Understanding Mid-Latitude Jet Variability and Change using Rossby Wave Chromatography: Wave-Mean Flow Interaction

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ABSTRACT

Rossby Wave Chromatography (RWC) is implemented in a linearized barotropic model as a tool to understand the interaction between the mid-latitude jet and the eddy momentum fluxes (UV) in an idealized GCM. Given the background zonal-mean flow and the space-time structure of the baroclinic wave activity source, RWC calculates the space-time structure of the upper tropospheric UV. It is found that UV reinforces imposed zonal-mean zonal wind (U) anomalies that are collocated with the centers of action of EOF1 of the GCM. Critical level dynamics are essential for the positive feedback when U is equatorward of the mean jet and “reflecting level” dynamics are essential for the positive feedback when U is poleward of the jet. When the imposed U is out of phase with EOF1, the eddies tend to shift the imposed U poleward (equatorward) for anomalies that are equatorward (poleward) of the poleward center of action of EOF1. Critical (reflecting) level dynamics is most important for the poleward shift in the subtropics (mid-latitudes). There is no baroclinic feedback in these experiments. The changes in UV due to changes in wave source phase speed have a broad monopole structure that is nearly independent of the details of the imposed U. In the imposed U experiments the momentum flux anomalies are focused in two separate latitude bands: one in the subtropics and one in the mid-latitudes. When U and UV are coupled, however, U subtly adjusts so that the mid-latitude UV dominates over the subtropical UV.

1. Introduction

This is the final paper of a three part series on the use of Rossby Wave Chromatography (Held and Phillips (1987)) to understand the response of the mid-latitude jet to external forcing and, in this paper in particular, the internal variability of the mid-latitude jet (Lorenz (2013a); Lorenz (2013b)). By Rossby Wave Chromatography (RWC), we mean that we calculate the space-time (i.e. phase speed-latitude-wavenumber) structure of the upper tropospheric eddy momentum fluxes given the background zonal-mean flow and the space-time structure of the baroclinic wave activity source (convergence of vertical EP flux in the upper troposphere) (Held and Hoskins (1985); Held and Phillips (1987); Randel and Held (1991)). The dynamics of the eddies in our implementation of RWC are governed by the linearized barotropic vorticity equation on a sphere. Unlike other studies of the forced barotropic vorticity equation (Vallis et al. (2004); Barnes et al. (2010); Barnes and Hartmann (2011); Kidston and Vallis (2012)), which specify the vorticity forcing directly, we specify the baroclinic wave activity source and find the forcing that is consistent with the wave source.

In our implementation of RWC, the wave activity source and the background flow are decoupled, so that one can be changed without impacting the other. By separating the contributions of the wave activity source and the horizontal propagation of the waves, we can identify the dynamical mechanisms responsible for changing the latitude of the jet in response to external forcing and the dynamical mechanisms responsible for selecting the leading “modes” of the jet’s internal variability. In Lorenz (2013b), we explored the mechanisms responsible for maintaining poleward shifted jets in response to external forcing that acts to make the jet stronger. We found that changes in the Index Of Refraction (IOR) are responsible for maintaining the poleward shifted jet via changes in wave reflection (Kidston and Vallis (2012)). In the control climate there exists a selective “reflecting level” (or turning latitude) on poleward flank of jet: for a given wavenumber, low phase
speed waves are reflected but high phase speed waves are absorbed at the critical level on the poleward flank of jet (Fig 1a, the arrows represent the horizontal wave activity flux, $F_y = \bar{u}v' \cos(\phi)$, where $\phi$ is the latitude). When the zonal-mean zonal wind increases on the poleward flank of the jet, the peak of the reflecting level extends to higher phase speeds and a wider range of poleward propagating waves encounter a reflecting level instead of a critical level on the poleward flank (Fig 1b, to focus attention on the essential dynamics, only the reflecting level changes in the schematic) (Kidston and Vallis (2012)). The increased wave reflection leads to increased equatorward propagating waves (and therefore poleward momentum flux) across the jet. The equation governing the phase speed of the reflecting level is simply the Rossby wave dispersion relation with the meridional wavenumber set to zero:

$$c_\omega = \frac{\bar{u}}{a \cos \phi} - \frac{\beta a \cos \phi}{m^2},$$

(1)

where $c_\omega$ is the angular phase speed, $\bar{u}$ is the zonal-mean zonal wind, $a$ is the radius of the earth, $\beta$ is the absolute vorticity gradient ($= a^{-1} \partial_\phi (f + \zeta)$, where $f$ is the Coriolis parameter and $\zeta$ is the zonal-mean relative vorticity) and $m$ is the integer zonal wavenumber. The reflecting level is most important on poleward flank of the jet because it extends to the highest values of $c_\omega$ there. The maximum $c_\omega$ is on the poleward flank because 1) the planetary portion of $\beta$ goes to zero toward the pole and 2) the relative vorticity gradient is proportional to minus the second derivative of $\bar{u}$, which means that the circulation induced portion of $\beta$ is smallest on the jet flanks and largest in the jet core. Both these effects decrease $\beta$ on the poleward flank and by (1) imply that the maximum (in $c_\omega$) of the reflecting level is on the poleward flank. Equation (1) also implies that increases in $\bar{u}$ on the poleward flank lead to more reflection while increases in $\beta$ on the poleward flank lead to less reflection. RWC experiments in Lorenz (2013b) support this inference.

The presence of a selective reflecting level on the poleward flank of the jet also has important consequences for the response to changes in phase speed of the wave activity sources (Lorenz (2013b)). In the literature, ideas on the effect of phase speed changes on momentum fluxes emphasize the equatorward propagating waves and the critical level on the equatorward flank (Chen et al. (2007)). The relevant fluxes of wave activity in the control climate look like Fig. 2a. An increase in wave phase speeds means the spectrum shifts to the right leading to increased (decreased) equatorward propagating waves at high (low) phase speeds (Fig. 2b). In shifting to the right, the waves are still constrained to stop at their critical latitude, which they reach slightly sooner compared to the control case. Therefore, there is a net decrease in equatorward wave activity at all latitudes where there is a critical latitude for the waves. This leads to negative $\bar{u}$ forcing directly equatorward of the jet and positive forcing deeper in the subtropics. In the presence of a selective reflecting level, however, higher phase speeds also imply more wave absorption and less wave reflection on the poleward flank of the jet (Fig. 2c). The net result is a reduction in momentum fluxes across the jet in addition to the reduction on the equatorward flank. Hence increases in phase speed actually oppose the poleward shift (Lorenz (2013b)).
The changes in IOR and the increase in phase speeds impact wave reflection in opposite ways. In Lorenz (2013b), we found that the IOR changes are more important, leading to increased reflection and positive momentum fluxes across the jet. In order to understand the relative importance of \( \bar{u} \) and phase speed changes we need to better understand how the wave source phase speeds change relative to the reflecting level. Kidston et al. (2011) suggest that the horizontal scale of the waves increases as the jet shifts poleward. Increases in wave scale imply that wave phase speeds will decrease relative to \( \bar{u} \), which is in the correct direction to explain the dominance of IOR over phase speed. In Lorenz (2013a), however, we find that changes in wave scale are either very weak or essentially nonexistent among the GCM runs we considered, yet all runs showed a pronounced poleward shift of the jet. An alternative explanation for the dominance of IOR over phase speed changes is presented below.

In this paper, we will investigate the response of the RWC model to prescribed \( \bar{u} \) anomalies and the response when the RWC model is coupled to \( \bar{u} \). In addition to providing a better understanding of the response to external forcing, this analysis will enhance our understanding of the mechanisms that determine the “modes” of internal variability, given that a long-term positive feedback between \( \bar{u} \) anomalies and the eddy momentum fluxes select leading EOFs (Lorenz and Hartmann (2001); Lorenz and Hartmann (2003)). The proposed mechanisms for the positive feedback between \( \bar{u} \) and the eddies fall into two general categories: “baroclinic mechanisms” that depend on changes in the baroclinic wave activity source (Robinson (1996); Robinson (2000); Lorenz and Hartmann (2001); Chen and Plumb (2009); Zhang et al. (2012)) and “barotropic mechanisms” that depend on changes in meridional wave propagation independent of changes in the baroclinic wave source (Hartmann (1995); Hartmann and Zuercher (1998); Jin et al. (2006a); Jin et al. (2006b); Chen and Zurita-Gotor (2008); Barnes et al. (2010); Barnes and Hartmann (2011)). In addition to the positive feedback between EOF1 and the eddies, other feedbacks lead to the poleward and equatorward propagation of \( \bar{u} \) anomalies (James and Dodd (1996); Feldstein (1998); Lee et al. (2007); Sparrow et al. (2009)).

In this paper, we explore the response of the eddies to small \( \bar{u} \) perturbations about the Held and Suarez (1994) basic state. The effect of \( \bar{u} \) on the eddies is calculated using RWC and a simple model of the phase speeds of the wave activity source (see Lorenz (2013a)). We only consider wave-mean flow interaction for the case where the latitudinal profile of the baroclinic wave activity flux is constant because, as shown in Lorenz (2013a), barotropic processes appear to explain much of the structure and evolution of \( \bar{u} \) anomalies. Baroclinic feedbacks will be considered in future work. We begin this paper with a brief description of the GCM and the RWC model. In section 3, we explore the RWC response as a function of the latitude of imposed \( \bar{u} \) anomalies. In section 4, we isolate and understand the dynamical mechanisms involved in section 3. Next, we briefly discuss the RWC response as a function of the width of imposed \( \bar{u} \) anomalies (section 5). Finally, we discuss the modifications to the above picture when the...
eddy and \( \bar{u} \) are fully coupled (section 6).

2. GCM and RWC model

The dynamical core of the GCM is described in Lorenz (2013a) and all details regarding resolution and integration length can be found there. The standard idealized forcing given in Held and Suarez (1994) is used as the control run. The latitude of the climatological jet in the control run is 42\(^\circ\). The focus of this paper is on diagnosing and understanding the response of the RWC model to perturbations about a control \( \bar{u} \) and wave activity source spectrum from the GCM. For comparisons between the RWC model and the GCM see Lorenz (2013a).

Our implementation of RWC prescribes the covariance between the vorticity and the vorticity forcing for each wavenumber and phase speed under the assumption that the vorticity is related to the forcing by the linearized barotropic vorticity equation on a sphere (see Lorenz (2013a)). The background zonal wind and absolute vorticity field for the barotropic model are weighted vertical averages of the corresponding GCM fields (see Lorenz (2013a)). RWC calculates eddy momentum fluxes from the background \( \bar{u} \) and wave activity source spectrum. The wave activity source spectrum is defined as the convergence of the vertical EP flux (Edmon et al. (1980)) averaged from \( \sigma = 0.125 \) to \( \sigma = 0.525 \). The eddy momentum fluxes calculated from RWC are intended to simulate the GCM eddy momentum fluxes averaged over the same \( \sigma \)-level range. The RWC response is defined as the change in eddy momentum fluxes from the RWC simulation with perturbed \( \bar{u} \) and wave activity source spectrum minus the RWC simulation with control inputs. The perturbed wave activity source spectrum is taken from our simple phase speed model “forced” by the \( \bar{u} \) anomaly (see Lorenz (2013a)).

3. Response as a function of zonal wind latitude

In this section, we show results from a series of RWC experiments with imposed \( \bar{u} \) anomalies and we explore how the resulting \( \bar{u} \) anomalies feedback onto the imposed \( \bar{u} \) anomalies. In particular, we quantify the degree of positive feedback and/or poleward/equatorward propagation as a function of the latitude of the imposed \( \bar{u} \) anomaly.

In Fig. 3, we show the results of experiments with imposed \( \bar{u} \) anomalies of the form \( 2 \exp \left( -((\phi - \phi_0)/14^\circ)^2 \right) \) m s\(^{-1}\). We run multiple experiments with the latitude of the maximum \( \bar{u} \) anomaly, \( \phi_0 \), varying in increments of 2\(^\circ\) from the equator to the pole. The profile of the \( \bar{u} \) response is shown on the \( x \)-axis, \( \phi_0 \) is shown on the \( y \)-axis, and the gray line is the line \( y = x \). The total \( \bar{u} \) response tends to be concentrated in two main latitude bands centered at 22\(^\circ\) and 42\(^\circ\) (Fig. 3a). When \( \phi_0 \) is equatorward (poleward) of 38\(^\circ\), the 22\(^\circ\) anomalies are positive (negative) and the 42\(^\circ\) anomalies are negative (positive). The climatological jet is at 42\(^\circ\).

In Fig. 3b and 3c, we separate the effects of IOR changes (i.e. change background flow with no change in wave source phase speeds) and the effects of phase speed changes on the \( \bar{u} \) response. While the total \( \bar{u} \) response is somewhat symmetric about \( \phi_0 = 38^\circ \), the IOR \( \bar{u} \) response is instead dominated by positive anomalies in two separate latitude bands. The responses in these two bands do not occur together, instead the subtropical \( \bar{u} \) are “excited” when \( \bar{u} \) is equatorward (approximately) equatorward of the mean jet (\( = 42^\circ \)) and the mid-latitude \( \bar{u} \) are “excited” when \( \bar{u} \) is poleward of the mean jet. The phase speed induced \( \bar{u} \) on the other hand, are broad and negative regardless of the location the imposed \( \bar{u} \) anomalies. Therefore, the total \( \bar{u} \) response to a positive \( \bar{u} \) anomaly is the sum of a localized and positive IOR response and a broad and negative phase speed response (linear superposition nearly holds). Note that the broad phase speed response is not an artifact of the smoothing in the simple model of phase speed changes, because the response is also broad and negative when the smoothing is turned off (Fig. 3f). We offer an explanation for the broad phase speed response to local \( \bar{u} \) changes in section 4d.

When we impose the \( \bar{u} \) and \( \beta \) effects on the IOR separately (Fig. 3d), we see that the \( \bar{u} \) changes in the subtropics are almost entirely due to \( \bar{u} \) while the changes poleward of 30\(^\circ\) include contributions from both \( \bar{u} \) and \( \beta \). \( \beta \) has little effect on the \( \bar{u} \) in the subtropics because, as we show below, the movement of the critical level on the equatorward flank of the jet explains most of these \( \bar{u} \) anomalies. Poleward of 30\(^\circ\), we will see that the dynamics tend to be dominated by changes in the reflecting level as described in the Introduction. The effect of \( \beta \) is at a smaller “wave length” in \( \phi_0 \) compared to \( \bar{u} \) because the \( \beta \) anomalies are proportional to minus the second derivative of \( \bar{u} \). When \( \bar{u} \) is in phase with the latitude of the peak in the reflecting level of the dominant zonal wavenumbers (\( \approx 52^\circ \), see Lorenz (2013b)), \( \bar{u} \) and \( \beta \) are both positive at 52\(^\circ\) and therefore \( \bar{u} \) increases wave reflection while \( \beta \) decreases wave reflection (see equation (1)). As a result, when \( \phi_0 = 52^\circ \), the \( \bar{u} \) anomalies are positive for \( \bar{u} \) (Fig. 3d) and negative for \( \beta \) (Fig. 3e). As the imposed
The RWC eddy momentum flux response ($x$-axis) to imposed $\bar{u}$ anomalies as a function of the latitude of the maximum in the imposed $\bar{u}$ ($y$-axis). The gray line is the line $y = x$. The units are $m^2 s^{-2}$. 
a) Total response. b) Response from changes in IOR. c) Response from changes in the phase speed of the wave activity source. d) Response from the effect of $\bar{u}$ alone on IOR. e) Response from the effect of $\beta$ alone on IOR. f) As in c) but for no smoothing of the background flow changes in the phase speed model.

$\bar{u}$ monopole shifts away from 52°, the value of $\beta$ at 52° switches sign due to the change in curvature on the flanks of the $\bar{u}$ anomaly. This explains the positive $u'v'$ anomalies in Fig. 3e when $\phi_0 = 32^\circ$ and 72°. More quantitative diagnostics of the effect of the reflecting level are included in the next section.

To summarize the effect of the eddies back onto the imposed $\bar{u}$, we project the eddy momentum forcing ($= \zeta'v'$) onto the normalized $\bar{u}$ anomaly for each $\phi_0$ (Fig. 4a). This quantifies the magnitude of the positive eddy feedback onto $\bar{u}$. The total eddy forcing is a positive feedback onto the wind anomalies when $\bar{u}$ is located at 28° or 52°. These latitudes are close to the centers of action of EOF1 (32° and 52°). Part of the discrepancy in the location of the equatorward center of action can be resolved if we force the RWC model with narrower $\bar{u}$ more representative of the internal variability. As implied by Fig. 3, the structure of the positive feedback is derived from the IOR induced eddy fluxes (Fig. 4a). The phase speed induced changes, on the other hand, change relatively little with the latitude of $\bar{u}$. This insensitivity to $\bar{u}$ is not due to the smoothing present in the phase speed model because the structure of the phase speed feedback hardly changes when the smoothing is eliminated.

In Fig. 4b, we summarize the results of a similar series of experiments but with an imposed $\bar{u}$ dipole of the
Figure 4: a) Projection of the RWC eddy vorticity flux response back onto the imposed Gaussian $\bar{u}$ anomaly as a function of the latitude of the maximum in the imposed $\bar{u}$ (x-axis). The vorticity flux projections are separated into components: the total (thin solid), IOR induced (thick dotted), the phase speed induced (thick dash-dotted) and the phase speed induced with no smoothing in the phase speed model (thin dash-dotted). The units are m s$^{-1}$ day$^{-1}$. b) As in a) but for a $\bar{u}$ dipole. The x-axis is the latitude of the nodal line of the imposed dipole. c) The thin solid and thick dotted lines are the same as in a). Also shown are the projections on the poleward propagation pattern ($\approx -d\bar{u}/d\phi$) for the total (thick dash-dotted) and IOR induced (thin dash-dotted) eddy vorticity flux. d) As in c) but for a $\bar{u}$ dipole. The x-axis is the latitude of the nodal line of the imposed dipole.

Considerably more attention has been given to the positive feedback between EOF1 and the eddies than another form of coupling between $\bar{u}$ and $u'v'$: the poleward (and equatorward) propagation of $\bar{u}$ anomalies (James and Dodd (1996); Feldstein (1998); Lee et al. (2007); Sparrow et al. (2009)). Therefore, in Fig. 4cd, we compare the projection of $\zeta'v'$ on the normalized $\bar{u}$ itself with the projection of $\zeta'v'$ on the normalized $-d\bar{u}/d\phi$. For infinitesimal poleward shifts, $-d\bar{u}/d\phi$ describes the structure of the resulting anomalies and therefore the projection on $-d\bar{u}/d\phi$ quantifies the degree of poleward propagation (in the Northern Hemisphere). When the imposed $\bar{u}$ monopoles are equatorward (poleward) of 52$^\circ$ the eddies act to shift the anomalies poleward (equatorward) (Fig. 4c). Hence, the latitude 52$^\circ$ is a center of attraction that anomalies at other latitudes tend to propagate toward. The latitude 52$^\circ$ is also the location of the poleward center of action of EOF1 and the peak of the reflecting level for the dominant waves (see Lorenz (2013b)). Note there is a relative minima in the degree of poleward propagation near the location of the equatorward center of action of EOF1 (32$^\circ$). The dependence of poleward propagation on the latitude of the nodal line for $\bar{u}$ dipoles is also consistent with the propagation of arbitrary $\bar{u}$ dipoles toward EOF1 (Fig. 4d).

Because it is generally believed that a positive feedback between the leading EOF and eddies is responsible for the selection of the leading EOF (Lorenz and Hart-
mann (2001); Lorenz and Hartmann (2003)), the results in this section suggest that the RWC model can be used to understand the structure of the internal variability given that RWC model implies a positive feedback for anomalies in phase with EOF1 and no feedback (on $\bar{u}$ magnitude) for anomalies out of phase. We have already alluded to the key role of the latitude of the peak of the reflecting level on determining where $\bar{u}$ anomalies are stationary. We will discuss this further below after we come to a better understanding of the role of the subtropical critical level on the response to $\bar{u}$ anomalies equatorward of the mean jet. Also, note that the magnitude of the wave activity source is fixed in these experiments, so in other words, there is no baroclinic feedback.

4. Dynamical Mechanisms

a. Critical level dynamics

In this section, we develop a mechanistic model to diagnose and separate the portion of the IOR $\bar{u}\bar{v}'$ response that is due to critical level dynamics on the equatorward flank of the jet. The key predictor of the $\bar{u}\bar{v}'$ changes in our model is the latitude of the critical level as a function of phase speed, $\phi_{CL}(c)$. Changes in $\phi_{CL}(c)$ determine the amount of expansion/contraction of the equatorward flank of the $\bar{u}\bar{v}'$ profile. Specifically, let $\xi(\phi, c, m) \equiv \bar{u}\bar{v}' \cos(\phi)$ (we multiply the momentum flux by $\cos(\phi)$ so that $\xi$ is proportional to the meridional wave activity flux) and $\Delta \phi_{CL}$ be the change in the critical level, then the change in $\xi$ in the Northern Hemisphere is approximated by

$$\Delta \xi(\phi, c, m) = \begin{cases} -\max \left( \frac{\partial \xi}{\partial \phi}, 0 \right) \Delta \phi_{CL} & \text{if } \frac{\partial \xi}{\partial \phi} > 0 \text{ at } \phi_{CL} \\ 0 & \text{otherwise}, \end{cases}$$

(2)

where we use the fact that the structure of an infinitesimal shift of a function is proportional to minus the first derivative of the function and we truncate the shift profile with the maximum function so that $\Delta \xi$ represents a "one-sided shift" (i.e. expansion/contraction) of only the equatorward flank, which is where the critical level induced dissipation is occurring. Note we only assign non-zero values to $\Delta \xi$ when the slope of the mean $\xi$ is consistent with wave dissipation at the critical level. Also, though not documented in (2), $\Delta \xi$ is allowed to be non-zero only in the region of continuously positive $\partial \xi/\partial \phi$ in the vicinity of $\phi_{CL}$: if a region of positive $\partial \xi/\partial \phi$ is separated from $\phi_{CL}$ by a region of negative $\partial \xi/\partial \phi$, then $\Delta \xi$ is always zero in that region. We also find that the critical level model is more accurate when $\Delta \phi_{CL}$ is smoothed in phase before applying (2). For the results below, we smooth with the kernel: $\exp \left( \frac{(c/w)^2}{2} \right)$, where $w = 6.43$ m s$^{-1}$. A generalization of (2) to the Southern Hemisphere is straightforward.

To test the critical level diagnostic, we explore the imposed Gaussian profile experiments described in section 3 for the case where $\phi_0 = 26^\circ$. A phase speed/latitude spectrum (Randel and Held (1991)) of the change in $\bar{u}\bar{v}'$ from IOR is shown in Fig. 5. The response is dominated by increased $\bar{u}\bar{v}'$ on the subtropical flank of the jet (Fig. 5a). The critical level model successfully captures the general response (Fig. 5b), which suggests that the response in the RWC model is dominated by an equatorward shift of the critical level in response to increased $\bar{u}$ on the subtropi-
cal flank of the jet. We find that there is some variability in the sensitivity of $\xi$ to $\Delta \phi_{CL}$ depending on the zonal wavenumber and phase speed but that the predicted latitudinal structure is more reliable. Therefore, before we use the critical level model to diagnose the RWC experiments, we scale the prediction in (2) by the linear regression slope separately for each $m$ and $c$. The scaled critical level model improves some of the biases in the raw model (Fig. 5). One feature that is not explained by the critical level model is the weak reduction in $u'v'$ immediately poleward of the main center of $u'v'$ increases. We believe this is a result of the equatorward movement of the reflecting level at this location, which is synonymous with a contraction of the wave propagation region at this location. We also should point out that the reason the response to this $\bar{u}$ perturbation seems so simple is that we have isolated the effect of IOR changes. In the GCM, a similar $\bar{u}$ perturbation would also change the phase speed of the waves, resulting in a significantly more complicated pattern in the $u'v'$ response.

b. Diagnosing the imposed zonal wind experiments

Here we use the scaled critical level model described above and the reflection diagnostic described in Lorenz (2013b) to diagnose the imposed $\bar{u}$ experiments in section 3. The diagnostics are applied to the $u'v'$ response from IOR (Fig. 3b). First, we remove the $u'v'$ portion related to the critical level on the equatorward flank. Next, we isolate the $u'v'$ portion related to reflection on the poleward flank of the jet. Movements in the critical level dominate the $u'v'$ response in the subtropics (Fig. 6a). In the mid-latitudes, the reflection diagnostic suggests that the reflectivity of the poleward flank accounts for about 70% of the $u'v'$ response in the mid-latitudes (Fig. 6b). The residual is largest in the extratropics but is nevertheless less important than the other two terms (Fig. 6c).

Note that the location of the $u'v'$ anomalies due to reflection are nearly stationary in latitude despite the large variation in the latitude of the imposed $\bar{u}$. This is because no matter why the peak of the reflecting level changes, the momentum flux convergence/divergence associated with it is always concentrated on the poleward flank of the jet. In the schematic (Fig. 1), where all wave dissipation is focused at the critical level, the momentum flux convergence/divergence is displaced equatorward and is approximately co-located with the peak in the reflecting level in this case (see Lorenz (2013b)). For the dominant zonal wavenumbers (4 - 8), the peak of the reflecting level varies from only 56$^\circ$ to 48$^\circ$, therefore, the momentum flux convergence/divergence associated with changes in the peak of the reflecting level are always focused at about 52$^\circ$. Anomalies at 52$^\circ$ are thus associated with a positive feedback. Also, any $\bar{u}$ anomaly that excites the peak of the reflecting level will tend to propagate toward 52$^\circ$. The above ideas assume that the meridional scale of the anomaly is large enough that $\bar{u}$ dominates over $\beta$ in (1).
The reflecting level on the poleward flank of the anomaly bation is due to the effect of the increases in zonal wind on about 52° activity leads to momentum flux convergence/divergence at the anomaly poleward because any change in the reflectivity of the poleward flank acts to shift other hand, reflection on the poleward flank acts to shift the subtropical critical level is responsible for the positive feedback at the EOF1 center of action equatorward of the mean jet. Projecting the responses on the poleward propagation pattern (= −δu/δψ).

To quantify the feedbacks from critical and reflecting levels, we project the ζ’v’ anomalies on the imposed u. The subtropical critical level is responsible for the positive feedback at the EOF1 center of action equatorward of the mean jet and for the weak negative feedback deep in the subtropics (Fig. 7a). The reflecting level, on the other hand, is responsible for much of the positive feedback at the EOF1 center of action poleward of the mean jet. Projecting the responses on −δu/δψ (Fig. 7b), we see that the subtropical critical level is responsible for much of the poleward propagating anomalies in the subtropics as suggested by James and Dodd (1996) and Lee et al. (2007).

When u is in phase with the time-mean jet (= 42°), on the other hand, reflection on the poleward flank acts to shift the anomaly poleward because any change in the reflectivity leads to momentum flux convergence/divergence at about 52°. The poleward shift for this in-phase u perturbation is due to the effect of the increases in zonal wind on the reflecting level on the poleward flank of the anomaly (1). The effect of β is nearly zero because the perturbation is not narrow enough for the negative β on the jet flanks to be in phase with the reflecting level maximum (≈ 52°) and thus does not act to increase the range of phase speeds encountering a reflecting level. We will consider effect of u scale in section 5.

\[ \delta \phi_{CL} = -\frac{\delta \bar{u}}{\Lambda}, \]  

(3) where Λ is the mean shear, δu/dψ, at the critical level. As before, let ξ(φ, c, m) be the spectral density of u*v multiplied by cos(ψ). The perturbation has width, δw, so the range of phase speeds affected by the perturbation is

\[ \delta c = \Lambda \delta w. \]  

(4)

Therefore, after substituting the expressions for δφ and δc, the perturbation momentum flux (over phase speed range, δc) from (2) is:

\[ \Delta \xi(\phi, c, m) \delta c = \begin{cases} \max \left( \frac{\partial \xi}{\partial \phi}, 0 \right) \delta \bar{u} \delta w & \text{if } \frac{\partial \xi}{\partial \phi} > 0 \text{ at } \phi_{CL} \\ 0 & \text{otherwise.} \end{cases} \]  

(5)

Note that the mean shear has dropped out of the expression because the effect of shear on the movement of the critical level is opposite the effect of shear on the range of phase speeds encountering an altered critical level. If δw is the width of the grid spacing, then the total momentum flux change can be found by summing (5) over all c and m for each grid point while also noting that the correct δu is the δ\bar{u} at the critical level for waves with phase speed c. If the scale of the anomalies is large enough, however, the error from using the local δ\bar{u} instead of the δ\bar{u} at the critical level is small and one can simply integrate \[ \max \left( \frac{\partial \xi}{\partial \phi}, 0 \right) \] over c and m beforehand, and then multiply by δ\bar{u} and δw. In this limit, the momentum flux response to small \u anomalies is simply δ\bar{u} times a climatological profile of “wave dissipation”, D(φ):

\[ \delta u*v(\phi) = \frac{D(\phi) \delta \bar{u}(\phi)}{\cos(\phi)}, \]  

(6)
where
\[ D(\phi) = \sum_{c,m} \max \left( \frac{\partial \zeta}{\partial \phi}, 0 \right) \delta m \delta w, \] (7)
and \( \delta w \) is the grid spacing. Note, there is no explicit \( \delta c \) in the sum in (7) because of (4). The climatological wave dissipation in the control run is centered at about 24° (Fig. 8a). We apply the simple critical level model given by (6) to the imposed \( \bar{u} \) experiments in section 3. The model correctly predicts the transitions from negative feedback to poleward propagation to positive feedback as one moves from the equator to the pole, although the amplitude of the poleward propagation is too large (Fig. 8bc). When the \( \bar{u} \) anomaly is in phase with \( D \), then \( \bar{u}'v' \) is also in phase with \( \bar{u} \) and therefore the momentum flux convergence acts to shift the anomaly poleward. Critical level dynamics only begins to project positively on the \( \bar{u} \) anomaly when the bulk of the climatological wave dissipation lies equatorward of the anomaly. In this case, the convergence of momentum flux that feedbacks positively on the anomaly is due to the convergence of the climatological \( D \) as represented by the first term in the differentiation of (6) via the product rule:
\[ -\frac{du'v'}{d\phi} = -\frac{dD}{d\phi} \bar{u} + \left( -\frac{d\delta \bar{u}}{d\phi} \right) D, \] (8)
where for simplicity we neglect the \( \cos(\phi) \) factors. This first term is positive over the entire anomaly if the anomaly is entirely in the region of decreasing \( D \). Likewise the second term represents different degrees of poleward shift depending on the amplitude of \( D \) (the small equatorward shifts far on the flanks of \( D \) come from the first term).

An analysis of the projection of \( \zeta'v' \) on \( \bar{u} \) hides the fact that \( \bar{u} \) evolution under critical level dynamics follows a simple conservation equation with transport velocity \( D \):
\[ \frac{\partial \bar{u}}{\partial t} = -\frac{\partial D \bar{u}}{\partial \phi} - \frac{\bar{u}}{\tau}, \] (9)
where for simplicity we neglect the \( \cos(\phi) \) factors and we have added Rayleigh damping (see equation (17) in Lorenz (2013a)). Therefore, critical level dynamics is always acting to shift the integrated \( \bar{u} \) toward the wave activity source, which is poleward in this case. In addition, when there is convergence (divergence) in \( D \), critical level dynamics causes the anomaly to decrease (increase) in meridional scale. Note that the approximation above assumes that a local \( \bar{u} \) change causes a local \( u'v' \) change. If the conditions of this approximation do not hold then the largest \( u'v' \) response to a local \( \bar{u} \) is slightly poleward of the \( \bar{u} \) anomaly and the poleward propagating effect of critical level dynamics is even stronger.

d. Response to changes in phase speed

In this section we offer an explanation for the broad structure of the \( u'v' \) response to changes in wave activity source phase speed that is also relatively insensitive of the details of the imposed \( \bar{u} \) anomalies.

Diagnostics in Lorenz (2013a) suggest that advection rather than \( \beta \) dominates the phase speed changes of the wave activity source. Hence the response to an imposed \( \bar{u} \) anomaly is non-local in phase speed: both high and low phase speed waves change their phase speeds by about the same amount. Consider the response to an increase in \( \bar{u} \).
If all waves increase phase speeds the same amount, then, after a Galilean transform, the response is like that of a decrease in winds everywhere while keeping phase constant. This implies both critical and reflecting level dynamics come into play but in the opposite direction from the effect of $\bar{u}$ increases on IOR. The combination of the subtropical critical level $\bar{u}' v'$ and the mid-latitude reflecting level $\bar{u}' v'$ gives rise to the the broad phase speed response seen above. The results in Fig. 3 and 4 and the response to poleward shifted jets (Lorenz (2013b)) suggest that the broad response of $\bar{u}' v'$ to changes in phase speed appears even when $\bar{u}$ is localized in latitude. We believe the response is non-local because the wave activity phase speed spectrum is broad in addition to the previously mentioned non-localness of the phase speed changes. Therefore a local $\bar{u}$ anomaly at $\phi_0$ leads to $\bar{u}' v'$ anomalies 1) across all latitudes with subtropical critical levels for the waves with sources at $\phi_0$ and 2) across the jet axis provided the distribution of phase speeds for the waves with sources at $\phi_0$ intersects the peak in the reflecting level.

If the effect of phase speed on $\bar{u}' v'$ is opposite the effect of IOR, why is the IOR effect larger in the experiments shown above and in the response to external forcing in Lorenz (2013b)? The dominance of IOR depends on the scale of the imposed $\bar{u}$. For example, when we impose a broad constant angular velocity perturbation in the RWC model ($\bar{u} = 2 \cos(\phi)$ m s$^{-1}$) the IOR and phase speed effects nearly cancel (not shown). When the $\bar{u}$ scale is smaller than the scale of the wave activity source region, however, the IOR response is larger. This effect is easiest to understand when considering the reflecting level. For sake of discussion, we will ignore the effect of $\beta$ anomalies. Therefore, to increase the peak of reflecting level by 1 m s$^{-1}$, we only need to increase $\bar{u}$ by 1 m s$^{-1}$ at the location of the peak in the reflecting level. This local change in the reflecting level affects all waves propagating toward the poleward flank of the jet. To increase the phase speed of all waves propagating poleward toward the reflecting or critical level by 1 m s$^{-1}$, on the other hand, we need to increase $\bar{u}$ across all source latitudes by 1 m s$^{-1}$. Therefore, a well placed local $\bar{u}$ anomaly will have a much bigger impact on the waves through the IOR than through the phase speeds of the wave sources. For local $\bar{u}$ anomalies on the subtropical flank of the jet, the local critical level moves according to the full local $\bar{u}$ while the net phase speed change of the waves impinging on the critical level changes by a significantly smaller amount because some of these waves have source latitudes far removed from the local $\bar{u}$ anomaly. In summary, IOR dominates because the net phase speed change is an integrator of $\bar{u}$ across the entire width of the wave source region while IOR can respond strongly to well-positioned local $\bar{u}$ anomalies.

The above discussion suggests that phase speed effects are most important for $\bar{u}$ anomalies at the latitude of the mean jet. Indeed the effect of phase speed cancels the positive IOR feedback when the imposed $\bar{u}$ is at 42° (Fig. 4a). However, this is also a region where the IOR leads to strong poleward propagation via changes in the reflecting level on the poleward flank (Fig. 4c). If, on the other hand, the $\bar{u}$ anomaly is narrow enough to not disturb peak of reflecting level then it is also too narrow to be maintained: the eddies will immediately act to broaden anomaly and will eventually shift the anomaly poleward via reflecting level dynamics (see next section).

In Lorenz (2013a), we noted that changes in wave scale (Kidston et al. (2011)) and/or $\beta$ can potentially change the phase speed of the wave activity sources relative to $\bar{u}$. While the wave scale changes are weak, the changes in $\beta$ act to reduce the phase speed of the waves in the jet core. The sense of this change is to decrease the effect of wave phase speeds on $\bar{u}' v'$ relative to IOR. To quantify this effect we redo the reduced friction RWC coupled experiments in section 6 of Lorenz (2013a) but with only advection affecting the wave source phase speeds. In this experiment there is essentially no change in the magnitude of the response. This suggests that for the type of $\bar{u}$ perturbations considered here, the effect of $\beta$ and wave scale on $\bar{u}' v'$ via changes in the relative phase speed are negligible compared to the integrating mechanism discussed above.

5. Response as a function of zonal wind width

In this section we briefly discuss the response of the RWC model to $\bar{u}$ anomalies of different meridional scales that are in phase with the time mean jet. In section 4, we noted that for our particular (positive) $\bar{u}$ anomalies, it is $\bar{u}$ rather than $\beta$ that causes the peak of reflecting level to increase when $\bar{u}$ is in phase with the mean jet. In this case, the negative $\beta$ anomaly (associated with the positive curvature of $\bar{u}$) is too far poleward relative to the peak in the reflecting level for the dominant zonal wavenumbers. This implies, however, that smaller scale anomalies will “excite” the reflecting level via changes in $\beta$. While this is indeed the case, the eddy feedback for these small scale anomalies is much better described as a negative feedback acting to broaden the anomaly (not shown). In other
words, while there are \( \bar{u}' \bar{v}' \) poleward of the anomaly acting to shift the anomaly poleward there are also similar magnitude \( \bar{u}' \bar{v}' \) equatorward of the anomaly acting to shift the anomaly equatorward. The fluxes equatorward of the anomaly appear to be the result of the development of a secondary reflecting level peak (at much lower phase speeds) in the subtropics that is a result of the strong and small-scale \( \beta \) anomalies associated with a narrow \( \bar{u} \) anomaly. This secondary reflecting level maximum appears to block the meridional propagation of wave activity on equatorward flank of the anomaly (not shown). While the initial eddy fluxes and \( \bar{u} \) tendency are strongly dependent on the scale of the imposed anomaly, the long-term response when \( \bar{u} \) and \( \bar{u}' \bar{v}' \) are coupled is not. For example, coupled experiments with different scale \( \bar{u} \) forcing but with the same latitudinally integrated \( \bar{u} \) force (i.e. forcing scale and amplitude are inversely related) give nearly the same final equilibrated \( \bar{u} \) response (not shown). The details are the subject of future research.

6. Coupled System

Above we found that when imposed \( \bar{u} \) anomalies are out of phase with EOF1, the eddies act to shift the anomalies either poleward or equatorward toward the EOF1 structure (Fig. 4cd). This suggests that the long-term response of the coupled RWC system will project on EOF1 regardless of the structure of the forcing. We find this is indeed the case. In Fig. 9, we show the equilibrium \( \bar{u} \) and \( \bar{u}' \bar{v}' \) profile as a function of the latitude, \( \phi_0 \), of imposed mechanical forcing with a Gaussian profile.\(^1\) As in Fig. 3a, \( \phi_0 = 38^\circ \), which is between the equatorward center of action of EOF1 and the time mean jet, is a key latitude separating the type of response. In this case, forcing equatorward (poleward) of 38\(^\circ\) leads to the negative (positive) phase of EOF1. These results are likely relevant for the response of the jet to El Niño versus global warming: in contrast to global warming which acts to strengthen the jet, El Niño is associated with increased temperature gradients deeper in the subtropics. The associated direct \( \bar{u} \) response is therefore equatorward of the time mean jet and is therefore associated with an equatorward shift of the jet (Sun et al. (2013)).

The \( \bar{u}' \bar{v}' \) response is interesting in that the mid-latitude \( \bar{u}' \bar{v}' \) dominates over the subtropical \( \bar{u}' \bar{v}' \) even in the case of forcing deep in the tropics (Fig. 9b). In the tropical forcing case, the \( \bar{u}' \bar{v}' \) decreases in the mid-latitudes are related to the changes in the reflecting level caused by the \( \bar{u} \) decreases at poleward center of action of EOF1. How is the reflecting level altered in the first place? Based on Fig. 3, the adjustment appears to proceed as follows: 1) critical level dynamics act to shift the imposed \( \bar{u} \) poleward resulting in positive anomalies at the equatorward center of action of EOF1. 2) At the same time, these anomalies increase the phase speeds of the waves, leading to decreased reflection and negative \( \bar{u} \) on the poleward flank of the jet. 3) The negative \( \bar{u} \) then excites the positive feedback between \( \bar{u} \) and the reflecting level on the poleward flank. This mechanism is consistent with the equilibrium response in a new series of experiments where the source phase speeds remain constant (9cd). When the latitude of the forcing, \( \phi_0 \), is less than 32\(^\circ\), the \( \bar{u} \) and \( \bar{u}' \bar{v}' \) responses are mostly restricted equatorward of the time mean jet (42\(^\circ\)). Also note that the \( \bar{u} \) response is smaller for the case where phase speed is fixed. This is consistent with Fig. 4a, which shows that phase speed changes are associated with a positive feedback for anomalies equatorward of 25\(^\circ\) (such anomalies happen to be in phase with the fixed location of subtropical drag associated with any increase in phase speed). When the latitude of the forcing is in the extratropics, on the other hand, the fixed phase speed experiments show a much stronger response because the negative phase speed feedback is no longer present. The general structure of \( \bar{u}' \bar{v}' \), however, is much the same as the experiment with changing phase speeds.

In the extratropical forcing case, the dominance of the mid-latitude \( \bar{u}' \bar{v}' \) in the long-term response (Fig. 9b) appears to be related to subtle wind changes present in the subtropics. This case is similar to the RWC response to imposed dipolar \( \bar{u} \) anomalies. For example, the RWC model response to EOF1 anomalies is initially a \( \bar{u}' \bar{v}' \) dipole that soon adjusts to a \( \bar{u}' \bar{v}' \) monopole (Fig. 10a). This monopole structure is like the monopole structure in lagged regressions of \( \bar{u}' \bar{v}' \) on EOF1 in the full GCM (when EOF1 leads \( \bar{u}' \bar{v}' \) (not shown). The initial dipole structure is much like the \( \bar{u}' \bar{v}' \) dipoles in the prescribed \( \bar{u} \) experiments (Fig. 3a). A dipolar \( \bar{u}' \bar{v}' \) forces a tripolar \( \bar{u} \) and this evident in Fig. 10b with the growth of \( \bar{u} \) anomalies in the subtropics and the slight poleward shift of the anomaly at 32\(^\circ\). The subtropical winds do not need to adjust much, however, before the subtropical \( \bar{u}' \bar{v}' \) anomalies disappear. By turning off the phase speed effect, we have verified that the feedback responsible for this adjustment does not involve changes in phase speed (recall that the effects of phase speed are

\(^1\)The precise form of the forcing, \( F_z \), in equation (17) of Lorenz (2013a) is \( F_z = 1.16 \cdot 2/\pi \exp \left(-((\phi - \phi_0)/12\circ)^2\right) \).
weak for $\bar{u}$ dipoles (Fig. 4)). We believe the adjustment involves critical level dynamics on the equatorward flank of the jet and this is a topic of future research. Note, if this is the case, then the limit associated with (6) and (7) does not hold. Finally, we believe that the presence of an adjustment in the first place is due to RWC biases and that the RWC model is simply adjusting to its own version of the most persistent mode. The adjusted $\bar{u}$ pattern is not very different from the GCM EOF1 (Fig. 10c), but it has a large effect on the structure of $u'v'$. The subtle details of the long-term adjustment are not entirely clear at this point and further research is required to better understand the coupled system.

7. Discussion and Conclusions

This is part III of a series of papers using Rossby Wave Chromatography (RWC) (Held and Phillips (1987)) to understand the variability and response to external forcing of an idealized GCM. By RWC, we mean that we calculate the space-time (i.e. phase speed-latitude-wavenumber) structure of the upper tropospheric eddy momentum fluxes given the background zonal-mean flow and the space-time structure of the baroclinic wave activity source (convergence of vertical EP flux (Edmon et al. (1980)) in the upper troposphere).

The focus in this paper is on documenting and understanding the coupling between $\bar{u}$ anomalies and the eddy fluxes predicted by RWC. This study is limited to the case of $\bar{u}$ anomalies that are essentially barotropic in the sense that 1) the $\bar{u}$ tendency is well approximated by the sum of the eddy $\zeta'v'$ and a simple Rayleigh damping term acting on $\bar{u}$ and 2) the wave activity source integrated over phase speed is constant. These assumptions will be relaxed in future work. We also consider changes in the phase speeds of the wave activity sources.

We run a series of RWC experiments with imposed $\bar{u}$ monopoles at different latitudes. We find that the eddies reinforce the imposed $\bar{u}$ when it is collocated with the centers of action of EOF1 of the GCM. Reflecting level dynamics (Kidston and Vallis (2012); Lorenz (2013b)) are essential for the positive feedback when $\bar{u}$ is poleward of the jet and critical level dynamics (that are independent of phase speed changes) are essential for the positive feedback when $\bar{u}$ is equatorward of the jet. When the imposed $\bar{u}$ is out of phase with EOF1, the eddies tend to shift the imposed $\bar{u}$ poleward (equatorward) for anomalies that are
Figure 10: a) Evolution of eddy momentum flux anomalies in the unforced coupled RWC model with a positive EOF1 initial condition minus the evolution for a negative EOF1 initial condition (m s$^{-2}$). The initial EOF1 pattern is from the GCM. b) As in a) but for $\bar{u}$ (m s$^{-1}$). c) The normalized $\bar{u}$ profile for the initial condition (thin solid) and for time = 31 days (thick dotted).

equatorward (poleward) of the poleward center of action of EOF1. Critical level dynamics is most important for the poleward shift in the subtropics while reflecting level dynamics is most important for the poleward shift in the mid-latitudes. We provide a simple model of critical level dynamics that explains the degree of poleward propagation versus positive feedback in this series of experiments. Also, there is no baroclinic feedback in these experiments.

Looking at the imposed $\bar{u}$ experiments as a whole, we find that all of the dramatic changes in the structure of the momentum fluxes that occur as one changes the $\bar{u}$ latitude are due to changes in IOR. The changes in momentum fluxes due to changes in phase speed, on the other hand, always have the same broad monopole structure that gradually changes in amplitude as the $\bar{u}$ latitude changes. This lack of structure occurs because a local $\bar{u}$ anomaly at latitude $\phi_0$ leads to $u'v'$ anomalies 1) across all latitudes with subtropical critical levels for the waves with sources at $\phi_0$ and 2) across the jet axis provided the distribution of phase speeds for the waves with sources at $\phi_0$ intersects the peak in the reflecting level. We also find that for local $\bar{u}$ anomalies, IOR effects on $u'v'$ tend to dominate over phase speed because the net phase speed change is an integrator of $\bar{u}$ across the entire width of the wave source region while IOR is not.

In the imposed $\bar{u}$ experiments the $\bar{u}v'$ anomalies are focused in two separate latitude bands: one in the subtropics and one in the mid-latitudes. When $\bar{u}$ and $\bar{u}v'$ are coupled, however, the $\bar{u}v'$ tends to adjust so that the mid-latitude $u'v'$ dominates over the subtropical $u'v'$. The monopole structure of the adjusted $\bar{u}v'$ is consistent with the response of $\bar{u}v'$ to EOF1 in the GCM (as diagnosed by lagged regressions). The changes in $\bar{u}$ that bring about this adjustment are relatively small but the effect on the subtropical $u'v'$ is pronounced. The dynamics of this adjustment are the subject of future work.

Given that the $\bar{u}v'$ adjusts so that the fluxes across the jet axis dominate, the key for understanding the structure of the leading EOF1 appears to be the location of the peak of the reflecting level on the poleward flank of the jet. While the peak in the reflecting level is wavenumber dependent, in our GCM the peak of the reflecting level only varies from 56° to 48° for the dominant wavenumbers 4 - 8. This explains why the structure of the reflecting level induced $u'v'$ is insensitive to the precise location of the $\bar{u}$ anomalies. As long as $\bar{u}$ “excites” the reflecting level, there will be strong momentum flux divergence/convergence near the “average” reflecting level peak ($\approx 52^\circ$, Fig. 6b). Therefore patterns of variability that are in phase with the reflecting level peak dominate over other patterns (as long as the meridional scale of the anomaly is large enough that $\bar{u}$ dominates over $\beta$ in (1)). The above ideas focus on the poleward center of action of EOF1. Although the location of the equatorward center of action of EOF1 is associated with reinforcing $\bar{u}v'$ anomalies in the subtropics for generic, prescribed $\bar{u}$ anomalies (Fig. 4), the coupled $\bar{u}$ subtly adjusts so that these subtropical $\bar{u}v'$ anomalies disappear. In addition, the mechanistic critical level model above implies that critical level dynamics is al-
ways associated with non-stationary \( \bar{u} \) anomalies that migrate toward the wave source. This suggests that critical level dynamics associated with anomalies at the equatorward center of action of EOF1 play a secondary role in the positive feedback.

The above ideas suggest that latitudinal shifts of the jet dominate the variability only if the peak of the reflecting level is on the poleward flank of the jet. Meanwhile, the reflecting level is on the poleward flank provided the jet is narrow enough so that variations in \( \beta \), which is large in the jet and weak on the flanks, are large enough compared to \( \bar{u} \) in (1). If \( \beta \) gets too small on the poleward flank, however, then the peak of the reflecting level could potentially extend to phase speeds beyond where there are significant wave activity sources, leading to a decrease in the positive feedback. This provides a slightly different explanation of the decrease in the positive feedback with latitude (Barnes et al. (2010); Barnes and Hartmann (2011)) in that it is the reflecting level itself that is responsible for the positive feedback.

If, on the other hand, \( \beta \) is not “large” compared to \( \bar{u} \) (for example, the jet is broad), then the peak of the reflecting level approaches the latitude of the mean jet and the situation is like that depicted in Fig. 11. In this case, the momentum flux convergence associated with changes in the peak of the reflecting level remains on the poleward flank, however, the peak of the reflecting level now responds most strongly to \( \bar{u} \) anomalies that are nearly in phase with the jet. This suggests that there is no dominant EOF in this case and that instead the dominant form of \( \bar{u} \) variability is poleward propagating anomalies. This is consistent with the results of Lee et al. (2007) who found that poleward propagating anomalies dominate when mean potential vorticity gradients are weak. The peak of the reflecting level might also be displaced when a strong “subtropical jet” is present on the equatorward flank of the jet. Indeed, in Barnes and Hartmann (2011) the relative minimum in \( aK^*(= \sqrt{l^2 + m^2} \) in our notation) approaches the latitude of the mid-latitude jet as the subtropical jet becomes more important and the positive eddy feedback decreases in strength (their Fig. 12).² Barnes and Hartmann (2011) also show interesting jet states where the distribution of jet latitude is bimodal (their Fig. 14). In these cases, the dominant flank for reflection (as diagnosed by their \( aK^* \) profiles) actually changes from poleward flank of the jet to the equatorward flank of the jet depending on the state of the jet! The bimodality is likely associated with a complete change in dominant direction of wave activity propagation (for a certain class of waves). In the future, we plan to diagnose a large number of GCM runs with different means states to understand the effect of critical and reflecting level configuration on the character of the internal variability.

Finally, the results of this paper suggest that barotropic dynamics can account for the spatial structure of the positive and propagating feedbacks as a function the \( \bar{u} \) latitude. However, as alluded to in section 6 of Lorenz (2013a), the amplitude of the EOF1 positive feedback and therefore the persistence of EOF1 are too small in the RWC model. For example, the e-folding time scale for EOF1 initial conditions (after the adjustment discussed in section 6) in the RWC model is 22 days compared to 35 days for the GCM (after the transient negative feedback). This discrepancy might be simply due to biases in the RWC because RWC has a tendency to under-predict the response to external forcing even when the full wave activity change is pre-

Fig. 11: Schematic: as in Fig. 1 but for the case where the peak of the reflecting level is close to the latitude of the climatological jet.

²The latitude of the relative minimum in the \( aK^* \) is approximately the latitude of the relative maximum in the reflecting level.
scribed (see Fig. 3 in Lorenz (2013a)). This discrepancy might also be the result of a positive baroclinic feedback acting with the barotropic feedbacks discussed here (e.g. Barnes and Thompson (2013)). Any theory for the baroclinic feedback, however, should also explain why only EOF1 is associated with a positive feedback.

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