MA(1) filtered 5mn log-returns, as detailed in the text. The table reports the Ljung-Box portmanteau test for up to 22nd order autocorrelation, Q_{22}, the 1% critical value is 40.289. The augmented Dickey-Fuller test for unit root involving 22 augmentation lags, a constant term and time trend, ADF, the 5% critical value is −3.432, and at 1% critical value is −3.998. The Geweke and Porter-Hudak estimate for the degree of fractional integration, d_{GPH}, the estimates are based on the first $m = 1{,}256^{9/10} = 615$ Fourier frequencies, the asymptotic standard error for all of the d_{GPH} estimates is $\pi(24m)^{-1/2} = 0.026$.

	RV $\text{IV}_{i,t}^{1/2}$		
Stock	Q_{22}	ADF	d_{GPH}
AA	6,779.920	−5.228	0.426
AXP	9,952.751	−4.593	0.411
BA	6,311.014	−4.306	0.435
C	9,523.458	−3.996	0.443
CAT	7,041.166	−4.863	0.376
DD	11,710.089	−4.919	0.476
DIS	6,667.096	−4.005	0.427
EK	2,887.476	−4.629	0.403
GE	7,605.602	−4.746	0.408
GM	5,933.452	−4.151	0.395
HD	6,528.205	−4.706	0.425
HON	3,779.464	−6.131	0.352
HPQ	7,065.823	−4.161	0.351
IBM	9,776.701	−5.002	0.435
INTC	11,462.067	−4.263	0.490
IP	11,690.472	−4.593	0.379
JNJ	7,343.736	−4.394	0.391
JPM	7,778.902	−4.041	0.469
KO	8,460.514	−4.649	0.404
MCD	4,030.329	−4.032	0.301
MMM	6,977.170	−5.486	0.383
MO	6,010.802	−4.205	0.290
MRK	5,957.340	−4.282	0.388
MSFT	9,815.678	−5.337	0.411
PG	6,842.934	−4.301	0.380
SBC	9,885.576	−2.686	0.387
T	5,233.879	−4.201	0.367
UTX	8,411.193	−4.694	0.473
WMT	11,060.979	−4.224	0.415
XOM	10,032.711	−3.635	0.453
Median	7,204.779	−4.350	0.406
Mean	7,751.883	−4.482	0.405
Min.	2,887.476	−6.131	0.290
Max.	11,710.089	−2.686	0.490

Table 44. Summary statistics of the time series dependence for the log-volatility for each of the thirty DJIA stocks, $\text{IV}_{i,t}^{1/2}$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. TSRV is computed using a slow time scale of 5mn. RV is compute using unfiltered 5mn log-returns, as detailed in the text. The table reports the Ljung-Box portmanteau test for up to 22nd order autocorrelation, Q_{22}, the 1% critical value is 40.289. The augmented Dickey-Fuller test for unit root involving 22 augmentation lags, a constant term and time trend, ADF, the 5% critical value is −3.432, and at 1% critical value is −3.998. The Geweke and Porter-Hudak estimate for the degree of fractional integration, d_{GPH}, the estimates are based on the first $m = 1{,}256^{9/10} = 615$ Fourier frequencies, the asymptotic standard error for all of the d_{GPH} estimates is $\pi(24m)^{-1/2} = 0.026$.

Stock	TSRV $\log(\mathrm{IV}_{i,t}^{1/2})$ Q_{22}	ADF	d_{GPH}	RV $\log(\mathrm{IV}_{i,t}^{1/2})$ Q_{22}	ADF	d_{GPH}
AA	7,550.563	−4.860	0.410	7,634.977	−4.804	0.418
AXP	16,293.398	−3.819	0.468	15,705.337	−3.810	0.420
BA	8,869.587	−3.587	0.445	8,416.448	−3.770	0.385
C	14,840.416	−3.775	0.470	14,499.974	−3.627	0.465
CAT	8,804.066	−4.416	0.433	9,078.940	−4.717	0.405
DD	13,870.986	−4.338	0.449	13,336.717	−4.602	0.432
DIS	10,777.573	−3.191	0.437	10,758.264	−3.414	0.409
EK	4,132.995	−4.157	0.401	4,038.436	−4.253	0.384
GE	12,362.117	−3.788	0.462	11,458.080	−4.024	0.417
GM	8,683.572	−3.682	0.439	7,753.796	−3.569	0.402
HD	11,647.458	−4.194	0.513	10,594.395	−4.190	0.454
HON	7,331.189	−5.492	0.432	7,389.815	−5.683	0.397
HPQ	11,518.356	−3.926	0.425	10,391.650	−3.911	0.389
IBM	14,475.916	−4.434	0.512	13,486.011	−4.420	0.459
INTC	13,858.023	−4.020	0.525	13,416.215	−4.029	0.452
IP	13,624.768	−4.347	0.440	13,290.840	−4.540	0.402
JNJ	9,910.933	−3.884	0.415	8,762.948	−4.098	0.356
JPM	13,962.795	−3.515	0.502	12,857.541	−3.453	0.462
KO	11,373.978	−4.456	0.473	10,466.345	−4.443	0.417
MCD	6,973.546	−3.421	0.375	7,094.902	−3.618	0.355
MMM	10,161.474	−5.314	0.425	9,073.716	−5.315	0.385
MO	6,950.912	−4.357	0.357	9,263.647	−4.004	0.363
MRK	8,079.525	−3.753	0.414	7,355.662	−3.867	0.372
MSFT	13,286.505	−4.352	0.524	13,157.981	−4.464	0.479
PG	12,407.376	−4.294	0.528	11,384.656	−4.225	0.456
SBC	12,091.896	−2.341	0.427	11,235.903	−2.453	0.417
T	7,631.442	−4.278	0.423	7,437.222	−4.081	0.388
UTX	11,606.290	−4.081	0.475	11,312.745	−4.210	0.451
WMT	15,196.601	−3.741	0.486	13,678.106	−3.852	0.419
XOM	12,754.516	−3.437	0.438	11,981.926	−3.522	0.395
Median	11,562.323	−4.051	0.440	10,676.330	−4.055	0.413
Mean	11,034.292	−4.042	0.451	10,543.773	−4.099	0.413
Min.	4,132.995	−5.492	0.357	4,038.436	−5.683	0.355
Max.	16,293.398	−2.341	0.528	15,705.337	−2.453	0.479

Table 45. Summary statistics of the time series dependence for the log-volatility for each of the thirty DJIA stocks, $\log(\mathrm{IV}_{i,t}^{1/2})$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. TSRV is computed using a slow time scale of 5mn. RV is compute using de-meaned MA(1) filtered 5mn log-returns, as detailed in the text. The table reports the Ljung-Box portmanteau test for up to 22nd order autocorrelation, Q_{22}, the 1% critical value is 40.289. The augmented Dickey-Fuller test for unit root involving 22 augmentation lags, a constant term and time trend, ADF, the 5% critical value is −3.432, and at 1% critical value is −3.998. The Geweke and Porter-Hudak estimate for the degree of fractional integration, d_{GPH}, the estimates are based on the first $m = 1{,}256^{9/10} = 615$ Fourier frequencies, the asymptotic standard error for all of the d_{GPH} estimates is $\pi(24m)^{-1/2} = 0.026$.

Stock	RV $\log(\text{IV}_{i,t}^{1/2})$		
	Q_{22}	ADF	d_{GPH}
AA	7,636.552	−4.803	0.418
AXP	15,705.157	−3.810	0.420
BA	8,416.708	−3.770	0.385
C	14,500.952	−3.628	0.465
CAT	9,077.868	−4.717	0.405
DD	13,338.138	−4.601	0.432
DIS	10,749.352	−3.413	0.410
EK	4,046.022	−4.251	0.385
GE	11,455.748	−4.024	0.417
GM	7,760.674	−3.570	0.401
HD	10,595.257	−4.190	0.454
HON	7,391.595	−5.682	0.397
HPQ	10,393.203	−3.910	0.389
IBM	13,485.489	−4.422	0.459
INTC	13,417.875	−4.025	0.453
IP	13,291.200	−4.541	0.402
JNJ	8,759.668	−4.099	0.356
JPM	12,856.815	−3.454	0.462
KO	10,465.716	−4.443	0.417
MCD	7,092.638	−3.618	0.355
MMM	9,071.674	−5.315	0.385
MO	9,264.656	−4.003	0.362
MRK	7,356.803	−3.867	0.372
MSFT	13,150.266	−4.465	0.479
PG	11,386.414	−4.225	0.457
SBC	11,235.840	−2.452	0.417
T	7,437.997	−4.080	0.388
UTX	11,313.531	−4.207	0.451
WMT	13,677.297	−3.853	0.419
XOM	11,981.852	−3.523	0.396
Median	10,672.305	−4.052	0.413
Mean	10,543.765	−4.099	0.414
Min.	4,046.022	−5.682	0.355
Max.	15,705.157	−2.452	0.479

Table 46. Summary statistics of the time series dependence for the log-volatility for each of the thirty DJIA stocks, $\log(\text{IV}_{i,t}^{1/2})$. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations. TSRV is computed using a slow time scale of 5mn. RV is compute using unfiltered 5mn log-returns, as detailed in the text. The table reports the Ljung-Box portmanteau test for up to 22nd order autocorrelation, Q_{22}, the 1% critical value is 40.289. The augmented Dickey-Fuller test for unit root involving 22 augmentation lags, a constant term and time trend, ADF, the 5% critical value is -3.432, and at 1% critical value is -3.998. The Geweke and Porter-Hudak estimate for the degree of fractional integration, d_{GPH}, the estimates are based on the first $m = 1{,}256^{9/10} = 615$ Fourier frequencies, the asymptotic standard error for all of the d_{GPH} estimates is $\pi(24m)^{-1/2} = 0.026$.

Stock	b_0	b_1	b_2	R^2
AA	0.060 (0.085)	— —	0.858 (0.047)	0.536
AXP	−0.109 (0.062)		0.932 (0.042)	0.660
BA	0.024 (0.088)		0.870 (0.049)	0.536
C	0.028 (0.055)		0.855 (0.038)	0.631
CAT	0.157 (0.101)		0.780 (0.064)	0.441
DD	0.001 (0.067)		0.886 (0.046)	0.657
DIS	−0.073 (0.074)		0.903 (0.039)	0.602
EK	−0.084 (0.081)		0.922 (0.045)	0.433
GE	−0.074 (0.064)		0.903 (0.042)	0.596
GM	0.057 (0.080)		0.829 (0.054)	0.487
HD	0.080 (0.082)		0.825 (0.048)	0.576
HON	0.099 (0.121)		0.814 (0.062)	0.418
HPQ	−0.063 (0.086)		0.882 (0.040)	0.551
IBM	−0.041 (0.066)		0.901 (0.049)	0.632
INTC	−0.075 (0.074)		0.935 (0.031)	0.704
IP	0.128 (0.078)		0.801 (0.051)	0.555
JNJ	−0.079 (0.074)		0.919 (0.056)	0.553
JPM	−0.050 (0.064)		0.901 (0.037)	0.645
KO	0.032 (0.076)		0.847 (0.058)	0.581
MCD	0.239 (0.117)		0.709 (0.070)	0.358
MMM	−0.001 (0.081)		0.893 (0.063)	0.528
MO	0.264 (0.095)		0.662 (0.062)	0.332
MRK	0.053 (0.091)		0.844 (0.063)	0.448
MSFT	−0.041 (0.077)		0.925 (0.052)	0.613
PG	0.105 (0.079)		0.789 (0.070)	0.536
SBC	−0.114 (0.089)		0.934 (0.052)	0.640
T	−0.007 (0.118)		0.836 (0.060)	0.446
UTX	0.045 (0.061)		0.861 (0.042)	0.548
WMT	0.052 (0.068)		0.830 (0.046)	0.638
XOM	0.003 (0.051)		0.883 (0.042)	0.642
Median	0.013 (0.078)		0.866 (0.049)	0.554
Mean	0.020 (0.080)		0.858 (0.051)	0.551
Min.	−0.114 (0.051)		0.662 (0.031)	0.332
Max.	0.264 (0.121)		0.935 (0.070)	0.704

Table 47. In-sample one day ahead forecasts of $IV^{1/2}$ using an AR(1) model with constant for IV for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 1) $IV^{1/2} = b_0 + b_2 RV^{1/2} + \text{error}$. RV is compute using de-meaned MA(1) filtered 5mn log-returns. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	−0.068 (0.065)	0.988 (0.038)	— —	0.574
AXP	−0.077 (0.051)	0.971 (0.037)		0.679
BA	−0.120 (0.075)	1.010 (0.047)		0.584
C	−0.063 (0.046)	0.966 (0.032)		0.676
CAT	0.012 (0.077)	0.942 (0.054)		0.505
DD	−0.012 (0.052)	0.951 (0.039)		0.670
DIS	−0.057 (0.063)	0.974 (0.035)		0.612
EK	−0.039 (0.072)	0.959 (0.042)		0.445
GE	−0.057 (0.051)	0.971 (0.037)		0.624
GM	−0.037 (0.071)	0.965 (0.053)		0.539
HD	−0.058 (0.054)	0.976 (0.036)		0.638
HON	−0.050 (0.091)	0.950 (0.049)		0.461
HPQ	−0.064 (0.073)	0.947 (0.037)		0.584
IBM	−0.010 (0.051)	0.945 (0.043)		0.663
INTC	−0.072 (0.062)	0.984 (0.029)		0.720
IP	0.053 (0.070)	0.904 (0.050)		0.584
JNJ	−0.044 (0.056)	0.969 (0.046)		0.576
JPM	−0.037 (0.049)	0.961 (0.030)		0.680
KO	−0.008 (0.049)	0.951 (0.041)		0.639
MCD	0.067 (0.072)	0.888 (0.046)		0.428
MMM	0.018 (0.063)	0.934 (0.053)		0.574
MO	0.074 (0.088)	0.846 (0.062)		0.349
MRK	0.033 (0.080)	0.917 (0.060)		0.489
MSFT	−0.075 (0.059)	0.996 (0.042)		0.647
PG	0.072 (0.072)	0.890 (0.070)		0.599
SBC	−0.045 (0.067)	0.967 (0.042)		0.677
T	−0.096 (0.071)	0.988 (0.039)		0.523
UTX	−0.019 (0.069)	0.947 (0.052)		0.569
WMT	0.052 (0.056)	0.911 (0.045)		0.677
XOM	−0.027 (0.043)	0.966 (0.037)		0.677
Median	−0.038 (0.064)	0.960 (0.042)		0.592
Mean	−0.025 (0.064)	0.951 (0.044)		0.589
Min.	−0.120 (0.043)	0.846 (0.029)		0.349
Max.	0.074 (0.091)	1.010 (0.070)		0.720

Table 48. In-sample one day ahead forecasts of $IV^{1/2}$ using an AR(1) model with constant for IV for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 2) $IV^{1/2} = b_0 + b_1 TSRV^{1/2} + \text{error}$. TSRV is compute using a slow time scale of 5mn. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	−0.068 (0.065)	1.035 (0.133)	−0.044 (0.129)	0.574
AXP	−0.060 (0.052)	1.174 (0.164)	−0.200 (0.163)	0.679
BA	−0.119 (0.076)	1.049 (0.144)	−0.036 (0.135)	0.584
C	−0.064 (0.047)	1.435 (0.158)	−0.436 (0.151)	0.681
CAT	0.018 (0.074)	1.106 (0.148)	−0.153 (0.133)	0.507
DD	−0.015 (0.055)	0.819 (0.200)	0.126 (0.202)	0.670
DIS	−0.076 (0.065)	0.711 (0.287)	0.250 (0.270)	0.613
EK	−0.058 (0.074)	0.794 (0.262)	0.164 (0.250)	0.445
GE	−0.046 (0.057)	1.088 (0.249)	−0.113 (0.246)	0.624
GM	−0.026 (0.072)	1.293 (0.165)	−0.306 (0.158)	0.543
HD	−0.055 (0.052)	1.275 (0.120)	−0.273 (0.106)	0.642
HON	−0.047 (0.087)	1.139 (0.170)	−0.176 (0.141)	0.462
HPQ	−0.052 (0.069)	1.038 (0.181)	−0.089 (0.160)	0.584
IBM	−0.007 (0.057)	0.970 (0.127)	−0.025 (0.131)	0.663
INTC	−0.080 (0.066)	0.863 (0.159)	0.117 (0.159)	0.720
IP	0.052 (0.069)	1.045 (0.197)	−0.131 (0.192)	0.585
JNJ	−0.039 (0.065)	1.025 (0.166)	−0.055 (0.180)	0.576
JPM	0.002 (0.051)	1.362 (0.187)	−0.392 (0.183)	0.683
KO	0.011 (0.047)	1.229 (0.145)	−0.267 (0.139)	0.642
MCD	0.080 (0.065)	1.181 (0.162)	−0.270 (0.146)	0.433
MMM	0.055 (0.064)	1.239 (0.176)	−0.312 (0.173)	0.577
MO	0.084 (0.080)	0.781 (0.255)	0.053 (0.226)	0.349
MRK	0.060 (0.086)	1.229 (0.153)	−0.309 (0.154)	0.492
MSFT	−0.082 (0.063)	0.907 (0.146)	0.088 (0.145)	0.647
PG	0.080 (0.070)	0.970 (0.113)	−0.079 (0.070)	0.599
SBC	0.032 (0.068)	1.460 (0.162)	−0.497 (0.158)	0.682
T	−0.057 (0.072)	1.190 (0.126)	−0.196 (0.117)	0.525
UTX	−0.019 (0.068)	0.873 (0.153)	0.070 (0.123)	0.569
WMT	0.072 (0.056)	1.163 (0.138)	−0.242 (0.128)	0.679
XOM	−0.018 (0.043)	1.313 (0.122)	−0.331 (0.115)	0.680
Median	−0.022 (0.065)	1.097 (0.160)	−0.142 (0.152)	0.592
Mean	−0.015 (0.065)	1.092 (0.169)	−0.135 (0.159)	0.590
Min.	−0.119 (0.043)	0.711 (0.113)	−0.497 (0.070)	0.349
Max.	0.084 (0.087)	1.460 (0.287)	0.250 (0.270)	0.720

Table 49. In-sample one day ahead forecasts of $\mathrm{IV}^{1/2}$ using an AR(1) model with constant for IV for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 3) $\mathrm{IV}^{1/2} = b_0 + b_1 \mathrm{TSRV}^{1/2} + b_2 \mathrm{RV}^{1/2} + \mathrm{error}$. TSRV is compute using a slow time scale of 5mn. RV is compute using de-meaned MA(1) filtered 5mn log-returns. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	0.048 (0.094)	——	0.925 (0.055)	0.538
AXP	−0.096 (0.061)		1.016 (0.046)	0.660
BA	−0.002 (0.101)		0.952 (0.064)	0.521
C	0.001 (0.064)		0.950 (0.044)	0.631
CAT	0.173 (0.095)		0.835 (0.066)	0.458
DD	0.019 (0.077)		0.934 (0.056)	0.637
DIS	−0.095 (0.087)		0.984 (0.051)	0.578
EK	−0.098 (0.101)		1.022 (0.064)	0.389
GE	−0.068 (0.067)		0.980 (0.049)	0.592
GM	0.037 (0.074)		0.919 (0.055)	0.501
HD	−0.026 (0.069)		0.958 (0.044)	0.611
HON	−0.020 (0.113)		0.969 (0.063)	0.446
HPQ	−0.033 (0.101)		0.960 (0.050)	0.551
IBM	−0.041 (0.050)		0.979 (0.038)	0.664
INTC	−0.030 (0.064)		0.968 (0.031)	0.709
IP	0.101 (0.070)		0.880 (0.049)	0.578
JNJ	−0.053 (0.086)		0.984 (0.073)	0.555
JPM	−0.090 (0.083)		1.005 (0.049)	0.651
KO	0.008 (0.081)		0.934 (0.066)	0.598
MCD	0.143 (0.098)		0.843 (0.065)	0.383
MMM	−0.040 (0.085)		1.007 (0.073)	0.533
MO	0.255 (0.079)		0.759 (0.060)	0.353
MRK	0.004 (0.109)		0.961 (0.082)	0.469
MSFT	−0.006 (0.065)		0.972 (0.044)	0.653
PG	0.061 (0.072)		0.904 (0.066)	0.588
SBC	−0.082 (0.085)		0.990 (0.055)	0.654
T	0.060 (0.111)		0.896 (0.062)	0.466
UTX	−0.002 (0.072)		0.964 (0.055)	0.532
WMT	0.005 (0.063)		0.927 (0.046)	0.662
XOM	−0.020 (0.064)		0.967 (0.056)	0.640
Median	−0.004 (0.080)		0.960 (0.055)	0.578
Mean	0.004 (0.081)		0.945 (0.056)	0.560
Min.	−0.098 (0.050)		0.759 (0.031)	0.353
Max.	0.255 (0.113)		1.022 (0.082)	0.709

Table 50. In-sample one day ahead forecasts of $\mathrm{IV}^{1/2}$ using an AR(1) model with constant for $\log(\mathrm{IV}^{1/2})$ for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 1) $\mathrm{IV}^{1/2} = b_0 + b_2\mathrm{RV}^{1/2} + \text{error}$. RV is compute using de-meaned MA(1) filtered 5mn log-returns. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	−0.071 (0.089)	1.048 (0.054)	— —	0.558
AXP	−0.093 (0.051)	1.063 (0.040)		0.681
BA	−0.088 (0.091)	1.061 (0.061)		0.563
C	−0.064 (0.059)	1.047 (0.042)		0.668
CAT	0.003 (0.078)	1.005 (0.057)		0.501
DD	−0.032 (0.066)	1.019 (0.051)		0.655
DIS	−0.099 (0.083)	1.065 (0.053)		0.578
EK	−0.051 (0.093)	1.055 (0.062)		0.404
GE	−0.058 (0.057)	1.043 (0.045)		0.619
GM	−0.032 (0.066)	1.033 (0.053)		0.542
HD	−0.084 (0.061)	1.057 (0.043)		0.644
HON	−0.079 (0.097)	1.058 (0.057)		0.470
HPQ	−0.049 (0.092)	1.024 (0.048)		0.575
IBM	−0.043 (0.039)	1.038 (0.034)		0.689
INTC	−0.057 (0.056)	1.027 (0.029)		0.723
IP	−0.002 (0.060)	1.001 (0.045)		0.608
JNJ	−0.018 (0.063)	1.022 (0.058)		0.581
JPM	−0.067 (0.063)	1.049 (0.040)		0.671
KO	−0.026 (0.058)	1.019 (0.051)		0.637
MCD	−0.015 (0.088)	1.016 (0.062)		0.433
MMM	−0.047 (0.072)	1.057 (0.064)		0.570
MO	0.030 (0.067)	1.000 (0.053)		0.380
MRK	−0.011 (0.094)	1.028 (0.076)		0.503
MSFT	−0.052 (0.044)	1.045 (0.033)		0.672
PG	−0.006 (0.051)	1.017 (0.052)		0.635
SBC	−0.041 (0.075)	1.030 (0.050)		0.682
T	−0.062 (0.085)	1.054 (0.054)		0.511
UTX	−0.050 (0.079)	1.036 (0.062)		0.544
WMT	−0.014 (0.052)	1.010 (0.043)		0.690
XOM	−0.029 (0.056)	1.031 (0.052)		0.663
Median	−0.048 (0.066)	1.035 (0.052)		0.595
Mean	−0.044 (0.069)	1.035 (0.051)		0.588
Min.	−0.099 (0.039)	1.000 (0.029)		0.380
Max.	0.030 (0.097)	1.065 (0.076)		0.723

Table 51. In-sample one day ahead forecasts of $\mathrm{IV}^{1/2}$ using an AR(1) model with constant for $\log(\mathrm{IV}^{1/2})$ for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 2) $\mathrm{IV}^{1/2} = b_0 + b_1 \mathrm{TSRV}^{1/2} + \mathrm{error}$. TSRV is compute using a slow time scale of 5mn. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	−0.065 (0.092)	0.855 (0.180)	0.179 (0.143)	0.559
AXP	−0.086 (0.048)	1.195 (0.231)	−0.130 (0.216)	0.681
BA	−0.084 (0.088)	1.142 (0.167)	−0.078 (0.131)	0.564
C	−0.059 (0.056)	1.276 (0.213)	−0.217 (0.176)	0.669
CAT	0.004 (0.079)	0.986 (0.164)	0.018 (0.132)	0.501
DD	−0.032 (0.067)	0.964 (0.224)	0.052 (0.197)	0.655
DIS	−0.120 (0.085)	0.523 (0.215)	0.512 (0.206)	0.585
EK	−0.071 (0.081)	0.901 (0.337)	0.156 (0.302)	0.404
GE	−0.050 (0.058)	1.161 (0.242)	−0.115 (0.231)	0.619
GM	−0.026 (0.063)	1.198 (0.184)	−0.157 (0.151)	0.542
HD	−0.081 (0.060)	1.157 (0.140)	−0.095 (0.117)	0.644
HON	−0.076 (0.095)	1.138 (0.215)	−0.077 (0.187)	0.470
HPQ	−0.030 (0.083)	1.285 (0.214)	−0.254 (0.178)	0.576
IBM	−0.044 (0.041)	1.016 (0.121)	0.021 (0.120)	0.689
INTC	−0.066 (0.058)	0.794 (0.177)	0.226 (0.171)	0.725
IP	−0.002 (0.059)	0.997 (0.211)	0.004 (0.196)	0.608
JNJ	−0.010 (0.060)	1.120 (0.175)	−0.098 (0.155)	0.582
JPM	−0.069 (0.066)	1.023 (0.161)	0.025 (0.158)	0.671
KO	−0.023 (0.055)	1.085 (0.168)	−0.064 (0.140)	0.637
MCD	−0.013 (0.085)	1.120 (0.228)	−0.096 (0.183)	0.433
MMM	−0.026 (0.065)	1.390 (0.211)	−0.333 (0.180)	0.572
MO	0.046 (0.068)	0.856 (0.202)	0.119 (0.169)	0.380
MRK	0.009 (0.091)	1.326 (0.195)	−0.295 (0.173)	0.505
MSFT	−0.058 (0.047)	0.817 (0.173)	0.220 (0.176)	0.674
PG	−0.003 (0.050)	1.155 (0.094)	−0.132 (0.086)	0.636
SBC	−0.011 (0.071)	1.312 (0.172)	−0.281 (0.149)	0.684
T	−0.058 (0.083)	1.138 (0.171)	−0.078 (0.147)	0.511
UTX	−0.051 (0.078)	0.784 (0.161)	0.242 (0.150)	0.545
WMT	−0.010 (0.049)	1.120 (0.207)	−0.105 (0.183)	0.690
XOM	−0.026 (0.057)	1.110 (0.105)	−0.077 (0.098)	0.663
Median	−0.038 (0.065)	1.120 (0.182)	−0.077 (0.170)	0.596
Mean	−0.040 (0.068)	1.065 (0.189)	−0.030 (0.167)	0.589
Min.	−0.120 (0.041)	0.523 (0.094)	−0.333 (0.086)	0.380
Max.	0.046 (0.095)	1.390 (0.337)	0.512 (0.302)	0.725

Table 52. In-sample one day ahead forecasts of $\mathrm{IV}^{1/2}$ using an AR(1) model with constant for $\log(\mathrm{IV}^{1/2})$ for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 3) $\mathrm{IV}^{1/2} = b_0 + b_1 \mathrm{TSRV}^{1/2} + b_2 \mathrm{RV} + \mathrm{error}$. TSRV is compute using a slow time scale of 5mn. RV is compute using de-meaned MA(1) filtered 5mn log-returns. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	0.123 (0.089)	— —	0.829 (0.048)	0.507
AXP	−0.030 (0.058)		0.891 (0.037)	0.620
BA	0.079 (0.098)		0.842 (0.053)	0.503
C	0.219 (0.107)		0.755 (0.052)	0.550
CAT	0.218 (0.094)		0.745 (0.059)	0.411
DD	0.058 (0.052)		0.854 (0.036)	0.624
DIS	0.331 (0.232)		0.718 (0.103)	0.494
EK	0.106 (0.082)		0.827 (0.042)	0.365
GE	0.050 (0.097)		0.834 (0.055)	0.536
GM	0.136 (0.079)		0.783 (0.052)	0.452
HD	0.162 (0.097)		0.783 (0.053)	0.540
HON	0.462 (0.271)		0.655 (0.123)	0.329
HPQ	0.020 (0.085)		0.849 (0.038)	0.521
IBM	0.028 (0.066)		0.857 (0.048)	0.600
INTC	−0.037 (0.077)		0.920 (0.032)	0.687
IP	0.160 (0.076)		0.783 (0.049)	0.534
JNJ	−0.022 (0.063)		0.879 (0.048)	0.523
JPM	0.113 (0.096)		0.824 (0.043)	0.584
KO	0.118 (0.096)		0.789 (0.069)	0.531
MCD	0.349 (0.123)		0.652 (0.070)	0.317
MMM	0.055 (0.082)		0.856 (0.063)	0.489
MO	0.328 (0.093)		0.625 (0.058)	0.304
MRK	0.123 (0.095)		0.800 (0.065)	0.410
MSFT	0.062 (0.107)		0.871 (0.065)	0.567
PG	0.244 (0.116)		0.687 (0.092)	0.457
SBC	−0.077 (0.087)		0.915 (0.051)	0.619
T	0.114 (0.123)		0.782 (0.061)	0.404
UTX	0.192 (0.084)		0.776 (0.049)	0.482
WMT	0.092 (0.070)		0.806 (0.047)	0.612
XOM	0.060 (0.047)		0.842 (0.037)	0.604
Median	0.114 (0.091)		0.815 (0.052)	0.522
Mean	0.128 (0.098)		0.801 (0.057)	0.506
Min.	−0.077 (0.047)		0.625 (0.032)	0.304
Max.	0.462 (0.271)		0.920 (0.123)	0.687

Table 53. Out-of-sample one day ahead forecasts of $IV^{1/2}$ using an AR(1) model with constant for IV for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 1) $IV^{1/2} = b_0 + b_2 RV^{1/2} + \text{error}$. RV is compute using de-meaned MA(1) filtered 5mn log-returns. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	0.006 (0.064)	0.951 (0.036)	— —	0.541
AXP	−0.006 (0.053)	0.932 (0.036)		0.643
BA	−0.011 (0.066)	0.951 (0.039)		0.540
C	0.088 (0.087)	0.882 (0.046)		0.607
CAT	0.105 (0.069)	0.885 (0.046)		0.463
DD	0.047 (0.042)	0.915 (0.031)		0.638
DIS	0.237 (0.156)	0.828 (0.074)		0.522
EK	0.119 (0.071)	0.874 (0.038)		0.388
GE	0.052 (0.078)	0.905 (0.048)		0.570
GM	0.062 (0.079)	0.903 (0.054)		0.493
HD	0.027 (0.066)	0.929 (0.039)		0.601
HON	0.073 (0.098)	0.891 (0.050)		0.420
HPQ	0.070 (0.095)	0.892 (0.042)		0.541
IBM	0.030 (0.049)	0.919 (0.041)		0.640
INTC	−0.022 (0.064)	0.963 (0.029)		0.699
IP	0.099 (0.068)	0.877 (0.048)		0.561
JNJ	0.013 (0.050)	0.926 (0.039)		0.546
JPM	0.140 (0.106)	0.872 (0.050)		0.611
KO	0.040 (0.048)	0.915 (0.040)		0.609
MCD	0.143 (0.074)	0.843 (0.046)		0.392
MMM	0.069 (0.060)	0.898 (0.051)		0.537
MO	0.222 (0.103)	0.752 (0.066)		0.299
MRK	0.092 (0.081)	0.877 (0.060)		0.457
MSFT	−0.015 (0.061)	0.963 (0.041)		0.616
PG	0.105 (0.070)	0.863 (0.068)		0.573
SBC	−0.016 (0.068)	0.951 (0.042)		0.657
T	−0.011 (0.075)	0.945 (0.040)		0.485
UTX	0.132 (0.087)	0.855 (0.056)		0.501
WMT	0.083 (0.056)	0.890 (0.045)		0.656
XOM	0.046 (0.050)	0.910 (0.038)		0.633
Median	0.065 (0.069)	0.900 (0.043)		0.553
Mean	0.067 (0.073)	0.899 (0.046)		0.548
Min.	−0.022 (0.042)	0.752 (0.029)		0.299
Max.	0.237 (0.156)	0.963 (0.074)		0.699

Table 54. Out-of-sample one day ahead forecasts of $\text{IV}^{1/2}$ using an AR(1) model with constant for IV for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 2) $\text{IV}^{1/2} = b_0 + b_1\text{TSRV}^{1/2} + \text{error}$. TSRV is compute using a slow time scale of 5mn. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	0.005 (0.065)	0.901 (0.114)	0.047 (0.119)	0.541
AXP	0.003 (0.058)	1.038 (0.166)	−0.105 (0.168)	0.643
BA	−0.030 (0.064)	0.745 (0.169)	0.202 (0.157)	0.543
C	0.073 (0.076)	1.472 (0.312)	−0.541 (0.311)	0.618
CAT	0.107 (0.071)	0.911 (0.118)	−0.024 (0.114)	0.463
DD	0.043 (0.045)	0.795 (0.193)	0.116 (0.193)	0.638
DIS	0.236 (0.162)	0.762 (0.322)	0.061 (0.312)	0.523
EK	0.129 (0.086)	0.950 (0.189)	−0.076 (0.199)	0.388
GE	0.072 (0.090)	1.127 (0.246)	−0.215 (0.259)	0.571
GM	0.065 (0.080)	0.976 (0.160)	−0.068 (0.146)	0.494
HD	0.031 (0.063)	1.167 (0.118)	−0.219 (0.107)	0.604
HON	0.073 (0.097)	0.962 (0.093)	−0.065 (0.058)	0.421
HPQ	0.043 (0.074)	0.742 (0.258)	0.150 (0.232)	0.542
IBM	0.031 (0.051)	0.932 (0.093)	−0.014 (0.090)	0.640
INTC	−0.039 (0.069)	0.745 (0.155)	0.214 (0.157)	0.700
IP	0.099 (0.069)	0.866 (0.168)	0.010 (0.166)	0.561
JNJ	0.014 (0.056)	0.931 (0.146)	−0.005 (0.155)	0.546
JPM	0.159 (0.101)	1.043 (0.179)	−0.168 (0.156)	0.612
KO	0.052 (0.048)	1.061 (0.103)	−0.141 (0.099)	0.611
MCD	0.153 (0.067)	1.165 (0.150)	−0.294 (0.137)	0.400
MMM	0.096 (0.062)	1.099 (0.175)	−0.207 (0.174)	0.539
MO	0.266 (0.100)	0.293 (0.228)	0.392 (0.194)	0.308
MRK	0.113 (0.087)	1.073 (0.123)	−0.196 (0.136)	0.458
MSFT	−0.018 (0.065)	0.924 (0.170)	0.038 (0.171)	0.616
PG	0.114 (0.068)	0.966 (0.089)	−0.100 (0.035)	0.574
SBC	0.038 (0.068)	1.283 (0.160)	−0.336 (0.153)	0.660
T	0.020 (0.075)	1.105 (0.144)	−0.155 (0.128)	0.487
UTX	0.132 (0.087)	0.796 (0.153)	0.056 (0.136)	0.501
WMT	0.106 (0.056)	1.154 (0.134)	−0.253 (0.126)	0.659
XOM	0.053 (0.047)	1.074 (0.229)	−0.158 (0.204)	0.634
Median	0.069 (0.069)	0.964 (0.160)	−0.072 (0.156)	0.553
Mean	0.075 (0.074)	0.969 (0.169)	−0.068 (0.160)	0.550
Min.	−0.039 (0.045)	0.293 (0.089)	−0.541 (0.035)	0.308
Max.	0.266 (0.162)	1.472 (0.322)	0.392 (0.312)	0.700

Table 55. Out-of-sample one day ahead forecasts of $\mathrm{IV}^{1/2}$ using an AR(1) model with constant for IV for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 3) $\mathrm{IV}^{1/2} = b_0 + b_1 \mathrm{TSRV}^{1/2} + b_2 \mathrm{RV}^{1/2} + \text{error}$. TSRV is compute using a slow time scale of 5mn. RV is compute using de-meaned MA(1) filtered 5mn log-returns. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	0.083 (0.097)	— —	0.907 (0.056)	0.516
AXP	−0.068 (0.059)		1.000 (0.045)	0.637
BA	0.032 (0.105)		0.934 (0.067)	0.493
C	0.053 (0.058)		0.919 (0.039)	0.599
CAT	0.212 (0.095)		0.812 (0.066)	0.432
DD	0.039 (0.076)		0.922 (0.056)	0.615
DIS	−0.051 (0.085)		0.962 (0.050)	0.548
EK	−0.007 (0.104)		0.972 (0.065)	0.349
GE	−0.035 (0.069)		0.960 (0.050)	0.570
GM	0.073 (0.075)		0.897 (0.055)	0.477
HD	0.011 (0.075)		0.937 (0.048)	0.590
HON	0.083 (0.135)		0.918 (0.074)	0.413
HPQ	0.014 (0.105)		0.939 (0.052)	0.529
IBM	−0.015 (0.053)		0.961 (0.040)	0.645
INTC	−0.006 (0.065)		0.958 (0.032)	0.696
IP	0.122 (0.070)		0.868 (0.048)	0.561
JNJ	−0.031 (0.089)		0.968 (0.076)	0.533
JPM	−0.049 (0.078)		0.984 (0.047)	0.624
KO	0.033 (0.084)		0.915 (0.068)	0.574
MCD	0.215 (0.101)		0.801 (0.065)	0.349
MMM	−0.007 (0.087)		0.983 (0.075)	0.507
MO	0.293 (0.078)		0.735 (0.058)	0.332
MRK	0.045 (0.114)		0.933 (0.086)	0.441
MSFT	0.039 (0.080)		0.947 (0.051)	0.629
PG	0.134 (0.103)		0.845 (0.089)	0.541
SBC	−0.059 (0.087)		0.978 (0.056)	0.637
T	0.117 (0.114)		0.867 (0.063)	0.439
UTX	0.038 (0.069)		0.940 (0.052)	0.503
WMT	0.030 (0.065)		0.910 (0.048)	0.644
XOM	−0.002 (0.065)		0.953 (0.057)	0.617
Median	0.033 (0.082)		0.936 (0.056)	0.545
Mean	0.044 (0.085)		0.921 (0.058)	0.535
Min.	−0.068 (0.053)		0.735 (0.032)	0.332
Max.	0.293 (0.135)		1.000 (0.089)	0.696

Table 56. Out-of-sample one day ahead forecasts of $IV^{1/2}$ using an AR(1) model with constant for $\log(IV^{1/2})$ for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 1) $IV^{1/2} = b_0 + b_2 RV^{1/2} + \text{error}$. RV is compute using de-meaned MA(1) filtered 5mn log-returns. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	−0.033 (0.090)	1.029 (0.055)	— —	0.533
AXP	−0.067 (0.050)	1.047 (0.040)		0.659
BA	−0.064 (0.095)	1.048 (0.064)		0.538
C	−0.015 (0.057)	1.018 (0.040)		0.634
CAT	0.041 (0.078)	0.981 (0.057)		0.475
DD	−0.012 (0.066)	1.006 (0.051)		0.633
DIS	−0.037 (0.084)	1.032 (0.052)		0.541
EK	0.031 (0.095)	1.008 (0.062)		0.364
GE	−0.025 (0.058)	1.021 (0.045)		0.596
GM	0.003 (0.067)	1.010 (0.053)		0.517
HD	−0.048 (0.062)	1.036 (0.044)		0.624
HON	−0.014 (0.103)	1.024 (0.060)		0.444
HPQ	−0.005 (0.096)	1.005 (0.050)		0.554
IBM	−0.026 (0.039)	1.026 (0.034)		0.674
INTC	−0.028 (0.057)	1.015 (0.030)		0.709
IP	0.022 (0.060)	0.987 (0.045)		0.590
JNJ	−0.001 (0.065)	1.009 (0.060)		0.562
JPM	−0.027 (0.059)	1.027 (0.038)		0.645
KO	−0.009 (0.061)	1.006 (0.053)		0.617
MCD	0.049 (0.087)	0.976 (0.061)		0.398
MMM	−0.020 (0.073)	1.037 (0.066)		0.544
MO	0.086 (0.069)	0.959 (0.054)		0.353
MRK	0.021 (0.100)	1.005 (0.080)		0.478
MSFT	−0.026 (0.046)	1.029 (0.034)		0.654
PG	0.012 (0.052)	1.002 (0.054)		0.617
SBC	−0.023 (0.077)	1.019 (0.052)		0.666
T	−0.017 (0.088)	1.030 (0.055)		0.485
UTX	−0.007 (0.072)	1.009 (0.058)		0.513
WMT	0.008 (0.054)	0.995 (0.045)		0.674
XOM	−0.012 (0.057)	1.018 (0.053)		0.642
Median	−0.013 (0.066)	1.016 (0.053)		0.576
Mean	−0.008 (0.071)	1.014 (0.051)		0.565
Min.	−0.067 (0.039)	0.959 (0.030)		0.353
Max.	0.086 (0.103)	1.048 (0.080)		0.709

Table 57. Out-of-sample one day ahead forecasts of $\mathrm{IV}^{1/2}$ using an AR(1) model with constant for $\log(\mathrm{IV}^{1/2})$ for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 2) $\mathrm{IV}^{1/2} = b_0 + b_1 \mathrm{TSRV}^{1/2} + \text{error}$. TSRV is compute using a slow time scale of 5mn. Robust standard errors in parenthesis.

Stock	b_0	b_1	b_2	R^2
AA	−0.027 (0.094)	0.794 (0.170)	0.217 (0.135)	0.535
AXP	−0.061 (0.048)	1.152 (0.219)	−0.102 (0.204)	0.659
BA	−0.060 (0.092)	1.119 (0.159)	−0.068 (0.121)	0.538
C	−0.013 (0.055)	1.115 (0.282)	−0.092 (0.245)	0.635
CAT	0.042 (0.079)	0.951 (0.158)	0.028 (0.127)	0.475
DD	−0.012 (0.067)	0.946 (0.227)	0.057 (0.199)	0.633
DIS	−0.071 (0.082)	0.409 (0.223)	0.593 (0.212)	0.553
EK	0.015 (0.086)	0.882 (0.343)	0.127 (0.311)	0.364
GE	−0.019 (0.060)	1.093 (0.205)	−0.070 (0.199)	0.596
GM	0.008 (0.064)	1.154 (0.182)	−0.137 (0.149)	0.518
HD	−0.044 (0.062)	1.131 (0.139)	−0.091 (0.122)	0.624
HON	−0.011 (0.102)	1.092 (0.203)	−0.065 (0.173)	0.444
HPQ	0.012 (0.088)	1.237 (0.204)	−0.225 (0.170)	0.555
IBM	−0.027 (0.041)	1.008 (0.109)	0.017 (0.107)	0.674
INTC	−0.041 (0.060)	0.750 (0.180)	0.258 (0.174)	0.711
IP	0.023 (0.059)	0.945 (0.202)	0.039 (0.187)	0.590
JNJ	0.010 (0.061)	1.133 (0.176)	−0.125 (0.152)	0.562
JPM	−0.030 (0.061)	0.995 (0.148)	0.032 (0.144)	0.645
KO	−0.006 (0.058)	1.070 (0.162)	−0.062 (0.133)	0.617
MCD	0.051 (0.084)	1.090 (0.217)	−0.106 (0.179)	0.399
MMM	0.002 (0.066)	1.375 (0.212)	−0.339 (0.182)	0.546
MO	0.109 (0.070)	0.756 (0.200)	0.169 (0.167)	0.355
MRK	0.042 (0.097)	1.304 (0.197)	−0.296 (0.170)	0.480
MSFT	−0.030 (0.049)	0.848 (0.206)	0.175 (0.207)	0.655
PG	0.013 (0.052)	1.121 (0.069)	−0.113 (0.046)	0.618
SBC	0.004 (0.074)	1.280 (0.166)	−0.260 (0.141)	0.668
T	−0.014 (0.086)	1.109 (0.167)	−0.073 (0.142)	0.486
UTX	−0.009 (0.072)	0.740 (0.169)	0.259 (0.159)	0.515
WMT	0.012 (0.051)	1.103 (0.208)	−0.103 (0.183)	0.674
XOM	−0.009 (0.057)	1.096 (0.117)	−0.076 (0.107)	0.642
Median	−0.009 (0.065)	1.092 (0.190)	−0.069 (0.169)	0.576
Mean	−0.005 (0.069)	1.027 (0.187)	−0.014 (0.165)	0.566
Min.	−0.071 (0.041)	0.409 (0.069)	−0.339 (0.046)	0.355
Max.	0.109 (0.102)	1.375 (0.343)	0.593 (0.311)	0.711

Table 58. Out-of-sample one day ahead forecasts of $IV^{1/2}$ using an AR(1) model with constant for $\log(IV^{1/2})$ for each of the thirty DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. The Mincer-Zarnowitz regression 3) $IV^{1/2} = b_0 + b_1 TSRV^{1/2} + b_2 RV + \text{error}$. TSRV is compute using a slow time scale of 5mn. RV is compute using de-meaned MA(1) filtered 5mn log-returns. Robust standard errors in parenthesis.

	Stock	b_0	b_1	b_2	R^2
RV 5mn MA(1)	Min.	−0.114 (0.051)		0.662 (0.031)	0.332
	0.10	−0.081 (0.062)		0.784 (0.038)	0.425
	0.25	−0.063 (0.067)		0.829 (0.042)	0.487
	0.50	0.013 (0.078)		0.866 (0.049)	0.554
	0.75	0.060 (0.088)		0.903 (0.060)	0.632
	0.90	0.142 (0.109)		0.929 (0.063)	0.651
	Max.	0.264 (0.121)		0.935 (0.070)	0.704
	Mean	0.031 (0.082)		0.844 (0.051)	0.541
	Std.	0.126 (0.024)		0.090 (0.013)	0.124
TSRV 5mn	Min.	−0.120 (0.043)	0.846 (0.029)		0.349
	0.10	−0.076 (0.049)	0.897 (0.034)		0.453
	0.25	−0.063 (0.052)	0.942 (0.037)		0.539
	0.50	−0.038 (0.064)	0.960 (0.042)		0.592
	0.75	0.012 (0.072)	0.971 (0.050)		0.670
	0.90	0.060 (0.079)	0.988 (0.057)		0.678
	Max.	0.074 (0.091)	1.010 (0.070)		0.720
	Mean	−0.021 (0.064)	0.945 (0.045)		0.571
	Std.	0.067 (0.016)	0.052 (0.013)		0.124
Both	Min.	−0.119 (0.043)	0.711 (0.113)	−0.497 (0.070)	0.349
	0.10	−0.078 (0.049)	0.807 (0.124)	−0.361 (0.116)	0.454
	0.25	−0.058 (0.056)	0.970 (0.144)	−0.273 (0.131)	0.543
	0.50	−0.022 (0.065)	1.097 (0.160)	−0.142 (0.152)	0.592
	0.75	0.032 (0.072)	1.229 (0.181)	−0.025 (0.180)	0.670
	0.90	0.076 (0.078)	1.337 (0.252)	0.122 (0.236)	0.682
	Max.	0.084 (0.087)	1.460 (0.287)	0.250 (0.270)	0.720
	Mean	−0.012 (0.064)	1.087 (0.180)	−0.132 (0.165)	0.573
	Std.	0.073 (0.015)	0.255 (0.061)	0.247 (0.064)	0.125

Table 59. In-sample one day ahead forecasts of $IV^{1/2}$ in percentage using an AR(1) model with constant for IV for the DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. Robust standard errors in parenthesis.

	Stock	b_0	b_1	b_2	R^2
RV 5mn MA(1)	Min.	−0.077 (0.047)		0.625 (0.032)	0.304
	0.10	−0.026 (0.061)		0.671 (0.037)	0.347
	0.25	0.055 (0.077)		0.776 (0.047)	0.452
	0.50	0.114 (0.091)		0.815 (0.052)	0.522
	0.75	0.192 (0.098)		0.854 (0.063)	0.584
	0.90	0.329 (0.123)		0.885 (0.081)	0.620
	Max.	0.462 (0.271)		0.920 (0.123)	0.687
	Mean	0.150 (0.110)		0.792 (0.062)	0.502
	Std.	0.179 (0.070)		0.102 (0.029)	0.131
TSRV 5mn	Min.	−0.022 (0.042)	0.752 (0.029)		0.299
	0.10	−0.013 (0.049)	0.849 (0.036)		0.406
	0.25	0.013 (0.060)	0.877 (0.039)		0.493
	0.50	0.065 (0.069)	0.900 (0.043)		0.553
	0.75	0.105 (0.081)	0.929 (0.050)		0.616
	0.90	0.142 (0.100)	0.951 (0.063)		0.649
	Max.	0.237 (0.156)	0.963 (0.074)		0.699
	Mean	0.075 (0.080)	0.889 (0.048)		0.531
	Std.	0.087 (0.036)	0.067 (0.015)		0.131
Both	Min.	−0.039 (0.045)	0.293 (0.089)	−0.541 (0.035)	0.308
	0.10	−0.008 (0.050)	0.745 (0.098)	−0.273 (0.094)	0.410
	0.25	0.031 (0.062)	0.866 (0.123)	−0.196 (0.126)	0.494
	0.50	0.069 (0.069)	0.964 (0.160)	−0.072 (0.156)	0.553
	0.75	0.113 (0.086)	1.099 (0.189)	0.047 (0.193)	0.618
	0.90	0.156 (0.098)	1.166 (0.252)	0.176 (0.245)	0.651
	Max.	0.266 (0.162)	1.472 (0.322)	0.392 (0.312)	0.700
	Mean	0.084 (0.082)	0.944 (0.176)	−0.067 (0.166)	0.533
	Std.	0.097 (0.037)	0.343 (0.079)	0.285 (0.086)	0.129

Table 60. Out-of-sample one day ahead forecasts of $IV^{1/2}$ using an AR(1) model with constant for IV for the DJIA stocks. AR(1) models are estimated on a rolling window of 100 days, from January 3, 2000 to December 31, 2004. Robust standard errors in parenthesis.

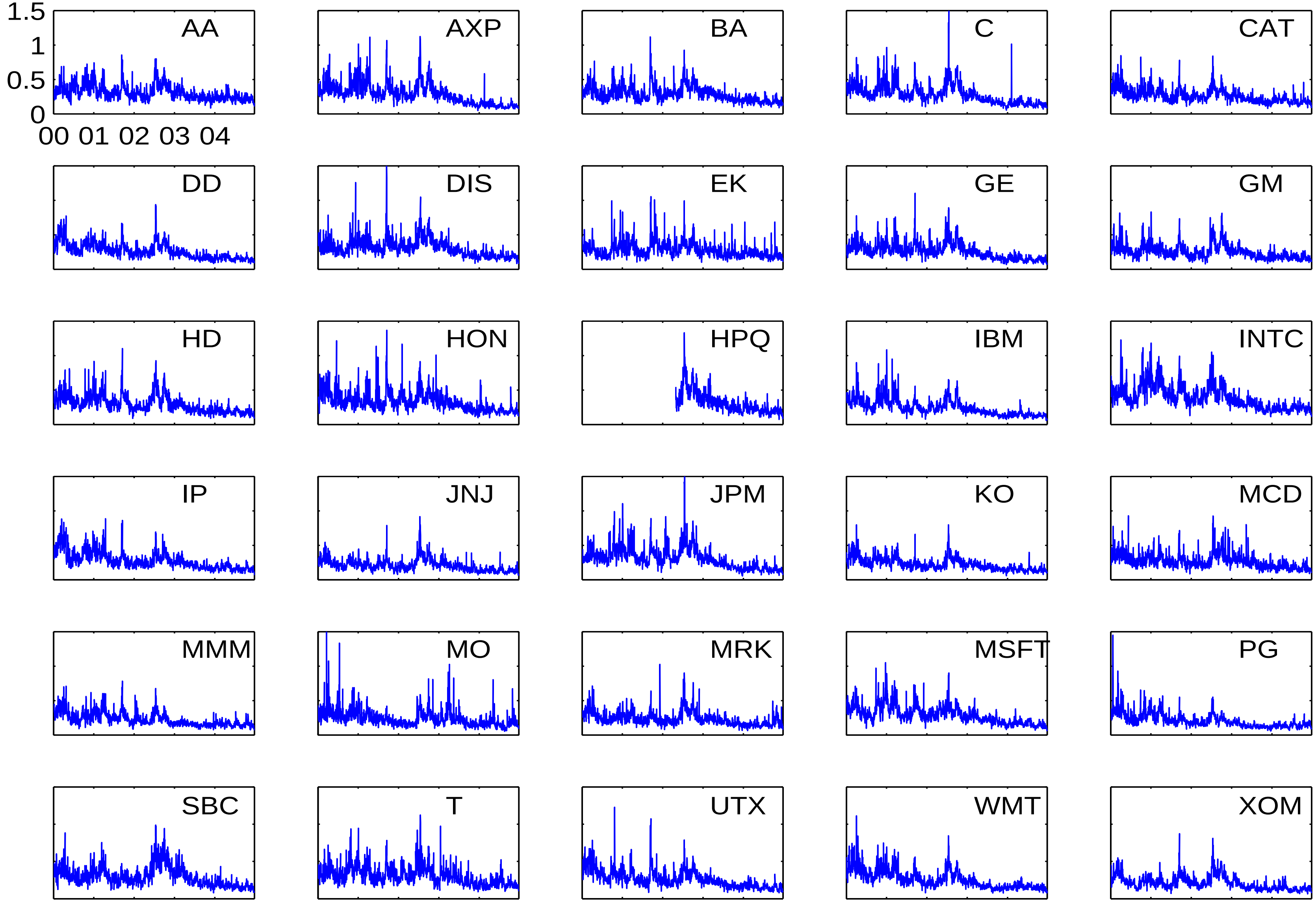

Fig. 1. Integrated volatility for each of the thirty DJIA, $IV_{i,t}^{1/2}$, on an annual base (252 days) estimated using TSRV with a slow time scale of five minutes. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations.

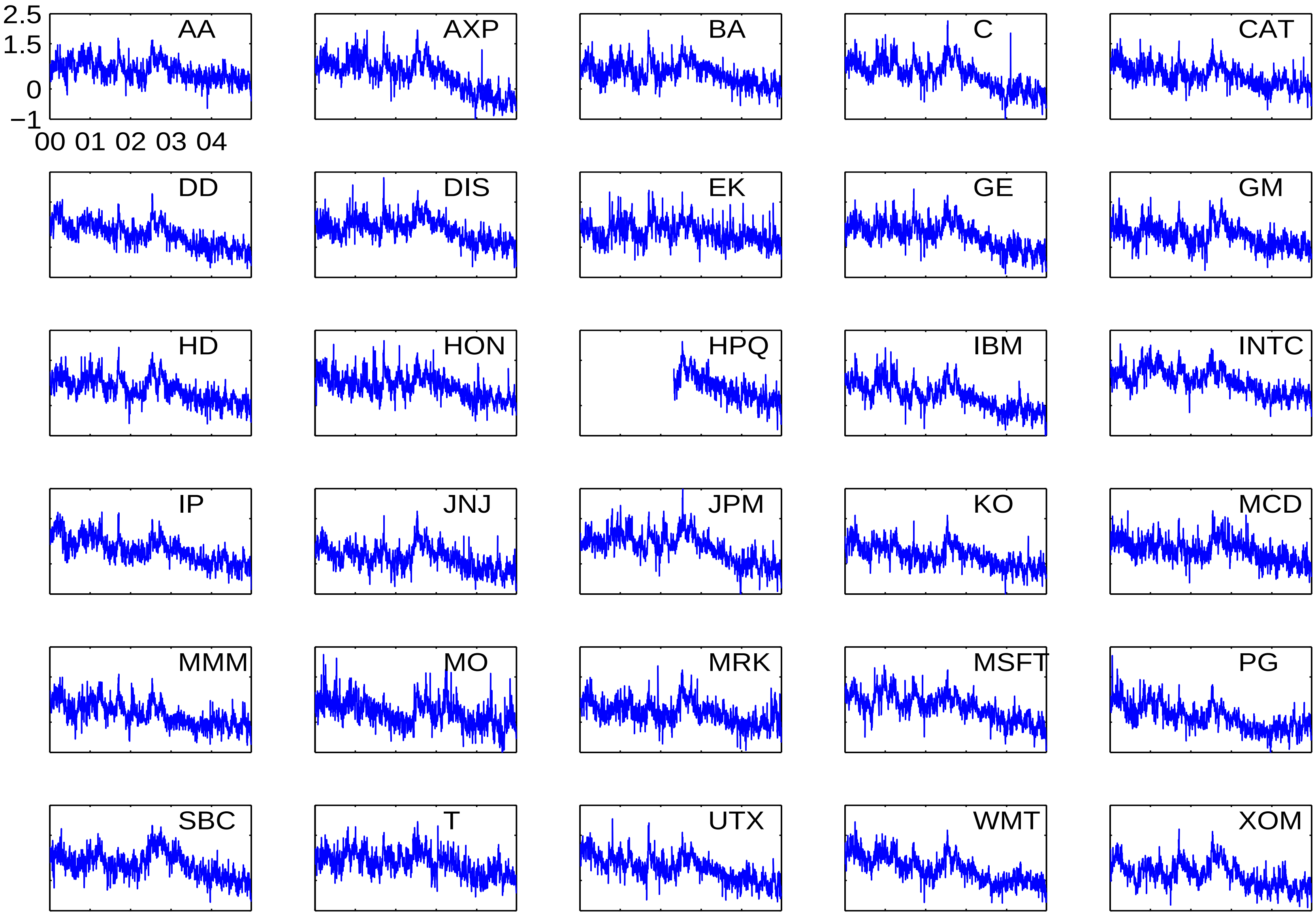

Fig. 2. Logarithmic integrated volatility for each of the thirty DJIA, $\log(\mathrm{IV}_{i,t}^{1/2})$, in percentage on a daily base estimated using TSRV with a slow time scale of five minutes. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations.

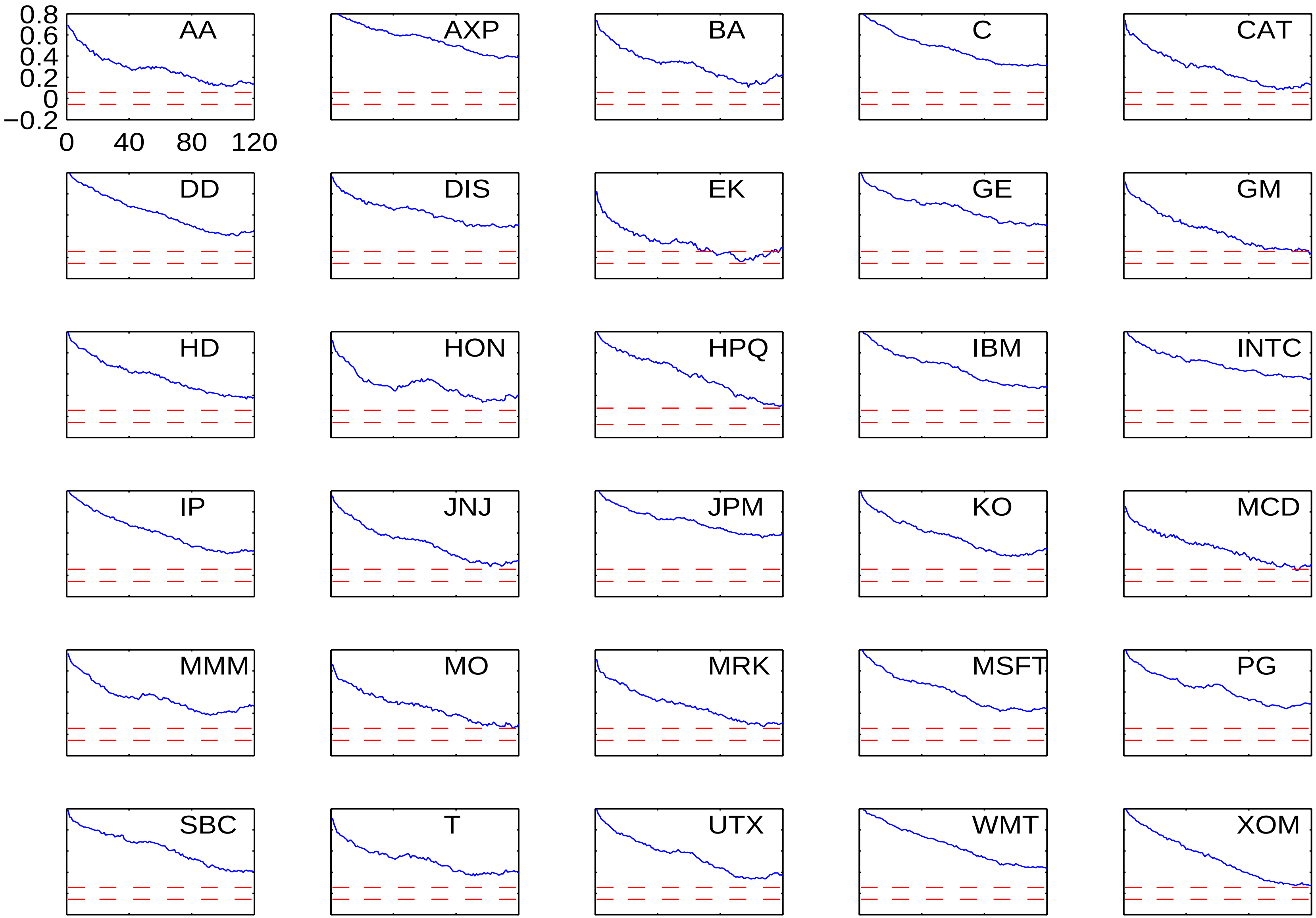

Fig. 3. Autocorrelation function of the logarithmic integrated volatility , $\log(\mathrm{IV}_{i,t}^{1/2})$, for each of the thirty DJIA estimated using TSRV with a slow time scale of five minutes. The sample extends from January 3, 2000 to December 31, 2004, for a total of 1,256 observations.

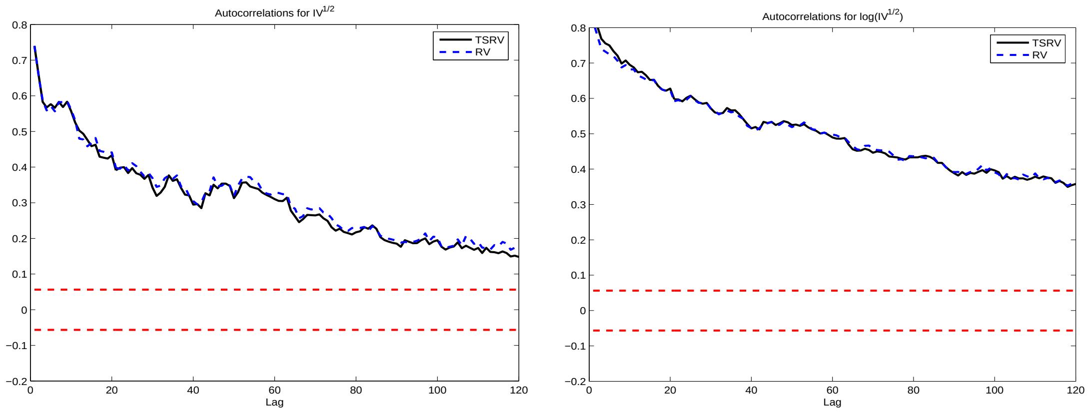

Fig. 4. Autocorrelations of $\mathrm{IV}^{1/2}$ (left plot) and $\log(\mathrm{IV}^{1/2})$ (right plot) given by TSRV and RV for the Intel stock (ticker INTC) based on the sample from January 3, 2000 to December 31, 2004. Lag is in trading days.

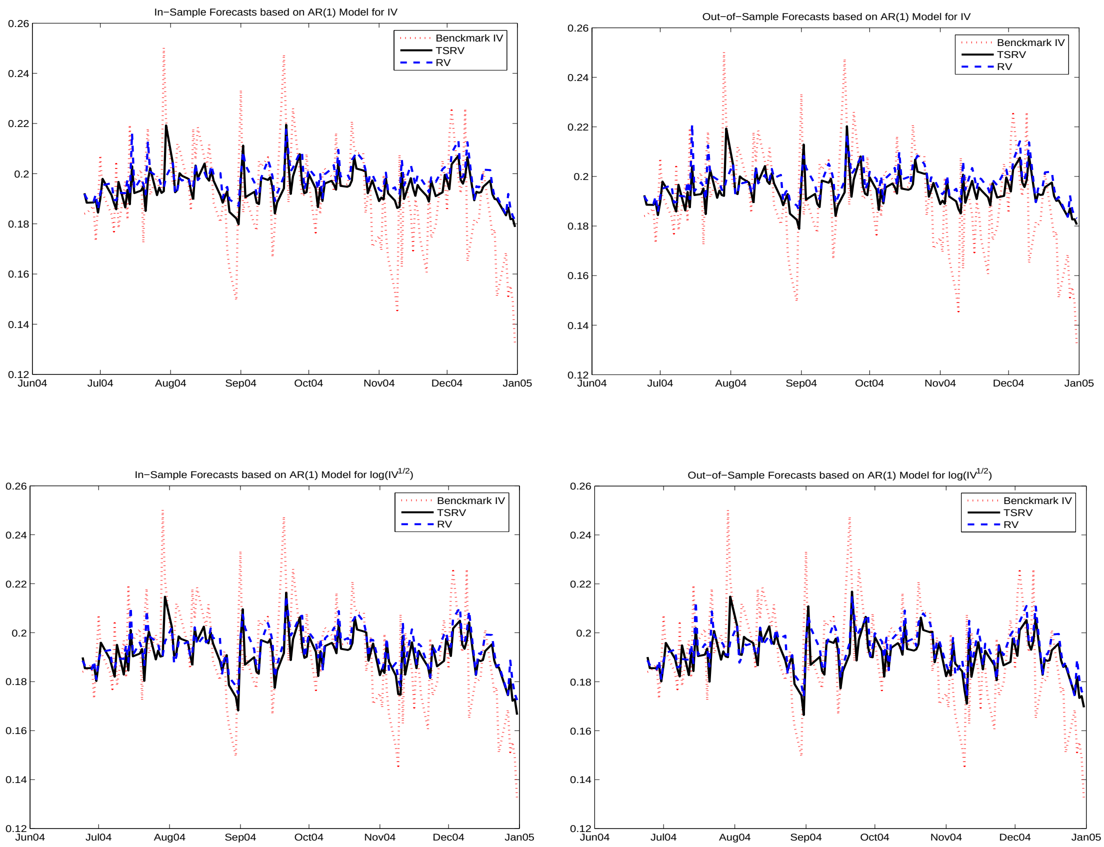

Fig. 5. Integrated volatility, $\mathrm{IV}^{1/2}$, forecasts for the Intel stock (ticker INTC) from June 24, 2004 to December 31, 2004, given by TSRV and RV. The $\mathrm{IV}^{1/2}$ is on annual base (252 days). The left plots show the in-sample forecasts based on the AR(1) model for IV (upper plot) and the AR(1) model for $\log(\mathrm{IV}^{1/2})$ (lower plot). The right plots show the out-of-sample forecasts based on the AR(1) model for IV (upper plot) and the AR(1) model for $\log(\mathrm{IV}^{1/2})$ (lower plot). AR models are estimated on a moving window of 100 trading days.