Pledgeability, Industry Liquidity, and Financing Cycles

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Abstract

Why are downturns following prolonged episodes of high valuations of firms so severe and prolonged? In this paper, we propose a different explanation as to why firms that overpromise external payments underperform. We account not just for traditional debt overhang but also overhang from more general financial contracts, and propose a theory of financial cycles. In our theory, the control rights used by external investors to enforce claims in an asset price boom (rights to sell assets) differ from the control rights used in more normal terms (rights over cash flows we term pledgeability). Firm management’s limited incentive to enhance the pledgeability of cash flows in an asset price boom can have long-drawn adverse effects in a downturn, which may not be resolved by renegotiation. This can also explain why asset turnover to outsiders is high in a downturn as well as why industry productivity falls.

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Why do downturns following episodes of prolonged high valuations of firms prove to be detrimental to growth and result in more severe recessions (see Krishnamurthy and Muir (2015) and López-Salido, Stein and Zakrajšek (2015))? One traditional rationale is based on the idea of “debt overhang” – the debt built up during the boom serves to restrict investment and borrowing during the bust. However, if everyone, including the debt holders, knows that debt is holding back investment, they have an incentive to write down the debt in return for a stake in the firm’s growth. For debt overhang to be a serious concern, the firm and debt holders must be unable to undertake value enhancing contractual bargains. Another view is that borrowers cannot be trusted to take only value enhancing investments, even in a downturn. So debt overhang is needed to constrain the borrower’s investment – overhang is a second best solution to a fundamental moral hazard problem (see Hart and Moore (1995)). The immediate question raised by such an analysis is why do we want to constrain borrowers more in bad times? Why is the moral hazard problem so much more serious in a downturn? Why isn’t debt overhang the tool of choice to constrain corporate mal-investment all the time (and casual empiricism suggests it is not)?

In this paper, we provide an explanation of the causes and consequences of financial contract overhang (including debt overhang) and explain why it is more acute when it follows a period of high valuations and rational optimism about the future values of firms. In doing so, we differentiate between the control rights that are due to high resale prices for assets, which enable external claims to be enforced in a boom, and control rights based on pledging of cash flows that facilitate the enforcement of external claims at other times, including downturns. It is the change in the operational control rights that causes the external claim build up to have long-drawn adverse effects in the downturn.

Let us be more specific. Consider an industry that requires special managerial knowledge. Within the industry, there are incumbents (those who are running firms) and industry insiders (those who know the industry well enough to be able to run firms as efficiently as the incumbents). Industry outsiders (financiers who don’t really know how to run industry firms but have general managerial/financial skills) are the other agents in the model.

We first examine the effects of financing with fully state-contingent financial contracts, and then we turn to standard debt with a constant payment in a given period. Financiers have two sorts of control rights; first, control through the right to repossess and sell the underlying asset being financed if payments are not made and, second, control over cash flows generated by the asset. The first right only requires the frictionless enforcement of property rights in the economy, which we assume. It has especial value when there are a large number of capable potential buyers willing to pay a full price for the asset. Greater wealth amongst industry insiders (which we term industry liquidity) increases the
availability of this asset-sale-based financing. Because we analyze a single industry, high levels of this industry liquidity can be interpreted as an economy-wide boom.

The second type of control right is more endogenous, and conferred on creditors by the firm’s incumbent manager as she makes the firm’s cash flows more appropriable or pledgeable – for example, by improving accounting standards and transparency, by setting up escrow accounts and monitoring arrangements, by including debt covenants and conditions on dividend payments, or even by standardizing managerial procedures so as to make herself more replaceable as a manager. From the incumbent manager’s perspective, enhancing cash flow pledgeability is a double-edged sword; while it makes it easier for the incumbent to sell the firm when she is no longer fit to run it (because new buyers can borrow against future pledgeable cash flows to finance the acquisition) it makes it easier for existing creditors to collect more when the incumbent stays in control. Low pledgeability also serves to entrench the incumbent, by reducing the ability of outsiders to outbid the incumbent. Thus cash flow pledgeability is subject to moral hazard, which as we shall see, limits the fund raising capacity of the firm. The advantages of high pledgeability for financial capacity have been studied by Holmström and Tirole (1998). We examine the tradeoff between the advantages and disadvantages of increased pledgeability for the incumbent.

When markets are buoyant and industry insiders have plenty of cash, repayment is enforced by the high resale value of assets and not by any pledging of cash flows by the incumbent. Industry assets trade for fundamental value (with no underpricing), as in Shleifer and Vishny (1992). The most efficient users hold the assets because they have enough cash and borrowing capacity up front to buy. This has an important implication: In such an environment of easy sales, incumbents have little reason to maintain cash flow pledgeability.

The high resale value of assets increases the amount of financing that a firm can credibly repay, increasing the potential leverage of the firm. If the firm uses this financing capacity by issuing standard debt, both high debt built up during the asset price boom which was expected to continue and the neglect of cash flow pledgeability can be counterproductive in a downturn. Industry insiders, also hit by the downturn, no longer have personal wealth to buy assets, nor does the low cash flow pledgeability allow them to borrow against future cash flows to pay for purchases. Asset prices plummet. Faced with large debt claims, incumbents see more value to reducing future pledgeability (so as to further reduce the payout to outside claim holders) than maintaining it.

With debt high, creditors will either agree to renegotiate debt down significantly, or seize assets and sell to financiers (industry outsiders). While industry outsiders have little ability to run the asset themselves, this may be a virtue – they have a strong incentive to improve pledgeability while the asset is under their control, because they want to sell the asset back to industry insiders at a high
price. Outsiders play an important role, therefore, not because they are flush with funds but because they are not subject to moral hazard over pledgeability.

Interestingly, anticipating such sales to outsiders as the industry turns down, current debt holders have little incentive to renegotiate down debt levels, even if it causes incumbent moral hazard over pledgeability; Short term improvements in pledgeability contribute little immediately to repayments, given the weak state of the industry, but improvements in long-term pledgeability after an asset sale will enhance the recovery of long term payments significantly. Consequently, in the downturn a larger number of the new asset owners will be less-productive industry outsiders, reducing average productivity. Eisfeldt and Rampini (2006, 2008) provide evidence consistent with this.

Eventually, as the economy recovers, outsiders sell the assets back to the more productive industry insiders, as the higher pledgeability increases the insiders’ ability to raise money against cash flows. Recoveries following periods of asset price inflation and high leverage are thus delayed, not just because debt has to be written down – and undoubtedly frictions in writing down debt would increase the length of the delay – but also because corporations have to restore the pledgeability of their cash flows to cope with a world where financing is more difficult. It is the latter which may make the debt hangover more prolonged.

Our paper explains why asset price booms based on a combination of liquidity and credit can be fragile (see, for example, Borio and Lowe (2002), Adrian and Shin (2010), and Rajan and Ramcharan (2015)). It also suggests a reason why credit cycles emerge, though a dynamic extension to the model is needed to explain the properties of such cycles fully (see, for example, Kiyotaki and Moore (1997)). More broadly, it suggests underpinnings for financial cycles (Borio (2012)).

Our paper builds on Shleifer and Vishny (1992), where the high net worth of industry participants allows assets to sell for their fundamental value because the best user of an asset can outbid less efficient users, which leads to efficient reallocation. In their paper, reallocation to inefficient users takes place only when industry insiders are less liquid than outsiders. Eisfeldt and Rampini (2008) develops a theory where capital reallocation is more efficient in good times, with key ingredients being private information about managerial ability and cyclical effects of labor market competition for managers. Good times lead to high required cash compensation to managers because reservation managerial wages become elevated. As a result, high ability managers can accept lower wages in return for the benefits of managing more assets. They use the differential compensation to bribe low ability managers to give up their assets. In bad times, managerial compensation is lower

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2 See Benmelech and Bergman (2011), Coval and Stafford (2007), and Shleifer and Vishny (2011) for comprehensive reviews.
and even if high ability managers accepted zero cash compensation, it would not be sufficient to bribe low ability managers to give up their assets. This leads to a more efficient reallocation of capital in good (high compensation) times and less in bad. In both Shleifer and Vishny (1992) and Eisfeldt and Rampini (2008), adjusting for current conditions (such as industry net worth or compensation), past conditions do not affect financial capacity or the efficiency of current reallocation of capital. This is unlike our model, where history matters, allowing us to explain prolonged downturns following booms, and sketch the possibility of financial cycles. Moreover, outsiders in our model are not necessarily more liquid, but still play an important role because they do not suffer from moral hazard over pledgeability. They take over the firm temporarily, even though they cannot generate cash flow, in order to raise future pledgeability.

The rest of the paper is as follows. In Section I, we describe the basic benchmark model of pledgeability choice and the timing of decisions in a two-period model (which establishes our main ideas in a simple setting). In Section II, we analyze the implications of pledgeability choice when financial contracts are fully state-contingent. The maximum amount that can be pledged to outside investors is characterized. In Section III we provide two important extensions, one where there is an additional period added to the model (which allows the possibility of buying the firm for resale) and the other where the incumbent can become disabled rather than fully incapacitated. In Section IV, we examine the implications of standard debt contracts rather than fully state-contingent payments. In Section V, we discuss the implications of the model and conclude.

1. The Framework

A. The Industry and States of Nature

Consider an industry with 3 dates (0, 1, 2) and 2 periods (period 1 and 2) between these dates. A period should be thought of as a phase of the financial cycle (see Borio (2012) for example), and extends over several years. Date t marks the end of period t. The state of the industry is realized at the beginning of every period. In the good state G the industry experiences prosperity and industry-wide distress occurs in the bad state B. The industry begins in one of the two states at date 0 – this state s_0 could be thought of as the state experienced in the previous (initially un-modeled) period. In period 1, the industry could be in either state (see Figure 1), with the probability of being in state G in period 1 being \( q^{s_0 G} \). In period 2, we assume the industry returns to state G for sure – this is meant to represent the long run state of the industry (we model economic fluctuations and not apocalypse). Note that a full description of the state in periods 1 and 2 includes the states that were realized in previous periods, but where a reference to past realized states is unnecessary we will skip it for convenience.
B. Agents and the Asset

There are two types of agents in the economy: High types (H) have high ability to manage an asset, which we will call the firm. Think of them as industry insiders. When the state is G, only a high ability manager in place at the beginning of a period $t$ can produce cash flows $C_t$ with the asset over the period; there is some mutual specialization established over the period between the manager and the firm (or more broadly, between the management team and the un-modeled organization that is needed to operate the firm) that creates a value to incumbency. In the B state, however, even a high ability manager cannot produce cash flows. A low ability manager, has no ability to produce cash flows regardless of the state. These could be financiers who try and manage the asset on their own, or industry insiders who have lost their ability (see below). Financiers have funds which they will lend to others managing the firm, if they expect to break even. All agents are risk neutral. We ignore time discounting, which is just a matter of rescaling the units of cash flows.

A high ability manager retains her ability into the next period only with probability $\theta^H < 1$. Think of this as the degree of stability of the industry. Intuitively, the critical capabilities for success are likely to be stable in a mature industry or in an industry with little technological innovation. However, in an industry which is young and unsettled, or in an industry with significant innovation, the critical capabilities for success can vary over time. A manager who is very appropriate in a particular period may be ineffective in the next. This is the sense in which an incumbent can lose ability and this occurs with higher probability in a young or changing industry.

The incumbent’s loss of ability in the next period becomes known to all shortly before the end of the current period. Loss of ability is not an industry wide occurrence and is independent across managers. So even if a manager loses her ability, there are a large number of other industry insider managers equally able to take her place next period. If a new high ability manager takes over at the
end of the current period, he will shape the firm towards his idiosyncratic management style, so he can indeed produce cash flows with the firm’s assets in the next period in good states.

C. Financial Contracts

Any manager can raise money from financiers against the asset by writing one period financial contracts. We will begin by analyzing an economy in which contracts are allowed to be state-contingent, so promised payments at the end of period $t$ are $D_t^s$.

Having acquired control of the firm, a manager would like to keep the realized cash flow for herself rather than share it with financiers. Two sorts of control rights force the manager to repay the external claims. First, the financier automatically gets a portion that we call pledgeable of generated cash flows over the period. Second, just before the end of the period, the financier gets the right to auction the firm to the highest bidder if he has not been paid in full. Below we will describe the two control rights in detail.

D. Control Rights over Cash Flow: Pledgeability

Let us define pledgeability as the fraction of realized cash flows that are automatically directed to an outside financier. In practice, it is determined by a variety of factors such as the information possessed by financiers and hence the nature of financiers (arm’s length or relationship), the nature of financing (for example, concentrated or dispersed), the quality of the accounting systems that are in place, the transparency of the organizational structure and the system of contracting (e.g., the absence of pyramids, the rules governing related party transactions, etc.), and the checks and balances that are imposed on the manager by the organization (the quality and independence of the board, the replaceability of the CEO, the independence of the auditor and the audit committee, etc.).

We assume a firm’s incumbent management can voluntarily choose general pledgeability, where general pledgeability is the fraction of next period’s cash flows an industry insider can commit to pay an outside financier if that insider replaces the incumbent and takes control over the firm’s assets at the end of this period. The incumbent’s choice of pledgeability will take time to embed (only by next period), and will then persist for some time (over the entire period). Therefore, in period $t$, the incumbent manager can set the general pledgeability of the firm’s assets for period $t+1$ at $\gamma_{t+1}$ where $\gamma_{t+1} \in [\underline{\gamma}, \overline{\gamma}]$ where $0 \leq \underline{\gamma} < \overline{\gamma} \leq 1$, and there is a small fixed cost $\varepsilon$ of setting $\gamma_{t+1} > \underline{\gamma}$. The cost $\varepsilon$ is that of taking actions like setting up a bond trustee, establishing a lending relationship by providing information to a lender or writing in detailed covenants that will ensure higher cash flow pledgeability. For the remaining analysis, our results will be presented primarily for the case where $\varepsilon \to 0$, and a positive $\varepsilon$ will only alter the results quantitatively. The economy or industry’s
institutions supporting corporate governance (such as regulators and regulations, investigative agencies, laws and the judiciary) will determine the range of possible values of pledgeability.

We assume the share the incumbent can pledge next period, incumbent pledgeability, is fixed at $\gamma^i$. It can differ from general pledgeability because the incumbent may have relationships with financiers or there may be types of public information about the incumbent which allow her to commit a different amount. For now, assume $\gamma^i$ is given. When we refer to pledgeability with no other modifier, we mean general pledgeability. If the incumbent is a low type, he cannot generate cash flows, but he can set next period’s pledgeability—he does not have industry-specific managerial capabilities but has governance capabilities.

E. Control Rights over Assets: Auction and Resale

If the financier has not been paid in full from the pledged cash flow and any additional sum the incumbent voluntarily pays, then the financier gets the right to auction the firm to the highest bidder at date $t$. One can think of such an auction as some form of bankruptcy. Therefore, the incumbent can retain control by either paying off the financier in full (possibly by borrowing once again against future pledgeable cash flows) or by paying less than the full contracted amount and outbidding other bidders in the auction.

F. Initial Conditions and Wealth

At date 0, the incumbent has initial wealth $\omega_0 \geq 0$. We assume that high ability industry insiders other than the incumbent start out with wealth $\omega_0^{H, r_0} \geq 0$. If the state is good in period $t$, both the wealth of industry insiders and that of the incumbent go up over the period by $\rho C_t$, with the wealth of the incumbent going up further by the unpledged cash flow he generates within the firm. We will refer to the wealth of an industry insider as industry liquidity. Intuitively, a string of good (G) states for the industry will mean that insiders—working as consultants, sub-contractors or employees—will increase their net worth, even if they do not own the firm. In any auction, bidders can pay cash from their wealth and any money they can raise from financiers against future cash flows.

G. Efficiency

The measure of unconstrained economic efficiency we use through the rest of this paper is the ability to keep the asset in the hands of the most productive owner. We do not model investment, instead assuming that the asset exists and is owned by an incumbent. If we put a floor on the value of asset which must be bid at date 0 (the value of real inputs to be assembled into the firm), real
investment would be possible at that date only if enough funding were available. Sufficiently weak incentives to make cash flows pledgeable or to transfer the firm to more efficient producers would reduce bids at date 0 below this floor, and this would result in underinvestment.

H. Timing

In the basic model with state-contingent financial contracts, the timing of events within period 1 is described in Figure 2. We assume that the incumbent learns the state, then sets general pledgeability, \( \gamma_2 \), knowing the amount of payment that is due at date 1. Next, her ability in period 2 is realized. Production takes place and the pledgeable fraction \( \gamma_1 \) of cash flows set in the previous period goes to financiers automatically. At date 2, she either pays the remaining due or enters the auction.

![Figure 2: Timing and Decisions in Period 1](image)

II. Solving the Basic Model

We solve the model backwards from date 2, with an emphasis on the decisions on date 1. We will characterize the choice of pledgeability and the maximum state-contingent payments to lenders, where there are two forms of moral hazard which interact to determine the maximum state-contingent payments to lenders. First, incumbents can withhold cash flows from financiers except for what they are forced to pay by pre-set pledgeability or the financier’s threat to seize and auction assets. Second, the incumbent can choose future pledgeability, potentially entrenching herself. In this setting where all outcomes are efficient, many of our positive implications will be clear. We will use these results in extensions to examine the possible inefficiency of real outcomes.

2.1 Date 2

Since the economy ends at date 2 and there is no uncertainty over the state in period 2, a high type industry insider who bids for control at date 1 can borrow up to \( D_2 \equiv \gamma_2 C_2 \) where \( \gamma_2 \) is preset by the incumbent in period 1. The incumbent can borrow up to \( \gamma' C_2 \) at date 1 if she remains a high type and bids to retain control into period 2.
2.2. Date 1

Let the promised payment to the financier at date 1 in state \( s_1 \) be \( D_{1}^{s_1} \), \( s_1 \in \{ G, B \} \). If the incumbent in period 1 is an industry insider and the state is G, she will generate pledgeable cash of \( \gamma_1 C_1 \) which goes directly to the financier (up to the value of her promised claim), where \( \gamma_1 \) is a parameter that is preset before date 0. The remaining payment due is \( \tilde{D}_{1}^{s_1} = D_{1}^{s_1} − \text{Min}[\gamma_1 C_1, D_{1}^{s_1}] \). If the state \( s_1 \) is B, \( \tilde{D}_{1}^{s_1} = D_{1}^{s_1} \).

In any date 1 auction for the firm, financiers will not bid to take direct control of the firm since the firm generates no cash flow in their hands in the last period and the firm has no residual value. Industry insiders will bid using their date-1 wealth and any amount borrowed at date 1 by pledging period 2’s output. Their wealth increases by \( \rho C_1 \) in state G, and remains unchanged in state B, i.e., \( \omega_1^{H,G} = \omega_0^{s_0} + \rho C_1 \) and \( \omega_1^{H,B} = \omega_0^{s_0} \). Together with the amount \( \gamma_2 C_2 \) they can borrow, the total amount that they can pay is \( \omega_1^{H,s_1} + \gamma_2 C_2 \). Of course, they will not bid more than the total value of cash flow, \( C_2 \). So the maximum auction bid at date 1 is \( B_1^{H,s_1}(\gamma_2) = \text{Min}[\omega_1^{H,s_1} + \gamma_2 C_2, C_2] \).

Let us define a variable which will prove to be important in our model: Potential underpricing is the difference between the present value of cash flows accruing to an industry insider and the amount that he can bid if the incumbent sets pledgeability to be low. It is \( \text{Max}[C_2 − B_1^{H,s_1}(\gamma)],0] \) at date 1. By choosing different levels of pledgeability, the incumbent can vary industry insiders’ bids between \( B_1^{H,s_1}(\gamma^G) \) and \( B_1^{H,s_1}(\gamma^B) \), thus altering the realized underpricing, which is the difference between the present value of future cash flows and the actual bid.

The incumbent will have to pay the financier in full or outbid others in an auction if she wants to retain control into period 2, so she pays \( \text{Min}[\tilde{D}_{1}^{s_1}, B_1^{H,s_1}(\gamma_2)] \). The cash she has at date 1 is the initial wealth level, \( \omega_0^{s_0} \), plus the non-pledgeable portion of cash flows generated during period 1. So at date 1, the incumbent has cash \( \omega_1^{i,G} = \omega_0^{i,s_0} + (1 − \gamma_1 + \rho) C_1 \) if the period 1 state is G, and \( \omega_1^{i,B} = \omega_0^{i,s_0} \) if the state is B. In addition, if she keeps her ability in period 2, she can also raise funds against period 2’s output, \( \gamma^i C_2 \). Therefore, the incumbent can pay as much as \( B_1^{i,s_1} = \text{min}\{\omega_1^{i,s_0} + \gamma^i C_2, C_2\} \) to the financier. The incumbent will retain control if the amount she
can pay is (weakly) greater than \( \min \left[ \bar{D}_1^{\psi}, B_1^{H,\psi} \right] \). Since the continuation value of the asset, \( C_2 \), is identical for the incumbent and the industry insiders, the incumbent is always willing to hold on to the asset if she is able to outbid. Of course, if the incumbent realizes she has lost her ability, or she is a low type to begin with, she will want to sell out since she cannot generate cash flow next period.

Regardless of who wins, the financier recoups \( \min[\gamma_1 C_1, D_1^{\psi}] + \min \left[ \bar{D}_1^{\psi}, B_1^{H,\psi} \right] \) if the incumbent in period 1 is a high type and the state is \( G \), and \( \min \left[ D_1^{\psi}, B_1^{H,\psi} \right] \) otherwise. The financier’s threat of seizing and selling assets is therefore a powerful instrument for him to extract repayment. The value of that threat depends on the bid \( B_1^{H,\psi} \) by industry insiders, which depends in turn on the wealth of industry insiders and the future pledgeability of the asset \( \gamma_2 \). Thus high future pledgeability is a way for the incumbent to commit to facing a high bid, and thus paying the financier a high sum, no matter who has control in period 2.

The incumbent’s choice of pledgeability, and thus the maximal credible payment, \( \bar{D}_1^{\psi,\text{Max}} \), are determined differently, depending on whether the incumbent can outbid industry insiders. We identify four cases. i) Pledgeability does not matter for repayment. ii) The incumbent can never outbid industry insiders. iii) The incumbent can always outbid industry insiders. iv) The incumbent can outbid industry insiders when pledgeability is low, but not when pledgeability is high. We solve explicitly for the maximal credible payment \( \bar{D}_1^{\psi,\text{Max}} \) in all these cases.

(i) Pledgeability does not matter for repayment

When \( B_1^{H,\psi}(\gamma) = C_2 \), industry liquidity is sufficiently high that high-type insiders can pay the full price of the asset, even if the incumbent has chosen low general pledgeability for period 2, so \( \bar{D}_1^{\psi,\text{Max}} = C_2 \). In this case, there is no potential underpricing and pledgeability does not matter for repayment. As a result, the incumbent will set pledgeability to be low. External payments are committed to through the high resale price of the asset. High pledgeability is neither needed nor desired by anyone in this case.

(ii) Incumbent cannot outbid industry insiders in an auction

When \( C_2 > B_1^{H,\psi}(\gamma) > B_1^{i\psi}(\gamma^i) \), the incumbent can never retain control if she enters into an auction, and will lose control unless the remaining debt to be repaid is lower than her own ability to bid, \( B_1^{i\psi}(\gamma^i) \). In this case, the incumbent is incentivized to sell the firm at the highest possible price.
Therefore, \(\hat{D}^{n,\text{Max}} = B^{H,n}_1(\mathcal{Y}) - \varepsilon\), and the incumbent sets next period’s pledgeability at the highest level (recouping the cost \(\varepsilon\) of setting pledgeability high by keeping the promised payment \(\varepsilon\) below the auction price).

(iii) Incumbent always retains control conditional on retaining ability

Consider \(B^{i,n}_1(\gamma^i) \geq \text{Min}[\hat{D}^{n}_1, B^{H,n}_1(\mathcal{Y})]\), so that the incumbent retains control even if general pledgeability is at its maximum. She will choose \(\gamma^2 = \mathcal{Y}\) iff

\[
\theta^H (C_2 - \text{Min}[\hat{D}^{n}_1, B^{H,n}_1(\mathcal{Y})]) + (1 - \theta^H)(B^{H,n}_1(\mathcal{Y}) - \text{Min}[\hat{D}^{n}_1, B^{H,n}_1(\mathcal{Y})]) - \varepsilon \\
\geq \theta^H (C_2 - \text{Min}[\hat{D}^{n}_1, B^{H,n}_1(\mathcal{Y})]) + (1 - \theta^H)(B^{H,n}_1(\gamma^2) - \text{Min}[\hat{D}^{n}_1, B^{H,n}_1(\mathcal{Y})])
\] (1)

The left hand side is the incumbent’s rents if she chooses \(\gamma^2 = \mathcal{Y}\), while the right hand side is the incumbent’s rents if she chooses \(\gamma^2 = \gamma\). The first term on each side of (1) is the residual amount the incumbent expects if she remains a high type in period 2. The second term on each side is the expected residual amount if she loses her ability and becomes a low type, and has to auction the firm at date 1. Note that a higher \(\gamma^2\) (weakly) increases the amount the financier gets and (weakly) decreases the first term, while it (weakly) increases the amount the incumbent gets in the auction and (weakly) increases the second term. The incumbent has the incentive to set \(\gamma^2 = \mathcal{Y}\) if by doing so she gets more, net of the cost \(\varepsilon\), than by setting \(\gamma^2 = \gamma\), and obtaining the expected amount on the right hand side.

The maximum level of promised payment \(\hat{D}^{n}_1\) that still gives her an incentive to choose \(\gamma^2 = \mathcal{Y}\) is easily checked to be \(D^{n,\text{PayIC}}_1 = \theta^H B^{H,n}_1(\mathcal{Y}) + (1 - \theta^H)B^{H,n}_1(\mathcal{Y}) - \varepsilon\). If the promised payment \(\hat{D}^{n}_1 > D^{n,\text{PayIC}}_1\), the incumbent sets \(\gamma^2 = \gamma\).

Note that the maximum incentive compatible payment, \(D^{n,\text{PayIC}}_1\), falls in stability, \(\theta^H\). Intuitively, higher is stability, lower the likelihood of a forced sale, and greater the attractiveness of choosing low pledgeability to reduce the enforceable payment, so lower the payment that can be sustained. Greater stability in an industry reduces the likelihood that different management
capabilities will be needed, and reduces management’s incentive to maintain high pledgeability for any debt level.3

(iv) Incumbent could lose control depending on the level of pledgeability

Now consider what happens when $B_{i, s}^H (\bar{\gamma}) \leq B_{i, s}^H (\bar{\gamma}) < \min[\tilde{D}_{i}^s, B_{i, s}^H (\bar{\gamma})]$ so that the incumbent retains control if she chooses low general pledgeability and continues to be a high type, because she lowers to $B_{i, s}^H (\bar{\gamma})$ the payment she has to make to retain control. By contrast, if she chooses high pledgeability, she loses control no matter what type she is because the high promised payment is enforceable and higher than what she can pay. So she chooses high pledgeability if

$$(B_{i, s}^H (\bar{\gamma}) - \min[\tilde{D}_{i}^s, B_{i, s}^H (\bar{\gamma})]) - \varepsilon \geq \theta (C_2 - \min[\tilde{D}_{i}^s, B_{i, s}^H (\bar{\gamma})])$$

(2)

This requires $\tilde{D}_{i}^s$ not to exceed $D_{i}^s$,Control IC $= B_{i, s}^H (\bar{\gamma}) - \theta (C_2 - B_{i, s}^H (\bar{\gamma})) - \varepsilon$. Intuitively, promised payments cannot be too high if the choice of high pledgeability means a certain loss of control – the incumbent needs to obtain adequate rents from sale to choose high pledgeability. It is easily checked that $D_{i}^{s, \text{PayIC}} \geq D_{i}^{s, \text{Control IC}} > B_{i, s}^H (\bar{\gamma})$. Note that the primary rationale for choosing low pledgeability differs between case (iii) and case (iv), hence the different superscripts on $D_{i}$. In case (iii) where the incumbent can always outbid insiders, the incumbent wants to keep general pledgeability low to reduce the amount she must pay to win the auction. In case (iv) where the incumbent can outbid industry insiders only when pledgeability is low, she wants to set pledgeability low so as to not lose control. Lemma 2.1 summarizes the results when $s_i = s \in \{G, B\}$.

Lemma 2.1

(i) If $B_{i, s}^H (\bar{\gamma}) = C_2$, there is no potential underpricing and $\tilde{D}_{i}^{s, \text{Max}} = C_2$ and $\bar{\gamma}_2 = \bar{\gamma}$. For any promised payment $\tilde{D}_{i}^s \leq \tilde{D}_{i}^{s, \text{Max}}$, the incumbent expects $V_{i, s}^s (\tilde{D}_{i}^s) = C_2 - \tilde{D}_{i}^s$.

If $C_2 > B_{i, s}^H (\bar{\gamma})$ and

---

3 There is a parallel here to Jensen (1986)’s argument that free cash flows in mature industries lead to greater waste. In his view, the paucity of investment needs in mature industries results in firms generating substantial free cash flows (and hence poorer governance because of a lower need to return to the market for funding). In our model, the lower probability of the need to sell the firm to managers with different capabilities (or equivalently, the lower need to issue financial claims to raise finance for unmodeled investment) in a mature or stable industry reduces the need to maintain better outside pledgeability.
(ii) if $B^H_i(\gamma^i) > B^i_i(\gamma^i)$, the incumbent can never outbid the insider, and
\[
\hat{D}^s_{1,\text{Max}} = B^H_{1,s}(\gamma) - \epsilon \quad \text{For any promised payment } \hat{D}^s_1 \leq \hat{D}^s_{1,\text{Max}}, \text{ the incumbent chooses } \\
\gamma_2 = \gamma, \text{ and expects } V^{i,s}_1(\hat{D}^s_1) = B^H_{1,s}(\gamma) - \hat{D}^s_1 - \epsilon \quad \text{if } B^i_i(\gamma^i) < \hat{D}^s_1 \leq \hat{D}^s_{1,\text{Max}}, \text{ and} \\
\text{expects } V^{i,s}_1(\hat{D}^s_1) = \theta^H C_2 + (1 - \theta^H)B^H_{1,s}(\gamma) - \hat{D}^s_1 - \epsilon \quad \text{if } \hat{D}^s_1 \leq B^i_i(\gamma^i).
\]

(iiiia) if $B^i_i(\gamma^i) \geq D^s_{1,\text{PayIC}}$, then $\hat{D}^s_{1,\text{Max}} = D^s_{1,\text{PayIC}}$. For any promised payment $\hat{D}^s_1 \leq \hat{D}^s_{1,\text{Max}}$, incumbent chooses $\gamma_2 = \gamma$ and expects $V^{i,s}_1(\hat{D}^s_1) = \theta^H C_2 + (1 - \theta^H)B^H_{1,s}(\gamma) - \hat{D}^s_1 - \epsilon$.

(iiiib) if $D^s_{1,\text{PayIC}} > B^i_i(\gamma^i) \geq D^s_{1,\text{ControlIC}}$, then $\hat{D}^s_{1,\text{Max}} = B^i_i(\gamma^i)$. For any promised payment $\hat{D}^s_1 \leq \hat{D}^s_{1,\text{Max}}$, incumbent chooses $\gamma_2 = \gamma$ and expects
\[
V^{i,s}_1(\hat{D}^s_1) = \theta^H C_2 + (1 - \theta^H)B^H_{1,s}(\gamma) - \hat{D}^s_1 - \epsilon.
\]

(iv) $D^s_{1,\text{ControlIC}} > B^i_i(\gamma^i) \geq B^H_{1,s}(\gamma)$, then $\hat{D}^s_{1,\text{Max}} = D^s_{1,\text{ControlIC}}$. For any promised payment $\hat{D}^s_1 \leq \hat{D}^s_{1,\text{Max}}$, the incumbent chooses $\gamma_2 = \gamma$, and expects
\[
V^{i,s}_1(\hat{D}^s_1) = \theta^H C_2 + (1 - \theta^H)B^H_{1,s}(\gamma) - \hat{D}^s_1 - \epsilon \quad \text{if } B^i_i(\gamma^i) < \hat{D}^s_1 \leq \hat{D}^s_{1,\text{Max}}, \text{ and} \\
\text{expects } V^{i,s}_1(\hat{D}^s_1) = \theta^H C_2 + (1 - \theta^H)B^H_{1,s}(\gamma) - \hat{D}^s_1 - \epsilon \quad \text{if } \hat{D}^s_1 \leq B^i_i(\gamma^i).
\]

Proof: Sketched in the text above.

Interestingly, the maximum credible payment $\hat{D}^s_{1,\text{Max}}$ does not increase monotonically with incumbent pledgeability $\gamma^i$. When $\gamma^i$ is low as in case (ii), the incumbent cannot retain control when debt levels exceed what she can pay. Since she always gets more by selling, she has all the incentive to set pledgeability high (except for the cost $\epsilon$), and hence promised payments can be set very high. By contrast, as $\gamma^i$ increases so that case (iv) applies and she has the chance to maintain control, her incentives start mattering, resulting in a lower ability to promise payments. The maximum credible promised payment now falls as $\gamma^i$ rises (from case (ii) to (iv)) because of moral hazard.
Figure 3a illustrates this non-monotonicity by plotting $\tilde{D}_{i,\text{max}}^{b}$ as a function of $\gamma^j$. $\tilde{D}_{i,\text{max}}^{b}$ is the highest when $\gamma^j < 0.2$. After a dramatic drop, $\tilde{D}_{i,\text{max}}^{b}$ starts to increase when $\gamma^j$ increases from 0.2 to 0.23 and stays flat after that.  

What about the effects of industry liquidity, or equivalently, the wealth of industry insiders, $\omega_i^{H,s_i}$? An increase in industry liquidity will push up the amount industry insiders can pay,

---

4 The numerical parameters for this example are:

$\bar{\gamma} = 0.4$, $\gamma = 0.3$, $\omega_i^{H,0} = 0.5$, $\omega_i^{s} = 0.6$, $\rho = 0.3$, $C_1 = C_2 = 1$, $\theta^H = 0.7$, $\epsilon = 0$. 

15
\[ B_1^{H,\gamma} (\gamma_2) \], for any level of pledgeability. This will increase the maximum pledgeable payment whenever there is potential underpricing. Figure 3b illustrates this by plotting \( \hat{D}_{i,\max}^G \) against \( \omega_i^{H,G} \). It follows that

**Corollary 2.1:** \( \hat{D}_{i,\max}^s \) is weakly increasing in \( \omega_i^{H,s} \), and strictly increasing if and only if there is potential underpricing.

Also, because moral hazard in setting pledgeability increases as the incumbent’s likelihood of losing ability decreases, the amount the industry insider incumbent can raise falls in industry stability, \( \theta^H \).

**Corollary 2.2:** \( \hat{D}_{i,\max}^s \) is weakly decreasing in industry stability \( \theta^H \).

We end this section by discussing the incumbent’s incentive to store cash in a place the lender cannot access. The incumbent has no incentive to store any cash at date 1 when contracts are fully state-contingent. To see this, note that given the state, promised payments, and bids,

\[ (s, \hat{D}_1^s, B_1^{H,s} (\gamma_2)) \], storing cash reduces the incumbent’s bid \( B_1^{i,s} (\gamma^i) \) and increases her date 2 consumption unit for unit, if she is still able to maintain control. However, storing cash makes the incumbent weakly more likely to lose control. Therefore, storing cash always makes the incumbent worse off if there is underpricing, and indifferent if not.

**Proposition 2.1:** During period 1, low general pledgeability \( \gamma_2 = \gamma \) is chosen only when there is no potential underpricing. At date 1, the incumbent does not have the incentive to carry cash forward to period 2.

### 2.3 Date 0

We will subsequently add a period 0 between dates -1 and 0, and for that purpose we now also keep track of the date-0 bid by the industry insiders. Once again, this will be the minimum of the funds the bidder can pay at date 0, and the cash flows he hopes to generate through ownership.

\[
B_0^{H,s} (\gamma_1) = \max_{\hat{D}_0^s \leq \hat{D}_0^{s,\max}} \min_{\hat{D}_1^s \leq \hat{D}_1^{s,\max}} \left[ \omega_0^{H,s_0} + q_1 \gamma G \left( \gamma_1 C_1 + \hat{D}_1^G \right) + (1 - q_1 \gamma G) \hat{D}_1^B, \right.
\]

\[
q_1 \gamma G \left( C_1 + \hat{D}_1^G + V_1^{i,G} (\hat{D}_1^G) \right) + (1 - q_1 \gamma G) \left( \hat{D}_1^B + V_1^{i,B} (\hat{D}_1^B) \right) \right].
\]

Financiers who have no managerial abilities may also bid at date 0, just to resell one period later. They require a residual value of only \( \epsilon \) to set \( \gamma_2 = \overline{\gamma} \) (they want to maximize sale value since they have no value from retaining the firm in period 2, so they set pledgeability high), their bids are:
\[ B_0^{L,G} = q_{sG}^T B_1^{H,G}(\overline{\gamma}) + (1 - q_{sG}^T) B_1^{H,B}(\overline{\gamma}) - \epsilon. \]

### 2.4 Financing Cycles with State-contingent Contracts

We now apply the above analysis to study the variations in pledgeability, financing capacity, as well as management turnover across the business cycle. To do this, we explicitly take account of the initial state of industry liquidity at date 0, and consider beginning in either \( s_0 = G \) and \( s_0 = B \). The initial state \( s_0 \in \{G, B\} \) summarizes the entire history before date 0, and represent the industry having experienced a boom or a recession respectively. We compare the outcomes in four states, denoted by \( s_0 s_1 \).

#### A. Pledgeability Choice

Proposition 2.1 tell us that low pledgeability is chosen if and only there is sufficient industry liquidity such that there is no potential underpricing. Simple comparison tells us that industry-wide liquidity is unambiguously the highest in state GG, which is meant to capture large and long-term booms. Therefore, pledgeability is most likely to be low in state GG.

For the remaining analysis, we make the following parametric restrictions such that there is no potential underpricing if and only if the economy is in state GG.

\[ \omega_0^{H,G} + \rho C_1 + \gamma C_2 > C_2, \]
\[ \omega_0^{H,G} + \gamma C_2 < C_2, \]
\[ \omega_0^{H,B} + \rho C_1 + \gamma C_2 < C_2. \]

This immediately implies Proposition 2.2.

**Proposition 2.2:** When state-contingent contracts are used to raise funds for bidding for the firm, pledgeability is set low only after large and long-term booms.

#### B. Management Turnover and Financing Capacity

When is management turnover likely to happen with fully contingent contracts? If the incumbent only sells the asset when she loses her ability, the turnover rate is \( (1 - \theta^H) \), which is the normal rate of industry instability. Note that the incumbent’s capacity to pay for the assets at date 1 is more volatile than that of industry insiders. The incumbent’s capacity to pay increases more than that of industry insiders when \( s_1 = G \), because not all of the cash flow generated by the firm is pledged. So turnover will be at the normal rate, since the incumbent can match outsider bids. In bad times (when \( s_1 = B \),...
$B_i^{s, s_i} (\gamma^i)$ could be below $B_i^{H, s_i}(\gamma)$ or $D_i^{s_i, \text{ControlIC}}$. In either case, the incumbent is replaced even if she retains her ability, so the 100% management turnover is more frequent than the normal rate, $\theta^H$. Intuitively, the incumbent in these cases sets pledgeability high because she gets enough to pay the cost of pledgeability in doing so, but gets nothing if she sets pledgeability low.

To see the conditions under which the incumbent always turnover, we compare $B_i^{s, s_i} (\gamma^i)$ with $B_i^{H, s_i}(\gamma)$ and $D_i^{s_i, \text{ControlIC}}$ in Table 1 below in different states. State GG is omitted since there is no potential underpricing. We assume there always exists underpricing in BG and BB even if high pledgeability is set. In addition, we assume that the incumbent and industry insiders start with the same wealth $\omega_1$ at the beginning of period 0. The incumbent pays $B_{-1}$ upfront to acquire the asset, and receives $(1 - \gamma_0) C_0$ if $s_0 = G$, but nothing if $s_0 = B$. Therefore,

$$\omega_0^G = \omega_1 - B_{-1} + \rho C_0 + (1 - \gamma_0) C_0, \quad \omega_0^B = \omega_1 - B_{-1} \quad \text{and} \quad \omega_0^{H,G} = \omega_1 + \rho C_0 - \omega_0^{H,B} = \omega_1$$

### Table 1: Wealth Comparison in Different States ($\epsilon = 0$)

<table>
<thead>
<tr>
<th>State</th>
<th>GB</th>
<th>BG</th>
<th>BB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_i^{s, s_i} (\gamma^i)$ - $B_i^{H, s_i}(\gamma)$</td>
<td>$(1 - \gamma_0) C_0 + (\gamma^i - \gamma) C_2 - B_{-1}$</td>
<td>$(1 - \gamma_1) C_1 + (\gamma^i - \gamma) C_2 - B_{-1}$</td>
<td>$(\gamma^i - \gamma) C_2 - B_{-1}$</td>
</tr>
<tr>
<td>$B_i^{s_i, \text{ControlIC}}$ - $D_i^{s_i, \text{ControlIC}}$</td>
<td>$(1 - \gamma_0) C_0 + (\gamma^i - \gamma) C_2 - B_{-1}$</td>
<td>$(1 - \gamma_1) C_1 + (\gamma^i - \gamma) C_2 - B_{-1}$</td>
<td>$(\gamma^i - \gamma) C_2 - B_{-1}$</td>
</tr>
<tr>
<td></td>
<td>$+ \theta^H \left[ (1 - \gamma) C_2 - \omega_1 - \rho C_0 \right]$</td>
<td>$+ \theta^H \left[ (1 - \gamma) C_2 - \omega_1 - \rho C_1 \right]$</td>
<td>$\theta^H \left[ (1 - \gamma) C_2 - \omega_1 \right]$</td>
</tr>
<tr>
<td></td>
<td>$+ \max \left{ C_2 - (w_{-1} + \rho C_0 + \tau C_2), 0 \right}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Let $C_0 = C_1 = C_2$ and $\gamma_0 = \gamma_1$. We further assume $1 - \gamma_1 - \theta^H \rho > 0$ so that the non-pledged cash flow exceeds the increased liquidity of industry insiders. A comparison across different rows in Table 1 shows that both $B_i^{s_i} (\gamma^i) < D_i^{s_i, \text{ControlIC}}$ and $B_i^{s, s_i} (\gamma^i) < B_i^{H, s_i}(\gamma)$ are most likely to hold in state BB, and least likely to hold in state GB. Moreover, these two equations are also more likely to hold for small $\theta^H$ and large $\rho$. In state BB, $B_i^{s_i, \text{BB}} (\gamma^i) < B_i^{H, s_i, \text{BB}} (\gamma)$ holds if and only if $(\gamma^i - \gamma) C_2 < B_{-1}$. In section 4.1, we extend the model by adding a previous period and show that this condition is not restrictive.

**Proposition 2.3:** When state-contingent contracts are used, management turnover (which is due to payment default and being outbid) is more frequent when the industry is in persistent distress. Financing capacity is highest in large and long-term booms.
Proof: Sketched in the text above.

Note that with state-contingent contracts, the capacity to borrow against future cash flows is likely to be high (compared to other states) in large and long-term booms. The industry is so liquid that there is no potential underpricing, the high type incumbent manager does not care who will manage the asset, removing the moral hazard over pledgeability. So ex-post financial capacity is high (and equal to the full value of the asset). Incumbents have as much liquidity as industry insiders, and turnover is at a normal level.

In intermediate liquidity states (states BG or GB), then there is potential underpricing and the incumbent may not find it incentive compatible to increase pledgeability unless required payments are set low.

By contrast, in deep and long-term recessions, as in state BB, the incumbent may not be able to hold on to control in an auction, even with low pledgeability. Intuitively, she has paid up front for buying the firm (a price which may include expected cash flow from the unrealized good state), and does not generate any internal cash flow in the bad state. So she may have a lower capacity to bid for continuing control than other industry insiders. Knowing she will lose control, she will experience lower moral hazard over pledgeability relative to the intermediate states. Even though liquidity is low, payment can be set very close to the highest possible outside bid (modulo the small cost, ε, of setting pledgeability high). Nevertheless, financing capacity is low in this case, since industry insiders themselves cannot bid much due to their limited wealth. Although financing capacities are low in all the three states GB, BG and BB, the reasons are not identical. In GB and BG, the incumbent is plagued by moral hazard on pledgeability choice. In BB, industry insiders simply have too little wealth.

We continue the numerical example.\(^5\) In particular, all agents start with initial wealth \(\omega_{-1}\) = 0.32 and one industry insider pays \(B_{-1} = 0.26\) at date -1 to acquire the firm. As a result, the incumbent has wealth \(\omega_{0}^{G} = 0.6\) if the period 0 state is G and \(\omega_{0}^{B} = 0.06\) if the period 0 state is B. The assumption in Proposition 2.3 holds so that in state BB, the incumbent always loses control. In state GG, \(B_{1}^{H,G} = 1 = C_{2}\) and there is no potential underpricing. In state GB and BG, the incumbent is constrained by the Pay IC constraint and the maximal credible payment are \(\tilde{D}_{1}^{GB,Max} = 0.83\) and \(\tilde{D}_{1}^{BG,Max} = 0.944\). In state BB, \(\tilde{D}_{1}^{BB,Max} = B_{1}^{H,BB} (\gamma_{BB}) = 0.72\). Pledgeability is set high in all states.

\(^5\) In addition to the parameters used in the example on which Figure 3a and 3b are based, we have \(C_{0} = 0.6, \gamma_{0} = \sqrt{\gamma}, q^{GG} = 0.8, q^{BG} = 0.1, \omega_{0}^{H} = 0.5 = \omega_{0}^{H,B} + \rho C_{0}, \omega_{0}^{G} = 0.6 = \omega_{0}^{B} + (1 - \gamma_{0} + \rho) C_{0}\).
except state GG. Management turnover rate equals to the normal rate of industry instability 
$(1 - \theta^H) = 0.3$ in all states except state BB. When state BB is realized, the incumbent manager is 
forced to leave and management turnover rate equals to 1.

2.5. Discussion

We have outlined two kinds of moral hazard – moral hazard over repayment, and moral 
hazard over setting pledgeability. The two are connected. When the economy is in a prolonged boom, 
industry insiders can pay full value for the firm even when pledgeability is set at a minimum level. 
There is no need to reduce the moral hazard over repayment by increasing pledgeability since 
creditors can extract full repayment through the threat of asset sales. However, when industry wide 
liquidity is lower, industry insider bids for the firm are lower than the cash flows it generates. This 
underpricing means that their bid can be raised by setting pledgeability higher. Not only does this 
raise what the incumbent can get if she has to sell the firm, it also increases the financier’s ability to 
extract repayment from her if she does not. Thus when the firm finances in the midst of industry 
illiquidity, increasing pledgeability is important to reduce moral hazard over repayment. At the same 
time, the incumbent also faces the possibility of moral hazard over pledgeability. Interestingly, this 
moral hazard is not only about fixed payments as in the standard debt overhang models. Instead, it 
stems from any committed payment.

The past and the future of the industry thus interact in interesting ways. When the industry 
experiences past good outcomes, and the future is also expected to be good, financing capacity is the 
highest. Not only is the firm likely to generate more in the future, but financiers can expect to recover 
what they lent through the threat of selling the fully priced asset. So they are willing to lend large 
amounts. For intermediate levels of past industry performance, industry bidders cannot bid full value 
for the firm’s future cash flows out of their accumulated liquidity, so pledgeability of future cash 
flows becomes important to getting high outside bids and repayment. But because moral hazard over 
pledgeability kicks in, committed payments cannot be too high so as to not discourage high 
pledgeability. A fall in industry performance therefore has the double whammy effect on financing of 
both increasing the underpricing of the firm’s asset by other industry bidders (because of their reduced 
liquidity) and also reducing the maximum possible committed payment as a fraction of that lower 
value (because of moral hazard over pledgeability).

Finally, there is an additional twist because the incumbent’s liquidity potentially fluctuates 
more with industry performance than other industry insiders, because she has paid out her cash up 
front to take control over the firm initially (as we will see later). This means that if the industry has a 
sequence of bad outcomes, the incumbent may be unable to retain control in the face of higher 
industry bids for the firm. Interestingly, this will reduce moral hazard over pledgeability since the
incumbent, with no hope of retaining control, focuses on getting the maximum bid for the firm. The payments that can be committed to lenders will now be a higher fraction of firm resale value, even though intrinsic firm value itself is low. Indeed, this aspect of the model is reminiscent of Jensen’s Free Cash Flow Theory (Jensen (1986)), where lower cash flow with the incumbent reduces moral hazard.

In the model thus far, we have shown that management turnover is higher in persistent downturns, because not only are there voluntary turnovers because the incumbent loses ability, but there are also involuntary turnovers because the incumbent is not able to outbid other industry insiders for control. We have, however, ignored another form of voluntary turnover – when the incumbent retains ability but wants to retire from the business at some time of their choosing. If incumbents can choose the timing of their leaving the business, they would certainly prefer to sell out when the asset is fully priced than when the asset is priced at a fraction of its fundamental value. This effect of exit through retirement would increase the turnover in times of high liquidity (persistent industry up turns) relative to other times. When we explore the effects of debt contracts, we will see another reason why aggregate turnover can be lower in downturns.

2.6. **Ex-ante Pledgeability Choice with State-Contingent Contracts**

What happens if fully state contingent contracts are written but the incumbent decides pledgeability *ex-ante*, i.e., before she knows the state this period? Figure 4 below shows the timing. The incumbent makes decision based on the probabilities of each state. If the cost, $\varepsilon$, is sufficiently small, there is no effect on real outcomes. High pledgeability is chosen *ex-ante* if the incumbent had the incentive to choose high pledgeability *ex-post* in at least one of the two subsequent states.

![Figure 4: Timing and Decisions with ex-ante Pledgeability Choice](image)

If the incumbent had the incentive to choose high pledgeability in both the subsequent states when she was making the choice ex post the knowledge of the state, then she would choose high pledgeability *ex ante*. The state continent payments would be identical even if the cost $\varepsilon$, is not small (because the payments set when choice is ex-post give an incentive to increase pledgeability in both
states). However, if liquidity is so high in one of the states that there is no potential underpricing (or potential underpricing of less than $\varepsilon$), and hence there was no incentive to increase pledgeability anticipating the outcome in that state, then there must be a lower state contingent payment in the other state so that the incumbent has sufficient rents to cover the cost of choosing high pledgeability before the state is known. This means the incumbent’s ability to raise funding will (weakly) fall relative to the expected funding when pledgeability is chosen ex post if the cost $\varepsilon$ is significant. If the probability of the fully liquid state and the cost $\varepsilon$ are both sufficiently high, the incumbent may even chose low pledgeability ex ante, since it is not worthwhile to lower the promised payment in the unlikely other state enough to give her the incentive to incur cost $\varepsilon$.

In the baseline case of our model, where $\varepsilon$ is small and contracts are state-contingent, the timing of the choice is not very important. We will revisit the implications of this timing choice when we study debt contracts.

III. Extensions

In this section, we introduce two extensions to the basic model. In section 3.1, we add one more period to the model, and show that at date 0, the asset can be sold to a financier who has no production capabilities. Such an allocation is more likely when the moral hazard on pledgeability limits the amount that the incumbent can credibly repay. In section 3.2, we relax the assumption that the incumbent cannot produce conditional on losing her ability. Instead, we assume that she can produce a fraction $\alpha C_T$ when the state is good. Interestingly, the maximal credible payment $\tilde{D}_{1,\text{Max}}$ can decrease significantly with $\alpha$, since the incumbent may want to retain control even if she loses some ability.

3.1. The Dynamic Model with fully state-contingent contracts

The allocations so far are always efficient. This is because high type managers always manage the asset and we have assumed that the economy starts in period 1 with a high-type manager in place. In this section, we add one more period upfront and show that in that period the asset could be sold to a financier, despite his having no ability to generate cash flows.

The setup is identical to that in Section I except that we formally introduce an additional period 0 and date -1. The economy has 4 dates—-1, 0, 1, 2— and 3 periods (period 0, 1 and 2). There is uncertainty over state realization in both period 0 and period 1. At date -1, the probability of a good state realized in period 0 is $q^G$. At date 0, the probability of a good state realized in period 1 is $q^G$. Figure 5 describes the state of nature in the dynamic model.
The analysis at date 1 remains unchanged. Here, we focus on date 0, when financiers may also bid successfully. Lemma 3.1 below is the date 0 analog of Lemma 2.1. We omit the payoff functions for simplicity.

Let $B^{I,s}_{0}$ and $B^{H,s}_{0}(\gamma)$ respectively be the bid by financiers and industry insiders. Let $B^{\text{min},s}_{0} = \max\{B^{I,s}_{0}, B^{H,s}_{0}(\gamma)\}$ be the minimum bid the incumbent will face.

**Lemma 3.1**

Let $s_0 = s_\perp$

(i) If $B^{\text{min},s}_{0} \geq q^G C + C_2$, then $\tilde{D}^{\text{Max}}_{0,s} = q^G C_1 + C_2$, and $\gamma_1 = \gamma$.

(ii) else if $q^G C + C_2 > B^{\text{min},s}_{0} \geq B^{H,s}_{0}(\overline{\gamma})$, then $\tilde{D}^{\text{Max}}_{0,s} = B^{\text{min},s}_{0}$ and $\gamma_1 = \gamma$.

(ii) else if $B^{\text{min},s}_{0} > B^{i,s}_{0}(\gamma')$ then $\tilde{D}^{\text{Max}}_{0,s} = B^{H,s}_{0}(\overline{\gamma}) - \varepsilon$ and $\gamma_1 = \overline{\gamma}$.

else if $B^{H,s}_{0}(\overline{\gamma}) > B^{\text{min},s}_{0}$ and if

(iii) $B^{i,s}_{0}(\gamma') \geq D^{\text{IC}}_{0}$, then $\tilde{D}^{\text{Max}}_{0,s} = D^{\text{IC}}_{0}$ and $\gamma_1 = \overline{\gamma}$.

(iii) else if $D^{\text{IC}}_{0} > B^{i,s}_{0}(\gamma') \geq D^{\text{Control IC}}_{0}$, then $\tilde{D}^{\text{Max}}_{0,s} = B^{i,s}_{0}(\gamma')$ and $\gamma_1 = \overline{\gamma}$.

\[ \text{23} \]
(iv) \( D^{\text{Control IC}}_0 > B^{L,s}_0(\overline{\gamma}) \geq B^{\text{min},s}_0 \) then \( \tilde{D}^{s,\text{Max}}_0 = D^{\text{Control IC}}_0 \) and \( \gamma'_1 = \overline{\gamma} \).

Proof: See Appendix for details, including the value function, \( D^{\text{Pay IC}}_0 \) and \( D^{\text{Control IC}}_0 \).

The cases in Lemma 3.1 are similar to those in Lemma 2.1. In case (ia), there is no potential underpricing so that low pledgeability is chosen. Case (1b) is unique to the dynamic model. It incorporates two different scenarios. In the first scenario, \( B^{L,s}_0 \geq B^{H,s}_0(\overline{\gamma}) \geq B^{H,s}_0(\gamma') \) so that industry insiders are constrained by the amount of liquidity they have, and by the potential moral hazard in setting future pledgeability. They are thus outbid by the financiers at date 0. The incumbent chooses low pledgeability in period 0 because she plans to sell the firm to a financier at date 0. Note that renegotiation does not resolve this issue.

Case (1b) holds in a second scenario. Here, \( q^{iG} C_1 + C_2 > B^{H,s}_0(\overline{\gamma}) = B^{H,s}_0(\gamma') \geq B^{L,s}_0 \). Raising pledgeability \( \gamma'_1 \) does not increase bids by industry insiders at date 0 although the price in the date 0 auction is less than the value of total future cash flows. This is because the successful bidder at date 0 is forced to sell out at date 1, and can pay for the interim rents he gets even with low pledgeability \( \gamma'_1 = \overline{\gamma} \). The price is low, but it is not underpriced from the point of view of the winning bidder. In this scenario, again, low pledgeability is chosen.

In contrast to period 1, the incumbent in period 0 therefore selects low pledgeability in two polar cases. As before, in a boom, acquirers have enough cash so that there is no underpricing and thus no potential rents to them. So pledgeability is set low. This is identical to the result at date 1. Interestingly, the period 0 incumbent also sets low pledgeability in a deep bust when a low type financier is able to outbid industry insiders. In that case, a fire sale occurs and the asset is acquired inefficiently by a financier. Although total output is not maximized, this outcome allows a larger amount to be pledged in the auction, since the financier is not subject to moral hazard over pledgeability. In a sense, his role is to change the nature of control rights from those based on asset sales to those based on cash flows. This also means that when faced with the possibility of extreme good and bad industry outcomes at date 0, the incumbent may choose low pledgeability even when able to contract state contingent payments.  

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6 We said earlier that \( (\gamma' - \gamma) C_1 < \omega^{H,B}_0 - \omega^{i,B}_0 = B_{-1} \) was not unusual. Here is why. Suppose that the incumbent starts with the same initial wealth \( \omega_{-1} \) as industry insiders. They acquire the firm by winning an initial auction at date -1 and needed to invest their initial wealth in the firm which ended up returning zero in state B at date 0. Hence, \( (\gamma' - \gamma) C_1 < \omega^{H,B}_0 - \omega^{i,B}_0 = B_{-1} \) will hold.
Example Continued:

We extend our previous example by assigning values to the new parameters in the full dynamic model. Let \( q^G = 0.7 \) and \( \omega, = 0.01 \). An industry insider wins the initial auction by bidding \( B^H_{-1} = 1.1606 \). A financier bids \( B^L_{-1} = 1.0308 \). The initial winner enjoys rents up to 0.4009 and thus she sets \( \left( \tilde{D}^G_0, \tilde{D}^G_0 \right) = \left( \tilde{D}^G_{0, \text{max}}, \tilde{D}^G_{0, \text{max}} \right) = (1.2280, 0.4400) \). The incumbent is able to stay in control if the state in period 0 turns out to be G. However, if state B occurs, he has no cash and other industry insiders are also poor (\( \omega^H, = \omega, \)). Therefore, the low-type financiers outbid everyone during the date 0 auction (\( B^L_0 = 0.44 > B^H_0 (\gamma) = 0.42 \)). As a result, the firm is sold inefficiently to a financier at date 0 if the state was bad. Note that expecting the inefficient sales, the initial date -1 incumbent will set \( \gamma_1 = \gamma \).

3.2. Inefficient Incumbency with State-contingent Contracts

A different extension of the basic model is to consider the incumbent losing ability with probability \( (1 - \theta^H) \) as before, but retaining the ability to produce \( \alpha C \) in the firm when the state is good, where \( \alpha \in (0,1) \), instead of \( \alpha = 0 \) as previously assumed. The disabled incumbent’s productivity now lies between that of the able industry insider, who can produce \( C \) when the state is good, and the financier, who can produce nothing. A new source of moral hazard emerges: the incumbent may want to retain the firm even when she loses some ability. This may necessitate still lower maximum payments so as to restore incentives.

An incumbent who is able in period 1 will remain high ability (H) in period 2 with probability \( \theta^H \) and be able to bid up to \( B^{H,s}_1 (\gamma) = \omega^{H,s}_1 + \gamma^i C \). The incumbent is disabled (L) with probability \( (1 - \theta^H) \) and can bid up to \( B^{L,s}_1 (\gamma) = \omega^{L,s}_1 + \gamma^i \alpha C \). To simplify notation, let us assume that, as before, the disabled incumbent can produce nothing if she leaves the firm.

Note that if \( \alpha C \leq B^{H,s}_1 (\gamma) \) or \( B^{L,s}_1 (\gamma) < B^{H,s}_1 (\gamma) \), the maximum debt capacity derived in Lemma 2.1 remains unchanged. Intuitively, the first inequality indicates that the disabled incumbent can get more by selling than by holding on, even after setting pledgeability low, so she will always sell when she loses ability. The second inequality indicates that the disabled incumbent cannot match the lowest possible outside offer, so once again she will have to sell even if she sets pledgeability low. Given that she sells when she loses ability, the analysis is then identical to that leading to Lemma 2.1.
Matters are different when $\alpha C_2 > B_1^{H,s}(\overline{\gamma})$ and $B_1^{\text{bl},s}(\gamma') > B_1^{H,s}(\gamma)$. First consider $\alpha C_2 > B_1^{H,s}(\overline{\gamma})$. Now it is impossible to provide incentives for the incumbent to choose high pledgeability. By retaining control, not only does she generate more cash flow than the highest possible outside bid, she can (weakly) reduce payout for any level of debt by choosing low pledgeability. She also retains control under all circumstances after choosing low pledgeability (because $B_1^{\text{bl},s}(\gamma') \geq B_1^{H,s}(\gamma)$). So low pledgeability is what she will always choose, and she will always retain control. This outcome is always inefficient since $\alpha C_2 < C_2$. Moreover, the maximum debt she can borrow will be $B_1^{H,s}(\gamma)$.

That leaves $B_1^{H,s}(\overline{\gamma}) > \alpha C_2 \geq B_1^{H,s}(\gamma)$. To determine $D_1^{\text{PayIC}}$, we need to first consider the case where $B_1^{\text{bl},s}(\overline{\gamma}) \geq B_1^{H,s}(\overline{\gamma})$, that is the incumbent can outbid industry insiders at date 1 if she retains her ability, even after choosing high pledgeability. For her to have the incentive to do so (and along the lines of our analysis for Lemma 2.1), it must be that

$$\theta^H(C_2 - D_1^{\text{PayIC}}) + (1 - \theta^H)(B_1^{H,s}(\overline{\gamma}) - D_1^{\text{PayIC}}) - \varepsilon \geq \theta^H(C_2 - B_1^{H,s}(\gamma)) + (1 - \theta^H)(\alpha C_2 - B_1^{H,s}(\gamma))$$

The left hand side is what the incumbent can get by choosing high pledgeability and selling when she loses ability, and the right hand side is what she gets by choosing low pledgeability and retaining control even after losing ability. It is easily seen that the inequality above holds if and only if

$$D_1 < D_1^{\text{PayIC}} = (1 - \theta^H)B_1^{H,s}(\overline{\gamma}) + \theta^H B_1^{H,s}(\gamma) - (1 - \theta^H)(\alpha C_2 - B_1^{H,s}(\gamma)) - \varepsilon.$$

Similarly, when $B_1^{H,s}(\overline{\gamma}) > B_1^{\text{bl},s}(\gamma') \geq B_1^{H,s}(\gamma)$, it can be shown that high pledgeability is chosen if and only if

$$D_1 < D_1^{\text{ControlIC}} = B_1^{H,s}(\overline{\gamma}) - \theta^H[C_2 - B_1^{H,s}(\gamma)] - (1 - \theta^H)[\alpha C_2 - B_1^{H,s}(\gamma)] - \varepsilon$$

Comparing with our earlier values for lemma 2.1, we can see that these values, indicating the maximum incentive compatible debt capacity under different circumstances, are lower by

$$(1 - \theta^H)[\alpha C_2 - B_1^{H,s}(\gamma)],$$

which is the expected rent the incumbent earns if she chooses low pledgeability and turns out to be of low ability.

Somewhat paradoxically, the higher the retention of ability $\alpha$ by the incumbent, the lower the incentive to pledge, and lower the debt she can raise. The consequences of “debt” overhang, even under state-contingent contracts, are thus even more serious – because “pledgeable” debt capacity is so low for an industry insider, the incentive for creditors to seize and sell assets to an industry outsider
when liquidity falls off increases. In a richer model with differentiated incumbents, the incumbents that lose the least ability on becoming disabled will be the first who has to sell out as liquidity falls off, based on how much debt the firm had taken on during the period of high liquidity. These need not be the most inefficient incumbents.

Example Continued:

Let $\alpha = 0.95$ so that $\alpha C_2 \in \left( B^{BG}_1 \left( \bar{\gamma} \right), B^{BG}_2 \left( \bar{\gamma} \right) \right)$. Meanwhile, $\alpha C_2 > B^{BB}_1 \left( \bar{\gamma} \right)$. As a result, $\tilde{D}^{BG}_{1,max} = 0.935 = D^{BG, PayIC}_{1} - \left( 1 - \theta^H \right) \left( \alpha C_2 - B^{BG}_1 \left( \bar{\gamma} \right) \right) = 0.944 - 0.3 \times (0.95 - 0.92)$. Therefore, the maximal credible payment that incumbent is able to raise is reduced when she can inefficiently stay control.

3.3. Discussion

The dynamics from adding an additional period introduce an interesting possibility in downturns. Control can move to outsiders because they do not suffer from moral hazard over pledgeability. While this is inefficient in the sense that outsiders cannot produce cash flows with the assets (and total surplus is not maximized), they can restore pledgeability of the firm. Anticipating restored pledgeability, and thus higher access to finance, initial bids may be higher. If these higher bids are beneficial, for example to permit a minimum quantum of investment to be raised, then temporary outsider control is constrained efficient. Nevertheless, capital will be allocated to produce less output in periods of low liquidity and recoveries from bad states will be slow.

Outsider control is reminiscent of leveraged buyout transactions (see, for example, Jensen (1989)), where firms in stable industries (where moral hazard over pledgeability is high) are taken over, and the management team, which is motivated by the prospect of going public soon, focuses on finding free cash flow that has been eaten up either through inefficiency or misappropriated by staff (the proverbial company jet). The management team does not really make fundamental changes to the firm’s earning prospects in the short time the firm is private, but it significantly enhances the pledgeability of future cash flows, and thus enhances the bids for the firm when it goes public.

The extension where incumbents lose some, but not all, ability suggests the intriguing possibility of entrenchment – because incumbents can generate more value for themselves running the firm, even after losing some ability, than the value of industry bids by more capable industry insiders, they prefer staying in control if possible. Left in charge, the incumbent would set pledgeability very low. So partial retention of ability by the incumbent increases moral hazard over pledgeability, reduces the payments the incumbent can commit to, and increases the likelihood that the firm will be taken over temporarily by industry outsiders in downturns.
It may be useful here to see the differences between our model and the seminal work by Shleifer and Vishny (1992) (henceforth SV). They focus only on liquidity varying over time. SV therefore emphasize control rights exclusively through asset sales while we introduce control rights over cash flow through the pledgeability channel, which itself suffers from moral hazard.

Our model therefore has different implications than Shleifer and Vishny (1992). As in SV, assets migrate in our model to agents who have lower ability to manage. However, the underlying rationale is different. In SV, asset gets inefficiently allocated because highly ability managers have less liquidity than outsiders. Debt, which was created to resolve a free cash problem, has the standard debt overhang effect which limits the amount of liquidity obtainable by industry insiders. Therefore, if financial contracts were state-contingent (or if debt could be renegotiated), the asset would never be sold to outsiders. In our model, the asset goes to low types precisely because they do not suffer from the moral hazard over pledgeability and not because they have more liquidity. Indeed, financiers are unwilling to renegotiate debt down because they know the asset will be sold to low types who can pay more by making the asset more pledgeable even when burdened with high debt. We will point out another important difference shortly.

IV. Debt Contracts

In this section, we turn to debt contracts. Simple debt contracts specify a constant promised payment on a given date in all states, that is, $D^*_t = D_t$ for all $s$. We do not add explicit frictions to make debt the optimal contract, such as costs of verifying the state at the relevant time. We assume there are impediments to renegotiation of debt contracts after the state is known but before pledgeability choices are made. Of course, one could also think of debt payments here as some non-renegotiable fixed costs. If fully renegotiable, debt in some cases becomes identical to state-contingent payments. We will subsequently analyze the model when pledgeability choice is made before the state is realized (but knowing the probabilities of each state). In this case, renegotiation does not occur even if allowed.

When debt contracts are used, it is possible that in bad states incumbents will not have incentives to increase pledgeability; there will be debt overhang on pledgeability choice. The access to financing will become even more volatile than the economic conditions. Intuitively, both fixed promised debt payments across states, and the act of choosing pledgeability before the state is known, have the effect of causing a spillover of effects between anticipated states.

4.1. Ex-post Pledgeability Choice

Because there is a single state in period 2, the promised payment when contracts are restricted to simple debt contracts will be identical to that for state-contingent contracts. At date 1, there are only
two candidates for debt level that will result in the maximum credible expected payment to financiers; the state-contingent promises which maximizes the credible payment in each of the two states, either 
\( \hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i \) or \( \hat{D}_1 \hat{\eta}_B, \max \). If there is no potential underpricing in state B, then 
\( D_1 \hat{\eta}_B, \max = \hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i \) as a high face value does not distort incumbent’s pledgeability choice.

When there is potential underpricing, the incumbent is able to raise 
\( q \hat{\eta}_G [\hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i ] + (1 - q \hat{\eta}_G) B_t^{H, \eta_B} (\gamma) \) at date 0 by setting \( D_1 = \hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i \) and \( \hat{D}_1 \hat{\eta}_B, \max \) by setting \( D_1 = \hat{D}_1 \hat{\eta}_B, \max \). Debt is risky in the former case as the incumbent defaults if state B is realized. In the latter case, debt is riskless. The incumbent can raise more by setting 
\( D_1 = \hat{D}_1 \hat{\eta}_B, \max + \gamma_1 C_i \) if and only if 
\( q \hat{\eta}_G [\hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i ] + (1 - q \hat{\eta}_G) B_t^{H, \eta_B} (\gamma) > \hat{D}_1 \hat{\eta}_B, \max \),

which is likely to hold when \( q \hat{\eta}_G \) is large.

Note that \( D_1 \in (\hat{D}_1 \hat{\eta}_B, \max, \hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i ) \) will not lead to a higher payment to lenders. It always delivers less than \( D_1 \hat{\eta}_B, \max = \hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i \) as it distorts pledgeability choice in state B but pays less than the maximum realizable in state G. Lemma 4.1 summarizes the results at date 1.

Detailed results about payoff functions are listed in the appendix.

**Lemma 4.1:**

1. If there is no potential underpricing even in state \( s_0 B \), \( D_1 \hat{\eta}_B, \max = \hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i \). Pledgeability choices are not distorted (relative to state-contingent contracting) in either state.

2. If there is potential underpricing in state \( s_0 B \) and 
\( q \hat{\eta}_G [\hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i ] + (1 - q \hat{\eta}_G) B_t^{H, \eta_B} (\gamma) > \hat{D}_1 \hat{\eta}_B, \max \), \( D_1 = \hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i \), then pledgeability choice is distorted in state B: \( \gamma_2 = \gamma \). Debt is risky in state B.

3. If there is potential underpricing in state B and 
\( q \hat{\eta}_G [\hat{D}_1 \hat{\eta}_G, \max + \gamma_1 C_i ] + (1 - q \hat{\eta}_G) B_t^{H, \eta_B} (\gamma) < \hat{D}_1 \hat{\eta}_B, \max \), \( D_1 = \hat{D}_1 \hat{\eta}_B, \max \). Pledgeability choice is not distorted in either state. Debt is riskless and leverage is set low so as to induce higher pledging.

Proof: Sketched in the text above.
We maintain the assumptions that there is no potential underpricing in state GG. Suppose states persist sufficiently so that \( q^{GG} \) is large and \( q^{BG} \) is small. Then
\[
q^{GG}[C_2 + \gamma_1 C_1] + (1 - q^{GG})B^{H,GB}_1(\gamma) > D^{GB,\text{Max}}_1 \quad \text{but} \quad q^{BG}[\tilde{D}^{BG,\text{Max}}_1 + \gamma_1 C_1] + (1 - q^{BG})B^{H,BB}_1(\gamma) < D^{BB,\text{Max}}_1.
\]
Low pledgeability is chosen in state GG, because there is no potential underpricing, and also in state GB, because the debt payment is in excess of that which provides the incumbent an incentive for choosing high pledgeability. High pledgeability is chosen in both state BG and BB.

**Proposition 4.1:** With standard debt contracts, if economic states are sufficiently positively autocorrelated then pledgeability will be low after the initial state G and high after the initial state B.

Example continued: Since \( q^{GG} = 0.8, \ D^{G,\text{max}}_1 = \tilde{D}^{GG,\text{max}}_1 + \gamma_1 C_1 = 1.3 \) and the IC constraint is violated in state GB. Therefore, \( \gamma_2 = \gamma \) is chosen in both state GG and GB. In state B, \( D^{B,\text{max}}_1 = \tilde{D}^{BB,\text{max}}_1 = 0.72 \) and \( \gamma_2 = \gamma \) in both BG and BB.

The point is that when the probability of a state is high, the debt contract which promises the most to investors is optimized for that state, causing seemingly sub-optimal outcomes if the state is not realized (of excessive debt, low pledgeability, and default in state GB and excessively low debt in state BG). Thus it may seem that the low probability state is ignored (see Shleifer et al), but our alternative explanation is that contractual rigidities prevent full adaptation. Note that if there is the possibility of renegotiation after the date 1 state is known but before pledgeability is chosen, financiers would be willing to reduce the face value in state GB to induce the incumbent to increase pledgeability. Also, in the context of our dynamic model, if debt caused the incumbent to set pledgeability low following a realization of a low probability state like GB, the cost of subsequently handing the firm over temporarily to an outsider to restore pledgeability would be low. So turnover to outsiders would be especially high (without an immediate increase in productive efficiency) in the unexpected bad times that ensue when hard-to-renegotiate debt is contracted anticipating good times.

**4.2. Ex-ante Pledgeability Choice**

In the previous section, the incumbent set pledgeability after the state in period 1 was already realized (ex-post choice). This reflects aspects of pledgeability which can be changed rather quickly (such as a more reputable accountant). Now let us see what happens when the incumbent chooses pledgeability, based on the probability distribution of the states, before the end of period state is known. This situation represents more durable pledgeability choices such as the specificity of the production technique or the internal organization of a firm, and implies rigidity of pledgeability across states in addition to the fixity of the debt contract. The choice of pledgeability turns out to interact
with traditional debt overhang in several interesting ways. The timing of events in the period is
equivalent to Figure 4 in Section 2.6, with the exception that $D^G_1 = D^B_1$.

When only debt contracts are allowed and pledgeability is decided \textit{ex-ante}, there is a single
incentive constraint. It is easily seen that the maximal debt payment $D^{s_0,IC}_1$ consistent with the
incumbent choosing high pledgeability lies in the range $D^{s_0,B,\text{Max}}_1$ and $D^{s_0,G,\text{Max}}_1 + \gamma_1 C_1$ if there is
potential underpricing in both states at date 1, and it lies below $D^{s_0,B,\text{Max}}_1$ if there is no potential
underpricing in state G only. If there is no potential underpricing in both states G and B, there is no
benefit of high pledgeability, so there is no reason or way to provide incentives for its choice.

To see this, let $\Delta s_0 (D_1)$ be the difference in state $s_0 s_1$ between the borrower’s payoff from
choosing high pledgeability and low pledgeability. When high pledgeability is to be implemented,
$D^{s_0,IC}_1$ satisfies $q^{s_0,IC} \left[ \Delta s_0 \left( D^{s_0,IC}_1 \right) - \gamma_1 (C_1) \right] + (1 - q^{s_0,IC}) \left[ \Delta s_0 \left( D^{s_0,B}_1 \right) \right] = 0$. For all $D_1$,
$\Delta s_0 (D_1) \equiv -\varepsilon$ if there is no underpricing in state $s_0 G$. If there is underpricing in state B, then
$\Delta s_0 (D_1) = 0$ for $D_1 = D^{s_0,B,\text{Max}}_1$ in $s_0 B$, negative for higher values of $D_1$ and strictly positive for all
lower values of $D_1$. So, to incentivize the choice of high pledgeability ex ante when there is no
underpricing in state $s_0 G$, it must be that $\Delta s_0 (D_1) > 0$, which means $D_1 < D^{s_0,B,\text{Max}}_1$. The other cases
are similarly shown.

The level of debt which provides incentives for high pledgeability, $D^{s_0,IC}_1$ may not be the face
value that enables the incumbent to credibly commit to repay the most. If $B^{H,G}_1 (\gamma) + \gamma_1 C_1 > D^{s_0,IC}_1$,
setting promised payment at $B^{H,G}_1 (\gamma) + \gamma_1 C_1$ will lead the incumbent to choose low general
pledgeability, and pay $B^{H,B}_1 (\gamma)$ in state B and $B^{H,G}_1 (\gamma) + \gamma_1 C_1$ in state G. This can provide a larger
expected payment if
\begin{align*}
q^{s_0,IC} \left[ B^{H,G}_1 (\gamma) + \gamma_1 C_1 - D^{s_0,IC}_1 \right] + (1 - q^{s_0,IC}) \left[ B^{H,B}_1 (\gamma) - \min \left\{ B^{H,B}_1 (\gamma), D^{s_0,IC}_1 \right\} \right] > 0. \quad (2)
\end{align*}

Intuitively, when there is no potential underpricing in state G, but high rents have to be given in state
B so as to offset the cost of choosing high pledgeability, $D^{s_0,IC}_1$ may have to be set very low (below
the payment which would provide incentives for high pledgeability if state B was known to occur).
Instead, the incumbent may be able to pay out more by setting debt high, committing to pay out
everything possible in the G state where pledgeability does not matter, and defaulting with low
payout in state B. Note that this occurs even when the cost $\varepsilon=0$. 31
Ex ante pledgeability across business cycles with debt contracts

When is low pledgeability chosen? We maintain the assumption that there is no potential underpricing if and only if the state of the economy is GG. Therefore, \( D_{1}^{G,JC} \leq \tilde{D}_{1}^{GB,Max} \). An examination of inequality (1) tells us that \( D_{1}^{G,JC} \) enables the incumbent to pledge more if and only if \( q^{GG} \) is low. When \( q^{GG} \) is large, \( B_{1}^{H,GG}(\gamma)+\gamma C_{1} = C_{2} + \gamma C_{1} \) can pledge more than \( \tilde{D}_{1}^{GB,Max} \) alone.

We interpret large \( q^{GG} \) as a boom that is likely to continue.

**Proposition 4.2:** When debt contracts are used and pledgeability choice is made *ex-ante*, low pledgeability is chosen in state G to maximize expected payments to financiers if the boom is sufficiently likely to continue.

Proposition 4.2 is very similar to Proposition 4.1, except that it holds even if renegotiation of debt is possible. Given that pledgeability choice is already made, lenders have no incentive to renegotiate debt down after the state is realized.

What if the economy begins in the B state? Recall that when state-contingent contracts were used, the incumbent has no chance to retain control in state BB, if \( \omega_{0}^{H,B} - \omega_{0}^{H,B} - (\gamma' - \gamma) C_{1} < 0 \). Therefore, high pledgeability is always preferred in state BB, as long as the \( \epsilon \) cost is compensated. As \( \epsilon \to 0 \), low pledgeability is never chosen *ex-ante*. To see this, note that when there is potential underpricing in state BG, high pledgeability is always preferred *ex-post*. When the incumbent is indifferent between high and low pledgeability in state BB (except for the \( \epsilon \) cost which converges to 0), \( D_{1}^{B,JC} \to \tilde{D}_{1}^{BG,Max} + \gamma C_{1} \) which exceeds \( B_{1}^{H,BG}(\gamma') + \gamma C_{1} \), the amount that can be committed by setting low pledgeability. As a result, the incumbent is able to payout less in both states once she violates the *ex-ante* incentive constraint. Therefore, in times other than booms, high pledgeability is selected for all probabilities of future outcomes, because all involve potential underpricing if good state realizes, and in very bad times the incumbent will be unable to hold on.

If \( \omega_{0}^{B,B} \geq \omega_{0}^{H,B} - (\gamma' - \gamma) C_{1} \), the incumbent may still retain control in state BB. In that case, the incumbent will choose high pledgeability for both large and small \( q^{BG} \). Clearly, \( D_{1}^{B,JC} \to \tilde{D}_{1}^{BG,Max} + \gamma C_{1} \) when \( q^{BG} \to 1 \), and \( D_{1}^{B,JC} \to \tilde{D}_{1}^{BB,Max} \) when \( q^{BG} \to 0 \). It is only at the intermediate level of \( q^{BG} \) that the incumbent might be able to commit to pay more by violating the incentive constraint. In this case, \( D_{1}^{B,JC} \) is significantly lower than \( B_{1}^{H,BG}(\gamma') + \gamma C_{1} \), and the incumbent can pledge more in BG by setting debt at \( B_{1}^{H,BG}(\gamma') + \gamma C_{1} \). Meanwhile, the benefit of low-
pledgeability, \( q^{BG} \left[ \left( B^{H,BG}_1(y) + y' C_1 \right) - D^{B,JC}_1 \right] \) need to exceed the cost or pledging less in state B, which equals to \( (1-q^{BG}) [D^{B,JC}_1 - B^{H,BB}_1(y')] \).

**Proposition 4.3:** When debt contracts are used and pledgeability choice is made ex-ante, if incumbent liquidity is sufficiently low, \( \omega^i_B < \omega^H_B - (y' - y') C_1 \), low pledgeability is never chosen in state B. Otherwise, low pledgeability might be chosen when \( q^{BG} \) is neither too high nor too low.

**Example Continued:**

Let us continue the example when debt contract is used. Since there is no potential underpricing in state GG, and the incumbent has no hope to maintain control, \( (D^{G,JC}_1, D^{B,JC}_1) \) are \( D^{G,JC}_1 = D^{GB,\text{max}}_1 = 0.83 \) and \( D^{B,JC}_1 = D^{BG,\text{max}}_1 + y_1 C_1 = 1.344 \). The result is consistent with Proposition 4.2 and 4.3. Since state G is very likely to continue, \( q^{GG} = 0.8 \), \( D^{G,\text{max}}_1 = D^{GG,\text{max}}_1 + y_1 C_1 = 1.3 \) and the IC constraint is violated. As a result, low pledgeability \( y_2' = y_2 \) is chosen in state G. High pledgeability would be chosen had \( q^{GG} \) been lower. Indeed, if \( q^{GG} = 0.01 \), \( D^{G,\text{max}}_1 = D^{G,JC}_1 = 0.83 \) and \( y_2' = y_2 \).

In state B, \( D^{B,\text{max}}_1 = D^{B,JC}_1 = 1.344 \) and \( y_2' = y_2 \). In fact, high pledgeability is always chosen for any \( q^{BG} \in [0,1] \). Figure 6 below describes the comparison between \( D^{1,\text{max}}_1 \) and \( D^{1,JC}_1 \) when \( q^{BG} \) varies between 0 and 1.

![Figure 6: \( D^{1,\text{max}}_1 \) and \( D^{1,JC}_1 \) for different \( q^{BG} \)](image)
4.3 Discussion

In a boom which is likely to continue, liquidity is high and when borrowers finance with debt, they will choose low pledgeability. Because with high probability the industry is liquid, higher pledgeability is not very useful, and there is little negative effect when a borrower promises so much that pledgeability is disincentivized. If the boom continues, this debt will be fully repaid and the level of turnover and capital reallocation will be at its normal level; in booms, it does not pay to worry about debt overhang on pledgeability, so leverage can be very high. If the boom does not continue, it is very likely that the incumbent will be able to hold on despite defaulting on the debt contract (such as where the incumbent management stays on after a bankruptcy filing similar to Chapter 11 of the United States bankruptcy law). However, access to finance will drop significantly if there is a negative shock after a boom. In contrast, in times of lower liquidity (normal or bad times), low pledgeability significantly reduces the amount recovered by financiers, implying that market forces lead borrowers to debt contracts which incentivize high pledgeability. In terms of observables, if we use loan contracts with many covenants as a proxy for high pledgeability, the prediction in bad to normal times is many covenants and relatively low levels of leverage when fresh capital structures are chosen (such as when the firm comes out of bankruptcy). In contrast, during booms we will see higher leverage (exceeding that which gives an incentive for high pledgeability) and few or no loan covenants (“covenant lite”). Boom periods with covenant lite loans and high leverage could also be interpreted as an increase in the fraction of market finance (bonds) as opposed to intermediated finance (loans).

For pledgeability choices which take some time to implement, we expect that decisions are made before the ex-post state is known. As a result, when there are distorted choices of pledgeability (few covenants and high leverage) the distortion cannot be resolved by renegotiation.

V. Discussion and Conclusion

We have focused on two kinds of moral hazard in this paper – moral hazard over appropriation of cash flows and moral hazard over pledgeability choice. When the bidder competing with the incumbent can pay full dollar up front for the asset, there is no need for pledgeability and the first moral hazard problem disappears, and so does the moral hazard over pledgeability. Low pledgeability is chosen. It is also chosen when the moral hazard over current pledgeability will be resolved by a sale to an outsider who will later restore high pledgeability. In contrast, when liquidity is low, and the future bidder can appropriate cash flows over and above what she pays, high pledgeability can reduce bidder rents. Since both incumbent rents and industry bidder rents are simultaneously reduced by high pledgeability, the incumbent suffers from moral hazard over pledgeability. Financing capacity is then limited by the need to retain incentives for pledgeability.
In good times the threat of ownership change is the means of enforcing debt contracts, and plentiful liquidity makes the threat credible. The seeds of distress are sown at such times, because incumbents have no incentive to maintain cash flow pledgeability – this alternative source of commitment seems unnecessary when times largely promise to be good. Also, institutions supporting pledgeability, such as forensic accountants, regulations, and regulators, may atrophy from disuse at such times. Moral hazard increases as bad times hit because incumbents have the incentive to enhance their own value by reducing the value of outside financial claimants. Hence financial capacity falls when good times are expected to continue but then end up in bad times, until outsiders take control. Cash flow pledgeability now becomes key to debt capacity, and industry outsiders have the incentive to increase it even in the face of high debt – it is precisely their ineffectiveness in managing the asset that makes them immune from moral hazard over pledgeability. As cash flow pledgeability increases and industry cash flows recover somewhat, industry insiders can once again bid large amounts and return to controlling firms. As liquidity among industry insiders increases further, the threat of asset sales once again becomes the source of debt enforcement. The incentive to maintain cash flow pledgeability wanes once again, and the cycle resumes.

Importantly, the change in effective creditor control rights, from cash-flow-based to asset-sale-based, occurs smoothly when economic conditions continue to improve. Incumbents simply neglect to maintain pledgeability since it is not needed to raise financing. However, when boom turns to bust, past neglect of pledgeability and the distortion to incentives caused by debt overhang ensure the transition from asset-sale-based to cash-flow-based enforcement is not smooth. Economic activity can be disrupted until outside capacity to control (and thus finance) is restored. Real investment, which we do not model, could fall significantly under these circumstances, even when it is positive net present value.

Another way of thinking about these financing cycles is that the pre-peak stage of the industry, where debt capacity relies on the creditors ability to threaten asset sales, may be associated with arm’s length debt. The post-crash stage, where debt capacity relies on cash flow pledgeability (and probably close monitoring), may be more associated with bank or intermediated credit. So our model suggests a pattern of change in the source of credit over time. It also suggests why assets that require management (such as mortgages or bank loans, or the securitized claims on such assets) may have different collateral haircuts associated with them over the cycle, unlike passively held assets such as equities. The haircuts fall in proportion to both the liquidity of industry insiders (on the upturn) and the restoration of pledgeability (in the downturn), with a possible steep increase as the state of the economy switches from upturn to downturn.

While we do not model investment, the point we make would become stronger still if we did. A greater share of the pie is more attractive when increasing the pie through new investment is difficult, so moral hazard over pledgeability increases still further in a downturn, over and above the effects of leverage.
Finally, the fluctuation in debt capacity may be larger if the range of possible pledgeability values is larger. To the extent that financial infrastructure such as accounting standards or collateral registries as well as contractual right enforcement are strong through the cycle, they may prevent large fluctuations in asset pledgeability. By allowing only moderate room to alter pledgeability, a strong institutional environment could lead to more stable credit. However, to the extent that the institutional environment is weak or responds to the cycle (forensic accountants retrain as loan brokers during the boom), asset pledgeability is more endogenous, and credit may vary more over the cycle. Credit booms and busts will be more pronounced in such cases, as are asset price booms and busts.

The model can, with some tweaking, be applied to areas where assets are not actively managed. For instance, an analogous argument to the one above can be made for real estate cycles. In the boom, the reliance on home repossession and resale as the basis for lending (and refinance) implies the lender reduces emphasis on undertaking due diligence on buyers, their income prospects, and their repayment capacity. New potential buyers are liquid because of home equity. In a downturn, repayment capacity becomes important, and the past lack of due diligence comes to haunt lenders. At such times, high debt overhang leads owners to neglect maintenance as there is little chance they will have any equity left in a sale. It may even make sense for a lender to repossess and leave the house vacant (or use the time to fix up the house) so as to get a better price when the recovery starts. The recovery starts as lenders restructure their lending procedures to focus on buyer income and repayment prospects until, as house prices boom, the threat of repossession becomes once again the basis of repayment.

This paper has focused on the choice of (general) pledgeability, assuming incumbent pledgeability to be costless or fixed. We develop implications for pledgeability enhancing devices such as accounting choice, routine production plans and bond covenants. Incumbent pledgeability could be thought of as exclusive relationship lending, which may have varying importance over the cycle. We plan to explore more of these implications in future work.
References


Appendix

Proof of Corrollary 2.1:
If \( \omega_1^{H^{s_i}} = 0 \) and there is potential underpricing, then
\[
D_1^{i, s} \text{ Pay IC} = \theta^H B_1^{H^{s_i}}(\gamma) + (1 - \theta^H) B_1^{H^{s_i}}(\bar{\gamma}) - \epsilon = \theta^H (\gamma C_2) + (1 - \theta^H) \bar{\gamma} C_2 - \epsilon \leq B_1^{i, s}(\gamma')
\]
because \( \bar{\gamma} \leq \gamma' \) and thus \( \hat{D}_1^{i, s, \text{Max}} = D_1^{i, s} \text{ Pay IC} - \epsilon \) according to case (iiiia) of Lemma 2.1. Apparently, \( D_1^{i, s} \text{ Pay IC} \) increases in \( \omega_1^{H^{s_i}} \) until it approaches \( B_1^{i, s}(\gamma') - \epsilon \). Eventually
\[
D_1^{i, s, \text{Max}} < B_1^{i, s}(\bar{\gamma}') \geq D_1^{i, \text{Control IC}}
\]
and according to case (iiib) of Lemma 2.1,
\[
\hat{D}_1^{i, s, \text{Max}} = B_1^{i, s}(\gamma') - \epsilon \text{. Here, } \hat{D}_1^{i, s, \text{Max}} \text{ is flat in } \omega_1^{H^{s_i}} \text{. If } \omega_1^{H^{s_i}} \text{ further increases such that}
\]
\[
D_1^{i, \text{Control IC}} > B_1^{i, s}(\bar{\gamma}') \geq B_1^{H^{s_i}}(\gamma'), \hat{D}_1^{i, s, \text{Max}} = D_1^{i, \text{Control IC}} - \epsilon \text{ according to case iv). Once again,}
\]
\[
\hat{D}_1^{i, s, \text{Max}} \text{ increases in } \omega_1^{H^{s_i}} \text{. With further increase in } \omega_1^{H^{s_i}} \text{, } B_1^{H^{s_i}}(\gamma') > B_1^{i, s}(\gamma') \text{ and according to case (ii)}, \hat{D}_1^{i, s, \text{Max}} = B_1^{H^{s_i}}(\bar{\gamma}') - \epsilon \text{, which also increases in } \omega_1^{H^{s_i}} \text{. Finally, if } \omega_1^{H^{s_i}} \text{ further increases, } B_1^{H^{s_i}}(\gamma') \geq C_2 \text{ and there is no potential underpricing. According to case (i) of Lemma 2.1,}
\]
\[
\hat{D}_1^{i, s, \text{Max}} = C_2 \text{ weakly increases in } \omega_1^{H^{s_i}}.
\]

Proof of Lemma 4.1:

Let \( s_0 \) and \( s_1 \) be the realized states at date 0 and date 1. Define \( \Delta_1^{s_i}(\hat{D}_1^{i, s_i}) \) as the excess of a borrower’s payoff from choosing high pledgeability over the payoff from choosing low pledgeability during period 1. Lemma 7.1 is useful for proving Lemma 4.1. It describes \( \Delta_1^{s_i}(\hat{D}_1^{i, s_i}) \) in various cases of Lemma 2.1.

**Lemma 7.1**: If \( B_1^{H, s_0}(\gamma') < B_1^{i, s_0}(\bar{\gamma}') < C_2 \text{, } \Delta_1^{s_i}(\hat{D}_1^{i, s_i}) > 0 \text{ for } \hat{D}_2^{i, s_i} < \hat{D}_2^{i, s_i, \text{max}} \) and
\[
\Delta_2^{s_i}(\hat{D}_2^{i, s_i}) < 0 \text{ for } \hat{D}_2^{i, s_i} > \hat{D}_2^{i, s_i, \text{max}} \text{. If } B_1^{H, s_0}(\gamma') = C_2 \text{, } \Delta_1^{s_i}(\hat{D}_1^{i, s_i}) = 0 \text{.}
\]

Proof: for notational convenience, we skip \( s_0 \). When \( s_1 = G \), \( \Delta_1^{s_i}(D_1^{i, s_i}) = \Delta_1^{s_i}(\hat{D}_1^{i, s_i} + \gamma C_1) \).

When \( s_1 = B \), we have \( \Delta_1^{s_i}(D_1^{i, s_i}) = \Delta_1^{s_i}(\hat{D}_1^{i, s_i}) \).

(i) If \( B_1^{H, s_0}(\gamma') = C_2 \), \( \hat{D}_1^{i, s, \text{Max}} = C_2 \) and \( \gamma_2 = \bar{\gamma}' \).
$V^i_{1,s}(\hat{D}_1^h, \gamma) = \begin{cases} 0 & \text{if } \hat{D}_1^h > C_2 \\ C_2 - \hat{D}_1^h & \text{if } \hat{D}_1^h \leq C_2 \end{cases}$

$V^i_{1,s}(\hat{D}_1^h, \overline{\gamma}) = \begin{cases} -\varepsilon & \text{if } \hat{D}_1^h > C_2 \\ C_2 - \hat{D}_1^h - \varepsilon & \text{if } \hat{D}_1^h \leq C_2 \end{cases}$

If $C_2 > B^H_{1,s_1}(\gamma')$ and

(ii) if $B^H_{1,s_1}(\gamma') \geq B^i_{1,s_1}(\gamma')$

$\hat{D}_1^{s_1,\text{Max}} = B^H_{1,s_1}(\overline{\gamma'}) - \varepsilon \cdot \gamma' = \overline{\gamma'}$ if $\hat{D}_1^s \leq \hat{D}_1^{s_1,\text{Max}}$ and $\gamma' = \overline{\gamma'}$ if $\hat{D}_1^s > \hat{D}_1^{s_1,\text{Max}}$.

$V^i_{1,s_1}(\overline{\gamma'}, \hat{D}_1^s) = \begin{cases} -\varepsilon & \text{if } \hat{D}_1^s > B^H_{1,s_1}(\overline{\gamma'}) \\ B^H_{1,s_1}(\overline{\gamma'}) - \hat{D}_1^s - \varepsilon & \text{if } B^H_{1,s_1}(\overline{\gamma'}) \geq \hat{D}_1^s > B^i_{1,s_1}(\gamma') \end{cases}$

$V^i_{1,s_1}(\gamma', \hat{D}_1^s) = \begin{cases} 0 & \text{if } \hat{D}_1^s > B^H_{1,s_1}(\gamma') \\ B^H_{1,s_1}(\gamma') - \hat{D}_2^s & \text{if } B^H_{1,s_1}(\gamma') \geq \hat{D}_1^s > B^i_{1,s_1}(\gamma') \end{cases}$

$\Delta^i_{1,\gamma}(\hat{D}_1^s) = \begin{cases} -\varepsilon & \text{if } \hat{D}_1^s > B^H_{1,s_1}(\overline{\gamma'}) \\ B^H_{1,s_1}(\overline{\gamma'}) - \hat{D}_1^s - \varepsilon & \text{if } B^H_{1,s_1}(\overline{\gamma'}) \geq \hat{D}_1^s > B^i_{1,s_1}(\gamma') \\ B^H_{1,s_1}(\gamma') - B^H_{1,s_1}(\gamma') - \varepsilon & \text{if } B^H_{1,s_1}(\gamma') \geq \hat{D}_1^s > B^i_{1,s_1}(\gamma') \\ (1 - \theta^H)[B^H_{1,s_1}(\gamma') - B^H_{1,s_1}(\gamma')] - \varepsilon & \text{if } \hat{D}_1^s \leq B^i_{1,s_1}(\gamma') \end{cases}$

(iii) $B^i_{1,s_1}(\gamma') \geq B^H_{1,s_1}(\overline{\gamma'})$, then $\hat{D}_1^{s_1,\text{Max}} = D_1^{s_1,\text{PayIC}} \cdot \gamma' = \overline{\gamma'}$ if $\hat{D}_1^s \leq \hat{D}_1^{s_1,\text{Max}}$ and $\gamma' = \overline{\gamma'}$ if $\hat{D}_1^s > \hat{D}_1^{s_1,\text{Max}}.$
\[ V_{1,i}^n (\gamma, \bar{D}_1^n) = \begin{cases} \theta^n [C_2 - B_{1;i}^{H,i,n}(\gamma)] - \epsilon & \text{if } \bar{D}_1^n > B_{1;i}^{H,i,n}(\gamma) \\ \theta^n C_2 + (1 - \theta^n) B_{1;i}^{H,i,n}(\gamma) - \bar{D}_1^n - \epsilon & \text{if } \bar{D}_1^n \leq B_{1;i}^{H,i,n}(\gamma) \end{cases} \]

\[ V_{1,i}^n (\gamma, \bar{D}_1^n) = \begin{cases} \theta^n [C_2 - B_{1;i}^{H,i,n}(\gamma)] & \text{if } \bar{D}_1^n > B_{1;i}^{H,i,n}(\gamma) \\ \theta^n C_2 + (1 - \theta^n) B_{1;i}^{H,i,n}(\gamma) - \bar{D}_1^n & \text{if } \bar{D}_1^n \leq B_{1;i}^{H,i,n}(\gamma) \end{cases} \]

\[ \Delta^n_i (\bar{D}_1^n) = \begin{cases} -\theta^n [B_{1;i}^{H,i,n}(\gamma) - B_{1;i}^{H,i,n}(\gamma)] - \epsilon & \text{if } \bar{D}_1^n > B_{1;i}^{H,i,n}(\gamma) \\ D_{1;i,\text{PayIC}}^n - \bar{D}_1^n & \text{if } B_{1;i}^{H,i,n}(\gamma) \geq \bar{D}_1^n > B_{1;i}^{H,i,n}(\gamma) \\ (1 - \theta^n) [B_{1;i}^{H,i,n}(\gamma) - B_{1;i}^{H,i,n}(\gamma)] - \epsilon & \text{if } B_{1;i}^{H,i,n}(\gamma) \geq \bar{D}_1^n \end{cases} \]

(iia) \( B_{1;i}^{H,i,n}(\gamma) > B_{1;i}^{H,i,n}(\gamma') \geq B_{1;i,\text{PayIC}}^n \), then \( \bar{D}_{1;i,\text{Max}}^n = D_{1;i,\text{PayIC}}^n \). \( \gamma_2 = \gamma \) if

\( \bar{D}_1^n \leq \bar{D}_{1;i,\text{Max}}^n \) and \( \gamma_2 = \gamma \) if \( \bar{D}_1^n > \bar{D}_{1;i,\text{Max}}^n \).

\[ V_{1,i}^n (\gamma, \bar{D}_1^n) = \begin{cases} -\epsilon & \text{if } \bar{D}_1^n > B_{1;i}^{H,i,n}(\gamma) \\ B_{1;i}^{H,i,n}(\gamma) - \bar{D}_1^n - \epsilon & \text{if } B_{1;i}^{H,i,n}(\gamma) \geq \bar{D}_1^n > B_{1;i}^{H,i,n}(\gamma') \\ \theta^n C_2 + (1 - \theta^n) B_{1;i}^{H,i,n}(\gamma) - \bar{D}_1^n - \epsilon & \text{if } \bar{D}_1^n \leq B_{1;i}^{H,i,n}(\gamma') \end{cases} \]

\[ \Delta^n_i (\bar{D}_1^n) = \begin{cases} -\theta^n [C_2 - B_{1;i}^{H,i,n}(\gamma)] - \epsilon & \text{if } \bar{D}_1^n > B_{1;i}^{H,i,n}(\gamma) \\ D_{1;i,\text{ControlIC}}^n - \bar{D}_1^n & \text{if } B_{1;i}^{H,i,n}(\gamma) \geq \bar{D}_1^n > B_{1;i}^{H,i,n}(\gamma') \\ D_{1;i,\text{PayIC}}^n - \bar{D}_1^n & \text{if } B_{1;i}^{H,i,n}(\gamma) < \bar{D}_1^n \leq B_{1;i}^{H,i,n}(\gamma') \\ (1 - \theta^n) [B_{1;i}^{H,i,n}(\gamma) - B_{1;i}^{H,i,n}(\gamma)] - \epsilon & \text{if } \bar{D}_1^n < B_{1;i}^{H,i,n}(\gamma) \end{cases} \]

(iib) \( D_{1;i,\text{PayIC}}^n > B_{1;i}^{H,i,n}(\gamma') \geq D_{1;i,\text{ControlIC}}^n \), then \( \bar{D}_{1;i,\text{Max}}^n = B_{1;i}^{H,i,n}(\gamma') - \epsilon \). \( \gamma_2 = \gamma \) if

\( \bar{D}_1^n \leq \bar{D}_{1;i,\text{Max}}^n \) and \( \gamma_2 = \gamma \) if \( \bar{D}_1^n > \bar{D}_{1;i,\text{Max}}^n \).
The proof of Lemma 4.1 follows apparently then. We prove the case when there is potential underpricing in both states. When \( D_1 = \tilde{D}_1^{B, \text{Max}} \), \( \Delta_1^{\delta \tilde{G}} (D_1) > 0 \) and \( \Delta_1^{\delta \tilde{B}} (D_1) = 0 \). Therefore,
Proof of Proposition 4.2:

Since we always assume that in GG, there is no potential underpricing, \( D_{1}^{G,IC} = \tilde{D}_{1}^{GB,Max} \). We also assume that \( \gamma_{1} = \frac{\gamma_{2}}{2} \) since date 0 state was G. The candidates for \( D_{1}^{G,Max} \) are either \( C_{2} + \gamma_{1}C_{1} \) or \( \hat{D}_{1}^{GB,Max} \in (B_{1}^{GB}(\gamma), B_{1}^{GB}(\overline{\gamma})) \).

- If \( D_{1} = C_{2} + \gamma_{1}C_{1} \), the incumbent can raise \( q_{GG}^{G} \left( C_{2} + \gamma_{1}C_{1} \right) + (1 - q_{GG}^{G})B_{1}^{GB}(\gamma) \).
- If \( D_{1} = \hat{D}_{1}^{GB,Max} \), the incumbent can raise \( \hat{D}_{1}^{GB,Max} \).

Therefore, \( D_{1}^{G,Max} = C_{2} + \gamma_{1}C_{1} \) if and only if \( q_{GG}^{G} \left( C_{2} + \gamma_{1}C_{1} \right) + (1 - q_{GG}^{G})B_{1}^{GB}(\gamma) > \tilde{D}_{1}^{GB,Max} \). We know that \( B_{1}^{GB}(\gamma) < \tilde{D}_{1}^{GB,Max} \leq B_{1}^{GB}(\overline{\gamma}) - \varepsilon < C_{2} + \gamma_{1}C_{1} \), the inequality holds for \( q_{GG}^{G} = 1 \) and fails for \( q_{GG}^{G} = 0 \). Since the LHS is strictly increasing in \( q_{GG}^{G} \), there exists \( q_{GG,IC}^{G} \) such that the inequality holds if and only if \( q_{GG}^{G} \geq q_{GG,IC}^{G} \).

Therefore,

- If \( q_{GG}^{G} \geq q_{GG,IC}^{G} \), \( D_{1}^{G,Max} = C_{2} + \gamma_{1}C_{1} \). \( \gamma_{2} = \gamma \) for \( D_{1}^{G} \in (\hat{D}_{1}^{GB,Max}, \tilde{D}_{1}^{GB,Max}] \) and \( \gamma_{2} = \overline{\gamma} \) for \( D_{1}^{G} \in (0, \hat{D}_{1}^{GB,Max}] \).
- If \( q_{GG}^{G} < q_{GG,IC}^{G} \), \( D_{1}^{G,Max} = \hat{D}_{1}^{GB,Max} \). \( \gamma_{2} = \overline{\gamma} \).

Proof of Proposition 4.3:

We always assume that in both BG and BB, there is underpricing even if \( \gamma_{2} = \overline{\gamma} \). We also assume that \( \gamma_{1} = \overline{\gamma} \) since the date 0 state was B.
1) If $\omega_0^b < \omega_0^H - (\gamma' - \gamma) c_1$, the incumbent loses control for sure in state BB.

$$D_{1,IC} = \tilde{D}_{1,BG,Max} + \gamma_1 c_1$$ if and only if $\tilde{D}_{1,BG,Max} + \gamma_1 c_1 \geq B_1^{H,BB} (\bar{\gamma}) = \omega_0^H + \bar{\gamma} c_2$. 

If $c_1 = c_2$, this translates into $(\rho + \bar{\gamma}) > \theta^H (\bar{\gamma} - \gamma)$ which always holds. Since

$$B_1^{H,BG} (\bar{\gamma}) < \tilde{D}_{1,BG,Max}$$, the only candidate for $D_{1,BG,Max}$ is $\tilde{D}_{1,BG,Max} + \gamma_1 c_1$. Therefore, for any level of $q^{BG}$, $\gamma_2 = \bar{\gamma}$ is chosen and $D_{1,BG,Max} = D_{1,IC} = \tilde{D}_{1,BG,Max} + \gamma_1 c_1$.

2) If $\omega_0^b > \omega_0^H - (\gamma' - \gamma) c_1$, the incumbent is either on the Pay IC (where the constraint is due to the effect on payment made given that control can be retained) or the control IC constraint (where pledgeability can cause the incumbent to lose control). We show the case that the incumbent is on the Control IC constraint in BB and on the Pay IC constraint in GB.

The result when she is on other IC constraints can be proved in a similar way.

Let’s first solve for $D_{1,IC}$.

**Fact 1:** $\tilde{D}_{1,BB,ControlIC} < D_{1,IC}$. Simple calculation shows that $\tilde{D}_{1,BB,ControlIC} < \tilde{D}_{1,BG,PayIC} + \gamma_1 c_1$.

Therefore, $q^G \Delta_{1,BG} (D_1 - \gamma_1 c_1) + (1 - q^G) \Delta_{1,BB} (D_1) > 0$ at $D_1 = \tilde{D}_{1,BB,ControlIC}$.

**Fact 2:** $D_{1,IC} < \tilde{D}_{1,BG,PayIC} + \gamma_1 c_1$. Simple calculation shows that $\tilde{D}_{1,BG,PayIC} + \gamma_1 c_1 > B_1^{H,BB} (\bar{\gamma})$.

Therefore, $q^G \Delta_{1,BG} (D_1 - \gamma_1 c_1) + (1 - q^G) \Delta_{1,BB} (D_1) < 0$ at $D_1 = \tilde{D}_{1,BG,PayIC} + \gamma_1 c_1$.

**Fact 3:** $B_1^{H,BG} (\bar{\gamma}) + \gamma_1 c_1 > B_1^{H,BB} (\bar{\gamma})$.

**Fact 4:** there exists $q^{BG,IC}$ such that

$$q^G \Delta_{1,BG} (B_1^{H,BG} (\bar{\gamma})) + (1 - q^G) \Delta_{1,BB} (B_1^{H,BB} (\bar{\gamma})) < 0$$ if and only if $q^{BG} < q^{BG,IC}$. Evaluate

$$q^G \Delta_{1,BG} (D_1 - \gamma_1 c_1) + (1 - q^G) \Delta_{1,BB} (D_1)$$ at $D_1 = B_1^{H,BG} (\bar{\gamma}) + \gamma_1 c_1$ shows that

$$q^G \Delta_{1,BG} (B_1^{H,BG} (\bar{\gamma})) + (1 - q^G) \Delta_{1,BB} (B_1^{H,BB} (\bar{\gamma})) = q^G (1 - \theta^H) (\bar{\gamma} - \gamma) c_2 - (1 - q^G) \theta^H (C_2 - \omega_0 - \gamma c_2)$$.
• If \( q^G < q^{G,JC} \), \( D_1^{B,JC} < B_1^{H,BG}(\gamma^G) + \gamma_1 C_1 \).
• If \( q^G > q^{G,JC} \), \( D_1^{B,JC} > B_1^{H,BG}(\gamma^G) + \gamma_1 C_1 \).

Therefore, \( \gamma_2 = \overline{\gamma} \) if \( q^{BG} > q^{BG,JC} \). For the remaining analysis, we discuss the when \( q^{BG} < q^{BG,JC} \).

**Fact 5:** \( D_1^{B,JC} < B_1^{H,BB}(\overline{\gamma}) \). Evaluate \( q^{BG} \Delta_1^{BG} (D_1 - \gamma_1 C_1) + (1 - q^{BG}) \Delta_1^{BB} (D_1) \) at

\[
D_1^{B,JC} = B_1^{H,BB}(\gamma^G) \text{ shows that}
\]

\[
q^{BG} \Delta_1^{BG} (B_1^{H,BB}(\overline{\gamma}) - \gamma_1 C_1) + (1 - q^{BG}) \Delta_1^{BB} (B_1^{H,BB}(\overline{\gamma})) = q^{BG} (1 - \theta^H) (\overline{\gamma} - \gamma_1) C_2 - (1 - q^{BG}) \theta^H (C_2 - \omega_0 - \gamma_2). 
\]

The expression takes on negative values if \( q^{BG} < q^{BG,JC} \).

**Fact 6:** \( D_1^{B,JC} < D_1^{H,BB}(\overline{\gamma}) < B_1^{H,BG}(\gamma) + \gamma_1 C_1 \). We can explicitly solve for \( D_1^{B,JC} \). It satisfies

\[
q^{BG} \left( 1 - \theta^H \right) (\overline{\gamma} - \gamma_1) C_2 + (1 - q^{BG}) \theta^H \left( \tilde{D}_1^{BB,ControlJC} - D_1^{B,JC} \right) = 0. 
\]

Solving this equation shows that,

\[
D_1^{B,JC} = \tilde{D}_1^{BB,ControlJC} + \frac{q^{BG} (1 - \theta^H)}{\theta^H (1 - q^{BG})} (\overline{\gamma} - \gamma_1) C_2.
\]

The question remaining is, given \( q^{BG} < q^{BG,JC} \), does \( B_1^{H,BG}(\gamma) + \gamma_1 C_1 \) pledge more than \( D_1^{B,JC} \)?

We know that \( B_1^{H,BG}(\gamma^G) + \gamma_1 C_1 \) pledges more if and only if

\[
q^{BG} \left( B_1^{H,BG}(\gamma) + \gamma_1 C_1 \right) + (1 - q^{BG}) B_1^{H,BB}(\overline{\gamma}) > D_1^{B,JC}. 
\]

Since \( D_1^{B,JC} \) also increases with \( q^{BG} \), it turns out that this inequality is non-monotonic w.r.t. \( q^{BG} \). In fact, it reduces to

\[
q^{BG} \left[ (\rho + \gamma_1) C_1 - \frac{1 - \theta^H}{\theta^H} \frac{1}{1 - q^{BG}} (\overline{\gamma} - \gamma_1) C_2 \right] > (1 - \theta^H) (\overline{\gamma} - \gamma_1) C_2 - \theta^H (C_2 - \omega_0 - \gamma_2). 
\]

The inequality is definitely violated for both large and small \( q^{BG} \). Low pledgeability might be chosen when \( q^{BG} \) is neither too high nor too low.
Proof of Lemma 3.1:

We have determined the maximum credible payment a bidder can make in each state at date 1. Now we solve who wins the auction in each state at date 0. We begin by solving all parties’ bids at date 0.

Industry insiders

The bid by industry insiders is easily arrived at. Each insider has the resources to bid up to

\[ \omega_0^{H,s_0} + q^{s_0G} \left( \gamma_1 C_1 + \tilde{D}_1^{s_0G} \right) + (1 - q^{s_0G}) \tilde{D}_1^{s_0B} \]  

where \( \omega_0^{H,s_0} \) is the cash she has at date 0 and

\[ \gamma_1 C_1 + \tilde{D}_1^{s_0G} \]  

(less than \( \gamma' C_1 + \tilde{D}_1^{s_0G,\text{Max}} \)) and \( \tilde{D}_1^{s_0B} \) (less than \( \tilde{D}_1^{s_0B,\text{Max}} \)) are the state contingent date-1 payments contracted at date 0. The additional surplus (before debt repayments) she hopes to get by acquiring the firm at date 1 (relative to staying an industry insider and collecting \( \rho C_1 \) in good states if she retains capability) is

\[ q^{s_0G} \left( (1 - \rho)C_1 + \tilde{D}_1^{s_0G} + V_1^{s_0G} (\tilde{D}_1^{s_0G}) \right) + (1 - q^{s_0G}) \left( \tilde{D}_1^{s_0B} + V_1^{s_0B} (\tilde{D}_1^{s_0B}) \right). \]

So the industry insider’s maximum bid with promised payments \( \left( \tilde{D}_1^{s_0G}, \tilde{D}_1^{s_0B} \right) \) and pre-set pledgeability \( \gamma_1 \) (by the incumbent) is

\[ B_0^{H,s_0}(\gamma_1) = \max_{\tilde{D}_1^{s_0G,\text{Max}} \leq \tilde{D}_1^{s_0G}, \tilde{D}_1^{s_0B,\text{Max}}} \min \left[ \omega_0^{H,s_0} + q^{s_0G} \left( \gamma_1 C_1 + \tilde{D}_1^{s_0G} \right) + (1 - q^{s_0G}) \tilde{D}_1^{s_0B}, \right. \]

\[ q^{s_0G} \left( C_1 + \tilde{D}_1^{s_0G} + V_1^{s_0G} (\tilde{D}_1^{s_0G}) \right) + (1 - q^{s_0G}) \left( \tilde{D}_1^{s_0B} + V_1^{s_0B} (\tilde{D}_1^{s_0B}) \right) \]

Note that higher \( \left( \tilde{D}_1^{s_0G}, \tilde{D}_1^{s_0B} \right) \) enable an industry insider to raise more at date 0, thus more likely to win an auction. Meanwhile, higher scheduled payments make her more likely to lose control at date 1, and lose the associated rents. The amount the industry insider raises from financiers at date 0, trades off these two effects.

Financiers
A low-type financier can also bid at date 0, with the objective of holding on to the firm over the period if he can get it cheaply, and selling at date 1. Given that the financier does not suffer from moral hazard (he has no desire to set \( \gamma_2 \) low because he wants to sell for certain), he can pay up to

\[ B^L_{0,s_0} = q^s \gamma^G B^H_{1,s_0} (\gamma^G) + (1 - q^s \gamma^G) B^H_{1,s_0} (\gamma^G)^G - \epsilon. \]

Note that the wealth of the financier coming into date 1 does not matter. Because he does not suffer from moral hazard, he can borrow the entire amount he realizes from the verifiable future sale (though he must keep a small rent to compensate himself for the cost of enhancing pledgeability).

**High-type Incumbent**

Consider now the high type incumbent at date 0. Her bid can be achieved in a similar way as industry insiders. She can afford to pay up to

\[ \omega \gamma^i + q^s \gamma^G \left( \gamma^C_1 + \tilde{D}^i_{1,iG,Max} \right) + (1 - q^s \gamma^G) \tilde{D}^{iB,Max}_1 \]

where \( i \) is an indicator variable for the period 1 state:

\[
\begin{align*}
B^i_{1,s} &= \max_{\tilde{D}^i_{1,iG,Max}, \tilde{D}^{iB,Max}} \min \left[ \omega \gamma^i + q^s \gamma^G \left( \gamma^C_1 + \tilde{D}^i_{1,iG,Max} \right) + (1 - q^s \gamma^G) \tilde{D}^{iB,Max}_1, \\
&\quad q^s \gamma^G \left( \gamma^C_1 + \tilde{D}^i_{1,iG,Max} + V^i_{1,iG} \left( \tilde{D}^{iG}_1 \right) \right) + (1 - q^s \gamma^G) \left( \tilde{D}^{iB}_1 + V^{iB,Max}_1 \left( \tilde{D}^{iB}_1 \right) \right) \right].
\end{align*}
\]

**Incumbent’s Choice of \( \gamma_1 \)**

Consider the incentive problem for the period-0 incumbent in setting \( \gamma_1 \). Let

\[ B^{min,s}_1 = \max \{ B^L_{1,s}, B^H_{1,s} (\gamma^G) \}. \]

This is the minimum bid the incumbent will face. Consistent with Lemma 2.1, we classify the analysis into four cases. i) Pledgeability does not matter for repayment. ii) The incumbent can never outbid industry insiders. iii) The incumbent can always outbid industry insiders. iv) The incumbent can outbid industry insiders when pledgeability is low, but not when pledgeability is high.

(i) Pledgeability does not matter for repayment
This case has two subcases. (a) If $B_{0,0}^{\min,s_0} \geq q^{s_0G}C_1 + C_2$ so that there is no potential underpricing. (b) $q^{s_0G}C_1 + C_2 > B_{0,0}^{\min,s} \geq B_0^{H,s}(\bar{\gamma})$ so that there is potential underpricing.

Subcase (b) also includes two scenarios. In the first scenario, $B_{0,0}^{L,s} \geq B_0^{H,s}(\bar{\gamma}) = B_0^{H,s}(\gamma)$ so that industry insiders are outbid by financiers. In the second scenario, $q^{s_0G}C_1 + C_2 > B_0^{H,s_0}(\bar{\gamma}) = B_0^{H,s_0}(\gamma) > B_0^{L,s_0}$, although there is potential underpricing in the date 1 auction. These rents cannot be pledged by the incumbent at date 0. In all categories above, the incumbent has no incentive to set pledgeability high since it does not affect the auction outcome.

(ii) The incumbent can never outbid industry insiders.

Along the lines of analysis in period 1, high pledgeability is set and $D_{0,0,\text{Max}} = B_0^{H,s}(\bar{\gamma}) - \varepsilon$.

(iii) Incumbent always retains control conditional on retaining ability

Along the lines of the analysis in period 1, when the incumbent’s choice does not lead to a change in control so long as she remains capable, the maximal promised payout $D_0^{s, \text{Pay IC}}$ solves equation $\theta_H V_{0,0}^{i,s_0}(\gamma^i, D_0^{s, \text{Pay IC}}) + (1 - \theta_H) (B_0^{H,s_0}(\bar{\gamma}) - D_0^{s, \text{Pay IC}}) - \varepsilon = \theta_H V_{0,0}^{i,s_0}(\gamma^i, B_0^{min,s_0})$, where $V_{0,0}^{i,s_0}(\gamma^i, d)$ is the maximum expected rent a high type incumbent can get if she wins the auction at date 0 after setting incumbent pledgeability at its maximum $\gamma^i$ and promised repayments enough to repay $d$. Specifically,

$$V_{0,0}^{i,s_0}(\gamma^i, d) = \max_{D_0^{s, \text{Pay IC}} \leq D_0^{s, \text{Max}}} \left[ q^{s_0G}(1 - \gamma^i)C_1 + V_1^{i,s_0G}(\tilde{D}_1^{s_0G}) + (1 - q^{s_0G})V_1^{i,s_0B}(\tilde{D}_1^{s_0B}) \right] \text{ such that}$$

$$\omega_0^{i,s_0} + q^{s_0G}(\gamma^i C_1 + \tilde{D}_1^{s_0G}) + (1 - q^{s_0G})\tilde{D}_2^{s_0B} \geq d.$$ If the payment to retain control does not dynamically affect future control, this reduces to $D_0^{s, \text{Pay IC}} = \theta_H B_0^{\min,s_0} + (1 - \theta_H) B_0^{H,s_0}(\bar{\gamma}) - \varepsilon$ which is analogous to the date 1 expression $D_1^{s, \text{Pay IC}}$. 

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Incumbent could lose control depending on the level of pledgeability

When the choice of pledgeability leads to a change in control, we have:

\[ D^0, \text{Control IC} = B_0^{H,s} (\bar{\gamma}) - \theta H V_{0^+}^{i,s} (\gamma' , B^0_{0^+, \text{Min,s}}) - \varepsilon. \]

Lemma 7.2 is the full-fledged version of Lemma 3.1.

**Lemma 7.2**

Let \( s_0 = s \),

(iia) If \( B^\text{Min,s}_0 \geq q^G C_1 + C_2 \), \( D_0^{r,\text{Max}} = q^G C_1 + C_2 \), and \( \gamma_1 = \bar{\gamma} \)

For any promised payment \( \tilde{D}_0^r \leq \tilde{D}_0^{r,\text{Max}} \), the incumbent expects \( V_{0^+}^{i,s} (\tilde{D}_0^r) = (q^G C_1 + C_2) - \tilde{D}_0^s \).

(ii) else if \( (q^G C_1 + C_2) > B^{H,s}_0 (\bar{\gamma}) \), \( D_0^{r,\text{Max}} = B^\text{Min,s}_0 \) and \( \gamma_1 = \bar{\gamma} \)

For any promised payment \( \tilde{D}_0^r \leq \tilde{D}_0^{r,\text{Max}} \), the incumbent gets \( V_{0^+}^{i,s} (\tilde{D}_0^r) = B^\text{Min,s}_0 - \tilde{D}_0^r \) if \( B^{i,s}_0 (\gamma') < \tilde{D}_0^r \). If \( B^{i,s}_0 (\gamma') \geq \tilde{D}_0^r \), then the incumbent gets

\[ V_{0^+}^{i,s} (\tilde{D}_0^r) = \theta H V_{0^+}^{i,s} (\gamma', D_0^r) + (1 - \theta H ) (B^\text{Min,s}_0 - \tilde{D}_0^r). \]

(iiia) else if \( B^{i,s}_0 (\gamma') \geq D_0^r \) \( \text{Pay IC} \), then \( D_0^{r,\text{Max}} = D_0^r \text{Pay IC} \). For any promised payment \( \tilde{D}_0^r \leq \tilde{D}_0^{r,\text{Max}} \), the incumbent chooses \( \gamma_2 = \bar{\gamma} \) and gets \( V_{0^+}^{i,s} (\tilde{D}_0^r) = \theta H V_{0^+}^{i,s} (\gamma', D_0^r) + (1 - \theta H ) (B^{H,s}_0 (\bar{\gamma}) - \tilde{D}_0^r) - \varepsilon. \)

Else if \( B^{H,s}_0 (\bar{\gamma}) > B^\text{Min,s}_0 \) and if

(iiiia) \( B^{i,s}_0 (\gamma') \geq D_0^r \) \( \text{Pay IC} \), then \( D_0^{r,\text{Max}} = D_0^r \text{Pay IC} \). For any promised payment \( \tilde{D}_0^r \leq \tilde{D}_0^{r,\text{Max}} \), the incumbent chooses \( \gamma_2 = \bar{\gamma} \) and gets \( V_{0^+}^{i,s} (\tilde{D}_0^r) = \theta H V_{0^+}^{i,s} (\gamma', D_0^r) + (1 - \theta H ) (B^{H,s}_0 (\bar{\gamma}) - \tilde{D}_0^r) - \varepsilon. \)
(iib) $D_0^s \text{ Pay IC} > B_0^{i,s}(\gamma') \geq D_0^s \text{ Control IC}$, then $\tilde{D}_0^{s,\text{Max}} = B_0^{i,s}(\gamma')$. For any promised payment $\tilde{D}_0^s \leq \tilde{D}_0^{s,\text{Max}}$, the incumbent chooses $\gamma_2 = \gamma$ and gets $V_0^{i,s}(\tilde{D}_0^s) = \theta_H V_{0^i}^{i,s}(\gamma, \tilde{D}_0^s) + (1 - \theta_H)(B_0^{H,s}(\gamma) - \tilde{D}_0^s) - \epsilon$.

(iv) $D_0^s \text{ Control IC} > B_0^{i,s}(\gamma') \geq B_0^{\text{min},s}$ then $\tilde{D}_0^{s,\text{Max}} = D_0^s \text{ Control IC}$. For any promised payment $\tilde{D}_0^s \leq \tilde{D}_0^{s,\text{Max}}$, the incumbent chooses $\gamma_2 = \gamma$ and gets $V_0^{i,s}(\tilde{D}_0^s) = B_0^{H,s}(\gamma) - \tilde{D}_0^s - \epsilon$ if $B_0^{i,s}(\gamma') < \tilde{D}_0^s$. Otherwise, the incumbent chooses $\gamma_2 = \gamma$ and gets $V_0^{i,s}(\tilde{D}_0^s) = \theta_H V_{0^i}^{i,s}(\gamma, \tilde{D}_0^s) + (1 - \theta_H)(B_0^{H,s}(\gamma) - \tilde{D}_0^s) - \epsilon$.

Full Analysis of Section 3.2

In this section, we provide a full characterization to the analysis of inefficient incumbency that we have discussed in Section 4.2. We relax the assumption that the incumbent cannot produce anything when she loses her ability. Instead, we assume the disabled manager is still able to produce $\alpha C_2$ if the industry is in good state in period 2.

Date 1

Abled Incumbent in Period 1

We start to analyze the decision of an abled incumbent in period 1. Let the state of period 1 be $S_1 = S$. As before, the industry insiders’ bids are $B_1^{H,s}(\gamma_2) = \text{Min}[\omega_1^{H,s} + \gamma_2 C_2, C_2]$.

Now, the incumbent’s bids depend on whether she retains her ability. Let $B_1^{H,s}$ and $B_1^{L,s}$ respectively be the bid when she retains and loses ability. Clearly, the incumbent’s bid is the minumum of the liquidity that she can raise and the firm’s continuation value in her hand: $B_1^{H,s} = \min\{\omega_1^{i,s} + \gamma' C_2, C_2\}$ and $B_1^{L,s} = \min\{\omega_1^{i,s} + \gamma' \alpha C_2, \alpha C_2\}$. When $\alpha = 0$, $B_1^{L,s} = 0$ and the disabled incumbent essentially becomes the financier who would not bid at date 1.

When $\alpha \in (0,1)$, $B_1^{L,s} > 0$. Below solve the date 1’s problem in three categories, depending on the comparison among $\alpha C_2$, $B_1^{H,s}(\gamma)$ and $B_1^{H,s}(\gamma)$.

a. $\alpha C_2 \leq B_1^{H,s}(\gamma)$

Lemma 7.3: When $\alpha C_2 \leq B_1^{H,s}(\gamma)$, the disabled incumbent will always sell the firm.
Proof: A disabled incumbent gets \( \max \left[ B^{H,s}_1(\gamma) - \hat{D}^s, 0 \right] \) if she sells the firm and gets 
\(-\min \left[ B^{H,s}_1(\gamma), \hat{D}^s \right] + \alpha C_2 \) if she does not sell. Here we are implicitly assuming that 
\( B^{L,s}_1 \geq \min \{ B^{H,s}_1(\gamma), \hat{D}^s \} \) so that she can afford to stay in control if she wants to. Selling the firm is preferable if 
\( \max \left[ B^{H,s}_1(\gamma) - \hat{D}^s, 0 \right] \geq -\min \left[ B^{H,s}_1(\gamma), \hat{D}^s \right] + \alpha C_2 \). Equivalently, 
\( \max \left[ B^{H,s}_1(\gamma) - \hat{D}^s, 0 \right] + \min \left[ B^{H,s}_1(\gamma), \hat{D}^s \right] \geq \alpha C_2 \Rightarrow B^{H,s}_1(\gamma) \geq \alpha C_2 \). A sufficient condition for this to hold is 
\( B^{H,s}_1(\gamma) \geq \alpha C_2 \). Therefore, if \( \alpha \) is sufficiently low such that 
\( \alpha C_2 \leq B^{H,s}_1(\gamma) \), the incumbent always sells the firm when she loses her ability, because the amount she can produce is less than the amount she can sell.

Lemma 7.4a is identical to Lemma 2.1 except for notation changes. We replace \( B^{i,s}_1(\gamma') \) with \( B^{H,s}_1(\gamma) \) and use \( V^{ih,s}_1 \) instead of \( V^{is}_1 \). We will use \( V^{il,s}_1 \) as the continuation value for an incumbent who starts with low ability in period 1.

**Lemma 7.4a**

Suppose \( \alpha C_2 \leq B^{H,s}_1(\gamma) \).

(i) If \( B^{H,s}_1(\gamma) = C_2 \),

\[ \hat{D}^{s,\text{Max}}_1 = C_2 \text{ and } \gamma_2 = \gamma. \]

For any promised payment \( \hat{D}^s_1 \leq \hat{D}^{s,\text{Max}}_1 \), incumbent expects:

\[ V^{ih,s}_1(\hat{D}^s_1) = C_2 - \hat{D}^s_1. \]

If \( C_2 > B^{H,s}_1(\gamma) \) and

(ii) if \( B^{H,s}_1(\gamma) > B^{H,s}_1(\bar{\gamma}) \)

then \( \hat{D}^{s,\text{Max}}_1 = B^{H,s}_1(\bar{\gamma}) - \epsilon \). For any promised payment \( \hat{D}^s_1 \leq \hat{D}^{s,\text{Max}}_1 \), the incumbent chooses

\[ \gamma_2 = \bar{\gamma} \text{ and expects } V^{ih,s}_1(\hat{D}^s_1) = B^{H,s}_1(\bar{\gamma}) - \hat{D}^s_1 - \epsilon \text{ if } \hat{D}^s_1 > B^{H,s}_1(\gamma') \text{ and } \]

\[ V^{ih,s}_1(\hat{D}^s_1) = \theta^H C_2 + (1 - \theta^H) B^{H,s}_1(\bar{\gamma}) - \hat{D}^s_1 - \epsilon \text{ if } \hat{D}^s_1 \leq B^{H,s}_1(\gamma'). \]

(iii) if \( B^{H,s}_1(\gamma') \geq D^{\text{PayIC}}_1 \), then \( \hat{D}^{s,\text{Max}}_1 = D^{\text{PayIC}}_1 \). For any promised payment \( \hat{D}^s_1 \leq \hat{D}^{s,\text{Max}}_1 \),

incumbent chooses \( \gamma_2 = \bar{\gamma} \) and expects

\[ V^{ih,s}_1(\hat{D}^s_1) = \theta^H C_2 + (1 - \theta^H) B^{H,s}_1(\bar{\gamma}) - \hat{D}^s_1 - \epsilon. \]
(iii) \( D_1^s \text{PayIC} > B_1^{H,s} (\gamma^r) \geq D_1^s \text{Control IC} \), then \( \hat{D}_1^{s, \text{Max}} = B_1^{H,s} (\gamma^r) \). For any promised payment \( \hat{D}_1^s \leq \hat{D}_1^{s, \text{Max}} \), incumbent chooses \( \gamma_2^r = \overline{\gamma} \) and expects 
\[
V_1^{H,s} (\hat{D}_1^s) = \theta^H C_2 + (1 - \theta^H) B_1^{H,s} (\overline{\gamma}) - \hat{D}_1^s - \varepsilon.
\]

(iv) \( D_1^s \text{Control IC} > B_1^{H,s} (\gamma^r) \geq B_1^{H,s} (\gamma) \) then \( \hat{D}_1^{s, \text{Max}} = D_1^s \text{Control IC} \). For any promised payment \( \hat{D}_1^s \leq \hat{D}_1^{s, \text{Max}} \), the incumbent chooses \( \gamma_2^r = \overline{\gamma} \) and expects 
\[
V_1^{H,s} (\hat{D}_1^s) = B_1^{H,s} (\overline{\gamma}) - \hat{D}_1^s - \varepsilon \text{ if } \hat{D}_1^s < B_1^{H,s} (\gamma^r) \text{ and } V_1^{H,s} (\hat{D}_1^s) = \theta^H C_2 + (1 - \theta^H) B_1^{H,s} (\overline{\gamma}) - \hat{D}_1^s - \varepsilon \text{ if } \hat{D}_1^s \leq B_1^{H,s} (\gamma^r).
\]

Proof: Apply Lemma 7.3 and the proof of Lemma 2.1

b. \( B_1^{H,s} (\overline{\gamma}) > \alpha C_2 > B_1^{H,s} (\gamma) \)

In this case, the disabled incumbent has incentives to keep control when pledgeability is set as low. However, when pledgeability is set as high, she strictly prefers to sell. We analyze this case in different subcases, depending on the interaction between the incumbent’s pledgeability choice and her affordability to maintain control, i.e., whether she can afford to maintain control with different pledgeability choices. We divide the analysis according to the affordability of keeping control.

1. **Pledgeability choice does not matter**

When \( B_1^{H,s} (\overline{\gamma}) = B_1^{H,s} (\gamma) = C_2 \), pledgeability choice does not matter. However, \( B_1^{H,s} (\overline{\gamma}) > \alpha C_2 > B_1^{H,s} (\gamma) \) would never hold.

2. **The disabled incumbent always maintains control**

1) **The abled incumbent always loses control**

This case happens if \( B_1^{H,s} \leq \min \{ B_1^{H,s} (\gamma), \hat{D}_1^s \} \).

By choosing high pledgeability, the incumbent gets \( \max \{ B_1^{H,s} (\overline{\gamma}) - \hat{D}_1^s, 0 \} - \varepsilon \). By choosing low pledgeability, the incumbent gets \( \max \{ B_1^{H,s} (\gamma) - \hat{D}_1^s, 0 \} \). As a result, the incumbent always chooses high pledgeability as long as \( \hat{D}_1^s \leq B_1^{H,s} (\overline{\gamma}) - \varepsilon \) and hence the
maximal promised payoff is \( \hat{D}_t^{\text{max}} = B_t^{H,t}(\gamma) - \epsilon \). The continuation value to the incumbent is \( V_t^{iH,t} = B_t^{H,t}(\gamma) - \hat{D}_t^t - \epsilon \) for \( \hat{D}_t^t \in (B_t^{iH,t}, \hat{D}_t^{iH,t,max}] \).

2) **The abled incumbent always maintains control**

This case happens if \( B_t^{iL,t} < \min\{B_t^{H,t}(\gamma), \hat{D}_t^t\} \) and \( B_t^{iH,t} \geq \min\{B_t^{H,t}(\gamma), \hat{D}_t^t\} \).

The incumbent will choose high general pledgeability if
\[
\theta^H(C_2 - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)]) + (1 - \theta^H)(B_t^{H,t}(\gamma) - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)]) - \epsilon \\
\geq \theta^H(C_2 - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)]) + (1 - \theta^H)(B_t^{H,t}(\gamma) - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)])
\]
\[
\Rightarrow \hat{D}_t^t \leq \theta^H B_t^{H,t}(\gamma) + (1 - \theta^H)B_t^{H,t}(\gamma) = D_t^{i,\text{PayIC}}.
\]

In this case, \( \hat{D}_t^{i,max} = D_t^{i,\text{PayIC}} \) and the value function is:
\[
V_t^{iH,t} = \theta^H C_2 + (1 - \theta^H)B_t^{H,t}(\gamma) - \hat{D}_t^t - \epsilon \text{ for } \hat{D}_t^t \in (B_t^{iH,t}, B_t^{iH,t}] \cap \hat{D}_t^t \in [0, \hat{D}_t^{i,max}] \).

3) **Pledgeability choice leads to change in control for the abled incumbent**

This case happens if \( B_t^{iL,t} < \min\{B_t^{H,t}(\gamma), \hat{D}_t^t\} \) and \( \min\{B_t^{H,t}(\gamma), \hat{D}_t^t\} \leq B_t^{iL,t} \leq \min\{B_t^{H,t}(\gamma), \hat{D}_t^t\} \).

The incumbent will choose high general pledgeability if
\[
(B_t^{H,t}(\gamma) - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)]) - \epsilon \geq \theta^H(C_2 - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)])
\]
\[
\Rightarrow \hat{D}_t^t \leq B_t^{H,t}(\gamma) - \theta^H(C_2 - B_t^{H,t}(\gamma)) - \epsilon = D_t^{i,\text{Control IC}}.
\]

In this case, the maximal promised payoff is \( \hat{D}_t^{i,max} = D_t^{i,\text{Control IC}} \) and the continuation value is \( V_t^{iH,t} = B_t^{H,t}(\gamma) - \hat{D}_t^t - \epsilon \text{ for } \hat{D}_t^t \in (B_t^{iH,t}, \hat{D}_t^{iH,t,max}] \).

3. **The disabled incumbent always maintains control**

This case happens if \( B_t^{iL,t} \geq \min\{B_t^{H,t}(\gamma), \hat{D}_t^t\} \). Since \( B_t^{iL,t} \leq B_t^{H,t} \), we know that the incumbent can also afford to stay in control if she retains her ability.

Clearly, she will choose high pledgeability if
\[
\theta^H(C_2 - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)]) + (1 - \theta^H)(B_t^{H,t}(\gamma) - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)]) - \epsilon \\
\geq \theta^H(C_2 - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)]) + (1 - \theta^H)(\alpha C_2 - \min[\hat{D}_t^t, B_t^{H,t}(\gamma)])
\]
\[
\Rightarrow \hat{D}_t^t \leq \theta^H B_t^{H,t}(\gamma) + (1 - \theta^H)B_t^{H,t}(\gamma) - (1 - \theta^H)(\alpha C_2 - B_t^{H,t}(\gamma)) - \epsilon \equiv D_t^{i,\alpha\text{PayIC}}
\]

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In this case, the maximal promised payoff is $\hat{D}_i^{s,\text{Pay}IC} = D_i^{s,\text{Pay}IC}$ and the continuation value is $V_i^{ill,s} = \theta^H C_2 + (1 - \theta^H) B_i^{ill,s}(\gamma) - \hat{D}_i^s - \epsilon$ for $\hat{D}_i^s \in [0, B_i^{ill,s}]$.

4. Pledgeability choice leads to change in control for the disabled incumbent

1) Pledgeability choice leads to change in control for the abled incumbent

This case happens if $\min\{B_1^{ill,s}(\gamma), \hat{D}_i^s\} \leq B_1^{ill,s} < \min\{B_1^{ill,s}(\gamma), \hat{D}_i^s\}$ and $\min\{B_1^{ill,s}(\gamma), \hat{D}_i^s\} \leq B_1^{ill,s} < \min\{B_1^{ill,s}(\gamma), \hat{D}_i^s\}$.

The incumbent will choose high pledgeability if

$B_1^{ill,s}(\gamma) - \text{Min}[\hat{D}_i^s, B_i^{ill,s}(\gamma)] - \epsilon \geq \theta^H (C_2 - \text{Min}[\hat{D}_i^s, B_i^{ill,s}(\gamma)]) + (1 - \theta^H)(\alpha C_2 - \text{Min}[\hat{D}_i^s, B_i^{ill,s}(\gamma)])$ \quad \text{In this case, the maximal promised payoff is } $\hat{D}_1^{s,\text{max}} = D_i^{s,\text{Pay}IC}$ and the continuation value is:

$V_1^{ill,s} = \theta^H C_2 + (1 - \theta^H) B_i^{ill,s}(\gamma) - \hat{D}_i^s - \epsilon$ for $\hat{D}_i^s \in (B_i^{ill,s}, \hat{D}_1^{s,\text{max}}]$.

2) The abled incumbent always maintains control

This case happens if $\min\{B_2^{ill,s}(\gamma), \hat{D}_i^s\} \leq B_1^{ill,s} < \min\{B_1^{ill,s}(\gamma), \hat{D}_i^s\}$ and $\min\{B_1^{ill,s}(\gamma), \hat{D}_i^s\} \leq B_1^{ill,s}$.

The incumbent will choose high pledgeability if

$\theta^H (C_2 - \text{Min}[\hat{D}_i^s, B_i^{ill,s}(\gamma)]) + (1 - \theta^H)(\alpha C_2 - \text{Min}[\hat{D}_i^s, B_i^{ill,s}(\gamma)]) - \epsilon \geq \theta^H (C_2 - \text{Min}[\hat{D}_i^s, B_i^{ill,s}(\gamma)]) + (1 - \theta^H)(\alpha C_2 - \text{Min}[\hat{D}_i^s, B_i^{ill,s}(\gamma)])$ \quad \text{In this case, the maximal promised payoff is } $\hat{D}_1^{s,\text{max}} = D_i^{s,\text{Pay}IC}$ and the continuation value is $V_1^{ill,s} = \theta^H C_2 + (1 - \theta^H) B_i^{ill,s}(\gamma) - \hat{D}_i^s - \epsilon$ for $\hat{D}_i^s \in (B_i^{ill,s}, B_i^{ill,s}]$.

Lemma 7.4b

Suppose $B_i^{ill,s}(\gamma) > \alpha C_2 > B_i^{ill,s}(\gamma)$.

1. If $B_i^{ill,s}(\gamma) > B_i^{ill,s}(\gamma)$,
then $\tilde{D}_i^{s,\text{Max}} = B_i^{H,s}(\tilde{\gamma}) - \varepsilon$. For any promised payment $\tilde{D}_i^s \leq \tilde{D}_i^{s,\text{Max}}$, the incumbent chooses $\gamma_2 = \tilde{\gamma}$ and expects $V_{i}^{H,s}(\tilde{D}_i^s) = B_i^{H,s}(\tilde{\gamma}) - \tilde{D}_i^s - \varepsilon$ if $\tilde{D}_i^s > B_i^{H,s}(\gamma')$ and

$V_{i}^{H,s}(\tilde{D}_i^s) = \theta^H C_2 + (1 - \theta^H)B_i^{H,s}(\tilde{\gamma}) - \tilde{D}_i^s - \varepsilon$ if $\tilde{D}_i^s \leq B_i^{H,s}(\gamma')$.

2. else

1) If $B_i^{IL,s}(\gamma') < B_i^{H,s}(\gamma)$,

(i) $B_i^{H,s}(\gamma') \geq D_1^{s,\text{PayIC}}$, then $\tilde{D}_i^{s,\text{Max}} = D_1^{s,\text{PayIC}}$. For any promised payment $\tilde{D}_i^s \leq \tilde{D}_i^{s,\text{Max}}$, incumbent chooses $\gamma_2 = \tilde{\gamma}$ and expects $V_{i}^{H,s}(\tilde{D}_i^s) = \theta^H C_2 + (1 - \theta^H)B_i^{H,s}(\tilde{\gamma}) - \tilde{D}_i^s - \varepsilon$

(ii) $D_1^{s,\text{PayIC}} > B_i^{H,s}(\gamma') \geq D_1^{s,\text{ControlIC}}$, then $\tilde{D}_i^{s,\text{Max}} = B_i^{H,s}(\gamma')$. For any promised payment $\tilde{D}_i^s \leq \tilde{D}_i^{s,\text{Max}}$, incumbent chooses $\gamma_2 = \tilde{\gamma}$ and expects

$V_{i}^{H,s}(\tilde{D}_i^s) = \theta^H C_2 + (1 - \theta^H)B_i^{H,s}(\tilde{\gamma}) - \tilde{D}_i^s - \varepsilon$.

(iii) $D_1^{s,\text{ControlIC}} > B_i^{H,s}(\gamma') \geq B_i^{H,s}(\gamma)$ then $\tilde{D}_i^{s,\text{Max}} = D_1^{s,\text{ControlIC}}$. For any promised payment $\tilde{D}_i^s \leq \tilde{D}_i^{s,\text{Max}}$, the incumbent chooses $\gamma_2 = \tilde{\gamma}$ and expects

$V_{i}^{H,s}(\tilde{D}_i^s) = B_i^{H,s}(\tilde{\gamma}) - \tilde{D}_i^s - \varepsilon$ if $\tilde{D}_i^s > B_i^{H,s}(\gamma')$ and

$V_{i}^{H,s}(\tilde{D}_i^s) = \theta^H C_2 + (1 - \theta^H)B_i^{H,s}(\tilde{\gamma}) - \tilde{D}_i^s - \varepsilon$ if $\tilde{D}_i^s \leq B_i^{H,s}(\gamma')$.

2) If $B_i^{H,s}(\gamma) \leq B_i^{IL,s}(\gamma') < B_i^{H,s}(\gamma)$,

(i) $B_i^{H,s}(\gamma') \geq D_1^{s,\text{aPayIC}}$, then $\tilde{D}_i^{s,\text{Max}} = D_1^{s,\text{aPayIC}}$. For any promised payment $\tilde{D}_i^s \leq \tilde{D}_i^{s,\text{Max}}$, incumbent chooses $\gamma_2 = \tilde{\gamma}$ and expects

$V_{i}^{H,s}(\tilde{D}_i^s) = \theta^H C_2 + (1 - \theta^H)B_i^{H,s}(\tilde{\gamma}) - \tilde{D}_i^s - \varepsilon$. 

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(ii) $D_{i}^s \alpha PayC > B_{i}^{H,s}(\gamma^i) \geq D_{i}^{s \alpha Control IC}$, then $\tilde{D}_{i}^{s,Max} = B_{i}^{H,s}(\gamma^i)$. For any promised payment $\tilde{D}_{i}^{s} \leq \tilde{D}_{i}^{s,Max}$, incumbent chooses $\gamma_2 = \overline{\gamma}$ and expects

$$V_{i}^{H,s}(\tilde{D}_{i}) = \theta^H C_2 + (1 - \theta^H)B_{i}^{H,s} (\overline{\gamma}) - \tilde{D}_{i}^{s} - \epsilon.$$

(iii) $D_{i}^{s \alpha Control IC} > B_{i}^{H,s}(\gamma^i) \geq \tilde{D}_{i}^{s,Max}$, then $\tilde{D}_{i}^{s,Max} = D_{i}^{s \alpha Control IC}$. For any promised payment $\tilde{D}_{i}^{s} \leq \tilde{D}_{i}^{s,Max}$, the incumbent chooses $\gamma_2 = \overline{\gamma}$ and expects

$$V_{i}^{H,s}(\tilde{D}_{i}) = B_{i}^{H,s}(\overline{\gamma}) - \tilde{D}_{i}^{s} - \epsilon \text{ if } \tilde{D}_{i}^{s} > B_{i}^{H,s} \gamma^i \text{ and}$$

$$V_{i}^{H,s}(\tilde{D}_{i}) = \theta^H C_2 + (1 - \theta^H)B_{i}^{H,s} (\overline{\gamma}) - \tilde{D}_{i}^{s} - \epsilon \text{ if } \tilde{D}_{i}^{s} \leq B_{i}^{H,s} \gamma^i.$$

3) If $B_{i}^{H,s}(\overline{\gamma}) \leq B_{i}^{H,d}(\gamma^i)$,

then $\tilde{D}_{i}^{s,Max} = D_{i}^{s \alpha PayC}$. For any promised payment $\tilde{D}_{i}^{s} \leq \tilde{D}_{i}^{s,Max}$, incumbent chooses $\gamma_2 = \overline{\gamma}$ and expects

$$V_{i}^{H,s}(\tilde{D}_{i}) = \theta^H C_2 + (1 - \theta^H)B_{i}^{H,s} (\overline{\gamma}) - \tilde{D}_{i}^{s} - \epsilon.$$

c. $\alpha C_2 \geq B_{i}^{H,s}(\overline{\gamma})$.

Now the incumbent strictly prefers to maintain control even when high pledgeability is chosen. This is because the amount that she can produce exceeds the highest amount that she can sell. Again, we classify the analysis into different cases and only explain the differences from the last category.

1. **Pledgeability choice does not matter**
2. **The disabled incumbent always loses control**
   1) **The abled incumbent always loses control**
   2) **The abled incumbent always maintains control**
   3) **Pledgeability choice leads to change in control for the abled incumbent**
3. **The disabled incumbent always maintains control**

This case happens if $B_{i}^{H,d} \geq \min \{B_{i}^{H,s}(\overline{\gamma}), \tilde{D}_{i}^{s}\}$ and since $B_{i}^{H,d} \leq B_{i}^{H,s}$, we know that the incumbent can definitely afford to keep control if she retains her ability.
Clearly, she will choose high pledgeability if
\[
\theta^H (C_2 - Min[D^s_1, B^H,s_s(\gamma)]) + (1 - \theta^H)(\alpha C_3 - Min[D^s_1, B^H,s_s(\gamma)]) - \varepsilon
\geq \theta^H (C_2 - Min[D^s_1, B^H,s_s(\gamma)]) + (1 - \theta^H)(\alpha C_3 - Min[D^s_1, B^H,s_s(\gamma)])
\]

The inequality above can never hold. In other words, the incumbent cannot commit to high pledgeability choice for any relevant debt level. Thus, low pledgeability is set and the maximal promised payoff is \( \hat{D}^{s,\text{max}}_1 = B^H,s_s(\gamma) \). The value is \( V^{H,s}_1 = \theta^H [C_2 - \hat{D}_1^s] + (1 - \theta^H)[\alpha C_3 - \hat{D}_1^s] \)
for \( \hat{D}_1^s \in [0, B^H,s_s] \).

4. **Pledgeability choice leads to change in control for the disabled incumbent**

1) **Pledgeability choice leads to change in control for the abled incumbent**

Same as 3, low pledgeability is set and the maximal promised payoff is \( \hat{D}^{s,\text{max}}_1 = B^H,s_s(\gamma) \). The value is \( V^{H,s}_1 = \theta^H [C_2 - \hat{D}_1^s] + (1 - \theta^H)[\alpha C_3 - \hat{D}_1^s] \).

2) **The abled incumbent always maintains control**

Same as 3, low pledgeability is set and the maximal promised payoff is \( \hat{D}^{s,\text{max}}_1 = B^H,s_s(\gamma) \). The value is \( V^{H,s}_1 = \theta^H [C_2 - \hat{D}_1^s] + (1 - \theta^H)[\alpha C_3 - \hat{D}_1^s] \).

**Lemma 7.4c**

Suppose \( \alpha C_2 \geq B^H,s_s(\gamma) \).

A. If \( \hat{D}_1^s \leq B^H,s_s(\gamma) \), \( \gamma_2 = \gamma \). Then \( \hat{D}^{s,\text{Max}}_1 = B^H,s_s(\gamma) \). For any promised payment

\[
\hat{D}_1^s \leq \hat{D}^{s,\text{Max}}_1, \text{ incumbent expects: } V^{H,s}_1(\hat{D}_1^s) = \theta^H C_2 + (1 - \theta^H)\alpha C_3 - \hat{D}_1^s.
\]

B. Else if \( \hat{D}_1^s > B^H,s_s(\gamma) \),

1. If \( B^H,s_s(\gamma) \geq C_2 \),

\[
\hat{D}^{s,\text{Max}}_1 = C_2 \text{ and } \gamma_2 = \gamma. \text{ For any promised payment } B^H,s_s(\gamma) < \hat{D}_1^s \leq \hat{D}^{s,\text{Max}}_1, \text{ incumbent expects: } V^{H,s}_1(\hat{D}_1^s) = C_2 - \hat{D}_1^s.
\]

If \( C_2 > B^H,s_s(\gamma) \) and

2. if \( B^H,s_s(\gamma) > B^H,s_s(\gamma) \)
then $\tilde{D}_1^{s,\text{Max}} = B_1^{H,s}(\gamma) - \varepsilon$. For any promised payment $B_1^{H,s}(y') < \tilde{D}_1^s \leq \tilde{D}_1^{s,\text{Max}}$, the incumbent chooses $y'_2 = \gamma$ and expects $V_1^{H,s}(\tilde{D}_1^s) = B_1^{H,s}(\gamma) - \tilde{D}_1^s - \varepsilon$ if $\tilde{D}_1^s > B_1^{H,s}(y')$

and $V_1^{H,s}(\tilde{D}_1^s) = \theta^H C_2 + (1 - \theta^H) B_1^{H,s}(\gamma) - \tilde{D}_1^s - \varepsilon$ if $B_1^{H,s}(y') < \tilde{D}_1^s \leq B_1^{H,s}(y')$.

3. Else $B_1^{H,s}(y') \leq B_1^{H,s}(y')$

3.1. If $B_1^{H,s}(y') < B_1^{H,s}(y')$

(i) $B_1^{H,s}(y') \geq D_1^{s,\text{PayIC}}$, then $\tilde{D}_1^{s,\text{Max}} = D_1^{s,\text{PayIC}}$. For any promised payment $B_1^{H,s}(y') < \tilde{D}_1^s \leq \tilde{D}_1^{s,\text{Max}}$, incumbent chooses $y'_2 = \gamma$ and expects $V_1^{H,s}(\tilde{D}_1^s) = \theta^H C_2 + (1 - \theta^H) B_1^{H,s}(\gamma) - \tilde{D}_1^s - \varepsilon$

(ii) $D_1^{s,\text{PayIC}} > B_1^{H,s}(y') \geq D_1^{s,\text{Control IC}}$, then $\tilde{D}_1^{s,\text{Max}} = B_1^{H,s}(y')$. For any promised payment $B_1^{H,s}(y') < \tilde{D}_1^s \leq \tilde{D}_1^{s,\text{Max}}$, incumbent chooses $y'_2 = \gamma$ and expects $V_1^{H,s}(\tilde{D}_1^s) = \theta^H C_2 + (1 - \theta^H) B_1^{H,s}(\gamma) - \tilde{D}_1^s - \varepsilon$

(iii) $D_1^{s,\text{Control IC}} > B_1^{H,s}(y') \geq B_1^{H,s}(y')$ then $\tilde{D}_1^{s,\text{Max}} = D_1^{s,\text{Control IC}}$. For any promised payment $B_1^{H,s}(y') < \tilde{D}_1^s \leq \tilde{D}_1^{s,\text{Max}}$, incumbent chooses $y'_2 = \gamma$ and expects:

$V_1^{H,s}(\tilde{D}_1^s) = B_2^{H,s}(\gamma^g) + \theta^H \rho C_3 - \tilde{D}_1^s - \varepsilon$ if $\tilde{D}_1^s > B_1^{H,s}(y')$

and $V_1^{H,s}(\tilde{D}_1^s) = \theta^H C_2 + (1 - \theta^H) B_1^{H,s}(\gamma) - \tilde{D}_1^s - \varepsilon$ if $B_1^{H,s}(y') < \tilde{D}_1^s \leq B_1^{H,s}(y')$.

3.2. If $B_1^{H,s}(y') \leq B_1^{H,s}(y')$

then $\tilde{D}_1^{s,\text{Max}} = B_1^{H,s}(\gamma)$. For any promised payment $\tilde{D}_1^s \leq \tilde{D}_1^{s,\text{Max}}$, incumbent expects:

$V_1^{H,s}(\tilde{D}_1^s) = \theta^H C_2 + (1 - \theta^H) \alpha C_2 - \tilde{D}_1^s$. 
Disabled Incumbent in Period 1

So far we have solved the date 1 problem for an incumbent who starts this period with high ability. Now we turn to the same problem for an incumbent who starts this period with ability lost. In other words, the period-1 incumbent is only able to produce $\alpha C_1$ if the state is good. This scenario is needed for the full dynamic analysis if we bring back period 0. Below, we summarize her decision, maximal payout and continuation value $V^{iL,s}_1$. Note that the incumbent’s bid is also different: $B^{iL,s}_1 = \min\{\alpha^{iL,s}_1 + \gamma' \alpha C_2, \alpha C_2\}$.

Lemma 7.5

1. $\alpha C_2 \leq B^{H,s}_1 (\gamma)\]

In this case, the incumbent always sells the firm. The maximal payout
$\tilde{D}^{s,Max}_1 = B^{H,s}_1 (\gamma) - \varepsilon$. For any promised payment $\tilde{D}^{s}_1 \leq \tilde{D}^{s,Max}_1$, the incumbent chooses
$\gamma_2 = \gamma$ and expects $V^{iL,s}_1 (\tilde{D}^{s}_1) = B^{H,s}_1 (\gamma) - \tilde{D}^{s}_1 - \varepsilon$.

2. $B^{H,s}_1 (\gamma) < \alpha C_2 < B^{H,s}_1 (\gamma)\]

The maximal payout is
$\tilde{D}^{s,Max}_1 = B^{H,s}_1 (\gamma) - (\alpha C_2 - B^{H,s}_1 (\gamma)) - \varepsilon$. For any promised payment $\tilde{D}^{s}_1 \leq \tilde{D}^{s,Max}_1$, the incumbent chooses $\gamma_2 = \gamma$ and expects
$V^{iL,s}_1 (\tilde{D}^{s}_1) = B^{H,s}_1 (\gamma) - \tilde{D}^{s}_1 - \varepsilon$.

3. $B^{H,s}_1 (\gamma) \leq \alpha C_2\]

The maximal payout is
$\tilde{D}^{s,Max}_1 = B^{H,s}_1 (\gamma)$. For any promised payment $\tilde{D}^{s}_1 \leq \tilde{D}^{s,Max}_1$, the incumbent chooses $\gamma_2 = \gamma$ and expects $V^{iL,s}_1 (\tilde{D}^{s}_1) = \alpha C_2 - \tilde{D}^{s}_1 - \varepsilon$.

Proof: To see the second case clearly, note that the incumbent gets
$B^{H,s}_1 (\gamma) - \min \left[ \tilde{D}^s, B^{H,s}_1 (\gamma) \right] - \varepsilon$ with high pledgeability choice and gets
$\alpha C_2 - \min \left[ B^{H,s}_1 (\gamma), \tilde{D}^s \right]$ with low pledgeability. Choosing high pledgeability and selling the firm is more profitable if and only if $\tilde{D}^s \leq B^{H,s}_1 (\gamma) - (\alpha C_2 - B^{H,s}_1 (\gamma)) - \varepsilon$. 