Research Computing: Princeton Perspectives

Numerical Relativity

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Numerical Relativity

- numerical relativity is concerned with solving the field equations of general relativity using a computer

$$G_{\alpha\beta} = 8\partial T_{\alpha\beta}$$

- a “solution” tells one what the geometric structure of spacetime is in a particular physical scenario, and this is most conveniently encoded via a *metric tensor*

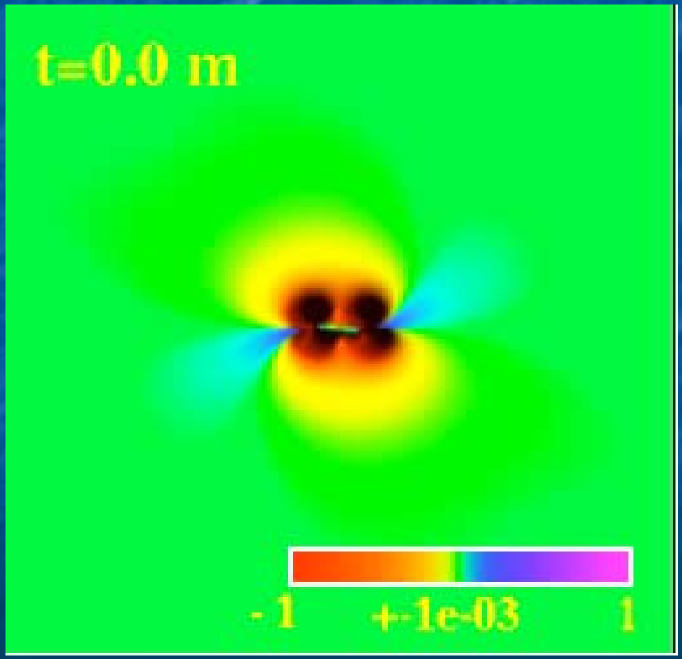
$$ds^2 = \sum_{\alpha,\beta} g_{\alpha\beta} dx^\alpha dx^\beta$$

Numerical Relativity

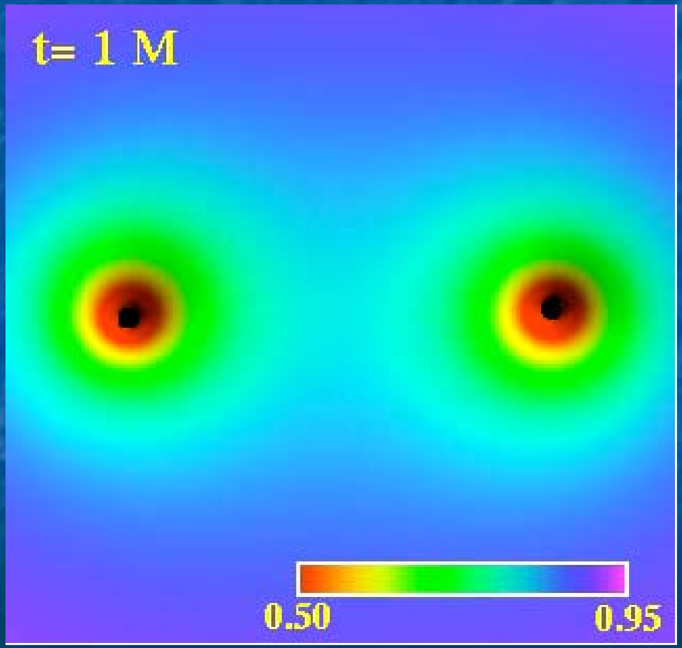
- when expressed in terms of the metric tensor, the Einstein equations form a *system of 10 coupled, non-linear, second order partial differential equations, each depending on the 4 (or more) spacetime coordinates*
  - thus, the computational problem of numerical GR is essentially solving PDEs
- difficulties
  - in all, the system of equations has of O(1000) terms that must be evaluated at each grid point in a computational domain
  - no unique formulation; well-behaved coordinates not usually known *a priori*; geometric singularities form inside black holes; typically several orders of magnitude of relevant spatio-temporal length scales that need to be resolved; constraints in system that could be problematic in a numerical evolution
- "easies"
  - away from the isolated singularities inside of black holes the metric is typically smooth, free of shocks, turbulence, etc., even when coupled to matter that exhibits these features.

Example: Black Hole Collisions

- With the advent of the modern generation of gravitational wave detectors we are on the verge of a new era in astronomy where we should be able to observe the universe in gravitational waves for the first time
  - the first generation of such detectors (LIGO, VIRGO, GEO, ...) will require some knowledge of the source to extract interesting information from the signals (with matched filtering, for example), which is why simulation of such events is important for observation
  - one of the most promising sources of gravitational waves are the inspiral and collision of two black holes



relative time-dilation



gravitational wave emission

Computational Requirements

- Most important hardware: *FLOPS*
  - not only for "production" runs, but for development as well, as many of the difficulties do not arise in symmetry reduced situations
    - vacuum binary black hole simulations are teraflop scale; many interesting astrophysical scenarios involving matter will require petaflop computers
  - One of the most cost effective ways of achieving high flop-rates today is through Beowulf clusters, such as Woodhen and Della
    - parallelization of the discretized PDEs is straight-forward using domain decomposition, but this then requires a *relatively low-latency, high-bandwidth interconnect* between nodes in the cluster
  - huge amounts of data are produced --- need *fast, large compacity file storage*

Computational Requirements

- Most important "software": ***people!***
  - Computer systems at the bleeding edge of scientific computation are not, and possibly never will be usable in a stable and reliable manner without expert supervision
    - having support from facilities such as TIGRESS/PiCSIE/OIT and the knowledgeable people running them is *essential*
  - Having quick and interactive access to supercomputer resources is extremely important for doing research efficiently
    - this is only possible with a relatively small, courteous user base, and my experience on local machines in this regard has been very positive