Bruno Dupire: “Functional Ito Calculus and Robust Implied Volatility Hedge”

Abstract:

We present an extension of Ito calculus to functionals of price paths. When applied in a diffusion framework to the expectation of a path dependent claim conditional on the path so far, it leads to a Black-Scholes like PDE for path dependent options, even if the path dependency cannot be summarized by a finite number of state variables, with the classical Gamma/Theta (properly defined) trade-off. It also gives an alternative expression of the Clark-Ocone formula for the Martingale Representation Theorem. We apply the functional Ito Formula to obtain the difference of price of an exotic option in two different models and deduce the sensitivity of the price to local deformations of the implied volatility surface. It leads to a decomposition of the Vega across strikes and maturities and the associated hedge in terms of a portfolio of European options.