“Ambiguity” in Corporate Finance: Governance, Underinvestment, and Security Issuance*

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Abstract

Corporate decisions undertaken by a group of agents with heterogeneous beliefs, such as a corporate board, are de facto “ambiguous” to the group, even if each group member is Bayesian. We formally prove that Utilitarian, “Rawlsian” and Inertia-based governance rules are dynamically inconsistent. In the context of dynamic real investment problems, dynamic inconsistency may lead to underinvestment, i.e., multi-stage projects that would be undertaken by each group member are rejected by the group because of anticipated future disagreement. The issuance of securities such as risky and convertible bonds can be useful in generating unanimity and eliminating underinvestment.

Keywords: Ambiguity, Group decisions, Time inconsistency, Dynamic real investment, Security issuance.
1 Introduction

The word “corporation”, derived from the Latin corpus or body, refers to “a body formed and authorized by law to act as a single person”. The study of corporate decisions typically rests on the “common prior” doctrine according to which corporations are modeled as either (i) a single ‘manager’, whose choices are determined by maximizing expected utility with respect to a unique prior as in Savage’s (1954) Subjective Expected Utility (SEU) model, or (ii) Decision Making Groups (DMGs) consisting of SEU individuals with homogeneous prior beliefs, albeit possibly differentially informed. In reality, corporate boards, management teams, lending syndicates, and venture capital firms are examples of DMGs where individuals with different areas of expertise and opinions must collectively decide, as a single legal person, what the corporation is to do, often in the face of dramatically different views of whose model of the world is correct.

In this paper we study corporate decisions when the “common prior” assumption does not hold. A key idea underlying our analysis is that if group members have heterogeneous priors, the group as a whole faces an “ambiguous” decision: the probability of future states of the world are not objectively known by the group. The group is de-facto a “multi-prior” decision maker, even if each individual member is a rational single-prior Bayesian. We study the implications of this form of “ambiguity” for corporate finance by considering the dynamic choice problem of a group of agents that invest in a project with operational flexibility in the form of an option to continue or abandon the project in the future.

To describe choice in the presence of multiple priors we assume the Pareto condition, or unanimity: decisions supported by each group member are supported by the group as a whole. Unanimity, however, is an incomplete preference order: there might be alternatives that the DMG simply cannot rank. Yet, decisions eventually have to be made. We refer to any decision criterion that allows a DMG to express a preference over plans that are not rankable according to unanimity as an aggregation rule.

We study group decisions individuals whose preferences are described by the SEU model and who differ only in their beliefs about the probability of future outcomes – i.e. they have equal utility functions and common consumption. We consider three specific aggregation rules: (1) the

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1 Merriam-Webster Dictionary

2 Although we find this a natural staring point for the study of corporate decisions, recent studies question the Pareto criterion. See, for example, Gilboa, Samet, and Schmeidler (2004), Mongin (2005), Brunnermeier, Simek, and Xiong (2014), and Gilboa, Samuelson, and Schmeidler (2014).
Utilitarian rule, according to which the group attaches an index to each choice equal to a weighted average of each group members’ expected utility; (2) the Rawlsian rule, according to which the group attaches to each choice an index equal to lowest expected utility among all group members; and (3) the Inertia, or Status quo rule, according to which, in the absence of unanimity, the DMG identifies one of the alternatives (the status quo) that remains the default choice unless an alternative is unanimously seen to be better. All three aggregation rules are consistent with the Pareto criterion.

The aggregation rules that we consider generate behaviour that is similar to the behaviour generated by models of single person decision making in the presence of multiple priors. Under the utilitarian rule the DMG behaves as a fictitious individual with beliefs given by the (weighted) average of the beliefs of group members and whose static choice is described by the SEU model, similar to Harsanyi (1955). Under the Rawlsian rule the DMG behaves as a fictitious individual whose choice is described by the Minimum Expected Utility (MEU) model of Gilboa and Schmeidler (1989). Under the inertia aggregation rule, the DMG behaves as a fictitious individual whose choice is described by the Bewley Expected Utility model of Bewley (1986).

In practice corporate governance is a complex interplay of legal, sociological, and economic forces. While our model does not capture all of these forces, we feel that the aggregation rules we consider are consistent with features of actual governance processes. The utilitarian rule is conceptually connected to majority voting. The Rawlsian rule is conceptually connected to Rawls’s (1971) “veil of ignorance” argument and describes the decision making process of a “cautious group”. In the corporate context this cautiousness might result, for example, from the unlimited liability faced by directors in a corporate board. The Inertia rule describes typical “grid-lock” situations in which the absence of agreement leads to the supremacy of the status quo. Inertia can also be the outcome of “super-majority” voting rules (see, e.g., Krishna and Morgan (2012)).

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3Harsanyi’s (1955) result requires that each individual has identical (objective) beliefs and different utilities. In our paper we consider the case in which agents have identical utilities but different subjective beliefs. Mongin (1995) shows the impossibility of aggregation when individuals have different beliefs and utilities.

4Gilboa, Maccheroni, Marinacci, and Schmeidler (2010) provide a foundation of Gilboa and Schmeidler’s (1989) MEU model as a completion of the unanimity preference of an individual decision maker.

5The formal connection is provided by the Rae-Taylor theorem (see Rae (1969) and Taylor (1969) and Mueller (2004)) according to which if each individual has an equal prior probability of preferring each of two alternatives, majority rule maximizes each individual’s expected utility for the choice between these two alternatives. Brighouse and Fleurbaey (2010) provide a generalization of this result that allows for individuals to have different “stakes” in the decision, i.e., the utility difference between the preferred outcome and the non-preferred outcome varies across individuals.
Using these three aggregation rules, we study the decisions over time of a multi-agent DMG that has access to the invest-abandon-continue technology discussed above. Our analysis delivers three main theoretical implications for corporate investment, governance, and security design.

First, we show that all three aggregation rules can lead to a novel form of underinvestment where all members of the DMG, despite their different views of the world, agree that investment is best but nevertheless collectively decide not to invest. Prior research has shown that asymmetric information and/or conflicts of interest between debt and equity holders (e.g., Myers 1977) or old and new equity holders (e.g., Myers and Majluf 1984) can lead to underinvestment. We demonstrate that underinvestment can result even when all agents receive the same signals, the interests of all agents are aligned, and all agents have a common claim to the benefits of the investment. Our explanation rests on the dynamics of group decisions in the presence of heterogeneous beliefs; decision makers who are in agreement at one point in time recognize potential future conflict, anticipate how the conflict will be resolved, and, when they do not like the expected resolution, avoid the conflict by avoiding the investment.

Second, we show that the source of underinvestment in our model is dynamic inconsistency of all three aggregation rules considered, i.e., ex-ante preference between two alternatives can be reversed after learning occurs. It is this reversal that brings about future conflict that inhibits initial investment. Intuitively, dynamic inconsistency arises because learning may induce shifts in the individuals’ stake in the group decision. For instance, suppose that two choices $A$ and $B$ have identical payoffs in all future states of the world except state $s$, where $A$ has a much higher payoff than $B$. An individual who thinks that state $s$ is unlikely will have, ex-ante, a lower “stake” in the decision: the expected utility of $A$ is not much higher than that from $B$. If the state $s$ materializes, however, his stake in the decision becomes much larger. Aggregation rules reflect the stakes that group members have in various decisions. Therefore dynamic inconsistency can happen when members with the larger stakes in the decision before learning disagree with those who have larger stakes after learning. To the best of our knowledge, the fact that aggregation of heterogeneous beliefs leads to time inconsistency in the three aggregation rules we consider is a novel result.\footnote{The dynamic inconsistency of the Rawlsian rule parallels a similar result for the single-agent MEU models. Time inconsistency for non expected utility models is an area of active research in decision theory. Earlier contribution include Machina (1989), Epstein and Le Breton (1993), and Epstein and Schneider (2003). For more recent work on the dynamic inconsistency of the MEU model, see Al-Najjar and Weinstein (2009) and Siniscalchi (2011). In a different setting our result is similar in spirit to Jackson and Yariv (2011) who show that heterogeneity in discount rates also leads to time inconsistent behaviour for the DMG.}

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Third, when there is an operating policy that all members of the DMG would support, even if not all members see it as optimal, we show that underinvestment can be avoided with security issuance. Contract design can change the firm’s payout across different states of the world in a way that brings unanimity in operating decisions where disagreement would otherwise obtain.

Intuitively, the expected future conflict that causes initial underinvestment comes about in some cases when an optimistic group member becomes more influential in the decision making after learning. In such cases we say that the group will become ‘eager to continue’ (ETC) after learning. In other cases, underinvestment arises when a pessimist might become more influential in which case the group becomes ‘eager to abandon’ (ETA). We show that risky bonds can be designed to mitigate underinvestment due to eagerness to abandon and convertible bonds can be designed to mitigate underinvestment due to eagerness to continue. A risky bond can be designed to ensure default if the DMG chooses to abandon. In this case, unanimity is achieved because each group member, including the pessimists, sees that abandonment is equivalent to losing title to the asset so that there is nothing to lose from continuing. Similarly, a convertible bond can be designed in such a way that bondholders convert if the DMG chooses to continue. As a result, the benefit seen by the optimists from continuing is eliminated and abandonment is preferred. These findings raise an entirely new role for financial contracting, one that reflects the role of contracts in neutralizing conflicts that may arise among group members.

While our results are robust to various enrichments of the model, a critical assumption is that agents with heterogeneous expectations must make collective decisions. There are two components of this assumption.

First, we assume that rational individuals examining the same data can come to different conclusions about the implication of the data. Although this assumption is not common in corporate finance and economics, the reason appears to be that a common prior assumption is a convenient methodological device to focus on purely informational issues (see, e.g., Aumann (1987)). However, as emphasized by Morris (1995), “not all economic issues are informational and there are some cases in which differences in prior beliefs are essential to understanding economic phenomena”. We believe that the collective decision of corporate DMG is one such case.

The second component of our assumption is that individuals with differing models of how the world will unfold must come together to make decisions. In the typical corporate finance setting with heterogeneous expectations, the agents would agree to part company with the optimistic
agents buying out the pessimistic ones. We assume this is not possible and that agents must make decisions and continue operating the firm together. While this perspective is rare in corporate finance, it is common in the area of social learning where a number of studies have shown that the diversity of backgrounds and opinions increases firm performance and uncertainty induces agents to share information with others (see, e.g., Haunschild (1994)).

We contribute to several strands of literature in economics and finance. First, we explicitly recognize the ambiguity that results from group decision making in a corporate setting. Several recent studies have applied models of *individual decision making* under multiple priors to finance problems. For the most part, these applications use MEU preferences for individuals and focus on asset pricing and portfolio choice problems. Epstein and Schneider (2010) and Guidolin and Rinaldi (2012) provide excellent surveys. Fewer studies have examined BEU preferences for individuals in finance applications with a focus on static problems. Rigotti (2004) studies financing decisions, Rigotti and Shannon (2005) study risk sharing and allocative efficiency in general equilibrium, and Easley and O’Hara (2010) use BEU to study liquidity and market “freezes”.

In corporate finance, Nishimura and Ozaki (2007), Riedel (2009), and Miao and Wang (2011) examine the exercise decision of a real option for an individual MEU decision maker. Dicks and Fulghieri (2014) use the MEU model to study the relationship between strength of corporate governance and firm’s transparency. In all these studies, the dynamic consistency of investment decision is typically guaranteed by assuming that the set of priors is sufficiently rich (see Epstein and Schneider (2003)).

We differ from this prior literature by studying the dynamic real investment decision problem of a group of decision makers with heterogeneous beliefs and link the group’s aggregation rules to real investment decisions.

Second, we show that, when investment decisions are collectively made in a non market-mediated environment, differences of opinion and consequent disputes may lead to underinvestment. This is a new insight since existing rationalizations of corporate underinvestment rely typically on conflicts of interest among stakeholders, such as debt overhang (Myers (1977)), adverse selection (Myers and Majluf (1984)) and other agency frictions (Jensen and Meckling (1976)).

Third, we show that the source of underinvestment is the time inconsistency that arises with all of the three aggregation rules we consider. Our results on the group interpretation of ambiguity models are connected to the work of Crès, Gilboa, and Vieille (2011), and Nascimento (2012)
who study the aggregation of experts’ opinions. They focus on aggregation of groups with MEU members, assume a DMG with MEU preferences and study static problems. We add to this literature by focusing on the dynamic implications of a broader set of aggregation rules and provide the novel implication that all rules considered are subject to time inconsistency.

Fourth, we show how contracting can help resolve the underinvestment problem. It is well known (see, e.g., Leland and Pyle (1977) and Brennan and Kraus (1987)) that conflicts of interest can be solved by issuing securities. We point to a new role for financial contracts, one that works through the avoidance of internal disagreements within the DMG members.

We note that our assumption that agents must work together and cannot trade on their beliefs contrasts with studies where market trading plays an important part in investment. In decentralized markets, difference in beliefs are known to generate speculative trading (e.g., Miller (1977), Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003)). Speculative trading may induce stock overvaluation and overinvestment at the firm level (e.g., Shleifer and Vishny (1990), Blanchard, Rhee, and Summers (1993), Stein (1996), Baker, Stein, and Wurgler (2003), Polk and Sapienza (2009), Gilchrist, Himmelberg, and Huberman (2005), and Bolton, Scheinkman, and Xiong (2006)). By contrast, we argue that when investment decisions are collectively made in a non market-mediated environment, differences of opinions may lead to underinvestment.

The rest of the paper proceeds as follows. In Section 2 we provide a motivating example describing the decision problem of a DMG that we study throughout the paper. In Section 3 we introduce the three belief aggregation rules we use in the paper and study their dynamic consistency. In Section 4 we show how dynamic inconsistency may lead to underinvestment and in Section 5 we illustrate how security issuance can resolve the underinvestment problem. Section 6 discusses extensions, empirical implications, and limitations of our study and Section 7 concludes. Appendix A contains proofs for all propositions.

2 The problem of corporate group decisions: An Illustration

In this section we illustrate how aggregation of multiple priors may result in time inconsistent decisions and underinvestment: Investments that are seen as positive NPV by each group member are not undertaken by the DMG. Moreover, despite their different views, the group would
unanimously agree to invest if they could collectively precommit to a corporate plan but, in the absence of precommitment, they do not invest. The example further shows how issuing a risky bond can solve this underinvestment problem.

**Corporate structure and governance.** Two risk neutral friends, Olivia and Pietro, decided to form a corporation, O&P Ltd., with the objective of discovering new investment opportunities. They each contributed equally to a capital pool of $I$ in exchange for one share each. They agree not to trade their shares with each other or anyone else or to make any other side payment arrangements before the final cash flows are realized.

Olivia and Pietro have SEU preferences but have different beliefs about future states of the world. This heterogeneity led them to discover the investment opportunity they are now contemplating, but may also cause them to disagree about what is best for the firm. Despite their potential disagreements, however, they must govern the firm collectively as a single DMG. Accordingly, Olivia and Pietro agree to the following corporate governance mechanism: (i) When they agree that a particular course of action is best, they take that course; (ii) when they disagree, they construct a “utilitarian” index that assigns to each action the weighted average of the group members’ evaluations of that action and select the action that maximizes the utilitarian index. When facing a sequence of decisions Olivia and Pietro correctly anticipate the DMG’s future choices and have rational expectations about any subsequent decisions they have to make.

**Information and beliefs.** The problem unfolds over three dates, 0, 1, and 2. At time 2 the economy can be in one of three possible states, $s_1$, $s_2$, and $s_3$. We refer to $s_1$ as an ‘expansion’ and to the set of states $E = \{s_2, s_3\}$ as a ‘contraction event’, where $s_2$ represents ‘recovery’ and $s_3$ ‘recession’. At time 0 Olivia and Pietro have beliefs $\pi^O$ and $\pi^P$ about the state of the economy at time 2. Specifically,

$$\pi^O = (1/3, 1/3, 1/3), \quad \pi^P = (1/10, 8/10, 1/10). \quad (1)$$

In the event of a contraction at time 1 Olivia and Pietro update their beliefs according to Bayes’s rule, obtaining the posterior beliefs

$$\pi^O_E = (0, 1/2, 1/2) \quad \text{and} \quad \pi^P_E = (0, 8/9, 1/9). \quad (2)$$
Technology. At time 0 O&P’s corporate assets consist of $I = 985$ in cash and the opportunity to invest this amount in a factory that will last for two periods. For simplicity all cash flows are realized at time 2 and the discount rate is zero. At time 0 Olivia and Pietro have to decide collectively whether to invest in the project or not ($\mathcal{N}$). If they don’t, they are left with the initial $985$ in cash. If they do invest, and the economy is in a contraction phase at time time 1, then $O&P$ have the option to either abandon ($A$) or continue ($C$). If they abandon they receive a state independent value of $700$. If they continue they receive a state contingent cash flow of either $C(s_2) = 1,000$ in a recovery or $C(s_3) = 0$ in a recession.

To summarize, there are three different operating ‘plans’ at time 0: Do not invest, i.e. $\mathcal{N}$, invest and continue at time 1 if the economy contracts, i.e. $C$, and invest and abandon at time 1 if the economy contracts, i.e. $A$. The state dependent cash flows associated with each plan can be represented by a vector of payoffs in the states $(s_1, s_2, s_3)$ as follows: $\mathcal{N} = (985, 985, 985)$ $C = (2,000, 1,000, 0)$ and $A = (2,000, 700, 700)$.

Decisions. In order to correctly anticipate future decisions that they will make, Olivia and Pietro solve the investment problem recursively, starting from the continue-abandon decision at time 1.

Continue or Abandon at time 1. According to the governance mechanism in place, a plan will be accepted if it is unanimously supported by the group. Let $E_\pi^O(C)$ and $E_\pi^P(A)$ denote the subjective values at time 1 to Olivia and Pietro of the actions $C$ and $A$, respectively, if a contraction event, $E$, is underway. Using the conditional beliefs (2), the conditional values for Pietro and Olivia are:

$$E_\pi^O(C) = 500, \ E_\pi^P(A) = 700, \ E_\pi^P(C) = 889, \ E_\pi^P(A) = 700.$$  

(3)

There is disagreement between Pietro, who sees more value in continuing, and Olivia, who sees more value in abandoning. Given the lack of unanimity, the agreed upon governance rule requires that the group construct, for each action, a “utilitarian” index by equally weighting the valuations (3) at time 1: $E_\pi^{0.5}(C) = 694.50 < E_\pi^{0.5}(A) = 700$, where $\pi^{0.5} = 0.5 \pi^O + 0.5 \pi^P$. Therefore the group will decide to abandon the investment (i.e. $A$). For this decision the corporate decision is consistent with Olivia’s personal choice and therefore the group behaves as if the group decision is determined by her preference.

\footnote{In the analysis that follows we treat Olivia and Pietro as total firm value maximizers. Since they each own identical claims to the firm accounting for actual shares of the firm would simply involve dividing aggregate firm’s cash flow by 2 without affecting any of our results.}
Initial Investment at time 0. At time 0 both Olivia and Pietro agree on the value of not investing, \( \mathbb{E}_{\pi_O}(N) = \mathbb{E}_{\pi_P}(N) = 985 \). The value of investing depends on how the firm will handle the abandonment option. Using the two agents’ unconditional priors (1), the time 0 value to each agent, taking the time 1 action of \( C \) or \( A \) as given, is:

\[
\mathbb{E}_{\pi_O}(C) = 1,000, \quad \mathbb{E}_{\pi_O}(A) = 1,133, \quad \mathbb{E}_{\pi_P}(C) = 1,000, \quad \mathbb{E}_{\pi_P}(A) = 830.
\]

(4)

Since \( \mathbb{E}_{\pi_O}(C) = \mathbb{E}_{\pi_P}(C) = 1,000 > 985 \) the group would unanimously support investment if the firm were to continue operations in the face of an economic contraction. However, as shown above, \( P&O \) will abandon the project if an economic contraction takes place. Rationally anticipating this, the group will therefore rule out \( C \) as a possible choice. Because \( \mathbb{E}_{\pi_O}(A) = 1,133 > \mathbb{E}_{\pi_O}(N) = \mathbb{E}_{\pi_P}(N) = I = 985 > \mathbb{E}_{\pi_P}(A) = 830 \), Pietro and Olivia disagree about the unconditional choice between \( N \) and \( A \).

Given this disagreement the group will invoke the utilitarian governance mechanism which gives a project valuation of \( \mathbb{E}_{\pi^{0.5}}(A) = (1,133 + 830)/2 = 983 < I \). Hence Pietro and Olivia will decide that \( P&O \) will not invest even though both members individually would invest and the group would unanimously support investment if they could pre-commit to a future course of action.

Underinvestment and security issuance. Suppose that Olivia and Pietro decided that \( O&P \) will issue a zero-coupon bond at time 0 with maturity at time 2, face value \( X = 710 \) and price \( P \). If the project is abandoned by selling the factory for $700 it will be unable to repay the debt, leaving nothing for the shareholders of \( O&P \) in both states \( s_2 \) and \( s_3 \). If, instead, the group decides to continue operating the firm, it will leave nothing for the equity holders in state \( s_3 \) but the firm will be solvent in state \( s_2 \), where \( O&P \) is left with the payoff \( 1000 - 710 = 290 \). Therefore, the issuance of bonds at time zero makes continuation the unanimous choice at time 1. Anticipating this future decision and for bond prices \( P \) such that \( P + \mathbb{E}_{\pi^{0.5}}[\max(C-X,0)] > I \), i.e., \( P > 541.17 \), Olivia and Pietro unanimously support investment at time 0.

Summary. The above example is striking because it shows that, although each group member would invest if they made decisions on their own, the group decision is to not invest. This happens because, due to the disagreement on how to operate the firm in the future, the governance rule chosen by the group is time inconsistent, even when each member has standard SEU preferences.
This time inconsistency, in turns, induces underinvestment. The key departure from the standard corporate dynamic investment problem is the fact that the agents have to act collectively. As the example shows, this modification generates non-trivial consequences in the dynamic decision of a group. To formally analyze the complexity that the assumption of collective decision brings to the standard corporate problem it is necessary to explicitly define the concept of “aggregation rule” (governance mechanism) and its implication for dynamic decisions. In the next section we lay out the tools for this analysis which will allow us to discuss issues related to the dynamic consistency of aggregation rules. The section contains the main theoretical results of this paper which apply more broadly than the corporate example considered in this section. The reader who is interested only in the applications of these general principles in a corporate context can skip directly to Section 4.

3 Governance and dynamic group choice

In this section we formally describe and study belief aggregation and dynamic group choice. The main theoretical result of this section is to provide the novel insight that collective decision making with heterogeneous beliefs leads, almost inevitably, to time inconsistent behavior.

3.1 Aggregation rules

The DMG consists of \( J \) individuals with SEU preferences, a common non-decreasing utility function \( u(\cdot) \) over the set of outcomes \( X = \mathbb{R} \), and heterogeneous priors, \( \pi^1, \ldots, \pi^J \), over a finite and common state space \( S = \{s_1, \ldots, s_N\} \). A generic choice, or act, \( f \) is a function mapping \( S \) onto the set of outcomes \( X \). Individual \( j \)'s subjective expected utility derived from choosing act \( f \) is

\[
\mathbb{E}_{\pi^j}[u(f)] = \sum_{i=1}^{N} \pi^j(s_i)u(f(s_i)),
\]

where \( \pi^j(s) \) is \( j \)'s subjective probability of state \( s \) and \( f(s) \) is the outcome resulting from act \( f \) in state \( s \). Because the state space \( S \) and the utility \( u(\cdot) \) are common across agents, the beliefs \( \pi^1, \ldots, \pi^J \), completely characterizes the cardinal preferences of the \( J \) members of the DMG.

An aggregation rule is a procedure that the DMG utilizes to map individual subjective beliefs onto a collective decision when facing a choice between any pair of acts. We consider aggregation
rules that are consistent with the *unanimity* (UNA) or *Pareto* criterion that requires the DMG to weakly prefer act \( f \) to act \( g \) if every individual \( j = 1, \ldots, J \) weakly prefers \( f \) to \( g \). Using the notation \( \succeq \text{UNA} \) to define the order induced by unanimity, we have

\[ f \succeq \text{UNA} g \iff \mathbb{E}_{\pi^j}[u(f)] \geq \mathbb{E}_{\pi^j}[u(g)] \quad \text{for all } \pi^j, j = 1, \ldots, J. \quad (5) \]

The unanimity criterion induces an incomplete order. We consider aggregation rules that augment unanimity in ways that “complete” this order and determine collective choices. An aggregation rule can alternatively be thought of as a device to select a *pivotal* group member, i.e., a group member who, when acting alone, would make the same choice as that prescribed by the aggregation rule.\(^8\)

In what follows we consider three aggregation rules: (1) the Utilitarian rule, (2) the Rawlsian rule, and (3) the Inertia, or Status quo, rule.

**The utilitarian aggregation rule**

The aggregation rule used in the example of Section 2 belongs to the class of *utilitarian* aggregation rules.\(^9\) Formally, given a set of weights \( \lambda_1, \ldots, \lambda_J \geq 0, \sum_{j=1}^{J} \lambda_j = 1, \) and any pair of acts \( f \) and \( g \), the utilitarian group preference \( \succeq \text{Util} \) has the following representation:

\[ f \succeq \text{Util} g \iff \sum_{j=1}^{J} \lambda_j \mathbb{E}_{\pi^j}[u(f)] \geq \sum_{j=1}^{J} \lambda_j \mathbb{E}_{\pi^j}[u(g)]. \quad (6) \]

The weights \( \lambda_1, \ldots, \lambda_J \) can be interpreted as the influence each individual has on the group, possibly reflecting political power and/or the group’s confidence in or respect for the group member. A special case of the utilitarian aggregation rule is the *dictatorial rule* whereby an individual \( j^* \) (the dictator) makes decisions for the group based on his own belief and by ignoring other members’ beliefs, i.e., \( \lambda_{j^*} = 1 \) and \( \lambda_j = 0 \) for all \( j \neq j^* \).

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\(^{8}\)Note that the use of the term “pivotal” in our context is purely for expositional reasons and is not related to the notion of pivotal voter from the social choice literature.

\(^{9}\)The utilitarian aggregation rule we consider in this paper are conceptually linked to the seminal work on utilitarianism by Harsanyi (1955). Harsanyi’s theorem however, applies to the case of SEU individuals with common beliefs but heterogeneous utilities. When SEU individuals have heterogeneous utilities and heterogeneous subjective beliefs, it is impossible to obtain a SEU representation of the group’s preference that satisfy the Pareto criterion (see Hylland and Zeckhauser (1979) and Mongin (1995)). Gilboa, Samet, and Schmeidler (2004) show that an aggregation result can be achieved by weakening the Pareto criterion.
The utilitarian aggregation rule completes the unanimity criteria by representing the group’s preferences as SEU with the unique group belief $\Lambda^\pi$ given by

$$\Lambda^\pi(s) = \sum_{j=1}^{J} \lambda_j \pi^j(s) \text{ for all } s \in S.$$ \hspace{1cm} (7)

The utilitarian aggregation rule (6) can be equivalently expressed as follows:

$$f \succeq \text{Util} g \iff \sum_{j=1}^{J} \lambda_j (\mathbb{E}_{\pi^j}[u(f)] - \mathbb{E}_{\pi^j}[u(g)]) \geq 0,$$ \hspace{1cm} (8)

where the difference $\mathbb{E}_{\pi^j}[u(f)] - \mathbb{E}_{\pi^j}[u(g)]$ represents agent $j$’s intensity of preferences of the act $f$ over the act $g$. Equation (8) implies that a group member who has a stronger intensity of preferences relative to other group members or a larger utilitarian weight is more likely to see his preference reflected in the group’s choice, i.e., he is more likely to be pivotal.

The Rawlsian aggregation rule

Under the so called Rawlsian aggregation rule, the group attaches an index to each act equal to the lowest expected utility among all group members and then ranks acts according to these indices. This rule is conceptually linked to Rawls’s “maxmin rule” of distributive justice (see Rawls (1971)). Formally, the group preference $\succeq \text{Rawl}$ has the following representation:

$$f \succeq \text{Rawl} g \iff \min_{j \in \{1, \ldots, J\}} \mathbb{E}_{\pi^j}[u(f)] \geq \min_{j \in \{1, \ldots, J\}} \mathbb{E}_{\pi^j}[u(g)].$$ \hspace{1cm} (9)

As can be seen from (9), under the Rawlsian aggregation rule, the DMG’s preferences are identical to those of a multi-prior ambiguity averse individual that satisfies the axioms of Gilboa and Schmeidler’s (1989) Minimum Expected Utility (MEU) model.\textsuperscript{10} Gilboa, Maccheroni, Marinacci, and Schmeidler (2010) provide a novel foundation of the MEU model which is obtained from the unanimity criterion over the individual multiple priors, after invoking an axiom of “cautiousness” or “ambiguity aversion”.\textsuperscript{11} This axiom has a natural group interpretation: when facing a choice

\textsuperscript{10} To be consistent with Gilboa and Schmeidler (1989) axioms, the set $\Pi$ must be convex. We must then specify the set $\Pi$ as the the convex hull of the set $\{\pi^1, \ldots, \pi^J\}$. Because the SEU preferences are linear in probability, this alternative specification does not alter the choice implications for the preferences defined in (9).

\textsuperscript{11} According to this axiom, when an ambiguous act cannot be ranked with respect to a non-ambiguous act, the non-ambiguous one is chosen.
between an act $f$, on whose payoff distribution group members disagree, and an act $g$, on whose payoff distribution group members agree, the act $g$ is chosen whenever $f$ is not unanimously preferred to $g$.

Note finally that the Rawlsian aggregation rule selects as pivotal the individual(s) that derives the lowest utility from the act that is not chosen by the DMG. To see this, note that, if member $j_f$ derives the lowest utility from $f$, member $j_g$ derives the lowest utility from $g$, and $E_{π_jg}[u(g)] < E_{π_jf}[u(f)]$, then, by (9), the DMG will choose $f$. Because agent $j_f$’s has the lowest valuation of $f$ it must be the case that $E_{π_jg}[u(g)] < E_{π_jf}[u(f)] \leq E_{π_jg}[u(f)]$, and hence the DMG choice corresponds to the choice that would be made if $j_g$ were acting alone.

The Inertia (or Status quo) aggregation rule

The so called Inertia (or Status quo) aggregation rule consists of the unanimity (Pareto) criterion (5) augmented by the requirement that the DMG identifies one of the alternatives as the ‘status quo’, i.e., an action that will be taken unless an alternative is seen as unanimously better. Formally, the group preference $\succeq_{\text{Inertia}}$ resulting from the inertia rule, admits the following representation

$$f \succeq_{\text{Inertia}} g \text{ if either (i) } f \succeq_{\text{UNA}} g \text{ or (ii) } f \not\succeq_{\text{UNA}} g \text{ and } f \text{ is the status quo.}$$

The group choice under the inertia aggregation rule is identical to the behaviour of an individual whose choice is described by the Bewley Expected Utility (BEU) model of Bewley (1986). By relaxing Savage’s (1954) requirement that preferences are complete Bewley (1986) develops a theory of individual choice under which preferences can be represented through unanimity over a set of priors.$^{12}$ Unanimity is completed in Bewley’s model by assuming that individual agents are subject to an inertia or status quo bias when they cannot rank alternatives unanimously across all priors.$^{13}$ The inertia aggregation rule can therefore be thought of as the corporate choice equivalent of the BEU model. When there is a choice between a pair of acts and the status quo is

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$^{12}$In general, incompleteness can be either about beliefs or tastes, or both. Incompleteness about beliefs leads to a multi-prior representation of preferences, while incompleteness about tastes leads to a “multi-utility” representation. In the seminal work of Bewley (1986), incompleteness is only about beliefs, not tastes. In more recent work, Ok, Ortoleva, and Riella (2012) and Galaabaatar and Karni (2013) explore the case of incompleteness in both beliefs and tastes. In our paper the focus is on incompleteness about beliefs since we assume that all group members have the same utility function.

$^{13}$For a discussion of the role of status quo in decision making, see Samuelson and Zeckhauser (1988). It is important to note that in BEU the status quo is a device to complete the model of choice and is not part of the preference specification. The treatment of the status quo is therefore conceptually different from that of the behavioural economics literature that relies on “reference-dependent” preferences to analyze the implications of
selected, any members who derives a larger utility from the status quo is pivotal: his individual choice is aligned with the DMG’s choice.

3.2 Learning and dynamic consistency

The aggregation rules described above refer to a static decision problem. In the example of Section 2 the DMG faces a dynamic problem with decisions both before and after the arrival of information. To formally study this problem, we need to specify the learning framework and the evolution of the aggregation rule over time.

We consider a three-date model, where a group of agents makes decisions at time 0 and time 1. All group members learn at time 1 whether the time 2 state of the world belongs to a set \( E = \{ s_{K+1}, \ldots, s_N \} \) with \( K \) an integer such that \( 1 \leq K \leq N - 2 \). Each group member receives the same information and updates his beliefs according to Bayes’s rule. For any \( \pi^j \in \Pi \) we denote by \( \pi^j_E \) the posterior belief of group member \( j \) obtained through Bayesian updating upon learning the event \( E \), i.e., \( \pi^j_E(D) = \frac{\pi^j(D \cap E)}{\pi^j(E)} \), for any subset \( D \) of \( S \), and by \( \Pi_E = \{ \pi^1_E, \ldots, \pi^J_E \} \) the corresponding set of updated beliefs. We refer to the preferences before the event \( E \) is revealed as unconditional and to the preferences after learning \( E \) as conditional. Group member \( j \)’s conditional utility of act \( f \) is

\[
E_{\pi^j_E}[u(f)] = \sum_{s \in S} \pi^j_E(s)u(f(s)) \equiv \sum_{s \in E} \pi^j_E(s)u(f(s))
\]

where the last equality follows from the fact that \( \pi^j_E(s) = 0 \) for all state \( s \) in the complement event \( E^c = E \setminus S \). Throughout the rest of the paper we make the following assumption

**Assumption 1.** The information received by the DMG at time 1 is described by an event \( E \) that is considered plausible under all priors, i.e., \( 0 < \pi(E) < 1 \) for all \( \pi \in \Pi \). Furthermore, the set \( \Pi_E \) of updated priors contains more than one element.

The requirement that the event \( E \) has a positive probability for all group members implies Bayes’s rule is well defined for each member. The requirement that the set \( \Pi_E \) contains more than one conditional belief implies that after learning there is still belief heterogeneity among group members. If all group members held the same updated beliefs, they would unanimously biases such as the endowment effect, loss aversion or framing. See, for example, Kahneman and Tversky (1979), Thaler (1980), Kahneman, Knetsch, and Thaler (1991), and Tversky and Kahneman (1991).
rank any choice and therefore all aggregation rules that are consistent with unanimity, as the one
we consider in this paper, will give the same group preference.

We further assume that the members of the DMG agree to be governed by an aggregation
rule that applies, unaltered, both before and after the information about the event \( E \) is revealed
at time 1 and without excluding any member from the aggregation procedure. This implies
that learning only impacts the set of priors that are fed into the aggregation rules but not the
aggregation rule itself.\(^{14}\) In the context of single-agent decision making under multiple prior, this
assumption is typically referred to as \textit{full Bayesian updating, prior-by-prior}.\(^{15}\)

We now formally define the concept of \textit{dynamic consistency} of an aggregation rule.

**Definition 1 (Dynamic consistency).** An aggregation rule is dynamically consistent if, for all
acts \( f \) and \( g \) such that \( f = g \) on the event \( E \) (or its complement \( E^c \)), the DMG’s choice between
\( f \) and \( g \) is the same before and after learning.

The definition states that aggregation rules are dynamically consistent if the time 1 conditional
DMG ranking of the acts \( f \) and \( g \) coincides with the time 0 unconditional DMG ranking of
the same acts. In order to make the comparison meaningful, the time consistency requirement
only concerns the acts \( f \) and \( g \) that have the same consequence on the complement \( E^c \). As a
natural extension from the static case, a dynamic aggregation consists of a criterion for picking a
current pivotal member for any time 0 unconditional choice and then a future pivotal member for
any time 1 conditional choice. Aggregation rules that are dynamically consistent select pivotal
members whose identities do not change from time 0 to time 1. When there is no single individual
who is pivotal both at time 0 and at time 1, there must be disagreement between these successive
pivotal members and as a result, the aggregation rule is \textit{time inconsistent}.

\(^{14}\)Note however that we allow the status quo at time 0 to be different from the status quo at time 1. This
flexibility is important for our application since a change in status quo happens quite naturally in many situations
that are relevant for corporate decisions such as the choice of whether or not to start a new factory considered in
the example of Section 2. Before the investment the status quo is to walk away from the investment while after the
status quo is to continue operating the plant.

\(^{15}\)The issue of updating in the presence of multiple prior is a topic of active research in the decision theory
literature that deals with individual choice. This literature showed that there is a tension between full Bayesian
updating and time consistency. In response to this tension the literature proposed alternative updating rules to the
full Bayesian updating rule that typically require that some priors be excluded from Bayesian updating (see, e.g.,
Gilboa and Schmeidler (1993) and Hanany and Kilbanoff (2007)). If applied to groups, these rules would imply the
exclusion of some group members from the decision process as the group learns. Because we want to preserve the
empirically relevant fact that each group member retains some influence over the group decision as the group learns
we do not consider alternatives to full Bayesian updating prior by prior (or group member by group member).
3.3 Time inconsistency of aggregation rules

In this subsection we show that inconsistency is a typical feature of the three aggregation rules introduced in Subsection 3.1. A DMG that relies on these aggregation rules to make decision over time may therefore reverse its unconditional preferences between acts after learning.

Utilitarian aggregation rule

The next proposition provides necessary and sufficient condition for under which the utilitarian aggregation rule is time consistent.

**Proposition 1.** When Assumption 1 holds, any non dictatorial utilitarian aggregation rule is dynamically consistent if and only if all group members agree on the probability of the event $E$, that is $\pi^i(E) = \pi^j(E)$ for all $i, j = 1, \ldots, J$.

When the condition of this proposition is not satisfied, i.e. when there exist $i$ and $j$ such that $\pi^i(E) \neq \pi^j(E)$, the aggregation rule is time inconsistent and so there must exist a pair of acts $f$ and $g$ for which the pivotal member at time 0 differs from that at time 1 and the two disagree on the ranking of $f$ and $g$. This result is somewhat surprising since the utilitarian weight assigned to each group member is constant over time. Recall however that the determination of pivotal members according to the utilitarian rule depends not only on his influence on the group, measured by the utilitarian weight, but also by his intensity of preference, measured by the difference in the expected utility of the two acts being evaluated.

To understand the result in Proposition 1 it is important to explain why the intensity of preference may change with learning. Consider an SEU individual who thinks that the event $E^c$ is almost certain to occur and who must pick one of two distinct acts, $f$ and $g$, with $f = g$ on the event $E^c$. Unconditionally, this agent is almost indifferent between the two acts because they deliver the same payout on an event $E^c$ that is very likely to occur. However, if the event $E$ happens, as a consequence of Bayesian updating of his prior, the agent may perceive the two acts as being conditionally very different. This shift in view may result in a very high intensity of preferences for one act over the other. That is, while, ex ante, the individual is almost indifferent between $f$ and $g$ and hence unlikely to be pivotal, ex post, he may become pivotal if learning increases his intensity of preference towards one of the two acts.
To formalize the above intuition, recall that at time 0, the DMG acts as a fictitious SEU agent with a prior $\Lambda^\pi$ equal to the weighted average of the unconditional beliefs of all group members (see equation (7)). At time 1, if the event $E$ is realized, the DMG acts as an SEU individual with belief equal to the weighted average of the group members’ conditional beliefs

$$\Lambda^{\pi E}(s) = \sum_{j=1}^{J} \lambda_j \tilde{\pi}^j_E(s) \equiv \sum_{j=1}^{J} \lambda_j \frac{\pi^j(s)}{\pi^j(E)} \text{ for all } s \in E. \quad (11)$$

Time inconsistency arises when the DMG’s “implicit” conditional beliefs $\Lambda^{\pi E}$ differs from the Bayesian update $\Lambda^E_E$ of the DMG’s “implicit” unconditional belief $\Lambda^\pi$, defined by

$$\Lambda^E_E(s) \equiv \frac{\Lambda^\pi(s)}{\Lambda^\pi(E)} = \frac{\sum_{j=1}^{J} \lambda_j \pi^j(s)}{\sum_{j=1}^{J} \lambda_j \pi^j(E)} \text{ for all } s \in E. \quad (12)$$

According to Proposition 1 when all group member have the same unconditional belief about the event $E$, the conditional belief $\Lambda^{\pi E}$ given in (11) is the Bayesian update of the unconditional belief $\Lambda^\pi$ given in formula (12). Moreover, when group members disagree on the probability of the event $E$ but the aggregation rule is dictatorial with the dictator $j^*$ at all times, $\lambda_j = 0$ for all $j \neq j^*$ and $\lambda_{j^*} = 1$ for all decision times. Equation (11) gives $\Lambda^{\pi E} = \pi^{j^*}_E$ and equation (7) shows that $\Lambda^\pi = \pi^{j^*}$. Therefore we have $\Lambda^\pi_E = \Lambda^{\pi E}$. In all other cases, i.e. when $\Lambda^\pi_E \neq \Lambda^{\pi E}$, the utilitarian aggregation rule is time inconsistent.

**Rawlsian aggregation rule**

The next proposition provides necessary and sufficient condition under which the Rawlsian aggregation rule is time consistent.  

**Proposition 2.** The Rawlsian aggregation rule is dynamically consistent if and only if for every $\pi’, \pi'' \in \Pi = \{\pi^1, \ldots, \pi^J\}$ and event $B = E, E^c$ the belief $\pi^R$ defined as

$$\pi^R(s) = \pi'(B) \pi''_B(s) + \pi'(B^c) \pi''_{B^c}(s), \text{ for all } s \in S \quad (13)$$

belongs to the convex hull of the set $\Pi$,

$$co(\Pi) = \left\{ \sum_{j=1}^{J} \mu_j \pi_j \mid \mu_j \geq 0 \text{ and } \sum_{j=1}^{J} \mu_j = 1 \right\}$$
Condition (13) is the “rectangularity” condition introduced by Epstein and Schneider (2003) to axiomatize an intertemporal, dynamically consistent version of Gilboa and Schmeidler’s (1989) MEU model in which an individual agent updates prior-by-prior via Bayes’s rule. As for the case of utilitarian aggregation rule, the intuition for the result in Proposition 2 rests on the idea that an aggregation rule is dynamically consistent if the identity of all pivotal members do not change over time.

To illustrate this point, consider the example in Section 2, but assume that $O&P$ is governed by the Rawlsian aggregation rule. From the individual conditional valuations (3) the Rawlsian rule select $A$ at time 1 and from the unconditional valuations (4), the Rawlsian rule selects $C$ at time 0. Because $A$ is Olivia’s conditional choice at time 1 and $C$ is Pietro’s unconditional choice at time 0 we have that Pietro is pivotal at time 0 and Olivia is pivotal at time 1. Since Pietro and Olivia disagree on the choice between the acts $C$ and $A$, the aggregation rule is time inconsistent.

It is easy to see that the set of priors $\Pi = \{\pi^O, \pi^P\}$ in the example does not satisfy condition (13) and that the set of priors $\Pi^R$ that satisfy (13) must include the following two priors:

$$\pi^{OP} = \left(\frac{1}{3} \times 1, \frac{2}{3} \times \frac{8}{9}, \frac{2}{3} \times \frac{1}{9}\right) = \left(\frac{1}{3}, \frac{16}{27}, \frac{2}{27}\right)$$

$$\pi^{PO} = \left(\frac{1}{10} \times 1, \frac{9}{10} \times \frac{1}{2}, \frac{9}{10} \times \frac{1}{2}\right) = \left(\frac{1}{10}, \frac{9}{20}, \frac{9}{20}\right),$$

where $\pi^{PO}$ ($\pi^{OP}$) is a “mixture” of the unconditional beliefs of Pietro (Olivia) and the conditional beliefs of Olivia (Pietro). In this case, group member $OP$ is pivotal both at time 0 and time 1 and group choice is to not invest in the first place.

**Inertia-based aggregation rule**

The next proposition provides necessary and sufficient conditions for the dynamic consistency of the inertia aggregation rule.

**Proposition 3.** The Inertia aggregation rule is dynamically consistent if and only if, or any pair of acts $f$ and $g$ that are not unanimously ranked by group members and such that $f = g$ on the event $E$ (or its complement $E^c$), the status quo act at time 0 is identical to the status quo act at time 1.
The intuition for this result is simple. Since agents evaluate choices relative to the status quo, if the status quo does not change, the expressed preferences will not change. The DMG reverses its choice over time if an only if the status quo changes and if there is some disagreement within the group. When the status quo changes over time in the presence of disagreement among group members, the inertia-based aggregation rule is time inconsistent with any set of priors, including the rectangular set discussed above.

The above three propositions show that time inconsistency emerges naturally for collective decision making even if each group member has SEU preferences and uses Bayes rule to update his subjective belief. For instance, dynamic consistency of the utilitarian aggregation rule holds in the “knife edge” cases in which: (i) the group is ruled by either an entrenched and self-interested dictator, i.e., a single decision maker, or (ii) all group members agree on the probability of the events occurring. Both assumptions fail to capture, for example, the behaviour of a board of directors or a policy committee. While it is certainly true that in committees, members may have different degrees of influence, assuming the existence of a dictator who makes decisions and disregards the belief of other members is an arguably extreme assumption. Moreover, assuming that all members agree on the probability of the events, as required in Proposition 1, restricts in a non trivial way the heterogeneity in beliefs across the group members.

The requirements for time consistency of the Rawlsian rule, stated in Proposition 2, also seem to impose significant restrictions to the structure of the DMG. The rectangularity condition (13) requires that the group contain members whose opinions are mixtures of the opinions of other group member. This can imply, for example the existence of group member whose opinion changes from pessimism to optimism upon receiving bad news and vice-versa, as well as members who never change their stance. Furthermore, rectangularity would also imply that as the complexity of the problem increases (more states and/or decision nodes) the number of group members with different beliefs has to grow accordingly to insure dynamic consistency. While at the individual level rectangularity is a justifiable requirement for dynamic consistency, a direct interpretation of this requirement at the group level is conceptually difficult.

Finally, Proposition 3 shows that the inertia aggregation rule is dynamically consistent if the status quo does not change over time. This assumption does not capture the behaviour of corporations where the status quo change naturally as the corporation invests.
In summary, the conditions under which the three aggregation rules we study in this paper are dynamically consistent do not seem to be of practical relevance for modelling the behaviour of groups such as partnerships, policy committees, corporate boards, or financing syndicates. We conclude that, when a group of members with heterogeneous beliefs must make a collective choice that affect all members uniformly, time inconsistency emerges as an inescapable phenomenon. In what follows, we show that time inconsistency of aggregation rule has real consequences in that it might lead to underinvestment: projects that are considered profitable by each group member will be passed on by the group.

4 Aggregation rules and underinvestment

To study the real implications of the dynamic inconsistency of the aggregation rules discussed in the previous sections we revisit the example of Section 2 and generalize it to an arbitrary state space, $S = \{s_1, \ldots, s_N\}$, set of priors $\Pi = \{\pi^1, \ldots, \pi^J\}$, and event $E = \{s_{K+1}, \ldots, s_N\}$ for $1 \leq K \leq N - 2$. In this general setting, we derive conditions under which the underinvestment phenomenon highlighted in the example of Section 2 emerges as a consequence of the time inconsistency of an aggregation rule.

As in Section 2 we consider a self-financed DMG whose members are risk neutral\textsuperscript{16} SEU agents with heterogeneous beliefs. The DMG has monopoly access to a project that consists of a unit of capital that delivers a state-dependent cash flow at time 2. We study decisions made by the DMG at two different points in time. The first decision, is whether to invest an amount $I$ to acquire the unit of capital at time 0. We denote by $N$ the choice to not invest at time 0. If the initial investment is made, the DMG faces a second decision at time 1 if at that time the members of the DMG learn that an event $E$ has occurred. In this case the DMG has to decide whether to continue ($C$) or abandon ($A$) the project. All the cash flows are realized at time 2. To simplify the exposition, we assume that if the event $E^c$ is revealed to be true, the DMG does not have any decisions to make and the project continues.\textsuperscript{17} In addition to Assumption 1, which, as discussed above, insures non-degenerate cases of learning, we require that the set of time 2 outcomes of the act $C$ and $A$ satisfy the following assumption:

\textsuperscript{16}Our qualitative results hold as long as all members have the same risk preferences.
\textsuperscript{17}The model can be easily generalized to also include decisions to expand or abandon in both nodes at time 1. This generalization will not change the key economic implications of the different aggregation rule used by a group to reach a collective choice.
Assumption 2. The time 2 outcomes of acts $A$ and $C$ are such that:

$$C(s_1) > C(s_2) > \ldots > C(s_{N-1}) > C(s_N),$$

(14)

$$A(s) = \begin{cases} 
C(s), & \text{for } s \in E^c \\
A > 0, & \text{for } s \in E
\end{cases}$$

(15)

where the constant $A$ satisfies such that

$$C(s_{M+1}) \leq A < C(s_M), \text{ for an integer } M \in \{K + 1, \ldots, N - 1\}.$$  

(16)

Condition (14) is without loss of generality and simply labels the states according to the payoff of the technology. Condition (15) captures the idea that in the “good states”, $E^c$, the DM does not make any decision and hence $C$ and $A$ coincide, while in the “bad states”, $E$, abandoning the project, delivers a constant outcome corresponding to the liquidation value of the project. The key difference between $A$ and $C$ is that the abandonment option, $A$, delivers a cash flow $A$ that is known with certainty.\footnote{Condition (15) can be relaxed to allow for a non constant $A$, provided we preserve the assumption that there is less disagreement about the payoff from $A$ than there is about that of $C$.} Finally, condition (16) implies that the payoff of $C$ does not dominate and is not dominated by the payoff of $A$ since these two acts would be ranked unanimously by all parties if one dominated the other.

We solve the DMG’s investment decision problem recursively determining first the decision to continue or abandon at time 1 and then determine the initial investment decision at time 0.

The main result of this section is to show that when a DMG uses an aggregation rule that is dynamically inconsistent when deciding on the investment opportunity described above, underinvestment may arise. To provide conditions under which underinvestment may arise, we start with a formal definition of underinvestment.

Definition 2 (Underinvestment). In the context of a dynamic investment problem with follow-on operating options, underinvestment occurs when: (i) each group members would invest if they were the single owner of the technology, but (ii) the DMG foregoes investment.

In the example of Section 2 there is underinvestment because both Pietro and Olivia would undertake the project when asked individually, but they would not as a group. The DMG forgoes the investment because Pietro and Olivia disagree about how to operate the firm in the event of
an economic downturn: Pietro would prefer to continue operating the project while Olivia would abandon. Despite the difference of opinion, both would agree to invest if they expected that they would continue in a downturn - although Olivia sees more value in abandoning she is still better off investing and continuing. However, the future disagreement causes underinvestment because both parties recognize that the aggregation rule will result in abandonment. We refer to this form of underinvestment as being caused by the DMG’s ‘Eagerness to Abandon’ (ETA hereafter) the project. A different form of underinvestment may arise for the opposite reason, i.e., the future pivotal members prefers to continue operating the project while the current pivotal members prefer to abandon in the future. We refer to this form of underinvestment as being caused by the DMG’s ‘Eagerness to Continue’ (ETC hereafter).

In the rest of this section, we provide conditions under which ETC and ETA–underinvestment may arise as a consequence of the time inconsistency of the three aggregation rules covered in section 3.1. The nature of underinvestment depends critically on the aggregation rule that the DMG adopts; while the Utilitarian and Inertia rules can lead to both ETC and ETA–underinvestment, the Rawlsian rule can only lead to ETA underinvestment.

4.1 ETA–Underinvestment

Utilitarian rule. The next proposition provides sufficient conditions under which underinvestment occurs for a DMG who is governed by the utilitarian aggregation rule.

Proposition 4. Consider a DMG who is governed by the utilitarian aggregation rule with weights \( \lambda_1, \ldots, \lambda_J \), and suppose that

\[
\mathbb{E}_{\Lambda^{\pi \pi}}[C] < \mathbb{E}_{\Lambda^{\pi \pi}}[A] \equiv A \quad \text{and} \quad \mathbb{E}_{\Lambda^{\pi \pi}}[A] < \mathbb{E}_{\Lambda^{\pi \pi}}[C],
\]

(17)

where \( \Lambda^{\pi \pi} \) is the unconditional prior defined in (7) and \( \Lambda^{\pi \pi} \) is the conditional prior defined in (11). If the initial investment cost \( I \) is such that

\[
I < \max\{\mathbb{E}_{\pi j}[C], \mathbb{E}_{\pi j}[A]\}, \quad \text{for all } j = 1, \ldots, J,
\]

(18)

and

\[
\mathbb{E}_{\Lambda^{\pi \pi}}[A] < I,
\]

(19)

then time inconsistency of the utilitarian aggregation rule results in ETA–underinvestment.
Proof: Condition (17) implies that the DMG is time inconsistent: The first inequality in (17) implies that the DMG would choose the act $A$ at time 1 while the second inequality implies that the DMG chooses the act $C$ at time 0. Since the members of the DMG anticipate the future choice they realize that the act $C$ will not be selected at time 0. The time 0 choice is therefore between $A$ and $N$, and, by condition (19), the DMG does not invest. The $J$ inequalities (18) imply that each group member would invest if he/she was the sole owner of the technology. Hence there is ETA–underinvestment.

Notice that when condition (18) is replaced with the more stringent condition

$$I < \min_{\pi \in \Pi} E_\pi [C],$$  \hspace{1cm} (20)

underinvestment can potentially be resolved if the group members were able to pre-commit to the act $C$. Inequality (20) shows indeed that all group members would unanimously support investing in the project and operating it with the policy $C$ to not investing, $N$ and hence investment would take place at time 0. Note that, in the example of Section 2, conditions (17), (19) and (20) hold.

**Rawlsian rule.** The next proposition parallels the result of Proposition 4 to the Rawlsian aggregation rule.

**Proposition 5.** Consider a DMG that is governed by the Rawlsian aggregation rule and suppose that

$$\min_{\pi \in \Pi_E} E_\pi [C] < \min_{\pi \in \Pi_E} E_\pi [A] \equiv A \text{ and } \min_{\pi \in \Pi} E_\pi [A] < \min_{\pi \in \Pi} E_\pi [C].$$  \hspace{1cm} (21)

If the initial investment cost $I$ is such that condition (18) holds and

$$\min_{\pi \in \Pi} E_\pi [A] < I,$$

then, time inconsistency of the Rawlsian aggregation rule results in ETA–underinvestment.

Proof: The proof is identical to that of Proposition 4 after noticing that the interpretation of conditions (21) and (22) parallels the interpretation of conditions (17) and (19) in Proposition 4.

As for the utilitarian aggregation rule, if the “desirability” condition (18) is replaced with the more stringent condition (20), then the underinvestment problem can be solved if the group
members can pre-commit to the policy \( C \). The example following Proposition 2 above describes a situation where conditions (20), (21), and (22) are satisfied and where ETA–underinvestment occurs.

**Inertia rule.** Although theoretically possible, ETA–underinvestment does not seem realistic because it requires that following the investment the status quo is to abandon. In the next subsection we discuss the more appealing situation in which the status quo is to continue operation and ETC–underinvestment occurs.

### 4.2 ETC–Underinvestment

ETC–underinvestment occurs when the DMG prefers \( C \) to \( A \) conditional on learning the event \( E \) at time 1 but prefers \( A \) over \( C \) unconditionally at time 0.

**Utilitarian rule.** Suppose the DMG is governed by a utilitarian aggregation rule with weights \( \lambda_1, \ldots, \lambda_J \). Using the notation in Proposition 4, ETC–underinvestment occurs when the desirability condition (18) and the following conditions are satisfied:

\[
\begin{align*}
(i) & \ E_{A^*E}[C] > E_{A^*E}[A] \equiv A; \\
(ii) & \ E_{A^*}[A] > E_{A^*}[C]; \text{ and } (iii) \ E_{A^*}[C] < I.
\end{align*}
\] (23)

Conditions (i) and (ii) imply time inconsistency of the aggregation rule while condition (iii) indicates that the DMG, anticipating the future choice of \( C \), responds to the time inconsistency by not investing at time 0. If the desirability condition (18) is replaced with the more stringent condition \( I < \min_{\pi \in \Pi} E_{\pi}[A] \), the underinvestment problem can be solved when the utilitarian DMG can pre-commit to the chosen act \( A \) at time 1.

**Rawlsian rule.** When the DMG utilizes the Rawlsian aggregation rule, following the logic of Proposition 5, ETC–underinvestment requires conditional rejection of the “unambiguous” act \( A \) at time 1, i.e.,

\[
\min_{\pi \in \Pi_E} E_{\pi}[A] \equiv A < \min_{\pi \in \Pi_E} E_{\pi}[C] \text{ and } \min_{\pi \in \Pi} E_{\pi}[C] < \min_{\pi \in \Pi} E_{\pi}[A].
\] (24)

The above conditions are impossible under the Rawlsian aggregation rule. To see why, note that, as discussed above, under the Rawlsian aggregation rule the DMG behaves as an MEU ambiguity averse single decision maker. Because the act \( A \) is conditionally unambiguous, its rejection at
time 1 means that an unambiguous act has lower utility to an ambiguity averse agent than an ambiguous act. If this is the case, the act $A$ cannot be preferred to the ambiguous act $C$ at time 0.\footnote{Formally, suppose $A < \min_{\pi \in \Pi} E_{\pi}[C]$ as in (24) and consider any prior $\pi \in \Pi$. Then,

\begin{align*}
E_{\pi}(A) &= \pi(E)E_{\pi|E}(A) + \pi(E^c)E_{\pi|E^c}(A) \\
&= \pi(E)A + \pi(E^c)E_{\pi|E^c}(C) \\
&\leq \pi(E)E_{\pi|E}[C] + \pi(E^c)E_{\pi|E^c}(C) = E_{\pi}[C],
\end{align*}

where the first equality follows by the law of total probability, the second uses the fact that $A = A$ on $E$ and $A = C$ on $E^c$, and the inequality follows from $A < \min_{\pi \in \Pi} E_{\pi}[C]$. The above inequality shows that for any given $\pi \in \Pi$, $E_{\pi}(A) \leq E_{\pi}(C)$ and therefore, $\min_{\pi \in \Pi} E_{\pi}[A] \leq \min_{\pi \in \Pi} E_{\pi}[C]$ thus proving the impossibility of (24).}

**Inertia rule.** When the DMG is governed by the inertia aggregation rule, ETC–underinvestment occurs when: (i) the status quo at time 0 is the act $N$ initially and then it switches at time 1 to the act $C$, and (ii) time-inconsistency condition (24) is satisfied, and (iii) the initial investment cost $I$ is such that condition (18) and

$$\min_{\pi \in \Pi} E_{\pi}[C] < I$$

are satisfied. The two inequalities in condition (24) imply that the act $C$ and the act $A$ are not unanimously ranked at time 1 and therefore the DMG chooses the status quo act $C$ at time 1. Knowing this, inequality (25) and the inertia assumption imply that the DMG chooses the status quo act $N$ at time 0, i.e., not to invest. As in Proposition 4, condition (18) implies that each group members would like to invest as a sole owner of the technology. Therefore, the inertia aggregation rule induces underinvestment due to eagerness to continue. Finally, if condition (18) is replaced with the more stringent condition $I < \min_{\pi \in \Pi} E_{\pi}[A]$, the underinvestment problem can be solved if the DMG can precommit to the act $A$ at time 0.

5 **Eliminating underinvestment through security issuance**

The previous section shows that that when all group members agree that investing in the firm and running it according to a given operating policy is better than not investing, underinvestment arises when the DMG lacks the ability to pre-commit to such an operating policy. In this section we show that the issuance of securities is a mechanism through which group members can pre-commit to a commonly desired operating policy and hence resolve the underinvestment problem. A security can be designed to alter the payoffs in such a way as to “generate unanimity” with
respect to a desired operating policy. For sufficiently large issuance prices, group members will find it optimal to issue the security and invest in the technology. Notice, however, that security issuance is obviously not the only way in which a group can resolve the underinvestment problem. Section 6 discusses alternative solutions.

In the next two subsection we describe the types of contracts that are needed to resolve underinvestment due to eagerness to abandon (Subsection 5.1) and eagerness to continue (Subsection 5.2).

5.1 Risky bonds and ETA-underinvestment

ETA-underinvestment can be resolved by issuing securities that alter the project’s residual payoff in such a way as to induce all group members to prefer the continue option \( C \) to the abandon option \( A \) at time 1. In this subsection we show that risky bonds can achieve this goal, for sufficiently large issuance price. The next proposition addresses the case where the DMG is utilitarian.

**Proposition 6.** Consider a DMG that follows an utilitarian aggregation rule with weights \( \lambda_1, \ldots, \lambda_J \). Suppose that conditions (17), (19), and (20) hold so that the DMG faces an underinvestment problem caused by eagerness to abandon. Consider a risky bond that promises at time 2 a face value \( X \) satisfying

\[
A < X < C(s_M). \tag{26}
\]

If the price \( P \) that the financier is willing to pay for the risky bond satisfies

\[
I - P \leq \mathbb{E}_{\lambda^*}[C - X]^+, \tag{27}
\]

with \( \lambda^* \) the unconditional belief defined in (7) and \([x]^+ := \max(0, x)\), then the DMG: (i) issues the bond and invest in the project at time 0, and (ii) chooses to continue, \( C \), if the event \( E \) occurs at time 1.

To gain intuition, note that the left inequality of (26) implies that the bond triggers default with certainty if the DMG abandons at time 1. Note also that if the DMG continues at time 1, inequality (16) together with inequality (26) show that \( C(s_{M+1}) < X < C(s_M) \). Default is then triggered in states \( s_{M+1}, \ldots, s_N \) but the firm is solvent in states \( s_{K+1}, \ldots, s_M \). Therefore, group members are left with non-zero payoff in some states if the group decides to continue the project.
Because group members have nothing to lose from continuing and face certain default if they abandon, continuing at time 1 generates unanimity.\footnote{When the DMG is Rawlsian, the security must induce unanimity on the conditional choice of \( \mathcal{C} \) in order to solve the underinvestment problem. However, when the DMG is utilitarian, unanimity is not necessary since a security can potentially manipulate the intensity of preferences and reverse the DMG decision without altering the members’ conditional ranking of the acts \( \mathcal{C} \) and \( \mathcal{A} \). The assumption that the DMG is insolvent when the firm is abandoned (i.e. \( A < X \)) is not necessary to resolve the underinvestment problem. It is possible to construct bonds that have a face value \( X < A \) and yet provide the incentives to continue the firm at time 1.}

The bond price \( P \) in Proposition 6 is determined by the financier’s willingness to pay and it is net of possible transaction costs associated with the issuance. The left hand side of inequality of (27) is the investment required from the DMG net of funds issued by the financier to fund the project. Inequality (27) is a financing constraint that requires that the expected cash flow to equity be less than the initial investment required from equity based on the belief \( \Lambda^\pi \). Using the fact that \( I \leq \mathbb{E}_{\Lambda^\pi} [\mathcal{C}] \), it can be verified that the financing constraint (27) is satisfied when the financier uses the probability \( \Lambda^\pi \) to value the bond.

An equivalent to Proposition 6 also holds for the case of a DMG that follows the Rawlsian aggregation rule if we replace conditions (17) and (19) with, respectively, (21), and (22). In this case the result in Proposition 6 holds for a Rawlsian DMG, when the bond price \( P \) satisfies \( I - P \leq \min_{\pi \in \Pi} \mathbb{E}_\pi [\mathcal{C} - X]^+ \). This last condition is the equivalent of (27) for the case of a Rawlsian DMG. If we consider a Rawlsian DMG in the numerical example in Section 2, it can be easily verified that a bond with face value $710 satisfies condition (26) and can hence be used to solve the underinvestment problem if its price \( P \) is above \( I - \min_{\pi \in \Pi} \mathbb{E}_\pi [\mathcal{C} - X]^+ = $985 - $361 = $624. \)

5.2 Convertible bonds and ETC-underinvestment

ETC-underinvestment can be resolved by issuing securities that alter the project’s residual payoff in such a way as to induce all group members to prefer the abandonment option \( \mathcal{A} \) to the continuation option \( \mathcal{C} \) at time 1. In this subsection we show that convertible bonds can be designed to achieve this goal.\footnote{It is easy to verify that issuing risky bonds cannot mitigate ETC-underinvestment. Under our technology Assumption 2 a risky bond can only incentivize group members to choose the act \( \mathcal{C} \) as in Section 5.1.}

Consider a convertible bond issued by the DMG at time 0 to a financier. The bond promises a repayment of \( X \) at time 2 and, at the option of the bondholder, is convertible into a fraction \( \alpha \) of the firm’s equity at time 2. When the firm is operated under the policy \( \mathcal{P} = \mathcal{C} \) or \( \mathcal{A} \) at time 1 and the bond is converted in state \( s \), bondholders get \( \alpha \mathcal{P}(s) \) and the DMG get \( (1 - \alpha) \mathcal{P}(s) \). When
the bond is not converted in state \( s \), bondholders get \( \min(X, \mathcal{P}(s)) \) and the DMG get the residual claim \( \max(\mathcal{P}(s) - X, 0) \).

The next proposition shows that, when the DMG follows the inertia rule, a convertible bond can be designed in such a way as to “penalize” payoffs from choosing \( \mathcal{C} \) instead of \( \mathcal{A} \). Convertibles bonds are effective in this case, because in states where the project cash flows are poor, bondholders extract more value from the bond by not converting it. Not converting can be thought of as a threat that deter the most optimistic group member from continuing the project. The convertible bond induces unanimity on the choice of \( \mathcal{C} \) versus \( \mathcal{A} \) and, consequently, eliminates underinvestment provided that the issuance price is sufficiently large. Convertible bonds achieve the opposite effect of the risky bonds discussed in the previous subsection where the security was designed to neutralize the negative effect on investment from the most pessimistic priors.

**Proposition 7.** Consider a DMG that follows the inertia-based aggregation rule with status quo \( \mathcal{C} \). Suppose further that conditions (24), and (25) are satisfied and that \( I < \min_{\pi \in \Pi} E_{\pi} [\mathcal{A}] \) so that the DMG faces an ETC-underinvestment. Consider a convertible bond with maturity at time 2, face value \( X \), and conversion ratio \( \alpha \) that satisfy

\[
\alpha \mathcal{C}(s_{M+1}) < X < \min(\alpha A, \mathcal{C}(s_{M+1}))
\]

and

\[
\max_{\pi \in \Pi_E} \left[ \sum_{i=K+1}^{M} (1 - \alpha)\mathcal{C}(s_i)\pi(s_i) + \sum_{i=M+1}^{N} \max(\mathcal{C}(s_i) - X, 0)\pi(s_i) \right] \leq (1 - \alpha)A,
\]

If the price \( P \) that the financier is willing to pay for the convertible bond satisfies

\[
P \geq P =: \alpha \min_{\pi \in \Pi_{E}} E_{\pi} [\mathcal{A}],
\]

then the DMG (i) issues the bond, (ii) invests in the project at time 0, and (iii) chooses to abandon if the event \( E \) occurs at time 1.

Under the conditions of Proposition 7, the group member’s best response at time 1 to the bondholder’s conversion strategy at time 2 is to abandon when the event \( E \) occurs. To see this, assume first that the DMG abandons at time 1 in which case the firm’s payout is \( A \). The right hand side of (28) shows that \( X < \alpha A \) and thus bondholders will convert in all states \( s \in E \). Assume now that the DMG continues the project. By the right inequality in (28) and condition (16),
$X < \alpha C(s)$ for $s \in \{s_{K+1}, \ldots, s_M\}$. By the left inequality in (28) and the fact that the payoff $C(s)$ decrease in the state $s$ (see Assumption 2), $\alpha C(s) < X$ for $s \in \{s_{M+1}, \ldots, s_N\}$. Thus the bondholders convert at time 2 in states $s \in \{s_{K+1}, \ldots, s_M\}$ but not in states $s \in \{s_{M+1}, \ldots, s_N\}$. Anticipating these reactions from the bondholder in states $s \in E$, the DMG compares, at time 1, the payoff $\hat{C}(s)$ resulting from the status quo $C$ to that of the alternative $A$, i.e.,

$$\hat{C}(s) = \begin{cases} (1 - \alpha)C(s) & \text{if } s \in \{s_{K+1}, \ldots, s_M\} \\ \max(0,C(s) - X) & \text{if } s \in \{s_{M+1}, \ldots, s_N\} \end{cases}, \text{ versus } (1 - \alpha)A. \quad (31)$$

If condition (29) is satisfied, the DMG chooses to abandon the status quo $C$ and shuts down.

The price $P$ defined in (30) is the price offered by a financier with MEU preferences endowed with an identical set of priors $\Pi$ to the DMG in exchange of the bond. Condition (30) says that even if the convertible bond is priced by using the worst prior of the DMG members, issuing such a bond solves the ETC-underinvestment problem. As for the risky bond solution to underinvestment, the proof of Proposition 7 illustrates that there might be many convertible bonds differing in their face values $X$ and conversion ratios $\alpha$ that could mitigate the underinvestment problem.

The following example provides a numerical illustration of one such convertible bond. Consider a variation of the example of Section 2 where the DMG is composed of two members (Olivia and Pietro) who follow an inertia-based aggregation rule with status quo given by $N$ at time 0 and $C$ at time 1. Olivia and Pietro’s priors are $\pi^O = (1/3, 1/3, 1/3)$ and, $\pi^P = (1/10, 1/10, 8/10)$ The continue ($C$) and abandon ($A$) actions are characterized by the following payoffs $C = (2, 000, 1, 000, 580)$ and, $A = (2, 000, 780, 780)$. Direct computation show that for any investment cost $I$ between $764$ and $902$, ETC-underinvestment occurs. It can also be shown that issuing a convertible bond with face value $X = 77$ and conversion ratio $\alpha = 0.1$ solves the ETC-underinvestment even when the financier price it by making use of the worst belief $\pi^P$.

An equivalent to Proposition 7 holds for the case of a DMG that follows a utilitarian aggregation rule with weights $\lambda_1, \ldots, \lambda_J$, if in the proposition we replace conditions (24), and (25) with condition (23). In this case the result in Proposition 7 holds for a utilitarian DMG, when the bond price $P$ is such that

$$P \geq \alpha E_\Lambda^*[A]. \quad (32)$$
6 Empirical implications, limitations, and extensions

Our analysis rests on two critical assumptions: (i) agents have heterogeneous beliefs, and (ii) DMG members must act collectively as a single corporate body when they initiate and manage an indivisible investment opportunity.

We feel that these assumptions capture important aspects of corporate behavior but we also recognize that they are very strong. Indeed they rule out many go-to solutions found in corporate finance. For instance, the most common solution to conflicts caused by heterogeneous valuations is for the optimist to buy out the pessimist. In the context of the example of Section 2, rather than forgo the investment initially when future conflict is anticipated, Olivia would buy the firm from Pietro and operate it as the classical SEU manager. Even without such a trade, the underinvestment problem can be mitigated through precommitment to a partnership dissolution rule giving an exit option to all group members.\footnote{Commonly used mechanisms include the winner’s bid auction and the loser’s bid auction (e.g., Cramton, Gibbons, and Klemperer (1987)) as well as the ‘cake-cutting’ mechanism that leads to ‘shot-gun’ splitting rules (e.g., Morgan (2004)).} In the context of our simple model, all these solutions imply that the DMG evolve to a single decision maker when conflicts about a single project arises. In reality, many projects and decisions need to be evaluated and the value that diversity brings to the DMG would not be jettisoned when conflict arises about one of potentially many projects. We capture this through the simple assumption that the DMG group members can not part company.

Another common solution that we rule out is the possibility of an information technology that will allow Olivia and Pietro to converge to a single prior. Our analysis applies therefore to cases in which information acquisition is impossible or costly to acquire. This friction can be the outcome of a real direct cost in formation acquisition or an “indirect” cost that would be at play when agents have dogmatic beliefs, perhaps shaped by past experience, and do not update in a Bayesian way (see, e.g., Malmendier and Nagel (2011, 2014)).

**Empirical implications.** Our analysis highlight that the phenomena studied in this paper (time inconsistency, underinvestment, security issuance) are implied by the presence of disagreement among members of a decision making body. Some decisions are inherently more “uncertain” than others. For instance, assessing the value of expected oil production from a conventional reserve is likely to generate less disagreement than valuing a newly discovered enzyme whose commercial
applications are not yet known. Furthermore, each of these two projects can generate more or less disagreement depending on the underlying macroeconomic environment. Hence, both cross sectional measures of disagreement such as the dispersion of analyst forecasts (see e.g. Diether, Malloy, and Scherbina (2002)) and times series measures of macroeconomic uncertainty (see e.g. Baker, Bloom, and Davis (2012)) can also identify situations where the predictions of our model are more relevant.

A second fundamental empirical question raised by our assumptions is: How do DMGs such as boards actually make decisions? Are the utilitarian, Rawlsian and Inertia-based rules studied in this paper reasonable descriptions of how DMGs such as boards actually make decisions? As emphasized by Adams, Hermalín, and Weisbach (2008), and Hermalín and Weisbach (1998), we know very little about how boards actually make decisions. The theoretical literature on board behaviour typically assumes that all board members start with a common prior, and each member then receives different signals. We assume that group members receive the same information but start with heterogeneous priors. Although the common prior assumption is a convenient modelling device widely used in the economics literature (see Morris (1995)), the evidence of dispersion of opinions held by experts in areas such as the environment, health, and financial forecasting compels us to consider the multi-prior set up in corporate finance.

A third fundamental question raised by our assumptions is: What determines the composition of a DMG? In our model we take the composition of the DMG as given. As emphasized throughout our paper, decision making by a heterogeneous DMG is more complex than decision making by a SEU firm made up of like-minded individuals. Why, then, would DMGs intentionally create this more difficult decision making environment? It would be easy to create an “SEU group” by requiring that only agents with common views about the world are allowed to join the group. But the SEU group does not seem to be an apt description of actual DMGs such as modern board of directors where diversity is indeed prevalent. There is certainly some understanding of the consequences on boards’ behaviour of factor such as social learning, outside directors, cultural and gender diversity, and personal histories. But there is relatively little understanding of how, in the context of different priors, additions to and departures from boards are made.

**Limitations.** An important limitation of our analysis is that we have little power to predict which of many possible solutions to the underinvestment problem will be implemented in a particular setting. Solutions other than the debt contracts we identify are likely to exist. For instance,
incentive contracts written directly with group members would replicate the corporate securities that we examine. In the context of our model, however, these other solutions can do no better or worse than the solutions we identify and we have no explanation of why other equally effective solutions would not be used. This is because our simple model is based on the single friction that the group members with heterogeneous beliefs have to act collectively. In reality other solutions are used because other frictions exist.

Extensions. In order to provide sharper empirical predictions, therefore, we will have to enrich the problem studied to incorporate some of the other frictions present in the real world. Based on the SEU paradigm, corporate finance theory has made great strides in understanding various phenomena that arise in the presence of asymmetric information and agency conflicts. These include debt overhang (Myers (1977)), adverse selection (Myers and Majluf (1984)), the agency cost of equity and debt Jensen and Meckling (1976) and strategic default in financial distress (Leland (1994)). Each of these were developed in the context of simple SEU models that isolated the friction of interest. However, when considered in isolation, our theory can generate predictions about corporate behaviour that contradicts other theories. For example, our analysis shows that the issuance of debt contracts can solve underinvestment problems while, in contrast, the debt overhang literature predicts that debt can cause underinvestment. In our model debt overhang is eliminated by not allowing investment after initial debt is issued. In models that predict debt overhang, ambiguity is absent as is perfect renegotiation. Hence, future work to reconcile and sharpen our predictions will require extensions of our basic model to include other frictions.

7 Conclusion

In this paper we study corporate decisions made by a group of agents that hold heterogeneous beliefs and are bound by the constraint of acting collectively. The presence of heterogeneous beliefs make the group a de-facto multi-prior decision maker. We exploit the conceptual link between individual-agent models of decision making under ambiguity and models of collective decisions for groups and study a standard dynamic real investment problem.

Our main result is to show that aggregation, or governance, rules that are consistent with the Pareto criterion, such as Utilitarian, Rawlsian and Inertia-based rules, are dynamically inconsistent. Dynamic inconsistency of a governance rule may lead, in turn, to underinvestment:
projects that would be undertaken by each group member will be passed on by the group that anticipates future disagreement. The constraint of acting together as a group is a crucial friction that generates non-trivial consequences in the dynamic decision of a group. While there can be several solutions to the underinvestment problem, we highlight that security issuance can satisfy the demand for commitment and induce unanimity among group members. Interestingly, the model predicts that the presence of leverage can be beneficial in eliminating underinvestment, somewhat opposite to the role of leverage in the standard “debt overhang” argument.

By providing a bridge between ambiguity and corporate decision making, our analysis enlarges the scope of ambiguity models and enriches their predictive power. We believe that the group interpretation of ambiguity dictates the need to relax the Bayesian paradigm for the study of corporate decisions and represents a fruitful direction for both theoretical and empirical research in corporate finance.
A Proofs

Proof of Proposition 1

The following lemma gives a simple characterization of the dynamic consistency of the utilitarian aggregation rule.

Lemma 1. The utilitarian aggregation rule is dynamically consistent if and only if, for the events \( B = E, E^c \) and for all non-negative weights \( \lambda_1, \ldots, \lambda_J \), such that \( \sum_{i=1}^J \lambda_j = 1 \), the Bayesian update \( \Lambda_B^\pi \) of the weighted prior beliefs \( \Lambda^\pi \), defined in (12), is equal to the weighted posterior beliefs \( \Lambda^\pi_B \) defined, for each \( s \in B \), by \( \Lambda^\pi_B(s) = \sum_{j=1}^J \lambda_j \pi_j^B(s) \).

Proof: Sufficiency. Assume that \( \Lambda^\pi_E = \Lambda^\pi_E \) and consider two acts \( f \) and \( g \) such that \( f = g \) on \( E \). The case in which \( f = g \) on \( E^c \) is identical. If the DMG conditionally chooses the act \( f \) over the act \( g \), then \( E_{\Lambda^\pi_E}(u(f)) \geq E_{\Lambda^\pi_E}(u(g)) \) which in turn implies \( E_{\Lambda^\pi_E}(u(f)) \geq E_{\Lambda^\pi_E}(u(g)) \). Because \( f = g \) on \( E^c \) we have \( E_{\Lambda^\pi_E}(u(f)) = E_{\Lambda^\pi_E}(u(g)) \) and using the law of total probability, \( E_{\Lambda^\pi}(u(f)) = \Lambda^\pi(A)E_{\Lambda^\pi_E}(u(f)) + \Lambda^\pi(E^c)E_{\Lambda^\pi_E}(u(f)) \), gives \( E_{\Lambda^\pi}(u(f)) \geq E_{\Lambda^\pi}(u(g)) \). Thus the DMG unconditionally chooses the act \( f \) over \( g \) and the aggregation rule is dynamically consistent.

Necessity. Assume, by contradiction, that \( \Lambda^\pi_E \neq \Lambda^\pi_E \) so that there are two state \( s' \) and \( s'' \) in the event \( E \) such that

\[
\Lambda^\pi_E(s') > \Lambda^\pi_E(s') \quad \text{and} \quad \Lambda^\pi_E(s'') < \Lambda^\pi_E(s'').
\]

Consider a constant act \( f(s) = a > 0 \) for all \( s \in S \) generating the conditional expected utility

\[
E_{\Lambda^\pi_E}[u(f)] = E_{\Lambda^\pi_E}[u(f)] = u(a)
\]

Consider an act \( g \) defined by

\[
g(s) = \begin{cases} 
a, & \text{for } s \neq s', s'' \\
a + \epsilon & \text{for } s = s' \\
a - \eta & \text{for } s = s''
\end{cases}
\]

where \( \epsilon > 0 \) and \( \eta > 0 \) are arbitrary small real numbers. Using a first order expansion for small \( \epsilon \) and \( \eta \) and factorizing the first order term, we get the conditional expected utilities

\[
E_{\Lambda^\pi_E}[u(g)] = u(a) + u'(a) \left( \Lambda^\pi_E(s') \epsilon - \Lambda^\pi_E(s'') \eta \right) = u(a) + u'(a) \epsilon \Lambda^\pi_E(s') \left( \frac{\Lambda^\pi_E(s') - \eta}{\epsilon} \right)
\]
and

\[ \mathbb{E}_{\Lambda^\pi_E}[u(g)] = u(a) + u'(a) \left( \Lambda^\pi_E(s') \epsilon - \Lambda^\pi_E(s'') \eta \right) = u(a) + u'(a) \epsilon \Lambda^\pi_E(s'') \left( \frac{\Lambda^\pi_E(s')}{\Lambda^\pi_E(s'')} - \frac{\eta}{\epsilon} \right). \]

Inequalities (A1) imply that \( \frac{\Lambda^\pi_E(s')}{\Lambda^\pi_E(s'')} > \Lambda^\pi_E(s') \) and thus a choice of \( \epsilon \) and \( \eta \) such that \( \frac{\Lambda^\pi_E(s')}{\Lambda^\pi_E(s'')} > \frac{\eta}{\epsilon} \), together with the fact that \( u'(a) > 0 \), gives the ranking \( \mathbb{E}_{\Lambda^\pi_E}[u(g)] > u(a) \equiv \mathbb{E}_{\Lambda^\pi_E}[u(f)] \).

Using the law of total probability and recalling that \( f = g \) on the event \( E^c \), we obtain \( \mathbb{E}_{\Lambda^\pi}[u(g)] > \mathbb{E}_{\Lambda^\pi}[u(f)] \). The above choice of \( \epsilon \) and \( \eta \) also gives \( \mathbb{E}_{\Lambda^\pi_E}[u(g)] < \mathbb{E}_{\Lambda^\pi_E}[u(f)] \). This means that the DMG unconditionally prefers \( g \) to \( f \) while, upon learning \( E \), it prefers \( f \) to \( g \). The aggregation rule is therefore time inconsistent. This concludes the proof.

We can now proceed to the proof of Proposition 1.

**Sufficiency.** Let us first show that dynamic consistency obtains when all group members will agree on the probability of the event \( E \), that is \( \pi^i(E) = \pi^j(E) \) for all \( i, j \). The definition (7) of \( \Lambda^\pi \) implies that when there is agreement on the probability of the event \( E \), we have \( \pi^j(E) = \Lambda^\pi(E) \) for all \( j = 1, ..., J \). Equation (11) gives then

\[ \Lambda^\pi_E(s) = \frac{\sum_{j=1}^J \lambda_j \pi^j(s)}{\Lambda^\pi(E)} = \frac{\Lambda^\pi(s)}{\Lambda^\pi(E)} = \Lambda^\pi_E(s) \text{ for all } s \in E \]

and thus we have \( \Lambda^\pi_E = \Lambda^\pi_E \) which, by Lemma 1 implies dynamic consistency of the aggregation rule.

**Necessity.** Suppose that dynamic consistency holds. Then, by Lemma 1, \( \Lambda^\pi_E = \Lambda^\pi_E \) for any set of weights \( \lambda_1, \ldots, \lambda_J \). Let us consider the system of weights \( \lambda_i = \theta, \lambda_j = 1 - \theta, 0 < \theta < 1 \), and \( \lambda_k = 0 \) for \( k \neq i, j \). Dynamic consistency implies that, for any value of \( \theta \in (0, 1) \), it must be that

\[ \theta \pi^i_E(s) + (1 - \theta) \pi^j_E(s) = \frac{\theta \pi^i(s) + (1 - \theta) \pi^j(s)}{\theta \pi^i(E) + (1 - \theta) \pi^j(E)}. \quad (A2) \]

The above inequality implies that, for all \( \theta \in (0, 1) \) and \( s \in S \)

\[ \left( \pi^i_E(s) - \pi^j_E(s) \right) \left( 1 - \frac{\pi^j(E)}{\pi^j(E)} \right) (\theta^2 - 1) = 0. \quad (A3) \]

The above equality is true if (i) \( \pi^i_E \neq \pi^j_E \) and \( \pi^j(E) = \pi^j(E) \) or (ii) \( \pi^i_E = \pi^j_E \) and \( \pi^i(E) \neq \pi^j(E) \) or both. Because, by Assumption 1, the set \( \Pi_E \) of updated priors contains more than one element,
there will be a pair $i, j$ for which $\pi^i_E \neq \pi^j_E$ and therefore, (A3) is satisfied if, for all $i, j$ for which $\pi^i_E \neq \pi^j_E$, $\pi^i(E) = \pi^j(E)$. Note, however, that this condition implies that $\pi^i(E) = \pi^j(E)$ for all $i, j$. To see this, suppose there exists a $\pi^k_E$ such that $\pi^i_E \neq \pi^j_E$. By (A3), dynamic consistency implies that if $\pi^i_E \neq \pi^j_E$, $\pi^i(E) = \pi^j(E)$. Hence, because, $\pi^k_E = \pi^i_E$ and $\pi^i_E \neq \pi^j_E$ we have $\pi^k_E \neq \pi^j_E$. By A3 we then have $\pi^k(E) = \pi^j(E)$, and therefore $\pi^i(E) = \pi^j(E) = \pi^k(E)$. We conclude that dynamic consistency implies $\pi^i(E) = \pi^j(E)$ for all $i, j$.

\[\begin{align*}
\text{Proof of Proposition 2} \\
\text{Let us start with the following lemma:}
\end{align*}\]

**Lemma 2.** Assume that condition (13) is satisfied. For any act $f$, if the unconditional belief $\pi^f$ solves

$$
\pi^f \in \arg\min_{\pi \in \Pi} \mathbb{E}_\pi[u(f)]
$$

then the conditional beliefs $\pi^f_E$ and $\pi^f_{E^c}$ solve

$$
\pi^f_E \in \arg\min_{\pi \in \Pi_E} \mathbb{E}_\pi[u(f)] \text{ and } \pi^f_{E^c} \in \arg\min_{\pi \in \Pi_{E^c}} \mathbb{E}_\pi[u(f)]
$$

**Proof:** It is sufficient to prove that $\pi^f_E \in \arg\min_{\pi \in \Pi_E} \mathbb{E}_\pi[u(f)]$ because $E$ and $E^c$ play a symmetric role in the statement of the lemma. We proceed by contradiction and assume that (A5) is not true and that there exists a conditional prior $\pi' \in \Pi_E$ such that

$$
\mathbb{E}_{\pi'}[u(f)] < \mathbb{E}_{\pi^f}[u(f)].
$$

We now consider the prior $\pi^R$ defined for each state $s \in S$ by

$$
\pi^R(s) = \pi^f(E)\pi'(s) + \pi^f(E^c)\pi^f_{E^c}(s).
$$

Condition (13) implies that the prior $\pi^R$ belongs to $co(\Pi)$. Moreover,

$$
\begin{align*}
\mathbb{E}_{\pi^R}[u(f)] &= \pi^f(E)\mathbb{E}_{\pi'}[u(f)] + \pi^f(E^c)\mathbb{E}_{\pi^f_{E^c}}[u(f)] \\
&< \pi^f(E)\mathbb{E}_{\pi^f}[u(f)] + \pi^f(E^c)\mathbb{E}_{\pi^f_{E^c}}[u(f)] = \mathbb{E}_{\pi'}[u(f)],
\end{align*}
$$

(A7)
where the inequality follows from equation (A6). Because of the linearity in probabilities of the expectation operator, \( \min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)] = \min_{\pi \in \text{co}(\Pi)} \mathbb{E}_{\pi}[u(f)] \) and \( \pi^R \in \text{co}(\Pi) \), equation (A7) contradicts (A4).

We can now proceed to the proof of Proposition 2.

Sufficiency. Suppose that condition (13) holds. We show that this implies dynamic consistency of the Rawlsian aggregation rule. Consider two acts \( f \) and \( g \) such that \( f = g \) on \( E^c \) and such that the DMG unconditionally chooses the act \( f \) over the act \( g \), i.e., \( \min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)] > \min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(g)] \).

Consider a selection of an unconditional belief \( \pi^f \) from the set defined by equation (A4) associated to the act \( f \). The belief \( \pi^f \), generates conditionals beliefs \( \pi^f_{E^c} \) and \( \pi^f_{E^c} \). Similarly we select a belief \( \pi^g \) from the set \( \arg\min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(g)] \) and their conditionals \( \pi^g_{E^c} \) and \( \pi^g_{E^c} \). It is important to remember that Lemma 2 shows that the conditional beliefs \( \pi^f_{E^c} \), \( \pi^f_{E^c} \), \( \pi^g_{E^c} \) and \( \pi^g_{E^c} \) are all realizing the minimum conditional utility. Notice that since the act \( f \) is equal to the act \( g \) on the event \( E^c \), we can select conditional beliefs satisfying \( \pi^f_{E^c} = \pi^g_{E^c} \).

Because the DMG unconditionally chooses the act \( f \) over the act \( g \), we have

\[
\pi^f(E)\mathbb{E}_{\pi^f_{E^c}}[u(f)] + \pi^f(E^c)\mathbb{E}_{\pi^f_{E^c}}[u(f)] > \pi^g(E)\mathbb{E}_{\pi^g_{E^c}}[u(g)] + \pi^g(E^c)\mathbb{E}_{\pi^g_{E^c}}[u(g)]
\]  

(A8)

Since \( \pi^f_{E^c} = \pi^g_{E^c} \) and \( f = g \) on the event \( E^c \), we have \( E_{\pi^f_{E^c}}[u(f)] = E_{\pi^g_{E^c}}[u(g)] \). Let us denote by \( U \equiv E_{\pi^g_{E^c}}[u(g)] \) and rewrite (A8) as follows

\[
\pi^f(E)\mathbb{E}_{\pi^f_{E^c}}[u(f)] + (1 - \pi^f(E))U > \pi^g(E)\mathbb{E}_{\pi^g_{E^c}}[u(g)] + (1 - \pi^g(E))U.
\]  

(A9)

We will now consider all possible rankings of \( \mathbb{E}_{\pi^f}[u(f)] \), \( \mathbb{E}_{\pi^g}[u(g)] \), and \( U \) and show that all feasible configurations should imply \( \mathbb{E}_{\pi^f}[u(f)] > \mathbb{E}_{\pi^g}[u(g)] \).

1. Suppose \( \mathbb{E}_{\pi^f}[u(f)] \geq U \) and \( \mathbb{E}_{\pi^g}[u(g)] \geq U \). Then, the minima \( \min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)] \) and \( \min_{\pi \in \Pi} \mathbb{E}_{\pi}[u(g)] \) are achieved by choosing

\[
\pi^f(E) = \pi^g(E) = \pi(E) = \min_{\pi \in \Pi} \pi(E).
\]
To see this, assume for example that $\pi(E) < \pi^f(E)$ (the same arguments will also work for $\pi^g(E)$). Let us construct the prior $\hat{\pi}$ defined by

$$\hat{\pi}(s) = \pi(E)\pi^f_E(s) + \pi(E^c)\pi^{f_c}_E(s),$$

and observe that because $\mathbb{E}_{\pi_E}[u(f)] \geq U$, and $\pi(E) < \pi^f(E)$, we have

$$\mathbb{E}_{\hat{\pi}}[u(f)] = \pi(E)\mathbb{E}_{\pi_E^f}[u(f)] + (1 - \pi(E))U < \pi^f(E)\mathbb{E}_{\pi_E^f}[u(f)] + (1 - \pi^f(E))U = \mathbb{E}_{\pi^f}[u(f)].$$

By condition (13), the probability $\hat{\pi} \in \text{co}(\Pi)$ and therefore the last inequality violates the fact that $\pi^f \in \text{argmin}_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)] \equiv \text{argmin}_{\pi \in \text{co}(\Pi)} \mathbb{E}_{\pi}[u(f)]$. Inequality (A9) becomes then

$$\pi(E)\mathbb{E}_{\pi_E^f}[u(f)] + (1 - \pi(E))U > \pi(E)\mathbb{E}_{\pi_E^g}[u(g)] + (1 - \pi(E))U \quad \text{(A10)}$$

which implies $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)].$

2. Suppose $\mathbb{E}_{\pi_E^f}[u(f)] \leq U$ and $\mathbb{E}_{\pi_E^g}[u(g)] \leq U$. Then following a similar argument as above, the minima of min$_{\pi \in \Pi} \mathbb{E}_{\pi}[u(f)]$ and min$_{\pi \in \Pi} \mathbb{E}_{\pi}[u(g)]$ are achieved by choosing

$$\pi^f(E) = \pi^g(E) = \pi(E) = \max_{\pi \in \Pi} \pi(E),$$

and equality (A9) implies $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)].$

3. Suppose $\mathbb{E}_{\pi_E^f}[u(f)] < U < \mathbb{E}_{\pi_E^g}[u(g)]$. This would imply that the left hand side of inequality (A9) is strictly smaller that the right hand side of the same inequality for any choice of probabilities $\pi^f(E)$ and $\pi^g(E)$. This statement contradicts inequality (A8) and thus the the case $\mathbb{E}_{\pi_E^f}[u(f)] < U < \mathbb{E}_{\pi_E^g}[u(g)]$ is impossible.

4. Suppose finally that $\mathbb{E}_{\pi_E^f}[u(g)] \leq U \leq \mathbb{E}_{\pi_E^g}[u(f)]$. This case trivially implies that $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)]$ for any choice of probabilities $\pi^f(E)$ and $\pi^g(E)$.

To sum up, in all cases $\mathbb{E}_{\pi_E^f}[u(f)] > \mathbb{E}_{\pi_E^g}[u(g)]$. By Lemma 2, $\pi^f_E$ and $\pi^g_E$ minimize the conditional expected utility of acts $f$ and $g$. Hence when the DMG unconditionally prefers $f$ to $g$ and condition (13) holds, the DMG conditionally prefers $f$ to $g$, implying that the Rawlsian rule is dynamically consistent.
Necessity. We now show that if the Rawlsian aggregation rule is dynamically consistent, then the underlying set of priors satisfies condition (13). Without loss of generality, we first make the assumption that the preferences are risk neutral and the set of outcomes is the real line $\mathbb{R}$. We prove the case in which the event $B = E$. The case in which $B = E^c$ is identical. We proceed by contradiction and assume that the set of priors does not satisfy condition (13): there exist two beliefs $\pi'$ and $\pi''$ in the set of beliefs $\Pi$ such that the prior $\pi^R$ defined for each state $s \in S$ by

$$
\pi^R(s) = \pi'(E) \pi''_E(s) + \pi'(E^c) \pi''_E(s),
$$

does not belong to the convex hull of the set of priors $\text{co}(\Pi)$. We now construct a pair of acts such that if $f = g$ on the event $E^c$, the DMG conditionally chooses the act $f$ over the act $g$ but unconditionally it chooses the act $g$ over the act $f$.

Let us start by observing that, since $\pi^R \notin \text{co}(\Pi)$ and that the set $\text{co}(\Pi)$ is convex, the hyperplane separation theorem (Rockafellar (1997), Theorem 11.3, p. 97) shows that there exists a vector $g \in \mathbb{R}^S$ such that

$$
E_{\pi^R}[g] < E_{\pi}[g] \text{ for all } \pi \in \Pi.
$$

(A11)

From now on, we identify the vector $g$ with the act that delivers in each state $s$ an outcome equal to the $s^{th}$ component of the vector $g$.

We claim that the act $g$ must satisfy the following property:

$$
\min_{\pi \in \Pi_E} E_{\pi}[g] < \max_{\pi \in \Pi_E} E_{\pi}[g].
$$

(A12)

To see this, assume that inequality (A12) is in fact and equality so that all conditional priors evaluate the act $g$ identically: $E_{\pi}[g] = \text{constant}$ for all $\pi \in \Pi_E$. This last property cannot hold because it contradicts the property (A11): if we apply inequality (A11) to $\pi = \pi'$ we get

$$
E_{\pi^R}[g] \equiv \pi'(E)E_{\pi'_E}[g] + \pi'(E^c)E_{\pi''_E}[g] < \pi'(E)E_{\pi'_E}[g] + \pi'(E^c)E_{\pi''_E}[g] \equiv E_{\pi'}[g]
$$

which implies $E_{\pi'_E}[g] < E_{\pi''_E}[g]$. The last inequality contradicts the assumption that $E_{\pi}[g] = \text{constant}$ for all $\pi \in \Pi_E$. We have thus proven by contradiction that the act $g$ satisfies inequality (A12).

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23If the utility function is non linear, then we need to work on the space of utils rather than payout and we need then to assume that the span of utils is large enough to be able to find a vector within the span of $u$ that separates a convex set of probabilities from a given probability.
Let us now consider again the probability $\pi^R$ and note that its conditional update, $\pi^R_E$, belongs to the set $\Pi_E$ since, by construction, $\pi^R_E = \pi''_E$. Thus we have $\min_{\pi \in \Pi_E} \mathbb{E}_\pi(g) \leq \mathbb{E}_{\pi''_E}[g]$ and we will consider in the sequel both the case where this inequality is strict and the case where this inequality is an equality.

Case 1: $\min_{\pi \in \Pi_E} \mathbb{E}_\pi(g) < \mathbb{E}_{\pi''_E}[g]$. Let us define the act $f$ with

$$f(s) = \begin{cases} g(s) & \text{if } s \in E^c \\ A & \text{if } s \in E \end{cases}.$$ 

where $A$ satisfies

$$\min_{\pi \in \Pi_E} \mathbb{E}_\pi(g) < A < \mathbb{E}_{\pi''_E}[g]. \quad (A13)$$

The left inequality of (A13) shows that the Rawlsian DMG conditionally chooses the act $f$ over the act $g$. To establish that the same DMG unconditionally chooses the act $g$ over the act $f$, we need to prove that

$$\min_{\pi \in \Pi} \mathbb{E}_\pi[f] < \min_{\pi \in \Pi} \mathbb{E}_\pi[g].$$

Inequality (A11) shows that, in order to prove the last inequality, it is sufficient to prove that

$$\min_{\pi \in \Pi} \mathbb{E}_\pi[f] \leq \mathbb{E}_{\pi''_E}[g].$$

Since the belief $\pi'$ belongs to the set $\Pi$, the last inequality is implied by the more stringent inequality

$$\mathbb{E}_{\pi'}[f] \leq \mathbb{E}_{\pi''_E}[g]. \quad (A14)$$

Substituting the expressions of total probabilities shows that inequality (A14) is equivalent to

$$\pi'(E)\mathbb{E}_{\pi'_{E}}[f] + \pi'(E^c)\mathbb{E}_{\pi'_{E}}[f] \leq \pi'(E)\mathbb{E}_{\pi''_{E}}[g] + \pi'(E^c)\mathbb{E}_{\pi''_{E}}[g]. \quad (A15)$$

Recalling that $f = g$ on the event $E^c$ and simplifying by $\pi'(E)$, shows that inequality (A15) is equivalent to $\mathbb{E}_{\pi'_{E}}[f] \leq \mathbb{E}_{\pi''_{E}}[g]$. Because $f = A$ on the event $E$ and $\pi^R_E = \pi''_E$, this last inequality is equivalent to $A \leq \mathbb{E}_{\pi^R_E}[g]$ which, by the right hand side of inequality (A13) is true. We therefore showed that the DMG unconditionally chooses the act $g$ over the act $f$ which concludes the proof for Case 1.
Case 2: \( \min_{\pi \in \Pi_E} E_\pi(g) = E_{\pi^R}(g) \equiv E_{\pi^E_R}(g) \). Instead of using the belief \( \pi^R \) itself, we construct a slightly modified (fictitious) belief \( \pi^\mu \) defined for each \( s \in S \) by

\[
\pi^\mu(s) = \pi'(E)E_{\pi^\mu}(s) + \pi'(E^c)E_{\pi^\mu^c}(s),
\]

where \( 0 < \mu < 1 \), and the conditional belief \( \pi^\mu_E \) is given by

\[
\pi^\mu_E(s) = \mu \pi_E(s) + (1-\mu)\pi''_E(s)
\]

for all \( s \in E \), and where the belief \( \pi_E \) solves

\[
\pi_E \in \arg\max_{\pi \in \Pi_E} E_\pi[g].
\]

Inequality (A12) shows that \( E_{\pi^\mu}(g) = \min_{\pi \in \Pi_E} E_\pi(g) < \max_{\pi \in \Pi_E} E_\pi[g] = E_{\pi_E}(g) \). When the real number \( \mu \) converges to 0, the probability \( \pi^\mu \) converges uniformly to the probability \( \pi^R \).

Because the set \( co(\Pi) \) is closed and convex and \( E_{\pi^R}(g) < \min_{\pi \in \Pi} E_\pi[g] \), we can always choose \( \mu > 0 \) small enough so that \( E_{\pi^\mu}(g) < \min_{\pi \in \Pi} E_\pi[g] \). To see this, consider the function \( \psi \) defined over the interval \([0, 1]\) and defined by \( \psi(\mu) = E_{\pi^\mu}(g) \). The function \( \psi \) satisfies \( \psi(0) = E_{\pi^R}(g) < \min_{\pi \in \Pi} E_\pi[g] \). Because \( \psi \) is a continuous function, we can always choose \( \mu \) small enough so that \( \psi(\mu) = E_{\pi^\mu}(g) < \min_{\pi \in \Pi} E_\pi[g] \).

Because \( E_{\pi^E}(g) = \mu E_{\pi_E}(g) + (1-\mu)E_{\pi^E}(g) \), we have \( \min_{\pi \in \Pi_E} E_\pi(g) = E_{\pi^E}(g) \). To prove time inconsistency, the construction of the act \( f \) is identical to Case 1 with a selection of the real number \( A \) in the range

\[
\min_{\pi \in \Pi_E} E_\pi(g) < A < E_{\pi^R}(g).
\]  

The last inequality shows that the DMG conditionally chooses the act \( f \) over the act \( g \). Unconditionally, we need to prove that \( \min_{\pi \in \Pi} E_\pi[f] < \min_{\pi \in \Pi} E_\pi[g] \) and following similar arguments to Case 1, we can show that it is sufficient to prove that

\[
E_{\pi^R}(f) \leq E_{\pi^R}(g).
\]

Following similar arguments to case 1, it can be shown that the last inequality is equivalent to \( A \equiv E_{\pi^E}(f) \leq E_{\pi^E}(g) \) which, by inequality (A16), is satisfied. This concludes the proof.
Proof of Proposition 3

Let us first prove that if the status quo selected for the choice between any pair of act does not change over time, the Inertia rule is dynamically consistent.

**Sufficiency.** Consider two acts \( f \) and \( g \) such that \( f = g \) on \( E^c \). The case in which \( f = g \) on \( E \) is identical. Suppose the DMG chooses the act \( f \) over the act \( g \) and that the group members unconditionally disagree on the choice between these two acts. The DMG unconditionally chooses the act \( f \) over the act \( g \) because the act \( f \) is the status quo act. The absence of unanimity in the choice between the act \( f \) and the act \( g \) means that there exits al least one prior \( \pi^f \in \Pi \) for which

\[
\sum_{s \in S} u(f(s))\pi^f(s) \geq \sum_{s \in S} u(g(s))\pi^f(s) \quad (A17)
\]

and that there exists at least one prior \( \pi^g \in \Pi \) for which

\[
\sum_{s \in S} u(f(s))\pi^g(s) < \sum_{s \in S} u(g(s))\pi^g(s). \quad (A18)
\]

Dividing inequality (A17) by \( \pi^f(E) > 0 \) and noticing that \( f(s) = g(s) \) on the event \( E^c \) we see that inequality (A17) is equivalent to

\[
\sum_{s \in E} u(f(s))\pi^{f}_E(s) \geq \sum_{s \in E} u(g(s))\pi^{f}_E(s) \quad (A19)
\]

or equivalently

\[
E_{\pi^f_E}[u(f)] \geq E_{\pi^f_E}[u(g)]. \quad (A20)
\]

Similar steps also show that

\[
E_{\pi^g_E}[u(f)] < E_{\pi^g_E}[u(g)]. \quad (A21)
\]

Equations (A20)-(A21) show group members conditionally disagree on the choice between the act \( f \) to the act \( g \). Since the status quo act does not change, the DMG conditionally chooses the status quo act \( f \) over the act \( g \).

Suppose now that the DMG unconditionally chooses the act \( f \) over the act \( g \) because all group members prefer the act \( f \) to the act \( g \). Similar steps to equations (A17) and (A19) shows that unanimity is transmitted to the conditional preferences of group members and thus the DMG also chooses the act \( f \) over the act \( g \).
There is therefore no reversal in choice in both cases and the Inertia aggregation rule is dynamically consistent.

**Necessity.** Let us now prove the reverse implication. For that we proceed by contradiction by showing that if the status quo between two acts for which there is disagreement is changing over time, then the inertia aggregation rule is time inconsistent. We know from the steps above, that if there is unconditional disagreement between two acts then there will be conditional disagreement between these two acts. Therefore, the DMG will pick the status quo act both conditionally and unconditionally. If the status quo switches from the act \( f \) to the act \( g \), then we will have a reversal in choice and the inertia rule is time inconsistent.

**Proof of Proposition 6**

Let us first look at the choice between \( A \) and \( C \) conditional on the event \( E \). If the group member chooses the act \( A \), the project generate the cash flow \( A \) and the left inequality of condition (26) shows that the firm is insolvent. Because of limited liability the member receives a zero payoff and the bondholder receives the payoff \( A \). If the member chooses the act \( C \) the right inequality of condition (26) and property (16) implies that the firm is solvent for states \( s_{K+1}, \ldots, s_M \) and insolvent for states \( s_{M+1}, \ldots, s_N \). In the solvency states the payoff to the bondholder is the face value \( X \) and the payoff to the group member is the after debt repayment payoff \( C(s) - X \). In the default states the payoff to bondholder is \( C(s) \) while the group member receives a zero payoff. Overall the group members receives the non negative payoff \( (C - X)^+ \). Because the act \( A \) leads to a zero payoff, all group members unanimously choose the act \( C \). The DMG will then choose the act \( C \) when the event \( E \) is revealed to be true.

At time 0 group members contemplate starting the firm after issuing the bond or not invest and keep \( I \). The anticipated action is to continue the firm at time 1 and thus group members rule out the act \( A \). The bond is a security that yields the residual payoff \( (C - X)^+ \) to the group members and \( \min(X, C) \) to the financier. Assume that the financier is willing to pay the price \( P \) for the bond. The utilitarian DMG will invest if the amount \( I - P \) required to start the project, i.e., the DMG’s equity in the project, is smaller than the value to the DMG of the project net of payments promised to bondholder, i.e, inequality (27) holds. A similar result holds for the case in which the DMG follows the Rawlsian aggregation rule. In this case the DMG will invest if and only if \( I - P \leq \min_{\pi \in \Pi} \mathbb{E}_\pi [C - X]^+ \).
Proof of Proposition 7

Let us first look at the choice between $A$ and $C$ conditional on the event $E$. When the group member shuts down the firm, bondholders must compare the conversion outcome $(\alpha A)$ with the no conversion outcome $(X)$. The right inequality of condition (28) shows that, when the firm shuts down, it is optimal for the bondholders to convert the bond and the group member ends up with the residual payoff $(1 - \alpha)A$. When the DMG chooses $C$, the bondholder must compare the conversion outcome $\alpha C$ with the outcome of face value $X$ when the firm is solvent. When the firm is insolvent, bondholders will get the firm payoff $C$. Condition (28) shows that bondholders will convert an receive the payoff $\alpha C(s)$ in the solvent states $s_{K+1}, \ldots, s_M$, and not convert and receive the payoff $\min(X, C(s))$ in the insolvent states $s_{M+1}, \ldots, s_N$. As a result, the group member receives the residual payoff $(1 - \alpha)C(s)$ in solvent states and the payoff $C - \min(X, C(s)) = [C(s) - X]^+$ in insolvent states.

Denoting by $\pi^i$ the group member’s belief, the conditional expected payoff associated with the act $C$ is

$$
\sum_{i=K+1}^{M} (1 - \alpha)C(s_i)\pi^i_E(s_i) + \sum_{i=M+1}^{N} [C(s_i) - X]^+\pi^i_E(s_i)
$$

Because of Assumption 2, $A(s) = A$ for all $s \in E$. Hence the conditional expected payoff associated with the act $A$ is $A$. By condition (29), all group members choose to abandon the firm at time 1. The act $A$ generates unanimity at time 1 and is thus chosen by the DMG.

At time 0, each group member contemplates the choice between the actions $N, C, A$. If the bond is issued, the act $C$ is excluded from the menu of choice because all group members anticipates that the act $A$ will be chosen at time 1 by the DMG. Under any given prior $\pi \in \Pi$, recalling that $C = A$ on the event $E^c$, the expected payoff for the group member associated to that prior when the act $A$ is chosen is

$$
(1 - \alpha)E_\pi[A].
$$

Therefore the DMG making use of the inertia rule will invest at time 0 if

$$
I - P \leq (1 - \alpha)E_\pi[A] \text{ for all } \pi \in \Pi,
$$
or, equivalently,

\[ P \geq \alpha \min_{\pi \in \Pi} \mathbb{E}_\pi[A] - \left( \min_{\pi \in \Pi} \mathbb{E}_\pi[A] - I \right). \]

Since we assume that \( I < \min_{\pi \in \Pi} \mathbb{E}_\pi[A] \) the latter condition is implied by condition (30). \( \blacksquare \)
References


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